

Gate-model Quantum Computing.

A famous NP (non-deterministic) hard problem

Software Stack

Problem Definition e.g. traveling salesman problem

Quantum Algorithm e.g. QAOA — Quantum Approximate Optimization Algorithm

Quantum circuit gates & unitary operators

Quantum Compiler
→ actual set of gates
→ connectivity (2 qubits are not physically connected, but have some interactions between)

QPU Simulator

on laptop, we can simulate 20 to 22 qubits.

on supercomputer, around 50.

Then we will run out of the classical compute power.

Solovay-Kitaev theorem \Rightarrow Finite set of gates can approximate any unitary operation.

The gate model is universal, because it can transform any quantum states/qubits into any other quantum gates/qubits.

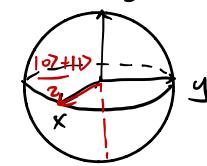
Quantum Circuits

A single-qubit gate moves a point on the surface of Bloch Sphere.

① X-gate $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \rightarrow \boxed{X}$

$|0\rangle$

② Hadamard gate $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \rightarrow \boxed{H}$



③ CNOT gate $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \rightarrow$ When control qubit is 1, then applies NOT.

$$\text{CNOT}|00\rangle = |00\rangle$$

$$\text{CNOT}|01\rangle = |11\rangle$$

when control qubit is 0, do nothing

Create $|\phi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$: $|0\rangle \xrightarrow{\boxed{H}} \left| \frac{|0\rangle + |1\rangle}{\sqrt{2}} \right\rangle \xrightarrow{\text{CNOT}} \left| \frac{|00\rangle + |11\rangle}{\sqrt{2}} \right\rangle$

Quantum Annealing for Optimization.

1. Adiabatic Quantum Computing

$\frac{1}{\hbar} \text{Euler's}$

Unitary evaluation and the Hamiltonian

Classical Ising model: $H = - \sum_{\langle i,j \rangle} J_{ij} \sigma_i^z \sigma_j^z - \sum_i h_i \sigma_i^z \rightarrow \text{Hermitian}$

Energy expectation value: $\langle H \rangle = \langle \psi | H | \psi \rangle$

Evolution: $U | \psi \rangle$

Schrödinger equation: $i\hbar \frac{d}{dt} |\psi(t)\rangle = H |\psi(t)\rangle$

Plank Constant

The temporal evolution of the system is described by the Hamiltonian applied to the time-dependent state

The fact that Hamiltonian is Hermitian implies that Operator U is unitary

Every Hamiltonian implies unitary operator.

(Solution) for time-independent H : $U = \exp(iHt/\hbar) \rightarrow \text{Unitary}$

Every gate has an underlying Hamiltonian.

(\bar{A} : conjugate of A . A^T transpose of A)

Hermitian matrix: $\bar{A}^T = A \Leftarrow$ the Hamiltonians (H)

summary Unitary matrix: $\bar{A}^T = A^{-1} \Leftarrow$ the time-evolution operator U .

The Adiabatic Theorem

$$\begin{cases} \text{2 Hamiltonians} \\ H_0 = \sum_i \sigma_i^x \rightarrow \text{transverse field} \\ H_1 = - \sum_{\langle i,j \rangle} J_{ij} \sigma_i^z \sigma_j^z - \sum_i h_i \sigma_i^z \end{cases}$$

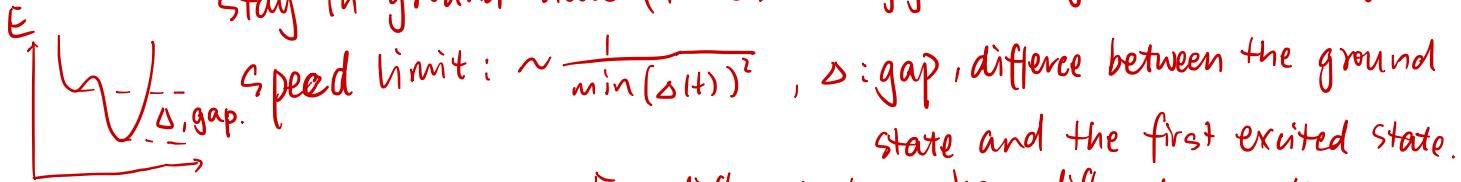
$$H(t) = (1-t)H_0 + tH_1, t \in [0, 1]$$

If we change the time t slowly, and start from the ground state of H_0 , end at the ground state of H_1 .

In classical Ising model, we can easily get stuck at local optimum.

↳ Solution: Adiabatic transition

Stay in ground state (lowest energy) throughout the change.



For different t , we have different gap $\Delta(t)$

The ground state, the lowest energy state of this, is equal superposition

However, if the gap is very small, the speed limit will be very bad.

i. It is not true to say we can solve a NP-hard problem faster or exponentially faster, because those problems have very small gap.

