

Quantum Computing

- △ Shor's algorithm: exponentially faster solve factoring problem
- △ block sphere representation.

Bloch sphere representation [edit]

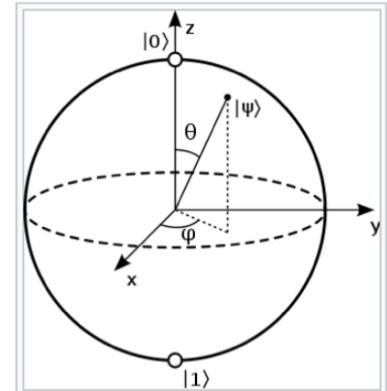
It might, at first sight, seem that there should be four degrees of freedom in $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, as α and β are complex numbers with two degrees of freedom each. However, one degree of freedom is removed by the normalization constraint $|\alpha|^2 + |\beta|^2 = 1$. This means, with a suitable change of coordinates, one can eliminate one of the degrees of freedom. One possible choice is that of Hopf coordinates:

$$\begin{aligned}\alpha &= e^{i\psi} \cos \frac{\theta}{2}, \\ \beta &= e^{i(\psi+\phi)} \sin \frac{\theta}{2}.\end{aligned}$$

Additionally, for a single qubit the overall phase of the state $e^{i\psi}$ has no physically observable consequences, so we can arbitrarily choose α to be real (or β in the case that α is zero), leaving just two degrees of freedom:

$$\begin{aligned}\alpha &= \cos \frac{\theta}{2}, \\ \beta &= e^{i\phi} \sin \frac{\theta}{2},\end{aligned}$$

where $e^{i\phi}$ is the physically significant relative phase.



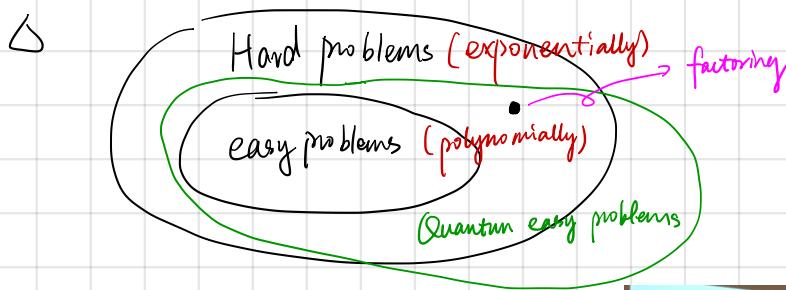
Bloch sphere representation of a qubit. The probability amplitudes for the superposition state, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, are given by $\alpha = \cos\left(\frac{\theta}{2}\right)$ and $\beta = e^{i\phi} \sin\left(\frac{\theta}{2}\right)$.

- △ Quantum circuit : measurement

- △ Classical: AND, OR, NOT, XOR

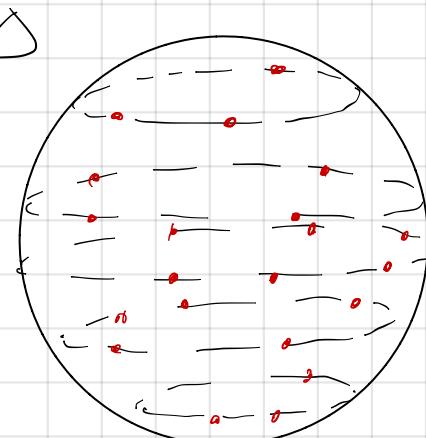
Quantum: { Hadamard } \Rightarrow Superposition + Entanglement
CNOT

△ application { Cryptography
Chemistry
ML }



△ factoring \Rightarrow factoring algorithm

2 logn · logn



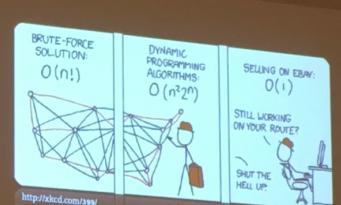
4) $3 \times 9 = ?$

$$= 3 \times \sqrt{81} = 3 \sqrt{81} = 3 \cdot 9 = 27$$

27
6
21
21
0

<http://xkcd.com/759/>

"easy"
(polynomial \Rightarrow efficient)
• Multiplying numbers
• Word processing

