Guangdong Technion
Israel Institute of Technology
广东以色列理工学院

First Homework Infi 1 (page 1/2)

(I) Show that the following real numbers are irrational.

a) $\sqrt{6}$

b) $\sqrt{2}+\sqrt{3}$

(II) Prove the following statements using induction.

a) For all $n \in \mathbb{N}$, we have

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

b) For all $n \in \mathbb{N}$, we have

$$1! \cdot 1 + 2! \cdot 2 + \cdots + n! \cdot n = (n+1)! - 1$$

(III) Let A, B be non-empty sets of real numbers that are bounded above. Prove or disprove the following statements:

a) Suppose there is $\varepsilon > 0$ such that for all $a \in A$, there is $b \in B$ with $a + \varepsilon < b$. Then $\sup A < \sup B$.

b) Suppose that for all $a \in A$, there is $\varepsilon > 0$ such that there is $b \in B$ with $a + \varepsilon < b$. Then $\sup A < \sup B$.

c) Assuming $A \cap B$ is non-empty, we have that $\sup(A \cap B) = \min\{\sup A, \sup B\}$.

d) We have that $\sup(A \cup B) = \max\{\sup A, \sup B\}$

e) There is a set $C \subseteq \mathbb{Q}$ such that $\sup A = \sup C$

(IV) Solve the following exercises regarding the concept of absolute value.

a) Show that, if $n \in \mathbb{N}$ and $x_i \in \mathbb{R}$, for all $1 \leq i \leq n$, then we have

$$\left| \sum_{i=1}^{n} x_i \right| \leq \sum_{i=1}^{n} |x_i|$$

b) Show that if $|x-3| < \frac{1}{2}$, then we have

$$\left| \frac{x^2 + x - 12}{x - 2} \right| < \frac{15}{2}$$

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First Homework Infi 1 (page 2/2)

(V) Consider the following functions

$$
\begin{aligned}
f : \mathbb{R} &\to \mathbb{R} \\
x &\mapsto \begin{cases} x^2, \text{ if } x \leq 0 \\ x - 3, \text{ if } x > 0 \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
g : \mathbb{R} &\to \mathbb{R} \\
x &\mapsto \begin{cases} 2x, \text{ if } x \geq 1 \\ -x + 3, \text{ if } x < 1 \end{cases}
\end{aligned}
$$

Determine $g \circ f$.