Please provide a solution for all the problems and send a single pdf file via Canvas by the due date. Points for all the problems are placed before problem description. If you have any questions do not hesitate to contact the course instructor or TA by e-mail or personally. Good luck!

1. (4 point) Let $\{\mathcal{C}_n\}$ be a family of regular $(J = 2, K)$ LDPC codes. The length of $\mathcal{C}_n$ is equal to n, the distance is d_n. Can this family be asymptotically good?
   (Hint: try to find an interconnection in between codewords and cycles in the Tanner graph)

2. (4 point) An LDPC code $\mathcal{C}$ of length N over $\mathbb{F}_q$ is a null-space of an $M \times N$ sparse parity-check matrix $\mathbf{H} = [h_{j,i}]$, $1 \le j \le M$, $1 \le i \le N$, over $\mathbb{F}_q$. The constructed code $\mathcal{C}$ can be described with the use of a bipartite graph, which is called the Tanner graph. The vertex set of the graph consists of the set of variable nodes $V = \{v_1, v_2, \dots, v_N\}$ and the set of check nodes $C = \{c_1, c_2, \dots, c_M\}$. The variable node v_i and the check node c_j are connected with an edge if and only if $h_{j,i} \neq 0$. The edge has a label $h_{j,i}$. Derive Sum-Product rules for non-binary LDPC codes (i.e. variable node and check node update rules). How would you represent messages in this case?

3. (2 point) Assume we have a parity-check matrix $\mathbf{H}$ of protograph based LDPC code

$$\mathbf{H} = \begin{bmatrix} \mathbf{P}_{1,1} & \mathbf{P}_{1,2} & \dots & \mathbf{P}_{1,n} \\ \mathbf{P}_{2,1} & \mathbf{P}_{2,2} & \dots & \mathbf{P}_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{P}_{m,1} & \mathbf{P}_{m,2} & \dots & \mathbf{P}_{m,n} \end{bmatrix} \in \mathbb{F}_2^{ms \times ns},$$

   where $\mathbf{P}_{i,j}$, $i = 1, \dots, m$, $j = 1, \dots, n$, is a permutation matrix of size $s \times s$. Give an upper bound on the distance of such code.