a) $\sum_{i=2}^{n}(1)$ where i is multiplied by itself each iteration

Thus, i increases with the exp 2^{2^t}

$$2^{2^t} = n$$

$$t = \log(\log(n))$$

so there are $\log(\log(n))$ iterations.

$$\Theta(\log(\log(n)))$$

b) We only care about how many times the inner for loop will trigger. The if statement triggers when $i = \sqrt{n}, 2\sqrt{n}, 3\sqrt{n}, 4\sqrt{n} \ldots \sqrt{n}\sqrt{n}$

Since each instance is cubed, the summation is

$$(\sqrt{n})^3 + (2\sqrt{n})^3 + (3\sqrt{n})^3 + \ldots (\sqrt{n}\sqrt{n})^3$$

$$= \sum_{k=0}^{\sqrt{n}} k^3 n^{3/2}$$

$$n^{3/2}\,\Theta(\sqrt{n}^4)$$

$$n^{3/2}\,\Theta(n^2)$$

$$= \Theta(n^{7/2})$$

c)

inner loop: $\sum_{1}^{\log(n)} 1 = \Theta(\log(n))$

$m = m + m$

$1, 2, 4, 8, 16$

$m = 2^t$

$2^t = n$

$t = \log(n)$

$$\sum_{i=1}^{n} \sum_{k=1}^{n} \left(\Theta(1) + O\left(\sum_{m=1}^{\log n} \Theta(1) \right) \right)$$

$$= \sum_{i=1}^{n} \left(\sum_{k=1}^{n} (\Theta(1)) + \sum_{n=1}^{\log n} \Theta(1) \right)$$

$$= \sum_{i=1}^{n} (n + \log(n))$$

$$= \Theta(n^2)$$

inner loop can only occur n times because there are only n spots in the array

d) $$\sum_{i=0}^{n} \left(\Theta(1) + O\left(\sum_{j=0}^{size} \Theta(1) \right) \right)$$

$$= \Theta(n) + \sum_{i=0}^{\log\left(\frac{n}{10}\right)} \left(\frac{3}{2}\right)^i$$

$$= \Theta(n) + \left(\frac{3}{2}\right)^{\log_{3/2}\left(\frac{n}{10}\right)}$$

$$= \Theta(n) + \frac{n}{10}$$

$$= \Theta(n)$$

$10, 15, 22, 33$

$n = 10 \times \left(\frac{3}{2}\right)^t$

$\frac{n}{10} = \frac{3}{2}^t$

$t = \log_{3/2}\left(\frac{n}{10}\right)$

if statement is triggered $\log_{3/2}\left(\frac{n}{10}\right)$ times.

Inner for loop is triggered $\left(\frac{3}{2}\right)^i$ times