INFORMATION AND CODING THEORY HOMEWORK 2 DUE TO 23:59 03/07/17

Please provide a solution for all the problems and send a single pdf file via Canvas by the due date. Points for all the problems are placed before problem description. If you have any questions do not hesitate to contact the course instructor or TA by e-mail or personally. Good luck!

1. (1 point) Let G_1 and G_2 be the generator matrices of $[n_1, k, d_1]$ and $[n_2, k, d_2]$ binary linear codes accordingly. Find the parameters of the code with generator matrix $G = [G_1|G_2]$.

2. (2 points) Is it possible to present $[8, 4]$ extended Hamming code as a cyclic code? Explain your answer.

3. (1 point) Let $\mathcal{C}$ be a binary cyclic code with length n and generator polynomial $g(x)$. Prove, that if n is an odd number and $(x + 1) \not| g(x)$, then $\mathcal{C}$ contains a codeword with all ones.

4. (1 point)
   a. Construct a finite field $GF(2^4)$ modulo a polynomial $\varphi(x) = x^4 + x^3 + 1$.
   b. Find the generator matrix of a primitive $[15, 7]$ BCH code over this field that can correct two errors.
   c. Decode a vector $\mathbf{y} = [\ 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1\ ]$.

5. (1 point)
   a. Construct a finite field $GF(2^3)$ modulo a polynomial $\varphi(x) = x^3 + x^2 + 1$.
   b. Construct a $[7, 3]$ Reed–Solomon code over $GF(2^3)$ and find its parameters
   c. Decode a vector $\mathbf{y} = [\ \alpha^4 \quad \alpha^2 \quad \alpha^4 \quad 0 \quad 0 \quad \alpha \quad \alpha^6\ ]$.

6. (2 points) Find a weight enumerator of a binary code $\mathcal{C}$ with parity-check matrix $\mathbf{H}$ of size $m \times \binom{m}{2}$ consisting of all possible columns of length m and weight 2, i.e.

$$\mathbf{H} = \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 1 \end{bmatrix}}_{\binom{m}{2}}.$$

   (Hint: find the weight enumerator of the dual code and apply the MacWilliams identity)

7. (2 points) Find a way to construct a trellis for a binary linear block code with a parity-check matrix

$$\mathbf{H} = [\mathbf{h}_1 \mathbf{h}_2 \dots \mathbf{h}_n],$$

   of size $(n - k) \times n$, where $\mathbf{h}_i$, $i = 1, \dots, n$, are the columns. Explain your method.
   a. How many states are in the trellis?
   b. What is the number of incoming and outgoing edges for each state?
   c. Draw a trellis for $[7, 4]$ Hamming code with a parity-check matrix

$$\mathbf{H}_3 = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$