Information and coding theory HW#2

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Problem 1

We have the $(J = 2, K)$ LDPC code C_n with length n. Let us estimate the minimum distance of this code. The Tanner graph T of this code have n variables nodes of degree 2 and k_n check nodes of degree K. From this Tanner graph we can construct K-regular graph $\tilde{T}$ over k_n vertices. Two vertices $v_1, v_2 \in V(\tilde{T})$ are connected by an edge iff in the graph T there exists a variable node v, s.t. edges (v_1, v) and (v_2, v) exist. So we have K-regular graph and it is easy to understand that codewords of the code C_n are determined by cycles in the graph $\tilde{T}$. If we denote the minimum distance of this code as d_n then it can be easily seen that there are not all verticies in $\frac{d_n}{2}$-neighborhood of any vertex. So we find that

$$1 + K + K(K-1) + \ldots + K(K-1)^{\frac{d_n}{2}} \leq k_n \leq n \tag{1}$$

$$1 + K\frac{(K-1)^{\frac{d_n}{2}+1} - 1}{K-2} \leq n \tag{2}$$

$$(K-1)^{\frac{d_n}{2}+1} \leq n \tag{3}$$

$$d_n \leq 2\log_{K-1} n \tag{4}$$

So we can easily find that

$$\frac{d_n}{n} \leq \frac{2\log_{K-1} n}{n} \longrightarrow 0 \tag{5}$$

Problem 2

Problem 3

We have $(ms \times ns)$ matrix consists of permutation $s \times s$ matrices. Let us choose vector $v \in \{0,1\}^{ns}$ in the following way

$$v = (\underbrace{11\dots1}_{2s}\underbrace{00\dots0}_{2ns-2s}) \tag{6}$$

It can be easily see that $H^T v = 0$, and that weight$(v) = 2s$. So we obtain an upper bound on the minimum distance of this code

$$d(C) \leq 2s \tag{7}$$