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WA.1

1) a)  $f_1(n) = \sum_{i=0}^n (2i)$  Apply  $R_1$

$$= 2 \sum_{i=0}^n (i)$$

$$= 2 \sum_{i=1}^n i = 2 \left( \frac{n(n+1)}{2} \right) \text{ Apply } R_7$$

$$= n(n+1)$$

$$= \boxed{n^2 + n}$$

Big-O bound:  $O(n^2)$

b)  $f_2(n) = \sum_{i=1}^n 2^i$

$$= \boxed{2^{n+1} - 1} \text{ apply } R_8$$

$$= 2 \cdot 2^n - 1$$

Big-O bound:  $O(2^n)$

c)  $f_3(n) = \sum_{i=1}^{\log(n)} 3(n) = 3 \sum_{i=1}^{\log(n)} (n)$  Apply  $R_1$

$$= \boxed{\frac{3}{2} (\log^2(n) + \log(n))}$$

$$= 3 \frac{\log(n)(\log(n)+1)}{2} \text{ Apply } R_7$$

Big-O bound:  $O(\log^2 n)$

$$= \frac{3}{2} \cdot (\log(n))^2 + \log(n)$$

d)  $f_4(n) = \sum_{i=1}^{\log(n)} i$

$$= \boxed{\frac{1}{2} (\log^2(n) + \log(n))}$$

$$= \frac{\log(n)(\log(n)+1)}{2} \text{ Apply } R_7$$

Big-O bound:  $O(\log^2 n)$

e)  $f_5(n) = \sum_{i=1}^{\log(n)} (O(1) 2^i)$

$$= O(1) \sum_{i=1}^{\log(n)} 2^i \text{ Apply } R_1$$

Big-O bound:  $O(1)$

$$= O(1) \cdot (2^{\log(n)+1} - 1)$$



$$f) f_6(n) = \sum_{i=1}^n i \log(n)$$

$$= \log(n) \cdot \sum_{i=1}^n i \quad \text{Apply } R_1$$

$$= \log(n) \cdot \frac{n(n+1)}{2} \quad \text{Apply } R_7$$

$$= \boxed{\frac{1}{2} (n^2 \log(n) + n \log(n))}$$

Big-O bound:  $O(n^2 \log n)$

$$g) f_7(n) = \sum_{i=1}^n \log(i)$$

$$= \log(1) + \log(2) + \dots + \log(n-1) + \log(n) \quad \text{Apply } R_5$$

for  $1 \leq n$

$$= n \log(n)$$

Big-O bound:  $O(n \log n)$

$$h) c < d < e < g < a < f < a$$