各位老师下午好，我们将要分享的工作是“Sparse signal detection and fingerprint feature recognition based on fast 2D DFRFT”。

Good afternoon teachers, the work we will share is "Sparse signal detection and fingerprint feature recognition based on fast two-dimensional discrete fractional Fourier transform ".

我们将要从研究动机、算法和仿真三个方面来描述。

We will describe it from three aspects: research motivation, algorithm, and simulation.

首先，我们将要分享研究的动机。

First, we will share the motivation for the research.

在当前信息化的社会，准确快速地识别个人身份是维护社会秩序的基本问题。然而传统的基于密码、令牌等方式的身份认证方式常会有丢失，遗忘，窃取的风险。指纹特征广泛应用于军事、民用等领域，因为独一无二的、稳定的、随身携带的、不易伪造的。一般的，指纹特征是分析指纹纹理的局部和细节结构提取出来的。然而，指纹节点特征的提取是复杂的。增强、二值化和细化处理过程计算量大。我们发现，指纹图像可以被视为二维啁啾图像的近似值。因此，我们提出了一种基于二维离散分数阶傅里叶变换（2D DFRFT）的指纹特征提取方法。还给出了二维DFRFT算法的有效实现。

In the current information-based society, accurate and rapid identification of individuals is a fundamental issue to maintain social order. However, traditional authentication methods (based on passwords, tokens, etc.) are often at risk of being lost, forgotten, and stolen. Fingerprint features are widely used in military and civilian applications due to being unique, stable, portable, and not easily forged. In general, fingerprint features are extracted by analyzing the local and detailed structure of the fingerprint texture. However, the extraction of fingerprint node features is complex. The enhancement, binarization, and refinement processes are computationally intensive. We find that the fingerprint image can be regarded as an approximation of a two-dimensional chirp image. Thus, we propose a fingerprint feature extraction method based on two-dimensional discrete fractional Fourier transform (2D DFRFT). Furthermore, an efficient implementation is also given.

分数阶傅里叶变换是处理非平稳信号的重要工具。我们这里给出了pei型二维离散分数傅里叶变换的定义。它是傅里叶变换的广义形式，由于增加了一个旋转角参数，它能够联合表征时间和频率。当旋转参数从0逐渐增加，分数阶傅里叶变换能够得到信号从时域逐步变化到频域的所有特征。因此分数阶傅里叶变换可以看作信号在时频平面上的旋转。因此，指纹图像经过FRFT后能够获取更多的特征。

The fractional Fourier transform (FRFT) is an important tool for dealing with nonstationary signals, especially chirp signals. The chirp signal is strongly aggregated and even sparse in the fractional Fourier domain. Here we give the definition of the pei-type two-dimensional discrete fractional Fourier transform. As a generalized form of the Fourier transform, the FRFT can jointly characterize time and frequency due to the addition of a rotation angle parameter. The FRFT can be regarded as the rotation of the signal in the time-frequency plane. When the rotation parameter increases from zero gradually, the FRFT can obtain all the features of the signal that change from the time domain to the frequency domain gradually. Therefore, the fingerprint image can acquire more comprehensive features after FRFT.

一般的，二维离散分数阶傅里叶变换的计算通过转化为一维实现。具体的，首先对二维信号的各行（列）进行1D DFRFT, 然后对得到的信号的各列（行）计算1D DFRFT.计算复杂度高，不能满足大数据和实时处理的需要。

In general, the computation of the 2D DFRFT is accomplished by converting to 1D DFRFT. Specifically, 1D DFRFT is first performed on each row of the two-dimensional signal, and then 1D DFRFT is calculated on each column of the obtained signal. Therefore, the complexity of 2D DFRFT istimes of that of 1D DFRFT, where the signal size is. For applications with large amounts of data, this calculation is very expensive. It also cannot meet the needs of real-time processing.

启发于全新的估计2D DFT算法MARS-SFT的思路，我们将进一步优化2D DFRFT的计算结构。

Inspired by the idea of a two-dimension sparse Fourier transform algorithm to estimate 2D DFT, we will propose the Sparse Two-Dimensional Fractional Fourier Transform (STDFRFT) algorithm to further optimize the computational structure of 2D DFRFT. This will lead to fast and effective fingerprint feature recognition.

