

# AI and Human Capital Accumulation: Aggregate and Distributional Implications\*

Yang K. Lu<sup>1</sup> and Eunseong Ma<sup>2</sup>

<sup>1</sup>HKUST

<sup>2</sup>Yonsei

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## Abstract

This paper examines how anticipated advances in artificial intelligence (AI) – which compress middle-skill wage premia but increase returns to high-level expertise – reshape human capital investment, labor supply, saving, and inequality. We build an incomplete-markets model with endogenous human capital and asset accumulation in general equilibrium, featuring three skill sectors and uninsurable idiosyncratic risk. We characterize household’s behavior using a two-period version, then calibrate an infinite-horizon model to U.S. data. Our findings reveal that AI induces a *voluntary job polarization* through both human capital investment and labor supply choices, reallocating workers away from the middle toward both tails. Human capital adjustments amplify AI’s positive effects on aggregate output and consumption while dampening its impact on employment. These adjustments also raise income and consumption inequality but mitigate the rise in wealth inequality that AI advancements would otherwise generate.

**Keywords:** AI, Job Polarization, Human Capital, Inequality

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\*Author emails: yanglu@ust.hk; masilver@yonsei.ac.kr

# 1 Introduction

A defining feature of recent AI advancements is their ability to perform complex, cognitive, non-routine tasks – capacities that once required substantial education and expertise. This fundamental difference sets AI apart from earlier waves of automation or computerization, which primarily replaced manual or routine labor.<sup>1</sup> In this paper, we make a central assumption – supported by a growing body of evidence – that AI adoption reduces the premium for middle-level skills while increasing the value of high-level expertise. Based on this assumption, we construct an incomplete markets economy with endogenous asset accumulation and general equilibrium to study how AI’s effects on skill premia interact with households’ human capital investment, and their subsequent impact on aggregate and distributional outcomes of the economy.

## 1.1 Evidences for AI’s effects on skill premia

Recent labor market data highlight the disproportionate impact of AI on entry-level employment opportunities. Bloomberg (2025) reports that, in the words of Matt Sigelman, president of the Burning Glass Institute, “Demand for junior hires in many college-level roles is already declining, even as demand for experienced hires in the same jobs is on the rise.” According to Revelio Labs (2025), postings for entry-level jobs in the US declined by about 35% since January 2023, with roles more exposed to AI experiencing even steeper reductions.

Recent experimental evidence reviewed by Calvino *et al.*, (2025) shows that workers’ productivity gains from AI depend on their skill levels and experience. On simpler tasks where AI performs well, the technology can narrow the productivity gap between experienced and less experienced workers. However, for more complex tasks that AI cannot yet perform effectively, those with greater digital proficiency or task-specific experience achieve higher productivity gains, as successful use of AI in these settings requires more advanced skills and experience that involves understanding AI’s capabilities and limitations.

Firm-level evidence reveals similar patterns. Aghion *et al.*, (2019) documents that the average worker in low-skilled occupations receives a significant wage premium when employed by a more innovative firm. Souza (2025) finds that the adoption of AI in Brazilian firms increases employment for low-skilled production workers but reduces employment and wages for middle-wage office workers. Asam and Heller (2025) report that GitHub Copilot enables software startups to raise initial funding 19% faster with 20% fewer developers, and that these productivity gains disproportionately benefit startups with more experienced founders.

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<sup>1</sup>For example, AI tools in medical diagnostics now assist radiologists in analyzing medical images, potentially reducing demand for entry-level radiologists while simultaneously increasing the productivity of senior professionals.

37 In anticipation of these changes, households are likely to adjust their human cap-  
38 ital investments. A 2022 report by Higher Education Strategy Associates finds that  
39 following decades of growth, dropping student enrollment in higher education has  
40 become a major trend in the Global North (Higher Education Strategy Associates,  
41 2022). In the U.S., the public across the political spectrum has increasingly lost  
42 confidence in the economic benefits of a college degree.<sup>2</sup>

43 On the other hand, demand for sector-based training and reskilling opportunities  
44 has been rising. The Oliver Wyman Forum (2024) study documents widespread and  
45 significant gaps between employees' desire for reskilling in generative AI and the  
46 opportunities their employers are willing to offer. The study estimates that, over  
47 the coming decade, billions of workers will need upskilling and millions may require  
48 complete reskilling.

## 49 *1.2 Overview of our model and results*

50 We consider an economy with three sectors, each requiring low, middle, or high levels  
51 of skill (human capital) and exhibiting increasing labor productivity. Households  
52 can invest in human capital to move up to more productive sectors; without such  
53 investment, their skills depreciate, causing them to shift toward less productive  
54 sectors over time. Human capital investment occurs at two levels: a basic level  
55 achievable while working, and a higher level that demands full-time commitment,  
56 such as pursuing higher education or reskilling training. Households face uninsurable  
57 idiosyncratic productivity shocks, affecting both their labor productivity and the  
58 returns to human capital investment.

59 We model AI advancements as increasing the productivity for the low and high  
60 sectors but not for the middle sector so that the skill premium of the middle sector  
61 decreases and the skill premium of the high sector increases.

62 Using a two-period partial equilibrium model, we show that the effects of AI  
63 on skill premia discourage human capital investment for households in the low sec-  
64 tor and encourage human capital investment for households in the middle sector,  
65 thereby increasing human capital inequality. Human capital investment via full-  
66 timing training crowds out households' labor supply so that households in the low  
67 sector supplies more labor whereas households in the high sector supplies less labor,  
68 in response to the AI advancements.

69 We also examine how human capital investment interacts with saving decisions.  
70 When households are able to adjust their human capital, changes in skill premia  
71 affect their exposure to idiosyncratic risk, since moving between sectors alters the  
72 level of their labor income. As AI reduces the skill premium for the middle sector,

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<sup>2</sup>Pew Research Center reports that about half of Americans say having a college degree is less important today than it was 20 years ago in a survey conducted in 2023 (Pew Research Center, 2024). A 2022 study from Public Agenda (2022), a nonpartisan research organization, shows that young Americans without college degrees are most skeptical about the value of higher education.

73 households in the low sector face less idiosyncratic risk and consequently decrease  
74 their precautionary saving. In contrast, because AI increases the skill premium for  
75 the high sector, households in the high sector become more exposed to risk and  
76 therefore increase their saving. For households in the middle sector, the effect of AI  
77 on saving is ambiguous.

78 At the economy level, the effects of AI advancements depend on the sectoral re-  
79 distribution of households and the general equilibrium effects via wage and capital  
80 return responses. We quantify these effects using a fully-fledged dynamic quanti-  
81 tative model that incorporates an infinite horizon, endogenous asset accumulation,  
82 and general equilibrium. The model is calibrated to reflect key features of the U.S.  
83 economy, capturing realistic household heterogeneity. The steady state distribution  
84 of human capital without AI advancements pins down the sectoral distribution of  
85 households. We then introduce fully anticipated AI advancements happening in the  
86 near future and study the transition dynamics from the current state of the economy  
87 to the eventual new steady state.

88 Our quantitative model demonstrates that AI induces a *voluntary job polariza-*  
89 *tion* through both human capital investment and labor supply choices. A substan-  
90 tial share of middle-sector households voluntarily reallocate to either the low or  
91 high sectors in the new steady state via human capital adjustments. During the  
92 transition, human capital accumulation becomes increasingly concentrated among a  
93 smaller segment of the population, reflecting growing inequality in skill acquisition.  
94 In addition to these population shifts, labor supply dynamics further contribute to  
95 job polarization: many middle-sector households reduce their labor supply as they  
96 engage in full-time training to upskill more rapidly, while labor supply in the low  
97 sector rises more than in the high sector.

98 Building on these labor dynamics, our model investigates how AI shapes the  
99 economy’s aggregate and distributional outcomes through both its direct impact on  
100 sectoral productivity and the endogenous adjustments in human capital investment.  
101 To highlight these mechanisms, we compare the transition dynamics of our bench-  
102 mark model – where households can adjust their human capital – with those of a  
103 counterfactual model where human capital remains fixed at its initial steady state.

104 Our findings reveal that human capital responses to AI amplify its positive effects  
105 on aggregate output and consumption, but mitigate its positive effect on employ-  
106 ment. While AI’s direct effect on sectoral productivity reduces income and con-  
107 sumption inequalities, job polarization resulting from human capital adjustments  
108 reverses this effect and increases both inequalities.

109 Regarding households’ saving, the indirect effect of AI through human capital  
110 adjustments has little impact on aggregate savings – both in terms of steady state  
111 and during the transition. However, these adjustments have a substantial impact  
112 on the distribution of wealth: while AI’s direct effect increases wealth inequality,

113 the indirect effect from human capital responses partially offsets this increase.

