0. A crash Course on Quantum Information

0.1 Classic VS Quantum Bits

Lecture 1. the Qubit

① Mathematical description of qubit

Classical bits

$0 \to |0\rangle \to \begin{pmatrix}1\\0\end{pmatrix}$

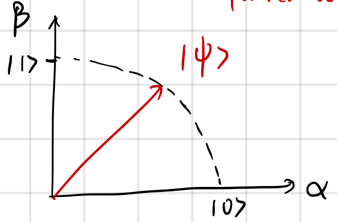
$1 \to |1\rangle \to \begin{pmatrix}0\\1\end{pmatrix}$

Qubits

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

Superposition of $|0\rangle$ and $|1\rangle$

For real vectors $\alpha^2 + \beta^2 = 1$



② What do qubits look like

e.g.1 In a maze, classically, a particle will either go left or go right.

quantumly, it can go left and right at the same time

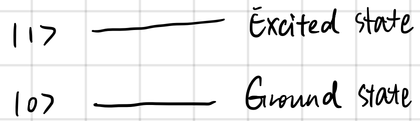
e.g.2 Physical implementation: represent bits by energy levels in an atom

classical bits:

in either ground or excited state

Qubits:

In two states at the same time



③ Basis and amplitudes

Standard basis/Computational basis:

$|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$

$|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$

Qubit:

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle \in \mathbb{C}^2$ → A qubit is a element of a complex vector space of dimention 2

amplitudes

Kets and bras:

ket ⇒ $|\psi\rangle = \begin{pmatrix}\alpha\\\beta\end{pmatrix}$

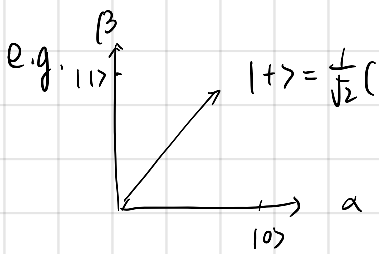
Conjugate

Transpose

bra ⇒ $\langle\psi| = (|\psi\rangle^*)^T = \begin{pmatrix}\alpha^*\\\beta^*\end{pmatrix}^T = (\alpha^* \ \ \beta^*)$

Inner product ⇒ $\langle\psi||\psi\rangle = \langle\psi|\psi\rangle = (\alpha^* \ \ \beta^*)\begin{pmatrix}\alpha\\\beta\end{pmatrix} = \alpha^*\alpha + \beta^*\beta = |\alpha|^2 + |\beta|^2 = 1$

e.g.



$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = \frac{1}{\sqrt{2}}\left(\begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

$\langle +|+\rangle = \frac{1}{2}(1 \ \ 1)\begin{pmatrix}1\\1\end{pmatrix} = \frac{1}{2}(1+1) = 1$

Lecture 2 More than One Qubit

① Standard / computational basis:

$x = x_1 x_2 \cdots x_n \in \{0,1\}^n$, $d = 2^n$ possible strings

$x \to |x\rangle \to \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$ Make the vector zero everywhere except at the position indexed by x.

e.g. $n=0 \Rightarrow |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\Leftarrow$ Quantum state of one qubits

② Quantum States of n qubits:

$|\psi\rangle = \sum_{x \in \{0,1\}^n} a_x |x\rangle$, $\sum_{x \in \{0,1\}^n} |a_x|^2 = 1$ $\leftarrow$ length 1 vector

$\therefore |\psi\rangle \in \mathbb{C}^d$ with $d = 2^n$, $\langle \psi | \psi \rangle = 1$

e.g.1 two qubits in equal superposition

Standard basis for 2 qubis:

$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $|01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ $|10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$ $|11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$

Equal superposition:

$$|\psi\rangle = \frac{1}{2}|00\rangle + \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$$

$$= \frac{1}{2}\left[\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right] = \frac{1}{2}\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

e.g.2. two qubits in an EPR pair

$$|\psi\rangle = |EPR\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}\left(\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

normalization

$$\langle \psi | \psi \rangle = \frac{1}{\sqrt{2}}(\langle 00| + \langle 11|)\left(\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\right)$$

$$= \frac{1}{2}(\underbrace{\langle 00|00\rangle}_{=1} + \underbrace{\langle 00|11\rangle}_{=0} + \underbrace{\langle 11|00\rangle}_{=0} + \underbrace{\langle 11|11\rangle}_{=1})$$

$$= \frac{1}{2}(1+1) = 1$$

e.g.3 Another two qubits

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |11\rangle) = \frac{1}{\sqrt{2}}\left(\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

0.2 Combining qubits using the tensor product

Lecture 1. Combining qubits using the tensor product

① How do we combine two qubits?