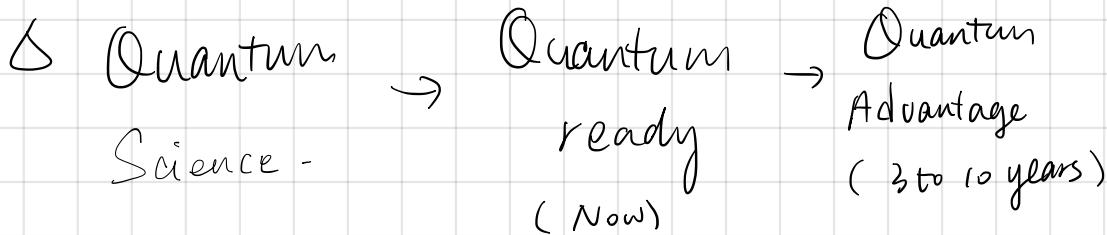
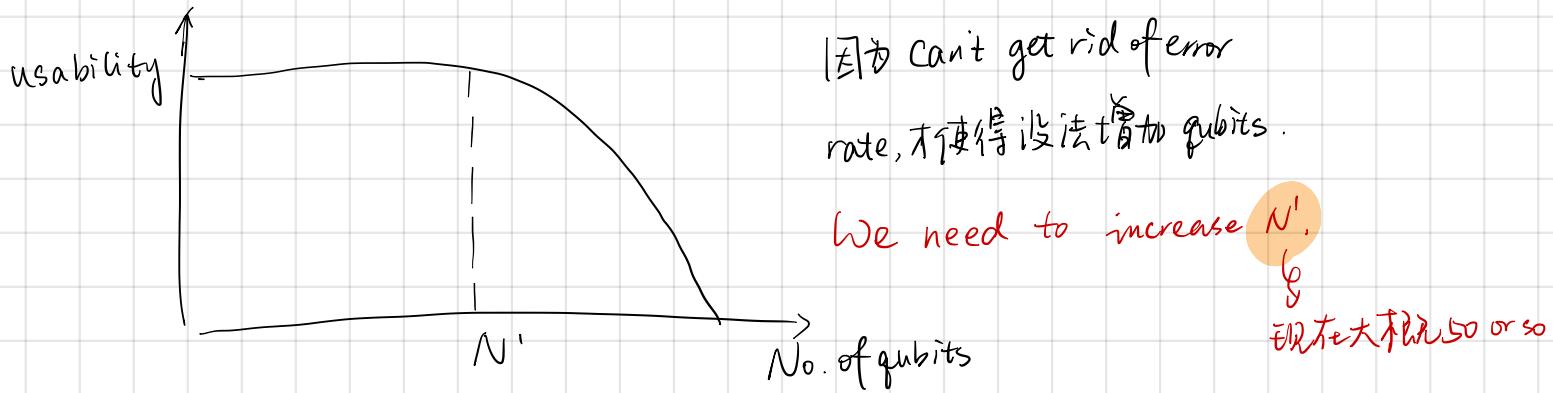


- "hard"
(exponential \Rightarrow intractable)
- Algebraic and Number Theoretic Algorithms (factoring)
 - Combinatorial optimization (traveling salesman)
 - Machine learning
 - Simulating quantum mechanics for chemistry

△ Fault-tolerance quantum Computer

quantum annealing $\frac{P}{Q} \& \frac{B}{K}$

△ A Quantum Computer's power depends on more than just adding qubits — error rate



△ Chemistry with QC:

H₂: 2 qubits

LiH: 4 qubits

BaH₂: 6 qubits

Caffeine (咖啡因): We can represent it using around 160 qubits.