### 114 1.3 Related Literature

115 This paper relates to the literature on how technological change, including AI and  
116 robotics, drives job polarization and affects the demand and supply of labor. Studies  
117 find that rising employment in both high- and low-wage occupations – at the expense  
118 of middle-skill jobs – characterizes job polarization across the UK, US, and Western  
119 Europe (Goos and Manning, 2007; Autor and Dorn, 2013; Goos *et al.*, 2014). Robots  
120 and automation have also been shown to reduce employment and wages across US  
121 regions (Acemoglu and Restrepo, 2020), with automation-induced job losses and  
122 declining labor force participation especially concentrated among vulnerable workers  
123 in highly automated sectors (Lerch, 2021; Faber *et al.*, 2022). Wang and Wong  
124 (2025) models AI as a learning-by-using technology and predicts large productivity  
125 gains and employment loss in the long-run.

126 Technological disruption also influences human capital accumulation. Faced with  
127 employment risks caused by automation, many affected workers invest in further  
128 education as a form of self-insurance, rather than relying solely on increases in the  
129 college wage premium (Atkin, 2016; Beaudry *et al.*, 2016). Consistent with this,  
130 Di Giacomo and Lerch (2023) and Dauth *et al.*, (2021) find that the adoption of  
131 industrial robots in the U.S. and Germany, respectively, has led to increased college  
132 and university enrollments.

133 Building on this literature, our paper develops a model that explicitly allows for  
134 a trade-off between labor supply and human capital investment. In our framework,  
135 job polarization emerges as a voluntary response to AI advancements: households in  
136 the middle sector may choose to either downskill to the low sector or upskill to the  
137 high sector, while an increasing number of middle-sector households opt for full-time  
138 training to accelerate their upskilling.

139 This paper also relates to the literature that studies human capital and physical  
140 capital in a unified framework. Chanda (2008) shows that the rise in returns to  
141 education reduces household savings. Waldinger (2016) finds that human capital  
142 is much more important than physical capital for innovation in both the short and  
143 long-run. Huggett *et al.*, (2011) develops a risky human capital model with incom-  
144 plete markets to estimate the source of lifetime inequality. Park (2018) investigates  
145 whether capital and human capital are over-accumulated in an incomplete market  
146 economy. Our model is most similar to Huggett *et al.*, (2011) in that human capital  
147 is risky and there is a trade-off between human capital investment and labor supply.  
148 Our analysis sheds light on the effect of AI-induced human capital adjustments on  
149 households labor supply and saving.

150 A growing body of literature suggests that AI and automation may contribute to  
151 rising inequality across income, consumption, and wealth (e.g., Sachs and Kotlikoff,

152 2012; Berg *et al.*, 2018; Prettner and Strulik, 2020; Hémous and Olsen, 2022).  
 153 Our model confirms that AI advancements indeed increase inequality in all three  
 154 dimensions. However, we find that the endogenous human capital responses to  
 155 AI amplify the rise in income and consumption inequality, while at the same time  
 156 mitigating the increase in wealth inequality.

157 The rest of the paper is organized as follows. Section 2 describes the model envi-  
 158 ronment. Section 3 solves the household’s problem using a two-period version of the  
 159 model. Section 4 solves the fully-fledged quantitative model and calibrates it to fit  
 160 key features of the U.S. economy, including employment rate, human capital invest-  
 161 ment, and household heterogeneity. Section 5 incorporates AI into the quantitative  
 162 model and examines its impacts on human capital adjustments. Section 6 analyzes  
 163 the aggregate and distributional effects of AI. Section 7 concludes.

## 164 2 Model Environment

165 Time is discrete and infinite. There is a continuum of households. Each house-  
 166 hold is endowed with one unit of indivisible labor and faces an idiosyncratic labor  
 167 productivity shock,  $z$ , and an idiosyncratic learning-ability shock,  $y$ . The labor  
 168 productivity shock follows an AR(1) process in logs:

$$\ln z' = \rho_z \ln z + \varepsilon_z, \varepsilon_z \stackrel{\text{iid}}{\sim} N(0, \sigma_z^2) \quad (1)$$

169 The learning-ability shock follows an AR(1) process in logs:

$$\ln y' = \rho_y \ln y + \varepsilon_y, \varepsilon_y \stackrel{\text{iid}}{\sim} N(0, \sigma_y^2) \quad (2)$$

170 Households observe  $(z_t, y_t)$  at the beginning of each period before making decisions.  
 171 The asset market is incomplete following Aiyagari (1994), and the physical capital,  
 172  $a$ , is the only asset available to households to insure against idiosyncratic labor  
 173 income risk. Households can also invest in human capital,  $h$ , which allows them to  
 174 work in sectors with different human capital requirement.

### 175 2.1 Production Technology

176 The production technology in the economy is a constant-returns-to-scale Cobb-  
 177 Douglas production function:

$$F(K, L) = K^{1-\alpha} L^\alpha \quad (3)$$

178  $K$  represents the total physical capital accumulated by households, while  $L$  denotes  
 179 the total effective labor supplied by households, aggregated across three sectors: low,

180 middle, and high. The marginal products of capital and effective labor determine  
181 the economy-wide wage rate,  $w$ , and interest rate,  $r$ .

182 These sectors differ in their technologies for converting labor into effective labor  
183 units and in the levels of human capital required for employment. The middle sector  
184 employs households with human capital above  $h_M$  and converts one unit of labor  
185 to one effective labor unit. The high sector, requiring human capital above  $h_H$ ,  
186 converts one unit of labor to  $1 + \lambda$  effective units, while the low sector, with no  
187 human capital requirement, converts one unit into  $1 - \lambda$  effective units. This implies  
188 a sectoral labor productivity  $x(h)$  that is a step function in human capital:

$$x(h) = \begin{cases} 1 - \lambda & \text{low sector if } h < h_M \\ 1 & \text{middle sector if } h_M < h < h_H \\ 1 + \lambda & \text{high sector if } h > h_H \end{cases} \quad (4)$$

189 A household  $i$  who decides to work thus contributes  $z_i x(h_i)$  units of effective labor,  
190 where  $z_i$  is his idiosyncratic productivity. Denote  $n_i \in \{0, 1\}$  as the indicator that  
191 takes one if the household works and zero if the household does not. The aggregate  
192 labor is

$$L = \int n_i z_i x(h_i) di, \quad (5)$$

193 assuming perfect substitutability of effective labor across the three sectors.

## 194 2.2 Household's Problem

195 Households derive utility from consumption, incur disutility from labor and effort of  
196 human capital investment. A household maximizes the expected lifetime utility by  
197 optimally choosing consumption, saving, labor supply and human capital investment  
198 each period, based on his idiosyncratic shocks  $(z_t, y_t)$ :

$$\max_{\{c_t, a_{t+1}, n_t, e_t\}_{t=0}^{\infty}} E_0 \left[ \sum_{t=0}^{\infty} \beta^t (\ln c_t - \chi_n n_t - \chi_e e_t) \right] \quad (6)$$

199 where  $c_t$  represents consumption,  $a_{t+1}$  represents saving,  $n_t \in \{0, 1\}$  is labor supply,  
200 and  $e_t$  is the effort of human capital investment.

201 If a household decides to work in period  $t$ , he will be employed into the appro-  
202 priate sector according to his human capital  $h_t$  and receive labor income  $w_t z_t x(h_t)$ .  
203 The household's budget constraint is

$$c_t + a_{t+1} = n_t (w_t z_t x(h_t)) + (1 + r_t) a_t \quad (7)$$

$$c_t \geq 0 \text{ and } a_{t+1} \geq 0 \quad (8)$$

204 We prohibit households from borrowing  $a_{t+1} \geq 0$  to simplify analysis.<sup>3</sup>

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<sup>3</sup>According to Aiyagari (1994), a borrowing constraint is necessarily implied by present value

Human capital investment can take three levels of effort:  $\{0, e_L, e_H\}$ . A non-working household is free to choose any of the three effort levels but a working household cannot devote the highest level of effort  $e_H$ , reflecting a trade-off between working and human capital investment. Hence:

$$e_t \in \{0, e_L, (1 - n_t)e_H\}. \quad (9)$$

Its contribution to next-period human capital is subject to the learning-ability shock:

$$h_{t+1} = y_t e_t + (1 - \delta_h) h_t \quad (10)$$

where  $\delta_h$  is human capital's depreciation rate. We interpret  $y_t e_t$  as effective human-capital investment, with  $y_t$  capturing a learning-ability shock.<sup>4</sup>

### 3 Household Decisions in a Two-Period Model

In this section, we solve the household's problem with two periods to gain intuition.

**Period-2 decisions** Households do not invest in human capital or physical capital in the last period. The only relevant decision is whether to work.