Adiabatic Quantum Computing

$$H = - \sum_{\langle i,j \rangle} J_{ij} \sigma_i^z \sigma_j^z - \sum_i h_i \sigma_i^z - \sum_{\langle i,j \rangle} g_{ij} \sigma_i^x \sigma_j^x$$

classical Ising model

interaction between transverse field.

(not transverse field Ising model, which don't have interactions)

This is universal!

(if it's able to implement a specific Hamiltonian)

2. Quantum Annealing

Adiabatic quantum Computing \rightarrow idealized

Quantum Annealing \rightarrow less idealized.

\rightarrow Minimum gate is hard to calculate \Rightarrow anneal repeatedly
(probably violating adiabatic conditions)

Pick the one with lowest energy.

(this isn't guaranteed to be the ground state and the global minimum, but will give you a better result.)

\rightarrow Solve classical Ising model.

\rightarrow Experimental conditions (finite temperature, electromagnetic interference).

Software Stack

Problem Definition e.g. TSP



Map to Ising model



Minor Embedding



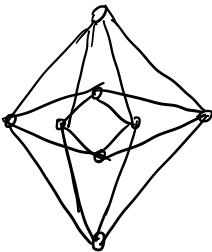
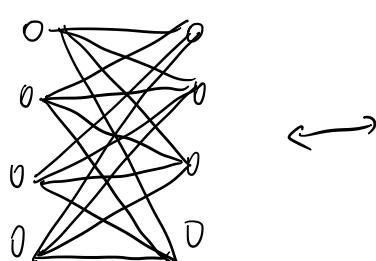
QPU

\rightarrow Similar to the compilation in the gate model architecture.

It maps the connectivity structure of the qubits to the hardware

Not all qubits are fully connected.

Chimera graph



Problem to solve: $\sigma_1 \sigma_2 + \sigma_2 \sigma_3 + \sigma_1 \sigma_3$

$$\sim$$

k_3

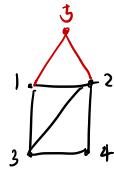
Implementations how to build a quantum computer?

Superconducting architecture

- △ Silicon-based
- △ Cooled to around 10mK
- △ Microwave pulses control the system → fast control speed

Disadvantages:

- △ 2D topology



good for 4 qubits.

but for a 5th qubit, it's difficult to establish between every qubit.
If we want to interact 3 and 5, a SWAP must be implemented.
and then SWAP gate.

SWAP:



∴ 7 gates for 1 gate.

- △ cooling requirements

- △ short coherence time.

time ↔ limited circuit depth. [10-20 gates]

Trapped ions ↗ operate at room temperature
have fully connectivity

- △ Individual ions (charged atomic particles)
confined using electromagnetic fields.

- △ lasers induce coupling

- △ long coherence time

Disadvantages:

- △ Scalability unclear [unclear how well it will scale to larger systems]
- △ speed of control is much slower.

Photonic systems

- △ e.g. photon polarization

- △ Room temperature

Disadvantages:

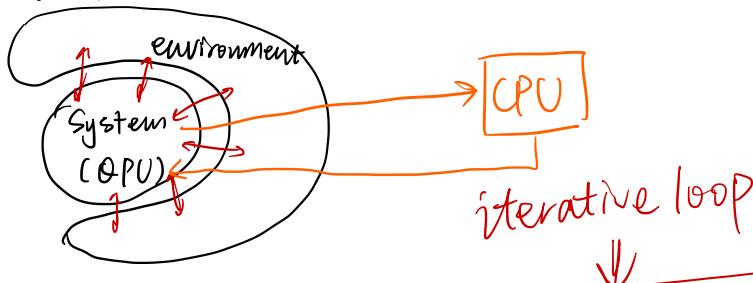
- △ photon loss

- △ photons can't be stored.

Variational Circuit

Quantum Approximate Optimization Algorithm.

