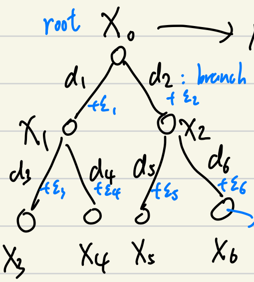
1. Evolution tree structure:



$\to N(\mu, \sigma_0^2)$

d_2: branch length. (Can be: num of Mutation)

x: Ground Truth variable

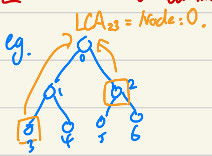
y: Observation.

一团细胞：的特征？？ Maybe base on Experiment.

⚠ LCA: lowest common ancestor.

$$\text{Claim: } \vec{x} = \begin{bmatrix} x_0 \\ \vdots \\ x_6 \end{bmatrix} = \mu + \underbrace{S}_{\text{mutation.}} \sim N(\mu I, \Sigma) \quad \Sigma \text{ a number.}$$

eg.



$\Sigma_{35} = Var(X_0) = \sigma_0^2$

$\Sigma_{34} = Var(X_1) = \sigma_0^2 + \sigma^2 d_1$

with $X_i \sim X_{p(i)} + N(0, d_i\sigma^2)$

While: $\vec{y} = \vec{x} + \underbrace{N}_{\text{noise}}$, $N \sim N(0, \delta I)$

scaled parameter for all branch.

$$\text{Theorem: } \Sigma_{ij} = Var(LCA_{ij}) = \sigma_0^2 + \sigma^2 \Big(\sum_{e \in path(r \to LCA_{ij})} d_e \Big) \quad [e: edge]$$

r: root.

proof: $\Sigma_{ij} = cov(X_i, X_j) = cov(X_{LCA} + \varepsilon_{i1} + \cdots + \varepsilon_{in}, X_{LCA} + \varepsilon_{j1} + \cdots + \varepsilon_{jm})$

$= cov(X_{LCA}, X_{LCA})$

since: $X_{LCA} = X_0 + \sum_{e \in path(r, LCA)} \varepsilon_e$

$$Var(X_{LCA}) = Var(X_0) + \sum_{e \in \cdots} Var(\varepsilon_e) = \sigma_0^2 + \sigma^2 \sum d_e$$

2. Now: $\begin{cases} \text{Known information: } Y_i (\forall i), \text{ All tree structure } (V, E, d) \\ \text{Unknown information: } \delta, \sigma, \mu, \sigma_0, \vec{X}_i \end{cases}$

We only need to estimate $\vec{X}_{leaf} = \begin{bmatrix} X_3 \\ X_4 \\ X_5 \\ X_6 \end{bmatrix} \sim N(\mu, \Sigma_{3:6, 3:6})$

$$y = \vec{X} + N, \; N \sim N(0, \delta_y I)$$

Then. We have two choice:

(1) Freq: $\max_{\vec{x},\delta_g,\mu,\sigma} \log P(\vec{y},\vec{x} \mid \delta,\mu,\sigma)$

$$= \max_{\vec{x},\delta_g,\mu,\sigma}\left[\sum_{g=1}^{G} \log P(\vec{y} \mid \vec{x};\delta_g) + \log P(\vec{x};\mu,\Sigma)\right)$$

set grad $=0$

(2) Bayesian: δ_g,μ,σ,

$$\max_{\delta_g,\mu,\sigma} P(\vec{y},\vec{x} \mid \delta_g,\mu,\sigma)$$

(3) Gaussian Process:

$Y = f(x) + \underline{N}$ $\quad N \sim \mathcal{N}(0,\delta^2 I)$

$f \sim GP(0,K)$

Kernel Design: linear Kernel. $k_{ij} = \sigma_0^2 + \sigma^2 \underbrace{(x_i - c)(x_j - c)}_{f(x_i,x_j)}$

or: RBf: $\sigma^2 \exp\left(-\frac{(x_i - x_j)^2}{2l^2}\right)$

Brovnian Motion.

Def: $\{B_t, t \geq 0\}$

1、初始条件：$B_0 = 0$

2、对于 $0 \leq s < t$，增量 $B_t - B_s$ 与 $\forall B_u,\ 0 \leq u \leq s$ 均独立.

3、$B_t - B_s \sim N(0, (t-s)\delta)$

性质：$\forall s < t$，有 $E[B_t | F_s] = B_s$，其中 F_s 为到 s 为止的信息.