The household works  $n = 1$  if and only if  $z \geq \bar{z}(h, a)$ , with  $\bar{z}(h, a)$  defined as

$$\ln(w\bar{z}(h, a)x(h) + (1 + r)a) - \chi_n = \ln((1 + r)a) \quad (11)$$

The household faces a trade-off between earning labor income and incurring the disutility of working. Given the sector-specific productivity  $x(h)$  specified in (4), the threshold for idiosyncratic productivity,  $\bar{z}(h, a)$ , takes on three possible values:

$$\bar{z}(h, a) = \begin{cases} \bar{z}(a) \frac{1}{1-\lambda} & \text{if } h < h_M \\ \bar{z}(a) & \text{if } h_M \leq h < h_H \\ \bar{z}(a) \frac{1}{1+\lambda} & \text{if } h > h_H \end{cases} \quad (12)$$

$$\text{where } \bar{z}(a) := \frac{(\exp(\chi_n) - 1)(1 + r)a}{w} \quad (13)$$

Households with higher human capital is more likely to work, whereas households with higher physical capital is less likely to work.

**Period-1 decisions** In addition to labor supply, period-1 decisions include saving and human capital investment, both of which are forward-looking and affected by

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budget balance and nonnegativity of consumption. Since the borrowing limit is not essential to our analysis, we set it to zero for simplicity.

<sup>4</sup>Separating  $y_t$  from the labor productivity shock  $z_t$  introduces an additional state variable relative to the special case in which learning ability is perfectly correlated with labor productivity. In the quantitative model, we discretize  $(z_t, y_t)$  jointly (allowing for correlation if desired).



the idiosyncratic risk associated with the productivity shock  $z'$ . Our model also features a trade-off between human capital investment and labor supply as a working household cannot devote the highest level of effort  $e_H$  in human capital investment. Therefore, human capital investment grants households the possibility of a discrete wage hike in the future but may entail a wage loss in the current period.

To see the implication of this trade-off and how it interacts with uninsured idiosyncratic risk, we proceed in two steps. We first derive the period-1 decisions without uncertainty by assuming that  $z'$  is known to the household at period 1 and  $z'$  is such that the household will work in period 2. We then reintroduce uncertainty in  $z'$  and compare the decision rules with the case without uncertainty.

### 3.1 *Period-1 Labor Supply and Human Capital Investment*

#### 3.1.1 **Consumption and saving without uncertainty**

The additive separability of household's utility implies that labor supply  $n$  and human capital investment  $e$  enters in consumption and saving choices only via the intertemporal budget constraint:

$$c + \frac{c'}{1+r'} = (1+r)a + n(wzx(h)) + \frac{w'z'x(h')}{1+r'}$$

with  $h' = ye + (1-\delta)h$ .

The log utility in consumption implies the optimality condition:

$$c' = \beta(1+r')c. \tag{14}$$

Combining it with the budget constraint, we obtain the optimal consumption as a function of labor supply  $n$  and human capital investment  $e$ :

$$c(n, e) = \frac{1}{1+\beta} \left[ (1+r)a + n(wzx(h)) + \frac{w'z'x(h' = ye + (1-\delta)h)}{1+r'} \right]. \tag{15}$$

#### 3.1.2 **Labor supply and human capital investment**

The optimal consumption rules in (15) and (14) allow us to express the household's problem as the maximization of an objective function in labor supply  $n$  and human capital investment  $e$ :<sup>5</sup>

$$\max_{n,e} (1+\beta) \ln c(n, e) - \chi_n n - \chi_e e \tag{16}$$

This maximization depends critically on the household's current human capital and achievable next-period human capital. Accordingly, we partition households into

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<sup>5</sup>This follows since  $c' = \beta(1+r')c$ , so  $\ln c' = \ln \beta + \ln(1+r') + \ln c$ .

248 three ranges of  $h$ :  $[0, h_M(1-\delta)^{-1})$ ,  $[h_M(1-\delta)^{-1}, h_H(1-\delta)^{-1})$ , and  $[h_H(1-\delta)^{-1}, h_{\max}]$ .

249 We now derive the decision rules for households  $h \in [0, h_M(1-\delta)^{-1})$  in detail, as the  
 250 decision rules for the other two ranges are similar. Conditional on a given learning  
 251 ability  $y$ , we define two cutoffs in human capital:

$$\underline{h}_M(y) := \frac{h_M - ye_H}{1 - \delta}, \quad \bar{h}_M(y) := \frac{h_M - ye_L}{1 - \delta} \quad (17)$$

252 These cutoffs divide households into three groups based on their ability to be em-  
 253 ployed in the middle sector in the next period. **Non-learners** are households with  
 254  $h < \underline{h}_M(y)$ . They cannot achieve  $h' > h_M$  with either  $e_L$  or  $e_H$  level of human  
 255 capital investment today. **Slow learners** are households with  $h \in (\underline{h}_M(y), \bar{h}_M(y))$ .  
 256 These households can reach  $h' > h_M$  in the next period only by investing  $e = e_H$   
 257 today. **Fast learners** are households with  $h > \bar{h}_M(y)$ . They can achieve  $h' > h_M$   
 258 in the next period if they invest  $e = e_L$  today.

259 Conditional on  $y$  and learner type, the optimal choices of labor supply and human  
 260 capital investment are characterized by cutoffs in  $z$ . The cutoff formulae varies with  
 261 households' current sectoral productivity, i.e, if  $h < h_M$  or not.

262 **Non-learners** will not invest in human capital,  $e = 0$ , and their future sectoral  
 263 productivity will be  $x(h') = 1 - \lambda$ . Non-learners work  $n = 1$  if and only if:

$$z \geq \bar{z}_{non}^L(a, h) = \frac{(\exp(\frac{\chi_n}{1+\beta}) - 1) \left[ (1+r)a + \frac{w'z'(1-\lambda)}{1+r'} \right]}{w(1 - \mathbb{1}_{h < h_M} \lambda)} \quad (18)$$

264 **Slow learners** choose between  $e = 0$  and  $e = e_H$ , since  $e = e_L$  incurs a cost  
 265 without any future benefit. Slow learners must trade off between working and human  
 266 capital investment: choosing  $e = e_H$  requires not working today ( $n = 0$ ), while  
 267 opting to work means forgoing investment in human capital ( $n = 1, e = 0$ ).<sup>6</sup>

268 Slow learners prefer  $(n = 1, e = 0)$  to  $(n = 0, e = e_H)$  if and only if:

$$z \geq \bar{z}_{slow}^L(a, h) = \frac{(\exp(\frac{\chi_n - \chi_e e_H}{1+\beta}) - 1) \left[ (1+r)a + \frac{w'z'}{1+r'} \right] + \lambda \frac{w'z'}{1+r'}}{w(1 - \mathbb{1}_{h < h_M} \lambda)} \quad (19)$$

269 **Fast learners** do not exert high effort  $e_H$  in human capital investment. They  
 270 choose among three options:  $(n = 1, e = 0)$ ,  $(n = 1, e = e_L)$ , and  $(n = 0, e = e_L)$ ,<sup>7</sup>

<sup>6</sup>The choice between  $(n = 0, e = e_H)$  and  $(n = 0, e = 0)$  does not depend on  $z$ . For  $e_H$  to be relevant,  $\lambda$  must be large enough so that  $(n = 0, e = e_H)$  is preferred to  $(n = 0, e = 0)$ . See Appendix A.2 for details on the lower bound for  $\lambda$ .

<sup>7</sup>Similar to the case of slow learners, the choice between  $(n = 0, e = e_L)$  and  $(n = 0, e = 0)$  does not depend on  $z$ . Moreover, since our model is set up so that  $(n = 0, e = e_H)$  dominates  $(n = 0, e = 0)$ , it implies that  $(n = 0, e = e_L)$  dominates  $(n = 0, e = 0)$ .