△ HHL algorithm

$$A \rightarrow A'$$

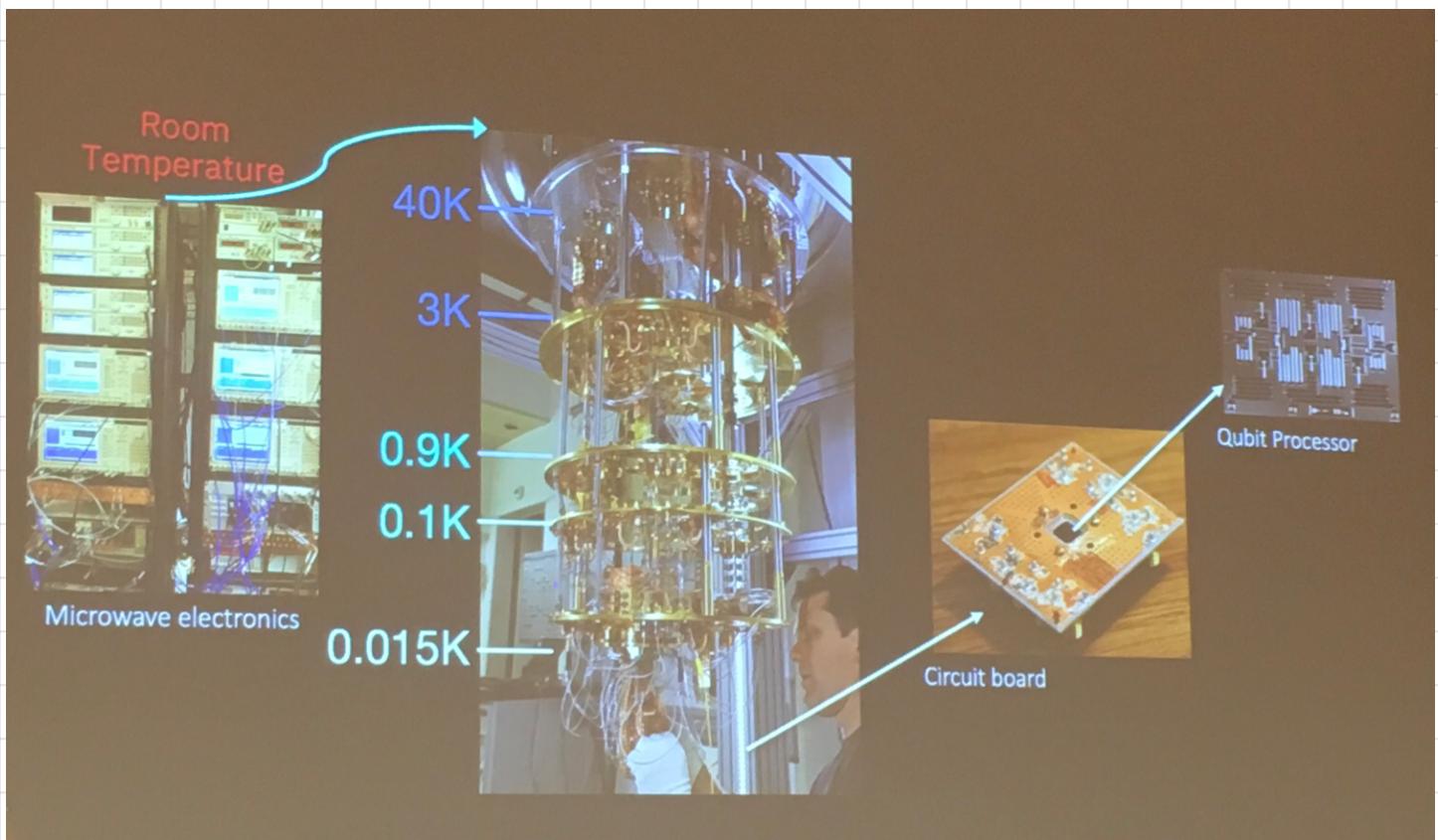
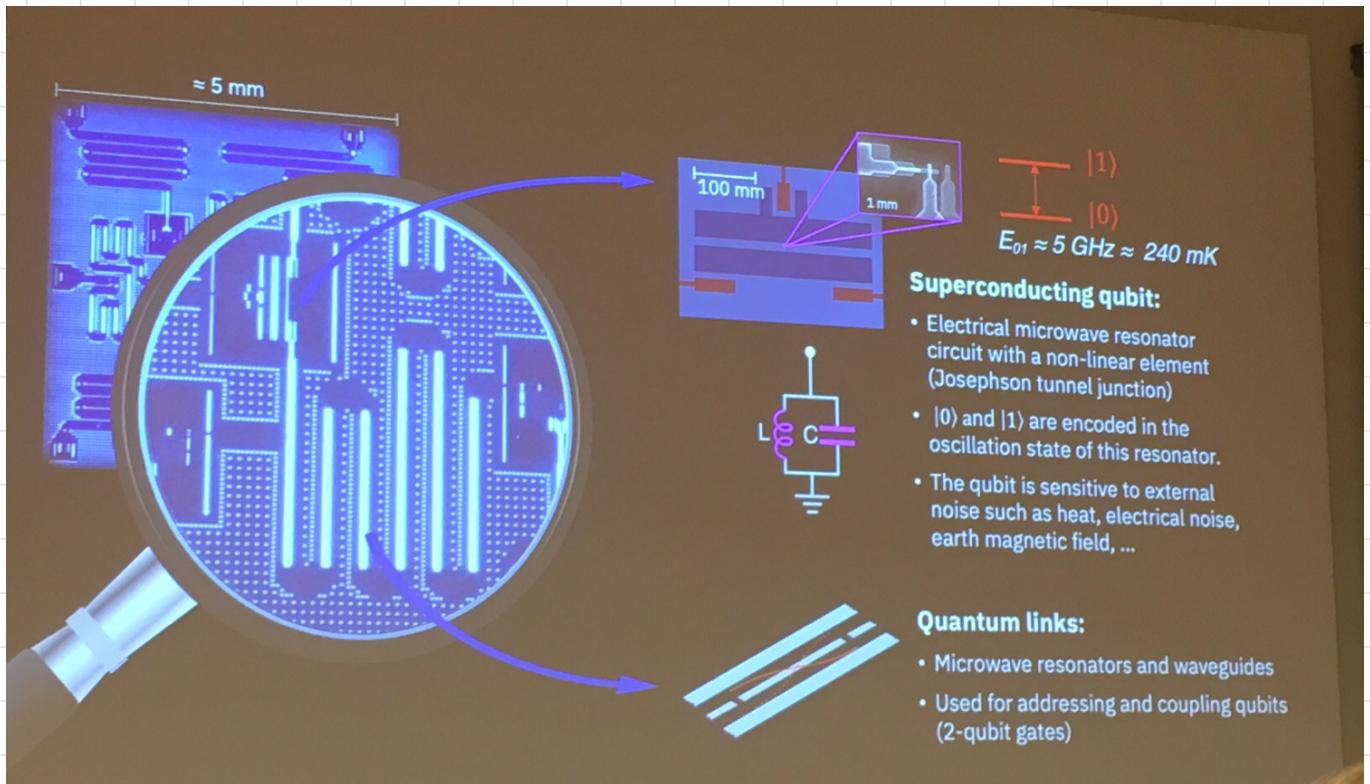
$O(N^3) \Rightarrow$ Computational cost of 矩阵求逆

\downarrow
 $O(\log N)$

b] (?) \tilde{f} ML, self-learning ML

△ No quantum memory yet

△ Quantum Computer im MHz -Bereich.



△ { Quantum register : qubits
Classical register : bits