Two seperate qubits:

(A) $|\psi\rangle_A = \alpha_A|0\rangle_A + \beta_A|1\rangle_A$

(B) $|\psi\rangle_B = \alpha_B|0\rangle_B + \beta_B|1\rangle_B$

What is $|\psi\rangle_{AB}$?

Tensor product:

$$|\psi\rangle_A \otimes |\psi\rangle_B = \begin{pmatrix} \alpha_A \\ \beta_A \end{pmatrix} \otimes |\psi\rangle_B = \begin{pmatrix} \alpha_A|\psi\rangle_B \\ \beta_A|\psi\rangle_B \end{pmatrix} = \begin{pmatrix} \alpha_A\alpha_B \\ \alpha_A\beta_B \\ \beta_A\alpha_B \\ \beta_A\beta_B \end{pmatrix}$$

e.g. Constructing the standard basis for two qubits:

Standard basis for 2 qubis:

$$|00\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad |01\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad |11\rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Another way to get ⇑, is to form the standard basis by combining the basis of two single qubits.

$\{|0\rangle_A, |1\rangle_A\}$, $\{|0\rangle_B, |1\rangle_B\}$

$$|0\rangle_A \otimes |0\rangle_B = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1\begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ 0\begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = |00\rangle$$

② Lazy notation

$$|\psi\rangle_A \otimes |\phi\rangle_B = |\psi\rangle_A|\phi\rangle_B$$

e.g. $|0\rangle_A \otimes |0\rangle_B = |0\rangle_A|0\rangle_B = |00\rangle_{AB}$

$$|\psi\rangle_1 \otimes \cdots \otimes |\psi\rangle_n = |\psi\rangle^{\otimes n}$$

③ Properties of tensor product

- △ distributive: $|\psi\rangle \otimes (|V_1\rangle + |V_2\rangle) = |\psi\rangle \otimes |V_1\rangle + |\psi\rangle \otimes |V_2\rangle$
- △ associative: $|\psi\rangle \otimes (|\Phi\rangle \otimes |\Gamma\rangle) = (|\psi\rangle \otimes |\Phi\rangle) \otimes |\Gamma\rangle$
- △ not commutative: $|\psi\rangle \otimes |\Phi\rangle \neq |\Phi\rangle \otimes (|\psi\rangle)$

e.g. 1 Combining two qubits

$|+\rangle_A = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|1\rangle_B$

① $|+\rangle_A \otimes |1\rangle_B = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \otimes |1\rangle_B = \frac{1}{\sqrt{2}}\begin{pmatrix}1\cdot|1\rangle_B\\1\cdot|1\rangle_B\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\0\\1\end{pmatrix}$

② $|+\rangle_A \otimes |1\rangle_B = \frac{1}{\sqrt{2}}(|0\rangle_A + |1\rangle_A) \otimes |1\rangle_B$

$= \frac{1}{\sqrt{2}}(|0\rangle_A \otimes |1\rangle_B + |1\rangle_A \otimes |1\rangle_B) = \frac{1}{\sqrt{2}}(|01\rangle_{AB} + |11\rangle_{AB})$

$= \frac{1}{\sqrt{2}}\left[\begin{pmatrix}0\\1\\0\\0\end{pmatrix} + \begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right] = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\0\\1\end{pmatrix}$

e.g. 2 Reconstructing equal superposition

$|+\rangle_A$, $|+\rangle_B$

$|+\rangle_A \otimes |+\rangle_B = \frac{1}{\sqrt{2}}(|0\rangle_A + |1\rangle_A) \otimes \frac{1}{\sqrt{2}}(|0\rangle_B + |1\rangle_B)$

$= \frac{1}{2}(|00\rangle_{AB} + |01\rangle_{AB} + |10\rangle_{AB} + |11\rangle_{AB})$