假设信号s是k稀疏的。我们算法的目标是估计k个重要频率的幅度和位置。

STDFRFT算法通过两次chirp调制和一次二维SFT实现。首先，为了弥补载波中chirp基的影响，我们将原始信号与chirp信号相乘，构造一个二维稀疏傅里叶变换（2D SFT）阶段的输入信号. 第二，我们使用迭代算法来估计频谱。每次迭代使用混叠滤波器分桶，只有少数的1D FFT和简单的映射被使用。最后，为了由傅里叶域调制到分数阶傅里叶域，我们将得到的估计结果乘以另一个chirp函数，得到STWFRFT的最终结果。

Suppose the signalis-sparse in the fractional Fourier domain. That is, the 2D DFRFTof the-point signalsatisfies that all butcoefficients are negligible. Whereis an integer much smaller than signal size. Without loss of generality, we suppose. The goal of our STDFRFT algorithm is to estimate the locationsand amplitudesof thesignificant frequencies. The STDFRFT algorithm is implemented by two chirp modulations and one 2D SFT. First, to compensate for the effect of the chirp basis, we multiply the original signal with a chirp signal and construct an input signal of two-dimensional sparse Fourier transform (2D SFT) stage. Second, we use an iterative algorithm to estimate the spectrum of. In each iteration, the aliasing filter is used to bucketing. And only a few 1D FFT and simple mappings are used. Finally, to modulate from the Fourier domain to the fractional Fourier domain, we multiply the above estimation by another chirp signal and get the final result of STDFRFT.

我们在c中提取相互平行的三个切片,分别进行fft运算后即可得到三组桶。减去之前迭代的结果。因为信号是稀疏的，所以要估计的频谱是稀疏的。 因此，一些桶只包括可忽略的噪声，并且必须将重要的频率划分为最大值的桶。 假设大值桶 h 仅包含一个有效频率，则可以估计该频率的位置和幅度。对于无噪声的信号，每次迭代经过上述分桶和估计过程即可很大概率的精确估计大部分重要频率。 对于含躁信号，18，19解码的位置是非整数，我们需要将其四舍五入。不幸的是，这可能引起解码误差。为了降低解码错误的概率，我们采用投票法。具体的，每次迭代中执行q次内循环，并记录解码的位置。如果位置被标记超过p次，我们认为它就是重要频率的位置。经过几次迭代以后，没有被解码的频率设置振幅为0。此外，如果频率被重复解码，振幅取和。

The analysis of an iteration of 2D SFT is as follows. We extract random parallel slices in. Three groups of buckets can be obtained after performing the FFT operation on slices. It is worth noting that the spectrumto be estimated in the iteration is obtained by subtracting the result estimated by previous iterations from the original spectrum.

Because the signalis-sparse, the spectrum to be estimated is sparse. Therefore, some buckets only include negligible noises and significant frequencies must be divided into largest-value buckets. Assuming that the large-value bucket h contains only one significant frequency, the location and amplitude of this frequency can be estimated.

For a noise-free signal, after the above bucketing and estimation process in each iteration, most of the significant frequencies can be estimated accurately with a high probability. For a noisy signal, the decoded positions are nonintegers and we need to round them up. Unfortunately, this may lead to decoding errors. To reduce the probability of decoding errors, the voting method is utilized. Specifically, the inner loop is executed q times in each iteration, and the decoded positions are recorded. If a location is marked more than p times, we consider it to be the location of a significant frequency.

After several times iterations, amplitude of the frequencies that are not decoded are set to zero. In addition, if frequencies are decoded repeatedly, the amplitudes are summed.

为了检验算法的有效性，我们构造分数域稀疏的随机信号。具体的， 我们算法检测的结果 我们设置信号的稀疏度为5，并且5个重要频率的位置和振幅是随机的。 分解为1D的检测结果 噪声下的结果 比较（时间、误差、样本）

In order to verify the effectiveness of the algorithm, we will construct a random signal with sparse fractional domain. Specifically, we set the sparsity of the signal to 5 in the fractional Fourier domain with orders (1.2566,1.2566). The positions and amplitudes of the five significant frequencies are random. At the same time, additive white Gaussian noise with SNR=26.9825dB is added. The constructed signal with a size of (256,256) is shown in Fig1. First, we detect the signal by decomposing 2D DFRFT to two groups of 1D DFRFT, which is implemented by direct method. And Fig. 2(a) shows the results of random sparse signal detection. Then, signal detection in the fractional Fourier domain is performed by the proposed STDFRFT algorithm. The program has one iteration in total. We set three loops in each iteration, and the voting threshold is 2. The obtained results are shown in Fig.2(b). In order to show more clearly, we give the corresponding top views in Fig. 2(c) and Fig. 2(d), respectively. As can be seen from Fig. 2, our algorithm accurately detects all the significant components in the noisy situation.

To visualize algorithm performance, we measured the computation time and the number of samples used in the time domain for different methods. Meanwhile, theerrors between the decoded results and the original fractional domain spectrum (no noise) are also computed. The table presents the time, error, and samples number for the methods clearly. And our method is optimal in every respect.