271 with the decision rule as follows:

$$n(z, h, a), e(z, h, a) = \begin{cases} n = 1, e = 0 & \text{if } z \geq \bar{z}_{fast}^L(a, h) \\ n = 1, e = e_L & \text{if } \underline{z}_{fast}^L(a, h) \leq z < \bar{z}_{fast}^L(a, h) \\ n = 0, e = e_L & \text{if } z < \underline{z}_{fast}^L(a, h) \end{cases} \quad (20)$$

272 where

$$\bar{z}_{fast}^L(a, h) = \frac{\left\{ \exp\left(\frac{\chi e_L}{1+\beta}\right) \lambda \left[ \exp\left(\frac{\chi e_L}{1+\beta}\right) - 1 \right]^{-1} - 1 \right\} \frac{w' z'}{1+r'} - (1+r)a}{w(1 - \mathbb{1}_{h < h_M} \lambda)} \quad (21)$$

273

$$\underline{z}_{fast}^L(a, h) = \frac{(\exp(\frac{\chi n}{1+\beta}) - 1)[(1+r)a + \frac{w' z'}{1+r'}]}{w(1 - \mathbb{1}_{h < h_M} \lambda)} \quad (22)$$

274 We set up our model so that  $\bar{z}_{fast}^L(a, h) > \underline{z}_{fast}^L(a, h)$ .<sup>8</sup>

275 **Decision rule diagram:** Figure 1 illustrates how the decision rules of labor supply  
 276  $n$  and human capital investment  $e$  vary with  $(z, h, a)$  for households with  $h < h_M \frac{1}{1-\delta}$   
 277 conditional on a given  $y$ . The human capital  $h$  changes along the horizontal axis  
 278 and the labor productivity shock  $z$  changes along the vertical axis. The two vertical  
 279 lines  $\underline{h}_M(y)$  and  $\bar{h}_M(y)$  defined in (17) separate the state space into non-learners  
 280 (unshaded area), slow learners (lightly shaded area), and fast learners (darkly shaded  
 281 area). Within each learner type, labor supply and investment choices vary with  $z$   
 282 and the dashed horizontal lines represents the cutoffs in  $z$ .

283 This decision rule diagram is representative for households in other ranges of  
 284 human capital. Figure 2 illustrates the regions in which households are able to make  
 285 positive human capital investments that move them across sectoral thresholds. Dark  
 286 shading denotes regions in which  $e_L$  is sufficient (fast learners), while light shading  
 287 denotes regions in which only  $e_H$  is sufficient (slow learners).

288 For households with  $h_M \frac{1}{1-\delta} \leq h < h_H \frac{1}{1-\delta}$ , conditional on  $y$  the boundaries for  
 289 state space division change to  $\bar{h}_H(y)$  and  $\underline{h}_H(y)$ :

$$\underline{h}_H(y) := \frac{h_H - y e_H}{1 - \delta}; \quad \bar{h}_H(y) := \frac{h_H - y e_L}{1 - \delta} \quad (23)$$

290 If  $h_M \frac{1}{1-\delta} \leq h < h_H$ , the four cutoffs that partition the decision regions for households  
 291 are denoted as  $\bar{z}_{non}^M(a)$ ,  $\bar{z}_{slow}^M(a)$ ,  $\underline{z}_{fast}^M(a)$ , and  $\bar{z}_{fast}^M(a)$  (see Appendix A.1 for the  
 292 explicit formulae).<sup>9</sup> If  $h_H \leq h < h_H \frac{1}{1-\delta}$ , the analogous cutoffs are given by  $\bar{z}_{non}^M \frac{1}{1+\lambda}$ ,  
 293  $\bar{z}_{slow}^M \frac{1}{1+\lambda}$ ,  $\underline{z}_{fast}^M \frac{1}{1+\lambda}$ , and  $\bar{z}_{fast}^M \frac{1}{1+\lambda}$ .

294 Households with  $h \geq h_H \frac{1}{1-\delta}$  are always non-learners, since their human capital

<sup>8</sup>Appendix A.2 provides the parameter restrictions such that the condition for  $(n = 0, e = e_H)$  to dominate  $(n = 0, e = 0)$  is sufficient for  $\bar{z}_{fast}^L(a, h) > \underline{z}_{fast}^L(a, h)$ .

<sup>9</sup>Appendix A.2 provides parameter restrictions for  $\bar{z}_{fast}^M(a, h) > \underline{z}_{fast}^M(a, h)$ .

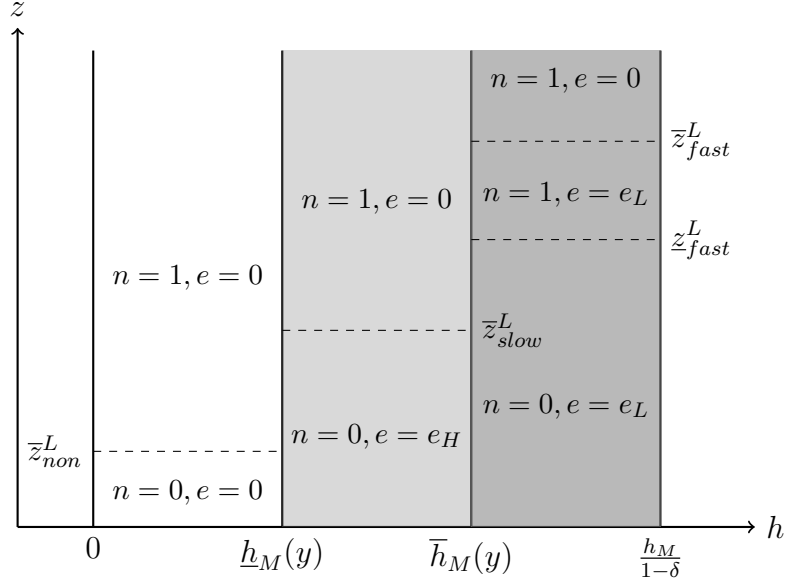


Figure 1: Decision Rule Diagram for  $0 \leq h < h_M(1 - \delta)^{-1}$

Conditional on a given learning ability  $y$ , the figure illustrates the decision rule  $(n, e)$  as a function of states  $(z, h, a)$  for households with  $0 \leq h < h_M \frac{1}{1-\delta}$ . The human capital  $h$  changes along the horizontal axis and the labor productivity shock  $z$  changes along the vertical axis. The two vertical lines  $\underline{h}_M(y)$  and  $\bar{h}_M(y)$  defined in (17) separate the state space into non-learners, slow learners, and fast learners. Within each learner type, labor supply and investment choices vary with  $z$ .

295 guarantees high-sector employment next period without further investment. For  
 296 them, only the cutoff  $\bar{z}_{non}^H(a) \frac{1}{1+\lambda}$  matters.

### 297 3.2 The Effects of Uninsured Idiosyncratic Risk

298 We now reintroduce the idiosyncratic risk to households in period 1 by assuming  
 299 that  $z'$  follows a log-normal distribution with mean  $\bar{z}'$  and variance  $\sigma_z^2$ .

300 Our previous analysis without uncertainty is a special case with  $\sigma_z^2 = 0$ . The  
 301 effects of uninsured idiosyncratic risk can be thought as how households' decisions  
 302 change when the distribution of  $z'$  undergoes a mean-preserving spread in the sense  
 303 of second-order stochastic dominance.

304 From a consumption-saving perspective, the uncertain  $z'$  is associated with future  
 305 labor income risk. It is well understood in the literature that idiosyncratic future  
 306 income risk raises the expected marginal utility of future consumption for households  
 307 with log utility and makes them save more. In our model, households can also supply  
 308 more labor to mitigate the effect of idiosyncratic income risk on the marginal utility  
 309 of consumption.

310 From the perspective of human capital investment, the uncertain  $z'$  is associated  
 311 with risk in the return to human capital. Conditional on working, households'  
 312 income increases with  $z'$ :  $c' = (1 + r')a' + w'x(h')z'$ .  $\ln(c')$  is increasing and concave

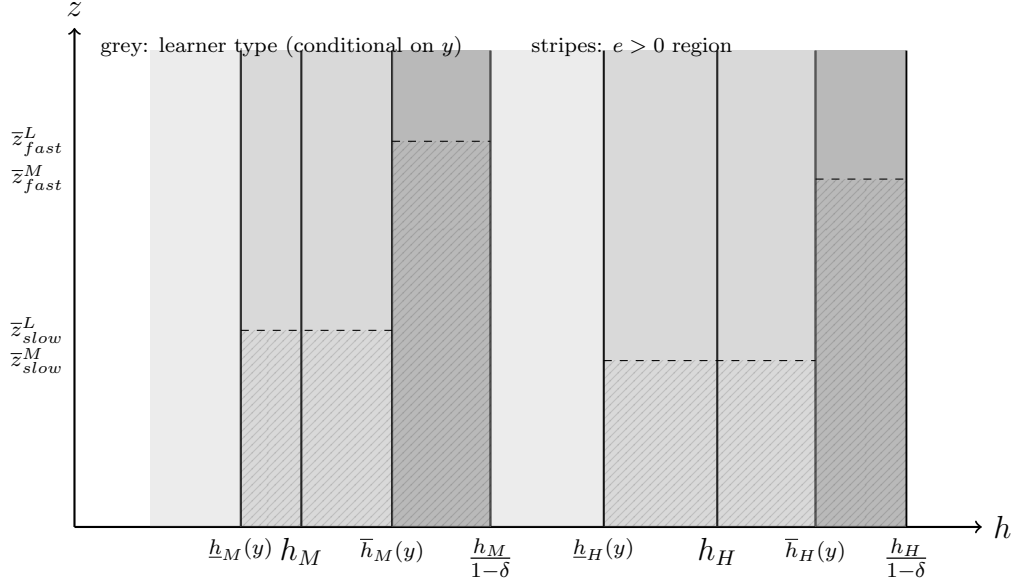


Figure 2: State Space for Human Capital Investment

With a separate learning-ability shock  $y$ , the figure illustrates how learner type (grey shading) and optimal investment ( $e > 0$ , striped areas) vary with the labor productivity shock  $z$  and human capital  $h$ . The cutoffs in  $h$  are conditional on  $y$ ; the cutoffs in  $z$  are scaled by current-sector productivity  $x(h)$ .

in  $z'$ , and a higher  $x(h')$  increases the concavity.<sup>10</sup> Consider two levels of  $h'$ ,  $\bar{h}' > \underline{h}'$ , a mean-preserving spread of  $z'$  distribution reduces the expected utility at both levels of  $h'$  but the reduction is larger for the higher level  $\bar{h}'$ . Hence, the expected utility gain of moving from  $\underline{h}'$  to  $\bar{h}'$  is smaller due to the idiosyncratic risk. Human capital investment is discouraged.