Variational circuits



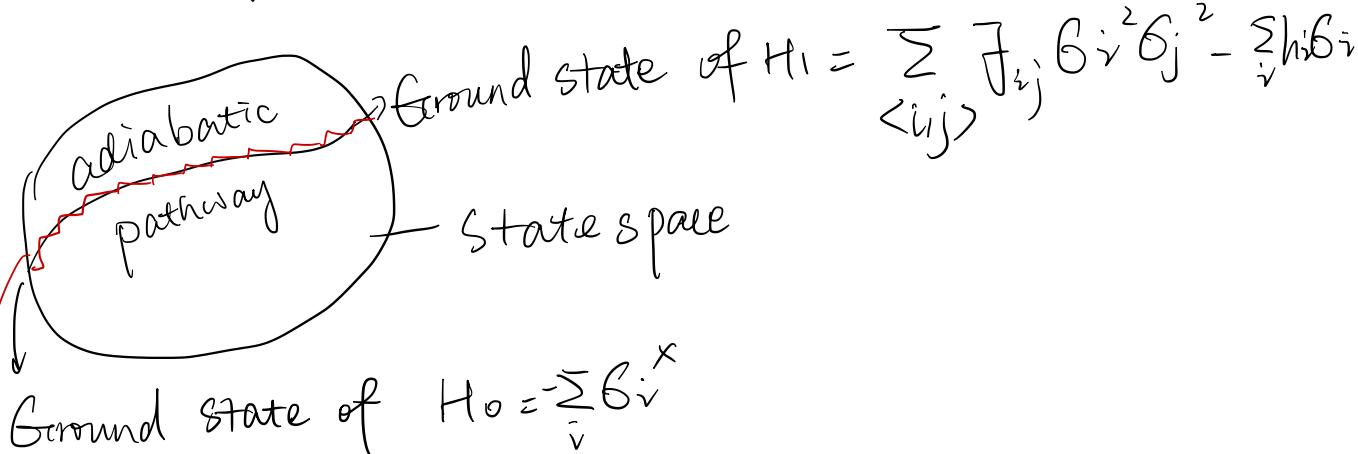
We have limited ability to control our system, and this limits our circuit depth, for instance.

So what we want is to run a short burst of calculation on the quantum processing unit, extract the result to a classical CPU, and then the circuit that we ran on the quantum computer is parameterized.

By understanding the result of a calculation, we can go back, adjust the parameters of the quantum processor, and again run a short burst of calculation.

QAOA: one of the most famous variational circuits.

draws inspiration from quantum annealing. approximate the adiabatic pathway



$$\text{Ground state of } H_0 = -\sum_i G_i^X$$

break the pathway up into discrete chunks and parameterize the circuit to have a more and more accurate approximation.

at the end we can read out the ground state just the same as the quantum annealer,

$$H(t) = (1-t) H_0 + t H_1$$

Discretize up to time t_0 :

$$U(t_0) \approx U(H_0, \beta_0) U(H_1, \gamma_0)$$

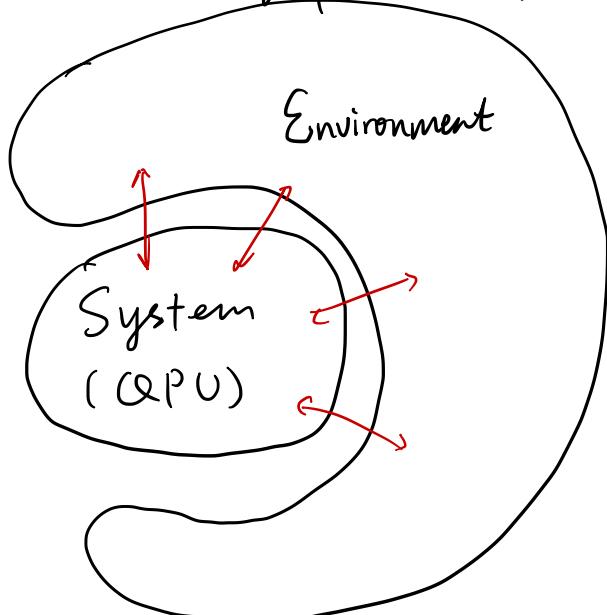
Then: \downarrow subsequent time step

$$U \approx U(H_0, \beta_0) U(H_1, \gamma_0) \cdots U(H_0, \beta_p) U(H_1, \gamma_p)$$

\hookrightarrow Trotterization

Sampling a Thermal State

Quantum annealing of finite temperature.



Let the interactions between environment happen and let this equilibration happen. In the end we'll end up in the thermal state.

$$H = - \sum_{\langle i,j \rangle} J_{ij} G_i^2 G_j^2 - \sum_i h_i G_i^2$$

Thermal State :

$$P(E_i) = \frac{\exp(-E_i/T)}{Z} \quad \text{Boltzmann Distribution}$$

Quantum approximate thermalization

$$\text{Recap: } |\psi\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

$$\text{Tr}[|\psi\rangle \langle \psi|] = \frac{1}{2}$$

① take an easy system with an ancilla. Trace out ancilla \Rightarrow thermal state

$$|\psi\rangle = \frac{1}{\sqrt{2}} (e^{-\frac{H_0}{2t}} |+\rangle |+\rangle + e^{\frac{H_0}{2t}} |-\rangle |-\rangle)$$

actual system ancilla system

$$\text{Tr}[|\psi\rangle \langle \psi|] = \frac{e^{-\frac{H_0}{T}}}{Z_0} = P_0 \Rightarrow \text{After trace out the ancilla system, we will get the thermal state of this Hamiltonian.}$$

② QAOA to approximate
In equilibrium with the environment

Read out an approximate thermal state P_A .