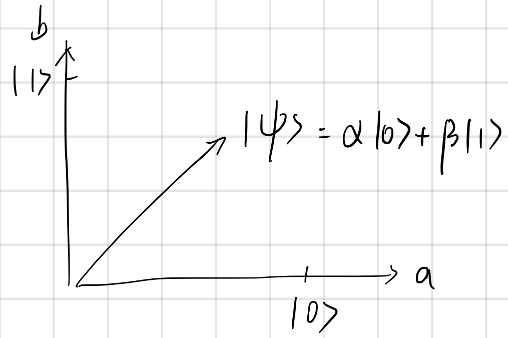
$= \frac{1}{2}\left[\begin{pmatrix}1\\0\\0\\0\end{pmatrix} + \begin{pmatrix}0\\1\\0\\0\end{pmatrix} + \begin{pmatrix}0\\0\\1\\0\end{pmatrix} + \begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right]$

$= \frac{1}{2}\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$

0.3 Measuring quantum bits

Lecture 1. Measuring qubits in standard basis

Measured in standard basis ↓



We can generate genuine randomness from a deterministic process.
(classical world 不可能)

⇑

lost the information of α and β

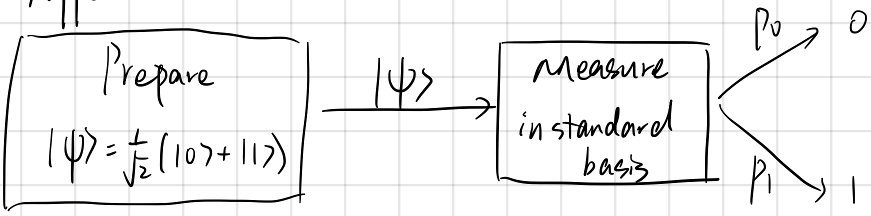
⇑

$P_0 = |\langle\psi|0\rangle|^2 = |(\alpha^* \ \beta^*)\begin{pmatrix}1\\0\end{pmatrix}|^2 = \alpha^2$ ⇒ outcome "$|0\rangle$" ⇒ collapse to $|0\rangle$

$P_1 = |\langle\psi|1\rangle|^2 = |(\alpha^* \ \beta^*)\begin{pmatrix}0\\1\end{pmatrix}|^2 = \beta^2$ ⇒ outcome "$|1\rangle$" ⇒ collapse to $|1\rangle$

$P_0 + P_1 = 1 = |\alpha|^2 + |\beta|^2$

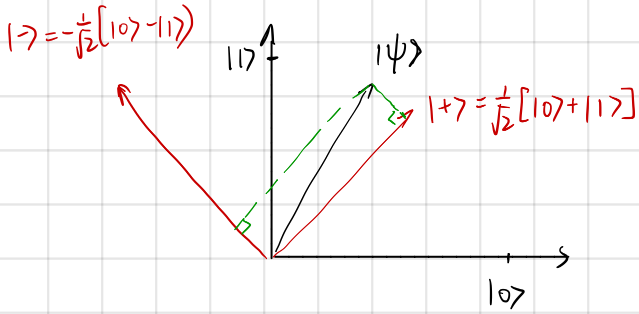
Application:



$P_0 = |\langle\psi|0\rangle|^2 = \left|\frac{1}{\sqrt{2}}\left(\langle 0| + \langle 1|\right)|0\rangle\right|^2 = \frac{1}{2}\left(\underset{=1}{\langle 0|0\rangle} + \underset{=0}{\langle 1|0\rangle}\right)^2 = \frac{1}{2}$ (because of standard basis, orthogonal.