Taking into account endogenous labor supply reinforces the discouragement of human capital investment by the idiosyncratic risk. Recall from Section 3 that households with  $z'$  lower than a cutoff do not work. The endogenous labor supply therefore provides insurance against the lower tail risk of the idiosyncratic  $z'$ . Moreover, the cutoff in  $z'$  is lower for those with higher human capital  $h'$ . This makes households with higher  $h'$  more exposed to the lower tail risk than those with lower  $h'$ , further reducing the gain of human capital investment.

**Proposition 1.** *The uninsured idiosyncratic risk in  $z'$  makes households in period 1 save more, work more and invest less in human capital.*

<sup>10</sup>The marginal effect of  $z'$  on  $\ln(c')$  is

$$\frac{\partial \ln(c')}{\partial z'} = \frac{w'x(h')}{(1+r')a' + w'x(h')z'} > 0$$

The second derivative is

$$\frac{\partial^2 \ln(c')}{(\partial z')^2} = - \left[ \frac{w'x(h')}{(1+r')a' + w'x(h')z'} \right]^2 < 0$$

and is more negative if  $x(h')$  is higher.

### 3.3 Period-1 Saving and Human Capital Investment

In this section, we study the impact of endogenous human capital investment on households' saving decisions. Specifically, we compare optimal saving behavior in two scenarios: one in which households can choose to invest in human capital, and an alternative scenario in which human capital is exogenously fixed. To facilitate the comparison, we assume in this section that there is no human capital depreciation.<sup>11</sup>

When the optimal choice of human capital investment is zero, optimal saving is identical in both scenarios. When the optimal human capital investment is either  $e_L$  or  $e_H$ , we compare the household's optimal saving to the case where human capital investment is exogenously fixed at zero, i.e.,  $(n = 1, e = 0)$ .<sup>12</sup>

To make the human capital relevant, we assume that  $n' = 1$  in period 2. The additive separability of work and human capital investment effort from consumption allows us to consider the optimal saving conditional on a given choice of labor supply and human capital investment.

In particular, the household maximizes expected lifetime utility:

$$\max_{a'} : \ln(c) + \beta \mathbb{E}_{z'}[\ln(c')], \quad (24)$$

subject to the budget constraints

$$c + a' = (1 + r)a + n(wzx(h)), \quad (25)$$

$$c' = (1 + r')a' + w'z'x(h'), \quad (26)$$

$$\text{with } h' = ye + (1 - \delta)h, e \in \{0, e_L, (1 - n)e_H\} \quad (27)$$

#### 3.3.1 Effect of on-job-training on saving

We now compare the optimal saving between  $(n = 1, e = e_L)$  and  $(n = 1, e = 0)$ , where  $e_L$  allows households to move to a higher sector in period 2 with higher sectoral productivity  $x(h')$ .

To simplify the notation while maintaining the key economic forces, we normalize  $(1 + r) = (1 + r') = 1$ ,  $w = w' = 1$ , the period-1 productivity shock  $z = 1$ , the period-1 learning-ability shock  $y = 1$ , and the period-2 productivity shock  $z'$  to

<sup>11</sup>If depreciation is allowed, the analysis proceeds similarly but involves more comparison paris.

<sup>12</sup>Why not compare to  $(n = 0, e = 0)$ ? Such a comparison is not meaningful when considering  $(n = 1, e = e_L)$  because the two scenarios involve different state spaces. To see it, suppose conditions are such that  $(n = 1, e = e_L)$  is optimal. If we were to fix  $e = 0$  exogenously, the household's lifetime income would fall, and as a result the household would have a greater incentive to work. Thus, it is not possible for the household to deviate from choosing  $n = 1$  when human capital is held fixed at  $e = 0$ . The comparison between  $(n = 0, e = 0)$  and  $(n = 0, e = e_L \text{ or } e_H)$  is similar to the comparison between  $(n = 1, e = 0)$  to  $(n = 1, e = e_L)$ , since human capital investment does not affect period-1 labor income in either case.

350  $\ln z' \sim \mathcal{N}(0, \sigma_z^2)$ . The budget constraints become:

$$c + a' = a + x, \quad c' = a' + txz' \quad (28)$$

351 where  $x$  is the household's period-1 labor income that reflects both productivity and  
 352 skill.  $t \geq 1$  represents the effect of human capital investment on period-2 income:  
 353  $t > 1$  if  $e = e_L$ ;  $t = 1$  if  $e = 0$ .

354 The optimal saving is determined by the FOC:

$$\frac{1}{a + x - a'} = \beta \mathbb{E}_{z'} \left( \frac{1}{a' + txz'} \right) \quad (29)$$

355 Denoting the mean and variance of  $z'$  as  $\mu$  and  $\Sigma$ , respectively:

$$\mu \equiv \mathbb{E}[z'] = e^{\sigma_z^2/2}, \quad \Sigma \equiv \text{Var}(z') = e^{\sigma_z^2}(e^{\sigma_z^2} - 1). \quad (30)$$

356 The second-order approximate solution to the FOC is:

$$a'^*(x, a; t) = \underbrace{\frac{\beta(a + x) - tx\mu}{1 + \beta}}_{\text{CE}} + \underbrace{\frac{t^2 x^2 \Sigma}{\beta(a + x + tx\mu)}}_{\text{Precautionary}} \quad (31)$$

357 The first term is the *certainty-equivalent* saving, which reflects the consumption  
 358 smoothing motive, increasing in the period-1 resources  $a + x$  and decreasing in the  
 359 period-2 expected labor income  $tx\mu$ . The second term is the *precautionary* saving,  
 360 which is increasing in the variance of period-2 labor income  $t^2 x^2 \Sigma$  and decreasing in  
 361 the expected total resources  $a + x + tx\mu$ .

362 The effect of on-job-training on saving can be decomposed into two components:

$$\frac{\partial a'^*}{\partial t}(x, a; t) = -\frac{x\mu}{1 + \beta} + \frac{x^2 \Sigma}{\beta} \frac{t[2(a + x) + tx\mu]}{(a + x + tx\mu)^2}. \quad (32)$$

363 The first term being negative captures the *crowd-out* effect on saving via consumption-  
 364 smoothing motive as on-job-training increases the expected period-2 labor income  
 365  $tx\mu$ . The second positive term captures the *crowd-in* effect via precautionary saving  
 366 motive as on-job-training exposes households to larger future income risk.

367 To capture the overall impact of on-job-training on saving, we define:

$$\Delta_{\text{on-job}}(x, a; t) = a'^*(x, a; t) - a'^*(x, a; 1) = \int_1^t \frac{\partial a'^*}{\partial u}(x, a; u) du, \quad (33)$$

368 where  $a'^*(x, a; t)$  is the optimal saving when households undertake on-job-training,  
 369 and  $a'^*(x, a; 1)$  is the optimal saving when human capital is kept exogenously fixed.

370 Whether on-job-training increases or decreases saving ultimately depends on  
 371 the balance between the crowd-out effect (via higher expected future income) and

the precautionary crowd-in effect (via heightened future income risk). The next proposition demonstrates that these effects can dominate differently depending on period-1 income  $x$ , so that the overall impact of on-job-training on saving can differ between low- and high-income households.