验证算法的收敛性。具体的，讨论算法最小的迭代次数，使得所有的频率被正确检测。分数域稀疏随机信号仍然被考虑，信号的稀疏度k取为[100--600]，信号大小N=N1\*N2为65536，其中分别令(N1,N2)等于(256，256),(512,128),(1024,64). 此外，信号被添加了SNR=34.1541的复高斯白噪声。图converge展示了仿真的结果。不难得到，STDFRFT算法是收敛的，且迭代次数随着稀疏度的增加而增加。

We have verified the convergence of the STDFRFT algorithm. Specifically, we have discussed the minimum number of iterations of the STDFRFT algorithm to detect all significant frequencies successfully. The fractional domain sparse random signals are still considered. And the sparsity of the signals is taken to be one hundred to six hundred. The signal size is 65536, and three different cases are chosen. In addition, the signals are added with complex Gaussian white noise with SNR = 34.1541dB. It is not difficult to obtain that the STDFRFT algorithm is convergent. Moreover, the number of iterations increases with the increase of sparsity and N/B.

已经介绍完了创新的算法。接下来，我们将展示仿真的效果。

The innovative STDFRFT algorithm has been introduced. Next, we will show the effect of the simulation.

线性调频信号在脉冲压缩、雷达载波、声纳等方面已经受到了广泛的研究和应用。二维线性调频信号同样也是非常重要的非平稳信号之一。 根据定义，分数阶傅里叶变换能够将信号用chirp正交基分解。因此FRFT对于处理线性调频信号具有天然的优势，并且具有很强的能量聚集性。用于实验的chirp信号如图所示。 为了校验仿真结果，我们记录了发射脉冲的雷达参数。具体的，脉冲幅度等于3。调频率和中心频率分别为(-16.4Hz/s,-7.8Hz/s) (640Hz,896Hz).

First of all, we have applied the proposed STDFRFT algorithm to detect two-dimensional chirp signals. The chirp signal has been studied and applied in pulse compression, radar carrier, sonar, and so on widely. The two-dimensional chirp signal is also one of the important nonstationary signals. By definition, the FRFT can decompose a signal with a chirp orthonormal basis. Therefore, FRFT has a natural advantage for processing chirp signals, and has strong energy concentration. Meanwhile, the fractional Fourier domain of the chirp signal is sparse at the optimal rotation angle. The chirp signal used for the experiment is shown in the Fig. 4. The sampling rate at discretization is 4096 hertz per second, and the pulse duration is two to the negative sixth power seconds. To check the simulation results, we recorded the radar parameters of the transmitted pulses. Specifically, the pulse amplitude is equal to 3. The modulation frequency and center frequency are (-16.4Hz/s, -7.8Hz/s) and (640Hz, 896Hz), respectively.

First, we use a discrete polynomial phase transform to estimate the frequency modulation in both dimensions of the signal. Then, the optimal rotation angle of the chirp signal is estimated using frequency modulation. Finally, the STDFRFT algorithm is used to estimate the fractional Fourier spectrum of the chirp signal. The program has one iteration in total. We set three loops in each iteration, and the voting threshold is 2. Fig.5 shows the estimated result and its top view. It is easy to get that the location of the dominant frequency is the center frequency of the chirp signal. The amplitude estimated by the algorithm is 2.999998, which is close to the true amplitude of the signal. Therefore, the detection of the 2D chirp signal is well done.

在这一小节，我们将考虑更复杂的信号，真实的指纹图像。一般的，指纹特征识别包含指纹的整体脊线流动模式、纹理构造的细节等.这是很耗时的分析过程。我们提出了分数傅里叶域的特征提取和识别，可以很快的获取指纹的主要特征。Gao提出，FRFT的相位具有相对时移不变性. 因此，我们提取的特征可以用于指纹的匹配。此外，指纹的纹理可以近似为多分量的2D chirp信号，因此在特定阶数下的分数阶傅里叶域是近似稀疏的。预处理后的指纹图像Fig5.

Next, we have considered more complex signals, real fingerprint images. Generally, fingerprint feature recognition includes the overall ridge flow pattern of the fingerprint, details of texture structure, etc. This is a time-consuming analysis process. We propose feature extraction and recognition in the fractional Fourier domain, which can quickly obtain the main features of fingerprints. Gao proposed that the phase of FRFT has relative time-shift invariance. Therefore, the extracted features in the fractional Fourier domain can be used for fingerprint matching. Furthermore, the texture of the fingerprint can be approximated as 2D chirp signals, and the fractional Fourier domain is almost sparse at a certain order. The preprocessed fingerprint signal is shown in Fig. We have acquired the FRFT by direct method to show that.

我们利用STDFRFT法实现了指纹的特征识别，如图所示。

We use the STDFRFT method to realize the feature recognition of fingerprints, as shown in Fig. 8(a). It is not difficult to find that our method can identify the main features of fingerprint signals in the fractional Fourier domain. And the effects of glitches and blurs are filtered out. Furthermore, we simulate the incomplete fingerprint signal as shown in Fig.6(b). The fingerprint features are identified by the STDFRFT method, and the result is displayed in Fig. 8(b). By comparing with Fig. 8(a), we can know that our algorithm can extract the main features even for incomplete fingerprints.

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That's all I'm sharing today. Thanks for listening and feel free to ask questions!