Similarly, $P_1 = |\langle\psi|1\rangle|^2 = \cdots = \frac{1}{2}$

Lecture 2. Measuring qubits in another basis



Outcome "$|+\rangle$" : $P_+ = |\langle\psi|+\rangle|^2$

Outcome "$|-\rangle$" : $P_- = |\langle\psi|-\rangle|^2$

General Rule for measuring qubits in a bis

△ Basis we measure in: $\{|b\rangle\}_b$, $|b\rangle \in \mathbb{C}^d$ $d = 2^n$

△ Probabilities of measurement outcomes: $P_b = |\langle\psi|b\rangle|^2$

△ After measurement: outcome $b \rightarrow$ post measurement state $|b\rangle$

e.g.1. $|\psi\rangle = \sqrt{\frac{1}{3}}|0\rangle + \sqrt{\frac{2}{3}}|1\rangle$

$P_0 = |\langle\psi|0\rangle|^2 = \left|\left(\sqrt{\frac{1}{3}}\langle 0| + \sqrt{\frac{2}{3}}\langle 1|\right)|0\rangle\right|^2 = \left|\sqrt{\frac{1}{3}}\underset{=1}{\langle 0|0\rangle} + \sqrt{\frac{2}{3}}\underset{=0}{\langle 1|0\rangle}\right|^2 = \frac{1}{3}.$

$P_1 = |\langle\psi|1\rangle|^2 = \cdots = \frac{2}{3}$

e.g. 2 Beware of complex amplitude!

$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + i|1\rangle)$, basis $\begin{cases} |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) \\ |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \end{cases}$

Note: Previously there is a "−", but it won't matter

$P_+ = |\langle\psi|+\rangle|^2 = \left|\frac{1}{2}(\langle 0| - i\langle 1|)(|0\rangle + |1\rangle)\right|^2 = \frac{1}{4}|\underset{=1}{\langle 0|0\rangle} - i\underset{=0}{\langle 1|0\rangle} - i\underset{=1}{\langle 1|1\rangle} + \underset{=0}{\langle 0|1\rangle}|^2 = \frac{1}{4}|1-i|^2 = \frac{1}{4}\underbrace{(1-i)(1+i)}_{=1+1} = \frac{1}{2}$

[bra: conjugate transpose ∴ $\langle\psi| = \langle 0| - i\langle 1|$]

Similarly, $P_- = \frac{1}{2}$.

e.g.3 Measuring two qubits.

$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

$P_{00} = |\langle\psi|00\rangle|^2 = |\frac{1}{\sqrt{2}}(\langle 00| + \langle 11|)|00\rangle|^2$

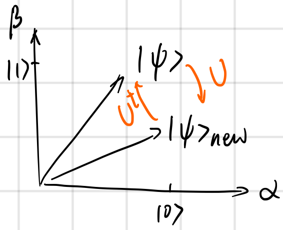
$= |\frac{1}{\sqrt{2}}(\underbrace{\langle 00|00\rangle}_{=1} + \underbrace{\langle 11|00\rangle}_{=0})|^2$

$= \frac{1}{2}$

Similarly, $P_{11} = \frac{1}{2}$

0.4 Performing Operations on qubits

Lecture1. Operations on qubits



$|\psi_{new}\rangle = U|\psi\rangle$

$\langle\psi_{new}| = \langle\psi|U^t$ [U^t: Conjugate transpose of U]

Need to make sure it remains a qubit!

Preserve normalization. ⟺ For all $|\psi\rangle$, $1 = \langle\psi_{new}|\psi_{new}\rangle = \langle\psi|U^tU|\psi\rangle$

⇓

$UU^t = U^tU$

This is only possible if: $U^tU = I$ [I is identity matrix]

$I\cdot|\psi\rangle = |\psi\rangle$

For one qubit, $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

∴ U^t also is the inverse of U.

Unitary transform

△ Unitary matrices:

U is unitary ⟺ $U^tU = UU^t = I$ $\quad U^t = (U^*)^T$ cojugate transpose U

e.g.1. Hadamard transform

$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, $H^t = H$

$H|0\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |+\rangle$

$H|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |-\rangle$

$|0\rangle$ and $|1\rangle$ are orthogonal, so after $\boxed{H}$, the $|+\rangle$ and $|-\rangle$ are also orthogonal e.g.2 X operation — Bit Flip

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \begin{cases} X^{\dagger} = X \\ X^{\dagger} X = XX = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I \end{cases}$$

Hermitian matrix

→ 主对角线上元素是实数. 所有元素关于主对角线对称:

e.g. $\begin{bmatrix} 3 & 2+i \\ 2-i & 1 \end{bmatrix}$

$$X|0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle$$

$$X|1\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

e.g.3. Z operation — Phase Flip

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \begin{cases} Z^{\dagger} = Z \\ Z^{\dagger} Z = ZZ = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I \end{cases}$$

$$Z|0\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

$$Z|1\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -|1\rangle$$

we always call this minus "phase"