**Proposition 2.** *If the idiosyncratic risk is large enough, i.e.,  $\frac{\Sigma}{\mu} > \sigma^*(t)$ , on-job-training crowds out saving for low-income households and crowds in saving for high-income households: for  $x < x^*(a, t)$ ,  $e = e_L$  lowers saving  $\Delta_{on-job}(x, a; t) < 0$ ; for  $x > x^*(a, t)$ ,  $e = e_L$  raises saving  $\Delta_{on-job}(x, a; t) > 0$ .*

*Proof.* See Appendix B. □

### 3.3.2 Effect of full-time training on saving

We next compare the optimal saving between  $(n = 0, e = e_L \text{ or } e_H)$  and  $(n = 1, e = 0)$ . Note that full-time training requires the households to give up their labor income in period 1, which is not the case for on-job-training. Following the same normalization and notation as in the previous subsection, we can write the budget constraints with full-time training and without training as:

$$e = e_H : \quad c + a' = a, \quad c' = a' + txz' \quad (34)$$

$$e = 0 : \quad c + a' = a + x, \quad c' = a' + xz' \quad (35)$$

where  $t > 1$  captures the effect of full-time training on period-2 income.

The second-order approximation to the optimal saving problem yields:

$$e = e_H : \quad a'_{e_H}^*(x, a; t) = \underbrace{\frac{\beta a - tx\mu}{1 + \beta}}_{\text{CE}} + \underbrace{\frac{t^2 x^2 \Sigma}{\beta(a + tx\mu)}}_{\text{Precautionary}} \quad (36)$$

$$e = 0 : \quad a'^*(x, a; 1) = \underbrace{\frac{\beta(a + x) - x\mu}{1 + \beta}}_{\text{CE}} + \underbrace{\frac{x^2 \Sigma}{\beta(a + x + x\mu)}}_{\text{Precautionary}} \quad (37)$$

The overall effect of full-time training on saving can be expressed as:

$$\begin{aligned} \Delta_{\text{full-time}}(x, a; t) &= a'_{e_H}^*(x, a; t) - a'^*(x, a; 1) \\ &= \Delta_{on-job}(x, a; t) + \Delta_H(x, a; t) \end{aligned} \quad (38)$$

$$\text{where } \Delta_H(x, a; t) \equiv x \left[ -\frac{\beta}{1 + \beta} + \frac{\Sigma}{\beta} \frac{t^2 x^2}{(a + x + tx\mu)(a + tx\mu)} \right] \quad (39)$$

Here,  $\Delta_H(x, a; t)$  captures the additional impact of full-time training on saving, over and above that of on-job-training. The first term reflects a further reduction in saving due to the need to forgo period-1 labor income. The second term shows



an increase in precautionary saving, as reduced current resources limit households' ability to self-insure against idiosyncratic risk in period 2.

The following lemma establishes some properties of  $\Delta_H(x, a; t)$ :

**Lemma 1.** *If  $\frac{\Sigma}{\mu} < \hat{\sigma}(t)$ ,  $\Delta_H(x, a; t) < 0$  and decreases in  $x$ . If  $\frac{\Sigma}{\mu} > \bar{\sigma}(t)$ ,  $\Delta_H(x, a; t) > 0$  if and only if  $x > \hat{x}(a, t)$ ; moreover, for  $x > \hat{x}(a, t)$ ,  $\Delta_H(x, a; t)$  increases in  $x$ .*

*Proof.* See Appendix B. □

Taken together, Proposition 2 and Lemma 1 imply that, when the idiosyncratic risk is large enough, full-time training *crowds out* saving for low-income households, but *crowds in* saving for high-income households.

**Proposition 3.** *If the idiosyncratic risk is large enough, i.e.,  $\frac{\Sigma}{\mu} > \max\{\sigma^*(t), \hat{\sigma}(t)\}$ , full-time training crowds out saving for low-income households and crowds in saving for high-income households: for  $x < \min\{x^*(a, t), \hat{x}(a, t)\}$ ,  $e = e_H$  lowers saving  $\Delta_{full-time}(x, a; t) < 0$ ; for  $x > \max\{x^*(a, t), \hat{x}(a, t)\}$ ,  $e = e_H$  raises saving  $\Delta_{full-time}(x, a; t) > 0$ .*

### 3.4 The Effects of an Anticipated Period-2 AI Shock

Suppose that an AI shock is anticipated to occur in period 2 and to increase the labor productivity for the low sector and the high sector but not the middle sector. The effect of AI shock on the sectoral productivity is captured by  $\gamma$  with  $0 < \gamma < 1$ :

$$x(h') = \begin{cases} 1 - \lambda + \gamma\lambda & \text{low sector if } h' < h_M \\ 1 & \text{middle sector if } h_M < h' < h_H \\ 1 + \lambda + \gamma\lambda & \text{high sector if } h' > h_H \end{cases} \quad (40)$$

In other words, the AI shock increases average labor productivity, reduces the earnings premium for the middle sector, and enlarges the earnings premium for the high sector relative to the middle sector.

#### 3.4.1 Effects on human capital investment

The AI shock lowers the incentive to work in the middle sector in period 2. Consequently, households with  $h < h_M/(1 - \delta)$  reduce their human capital investment, while those with  $h > h_M/(1 - \delta)$  increase it. More specifically, the upper bounds that determine whether households undertake positive human capital investment – denoted by  $\bar{z}_{slow}^L$  and  $\bar{z}_{fast}^L$  for  $h < h_M/(1 - \delta)$ , and  $\bar{z}_{slow}^M$  and  $\bar{z}_{fast}^M$  for  $h > h_M/(1 - \delta)$  – respond in opposite directions to the anticipated shock: the former decrease with  $\gamma$  and the latter increase. This relationship is formalized below.

**Proposition 4.** *An anticipated AI shock decreases human capital investment among households with  $h < h_M/(1 - \delta)$ , but increases it among those with  $h > h_M/(1 - \delta)$ . Specifically,  $\bar{z}_{slow}^L$  and  $\bar{z}_{fast}^L$  decrease with  $\gamma$ , while  $\bar{z}_{slow}^M$  and  $\bar{z}_{fast}^M$  increase with  $\gamma$ .*

425 *Proof.* See Appendix B. □

### 426 **3.4.2 Effects on labor supply**

427 **via income:** The AI shock raises period-2 labor income for households who will  
428 work in the low or high sector, leading to a positive income effect that reduces their  
429 labor supply in period 1.

430 **via full-time training:** Because full-time training and labor supply compete for  
431 time, the AI shock affects their tradeoff through its impact on human capital invest-  
432 ment incentives. For  $h > h_M/(1 - \delta)$ , where AI makes investing in additional skills  
433 more attractive, households are more likely to engage in full-time training and thus  
434 reduce period-1 labor supply. In contrast, for  $h < h_M/(1 - \delta)$ , where the AI shock  
435 lowers the payoff to investing in skills, households shift away from full-time training  
436 and supply more labor in the first period.

### 437 **3.4.3 Effects on saving**

438 The AI shock increases sectoral labor productivity for the low and high sectors in  
439 period 2, while leaving the middle sector's labor productivity unchanged. Its effect  
440 on saving can be analyzed as if we are varying the parameter  $t$  in the functions  
441  $\Delta_{\text{on-job}}(x, a; t)$ , defined in (33), and  $\Delta_H(x, a; t)$ , defined in (39).

442 **Proposition 5.**  $\Delta_H(x, a; t)$  is increasing in  $t$ .  $\Delta_{\text{on-job}}(x, a; t)$  is convex in  $t$ :

- 443 • If  $\Delta_{\text{on-job}}(x, a; t) > 0$  and  $t > 1$ ,  $\Delta_{\text{on-job}}(x, a; t') > \Delta_{\text{on-job}}(x, a; t)$  for  $t' > t > 1$ .
- 444 • If  $\Delta_{\text{on-job}}(x, a; t) > 0$  and  $t < 1$ ,  $\Delta_{\text{on-job}}(x, a; t') < \Delta_{\text{on-job}}(x, a; t)$  for  $1 > t' > t$ .

445 *Proof.* See Appendix B. □

446 **Households who stay in the same sector** For middle-sector households, the  
447 AI shock leaves both their incomes and saving unchanged.

448 By contrast, low-sector and high-sector households experience an increase in  
449 period-2 labor income  $x'$  as a result of the AI shock. If they remain in the same  
450 sector without needing additional human capital investment or on-the-job training,  
451 their saving behavior in the absence of the AI shock can be compared to the scenario  
452 with fixed human capital. Following the AI shock, however, their situation resembles  
453 one with on-the-job training that enhances  $x'$  (i.e.,  $t > 1$ ). Thus, the effect of the  
454 AI shock on saving is captured by the on-the-job training impact,  $\Delta_{\text{on-job}}(x, a; t)$ .

455 As shown in Proposition 2,  $\Delta_{\text{on-job}}(x, a; t)$  has opposite signs for low-skill and  
456 high-skill households. This implies that the AI shock *crowds out* saving among  
457 low-sector households, while it *crowds in* saving for high-sector households.

458 For households who must undertake full-time training to remain in the high  
 459 sector,  $\Delta_H(x, a; t)$  captures the additional effect of such training on saving. In this  
 460 case, a higher  $x'$ —brought about by the AI shock—corresponds to an increase in  $t$ ,  
 461 further boosting  $\Delta_H(x, a; t)$  (Proposition 5). Consequently, the AI shock *crowds in*  
 462 saving for high-sector households in this scenario as well.

463 **Households who upskill** For low-sector households, saving behavior remains  
 464 unchanged, as the AI shock does not affect their future productivity after upskilling.

465 For the middle-sector households who upskill via on-job-training, the AI shock  
 466 boosts their future productivity gain from  $\lambda$  to  $(1 + \gamma)\lambda$ , which corresponds to a  
 467 higher  $t$  in  $\Delta_{\text{on-job}}(x, a; t)$  with  $t > 1$ . According to Proposition 5, if the pre-shock  
 468 effect of on-the-job training on saving is positive, the AI shock will *raise* saving.  
 469 However, if this effect is negative, the overall impact of the AI shock on saving  
 470 becomes ambiguous.

471 For the middle-sector households who upskill via full-time training, there is an  
 472 *additional positive effect* of the AI shock on their saving, because a higher  $x'$  increases  
 473  $\Delta_H(x, a; t)$  (Proposition 5).

474 **Households who downskill** Downskilling, which reflects human capital depre-  
 475 ciation, does not require any new investment in skills. For high-sector households  
 476 who transition downward, the AI shock leaves their future productivity – and thus  
 477 their saving – unchanged.

478 For middle-sector households who downskill to the low sector, their saving differs  
 479 from the fixed human capital scenario by  $\Delta_{\text{on-job}}(x, a; t)$  with  $t < 1$ . The AI shock  
 480 mitigates their future productivity loss by reducing it from  $\lambda$  to  $(1 - \gamma)\lambda$ , effectively  
 481 increasing  $t$  to a new value  $t' < 1$ . According to Proposition 5, if the pre-shock effect  
 482  $\Delta_{\text{on-job}}(x, a; t)$  is positive, the AI shock will *reduce* saving. If this effect is negative,  
 483 however, the overall impact of the AI shock on saving is ambiguous.

### 484 3.5 Limitations of the two-period model

485 Up to this point, our analysis has focused on how AI influences household-level  
 486 decisions regarding human capital investment, labor supply, and saving within the  
 487 framework of a two-period model. While this provides valuable insights into indi-  
 488 vidual behavioral responses, understanding the broader, economy-wide implications  
 489 of AI requires moving to a more comprehensive setting – a quantitative model with  
 490 an infinite time horizon, endogenous asset accumulation, and general equilibrium  
 491 feedback.

492 **General equilibrium (GE) effects** When households adjust their investment in  
 493 human capital, labor supply, and savings in response to AI, these changes aggregate

up to affect the total supply of effective labor and capital in the economy. As these aggregates shift, they exert downward or upward pressure on the wage rate and the interest rate, feeding back into each household’s optimization problem. Thus, general equilibrium effects capture the intricate loop by which individual decisions shape, and are shaped by, the macroeconomic environment.

**Composition effects** Endogenizing human capital investment injects dynamism into how households sort themselves among the three skill sectors. When an AI shock occurs, individuals may choose to retrain, upskill, or even move to lower-skilled work, reshaping the distribution of labor across sectors. This shifting composition changes the relative size of each sector, with significant consequences for both aggregate outcomes and the distributional effects of AI.

## 4 A Quantitative Model

We now solve the full dynamic model with infinite horizon, endogenous asset accumulation, and general equilibrium. We calibrate the model to reflect key features of the U.S. economy, capturing reasonable household heterogeneity.

### 4.1 Calibration

We calibrate the model to match the U.S. economy. For several preference parameters, we adopt values commonly used in the literature. Other parameters are calibrated to align with targeted moments. The model operates on an annual time period. Table I summarizes the parameter values used in the benchmark model.

The time discount factor,  $\beta$ , is calibrated to match an annual interest rate of 4 percent. We set  $\chi_n$  to replicate an 80 percent employment rate. We calibrate  $\chi_e$  to match the fact that around 30 percent of the population invests in human capital (`oecd2025adultlearning`).

We calibrate parameters regarding labor productivity process as follows. We assume that  $z$  follows the AR(1) process in logs:  $\log z' = \rho_z \log z + \epsilon_z$ , where  $\epsilon_z \sim N(0, \sigma_z^2)$ . The shock process is discretized using the `tauchen1986finite` method, resulting in a transition probability matrix with 11 grids. We set the persistence parameter to  $\rho_z = 0.948$  and the standard deviation to  $\sigma_z = 0.269$ , following the estimates reported in Chang and Kim (2006).

We deviate from the two-period model by assuming that the labor supply is a discrete choice between 0 and  $\bar{n} = 1/3$ . This change only rescales the two-period model without altering the trade-off facing the households. But such rescaling facilitates the interpretation that households are deciding whether to allocate one-third of their fixed time endowment to work. The high-level human capital accumulation effort,  $e_H$  is assumed to equal  $\bar{n}$ . The low-level effort,  $e_L$  is set to half of  $e_H$ . The skill

Table I: Parameters for the Calibration

Parameter	Value	Description	Target or Reference
$\beta$	0.91795	Time discount factor	Annual interest rate
$\rho_z$	0.948	Persistence of $z$ shocks	Chang and Kim (2006)
$\sigma_z$	0.269	Standard deviation of $z$ shocks	Chang and Kim (2006)
$\underline{a}$	0	Borrowing limit	See text
$\chi_n$	2.47	Disutility from working	Employment rate
$\chi_e$	1.48	Disutility from HC effort	See text
$\bar{n}$	1/3	Hours worked	Average hours worked
$e_H$	1/3	High level of effort	Average hours worked
$e_L$	1/6	Low level of effort	See text
$h_M$	0.41	Human capital cutoff for M	See text
$h_H$	0.96	Human capital cutoff for H	See text
$\lambda$	0.2	Skill premium	Earnings Gini
$\delta_h$	0.1	HC depreciation rate	Standard value
$\alpha$	0.36	Capital income share	Standard value
$\delta$	0.1	Capital depreciation rate	Standard value

premium across sectors,  $\lambda$ , is set at 0.2 to match the earnings Gini coefficient. Human capital cutoffs,  $h_M$  and  $h_H$ , are set so that the population shares in low, middle, and high sectors are, respectively, 20, 40, and 40 percent. This population distribution roughly matches the fractions of U.S. workers in 2014 who are employed in routine manual occupations (low sector), routine cognitive and non-routine manual (middle sector), and non-routine cognitive (high sector) (Cortes *et al.*, 2017).

On the production side, we set the capital income share,  $\alpha$ , to 0.36, and the depreciation rate,  $\delta$ , to 0.1. For simplicity, we assume that human capital depreciates at the same rate, i.e.,  $\delta_h = 0.1$ .

#### 4.2 Key Moments: Data vs. Model

In Table II, we present a comparison of key moments between the model and the empirical data. The model does an excellent job of replicating the 80% employment rate observed in the data. In this context, employment is defined as having positive labor income in the given year, consistent with the common approach used in the literature. According to **oecd2025adultlearning**, the share of the population investing in human capital—those who are actively engaged in skill acquisition or education—is approximately 30%, a figure well matched by the model’s predictions. This is an important metric because it reflects the model’s capacity to capture the dynamics of human capital formation, which plays a critical role in shaping long-run earnings and income inequality. Additionally, the model accurately captures the distribution of income and earnings, aligning closely with observed data. This suggests that the model effectively incorporates the key mechanisms driving labor market

Table II: Key Moments

Moment	Data	Model
Employment rate	0.80	0.80
Human capital investment ratio	0.29	0.29
Gini coefficient for wealth	0.78	0.76
Gini coefficient for earnings	0.63	0.62
Gini coefficient for income	0.57	0.58

outcomes and the corresponding distributional aspects of earnings. Although the model does not explicitly target the wealth Gini coefficient, it achieves a close match to the data: the empirical wealth Gini is 0.78, while the model produces a value of 0.76. This highlights the model’s ability to capture substantial wealth inequality in the economy.

### 4.3 Steady-state Distribution

Table III presents the steady-state distribution of population, employment, and assets across sectors. The population shares are calibrated to 20%, 40%, and 40% by adjusting the human capital thresholds that define sectors. The shares of employment and assets are endogenously determined by households’ labor supply and savings decisions. Notably, the high sector accounts for 46% of total employment—exceeding its population share—indicating that a disproportionate number of households choose to work in that sector. Asset holdings are even more skewed: the high sector holds 68% of total assets, while the low sector holds only 8%.