Another name for X and Z operation: Pauli X & Pauli Z

$$Z|+\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = |-\rangle$$

$$Z|-\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = |+\rangle$$

e.g.4 Combining X and Z

If we apply two unitary transformations consecutively, the resulting transformation will also be unitary.

$$U = U_2 U_1$$

$$U^{\dagger} U = \underbrace{U_1^{\dagger} \underbrace{U_2^{\dagger} U_2}_{I} U_1}_{I} = U_1^{\dagger} I U_1 = U_1^{\dagger} U_1 = I = UU^{\dagger}$$

$U_1 = Z$

$U_2 = X$. $\quad Y = iXZ$

$Y|0\rangle = iXZ|0\rangle = iX|0\rangle = i|1\rangle$ [Z do nothing to $|0\rangle$, and X flips the bit around]

$Y|1\rangle = iXZ|1\rangle = -iX|1\rangle = -i|0\rangle$

0.5 Why we can't copy qubits.

Lecture 1. Qubits can't be copied

What would it mean to copy a qubit?

If a copy procedure exists: $U|\psi\rangle_A|0\rangle_B = |\psi\rangle_A|\psi\rangle_B$ It would produce two copies of $|\psi\rangle$

U: copy transformation; $|\psi\rangle_A$: qubit to be copied; $|0\rangle_B$: empty qubit register

U should work for any $|\psi\rangle_A$!

e.g. when $\begin{cases} |\psi\rangle_A = |0\rangle_A, & U|0\rangle_A|0\rangle_B = |0\rangle_A|0\rangle_B \\ |\psi\rangle_A = |1\rangle_A, & U|1\rangle_A|0\rangle_B = |1\rangle_A|1\rangle_B \end{cases}$

also for Hadamard basis:

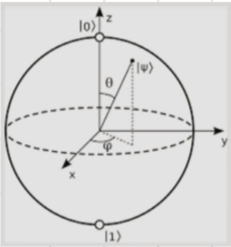
when $|\psi\rangle_A = |+\rangle_A$, $U|+\rangle_A|0\rangle_B = |+\rangle_A|+\rangle_B = \frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix} \neq \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$ ⇒ There can't be a copy operation U that copies all three of these states

$$U|+\rangle|0\rangle = U\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)|0\rangle = \frac{1}{\sqrt{2}}(U|0\rangle|0\rangle+U|1\rangle|0\rangle) = \frac{1}{\sqrt{2}}(|0\rangle|0\rangle+|1\rangle|1\rangle) = \frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$$

$$= \frac{1}{\sqrt{2}}\left(\begin{pmatrix}1\\0\\0\\0\end{pmatrix}+\begin{pmatrix}0\\0\\0\\1\end{pmatrix}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$$

0.6 The Bloch Sphere

Lecture 1. The Bloch Sphere Representation



⇒ Only works for one qubit

$$|\psi\rangle = \cancel{e^{i\gamma}}\left[\cos\left(\frac{\theta}{2}\right)|0\rangle + \sin\left(\frac{\theta}{2}\right)e^{i\varphi}|1\rangle\right]$$

We ignore it because if we measure the state, then the probabilities of getting measurement outcomes do not depend on this phase, so we can't see it.

∴ θ, φ give us a point on the sphere

e.g. $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle+|1\rangle) \Rightarrow \varphi = 0$ (Y dimension doesn't matter ⇒ XZ plane)

Further reading materials for this chapter

- Book: Quantum Computation and Quantum Information by M. Nielsen and I. Chuang provides much more information than we will ever need here! While a little dated by now, this book still provides a great overview over all aspects of quantum information, ranging from quantum computation, ideas for implementations all the way to the basics of quantum channel coding.
- Book: Quantum Processes, Systems and Information by B. Schumacher and M. Westmoreland gives a very nice introduction to qubits and basic concepts in quantum information. It is a beautiful introduction to quantum mechanics itself from the perspective of quantum information.
- Lecture notes: John Preskill's notes for Phys 219/CS 219 Quantum Computation are a great resource, and will sometimes provide you with a more "physics-y" take on things.