Table III: Distribution of Population, Employment and Assets

Sectors	Pop. Share (%)	Emp. Share (%)	Assets Share (%)
Low	20.76	18.58	8.07
Middle	38.87	35.35	23.92
High	40.35	46.07	68.01

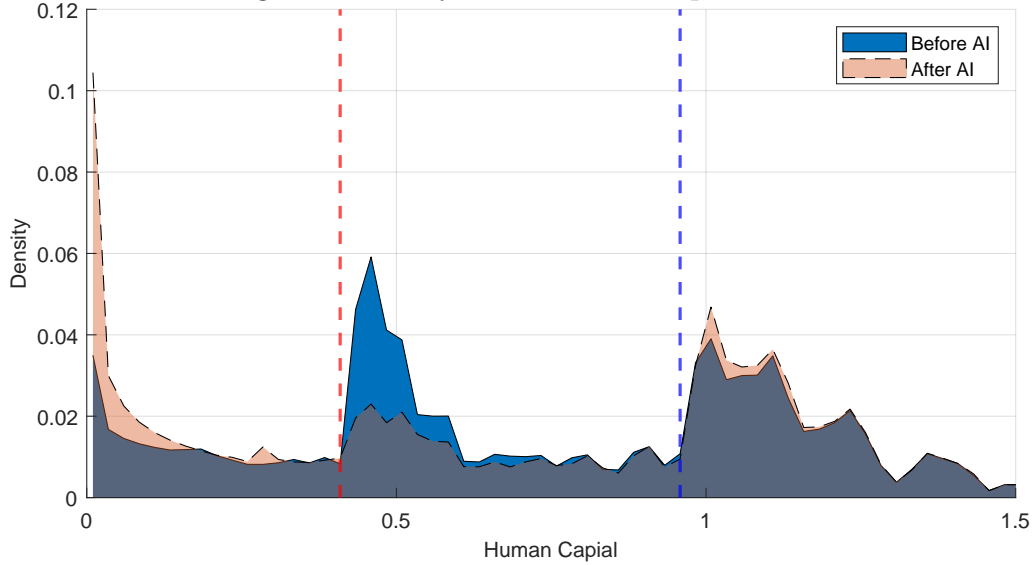
Note: Human capital cutoffs,  $h_H$  and  $h_M$ , determine the population share across sectors. Employment share and assets share are implied by households labor supply decisions and saving decisions.

## 5 AI’s Impact on Human Capital Adjustments

We now introduce AI technology into the quantitative model, assuming that it will be implemented in 10 years and that households have full information about its arrival. We examine both the transition dynamics and the differences between the initial and new steady states. This framework allows us to analyze how the economy adjusts in anticipation of, and in response to, the adoption of AI.

The effect of AI on the sectorial productivity is modeled as in (40) with  $\gamma = 0.3$ . That is, AI boosted the productivity of the low sector workers by 7.5% and the productivity of the high sector workers by 5%, leaving the middle sector intact.

Figure 3: Steady-state Human Capital Distribution



Note: The x-axis denotes the level of human capital, while the y-axis indicates the mass of households at each human capital level. The red vertical line marks the cutoff between the low and middle sectors, and the blue vertical line marks the cutoff between the middle and high sectors.

It captures the key idea that AI increases average labor productivity (Acemoglu and Restrepo, 2019), but reduces the earning premium for the middle sector, and enlarges the earning premium for the higher sector relative the middle sector.

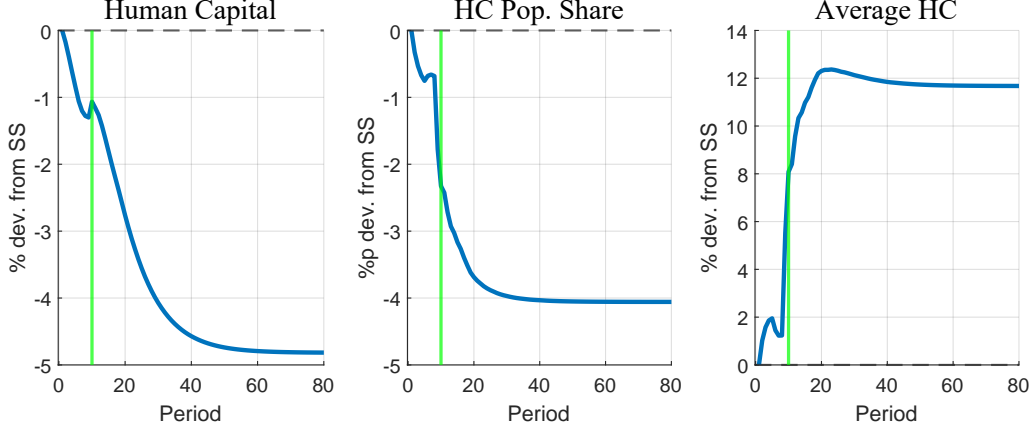
### 5.1 Human Capital Adjustments

Given the employment distribution in the initial steady state, AI is projected to increase the economy's labor productivity by 4% on average, assuming households do not alter their decisions in response. However, changes in earning premiums incentivize households to adjust their human capital investments.

**Steady-state human capital distribution:** Figure 3 illustrates how households reallocate across sectors in the new steady state relative to the initial one. The x-axis denotes the level of human capital, while the y-axis indicates the mass of households at each human capital level. The red vertical line marks the cutoff between the low and middle sectors, and the blue vertical line marks the cutoff between the middle and high sectors.

The gray shaded area shows the overlap between the two steady-state distributions. Within each sector, the distribution of households is skewed to the left, reflecting the tendency for human capital investment to be concentrated among those near the sectoral cutoffs. As shown in the decision rule diagram in Figure 2, some households seek to upgrade their skills, while others aim to remain in more skilled sectors. The blue shaded area highlights the mass of households who have exited the middle sector following the AI shock. The pink areas represent the additional mass of households in the new steady-state distribution, concentrated at the

Figure 4: Transition Path for Human Capital Investment



Note: The x-axis represents years, and the y-axis shows the percentage (or percentage point) deviation from the initial steady state. AI introduction is assumed to occur in period 10. “HC Pop. Share” denotes the fraction of households that make positive human capital investments, and “Average HC” denotes average human capital among those investing households.

lower end of the low sector and the lower end of the high sector.

**Transition path** Figure 4 reports the transition dynamics of aggregate human capital from the initial to the new steady state. The figure also displays its extensive margin (the share of households making positive human capital investments) and intensive margin (average human capital per household among those who invest).

As households reallocate from the middle sector to the low and high sectors, the net effect is a gradual decline in aggregate human capital along the transition path. This mirrors the steady-state change observed in Figure 3, where the increased mass at the lower end of the low sector outweighs the increase in the high sector.

Additionally, human capital accumulation becomes increasingly concentrated among a smaller share of the population. The proportion of households making positive human capital investments steadily declines, ultimately stabilizing at a level 4% lower than in the initial steady state. Meanwhile, the average human capital among those who invest rises, reaching a level 12% higher than the initial steady state in the long run.<sup>13</sup>

## 5.2 Job Polarization

An important implication of human capital adjustments to the AI shock is job polarization. Figure 5 illustrate the transition paths of population shares and employment rates in each sector. Notably, the middle sector experiences a significant decline, with its population share decreasing by approximately 13%. Additionally, employment within this sector plummets to a level 16% lower than the initial steady state. In contrast, both the low and high sectors see increases in their population

<sup>13</sup>The only exception to those patterns occurs at period 10 when the positive effects of AI on sectoral productivity are realized.



619 shares and employment rates. These dynamics indicate a reallocation of *workers*  
620 from the middle sector to the low and high sectors following the introduction of AI.

621 **Voluntary job polarization** This worker reallocation aligns with the phenomenon  
622 of “job polarization” (Goos *et al.*, 2014), where AI and automation technologies dis-  
623 proportionately replace tasks commonly performed by middle-skilled workers. How-  
624 ever, our model introduces a complementary mechanism to the conventional under-  
625 standing of this reallocation. Specifically, households in our model voluntarily exit  
626 the middle sector even before AI implementation by adjusting their human capital  
627 investments – many middle-sector workers opt for non-employment to invest in skills  
628 that will better position them for the post-AI labor market.<sup>14</sup> This mechanism is  
629 formally characterized in Proposition (4) in the two period model above.

630 **Employment flows more towards the low sector** Another intriguing finding  
631 in our model is the more pronounced employment effect in the low sector compared  
632 to the high sector. In the new steady state, the employment rate in the low sector  
633 increases by 12%, whereas in the high sector, it rises by only 0.5%. This asymmetry  
634 in employment rate changes suggests an unbalanced reallocation of workers from the  
635 middle sector, with a greater flow toward the low sector.

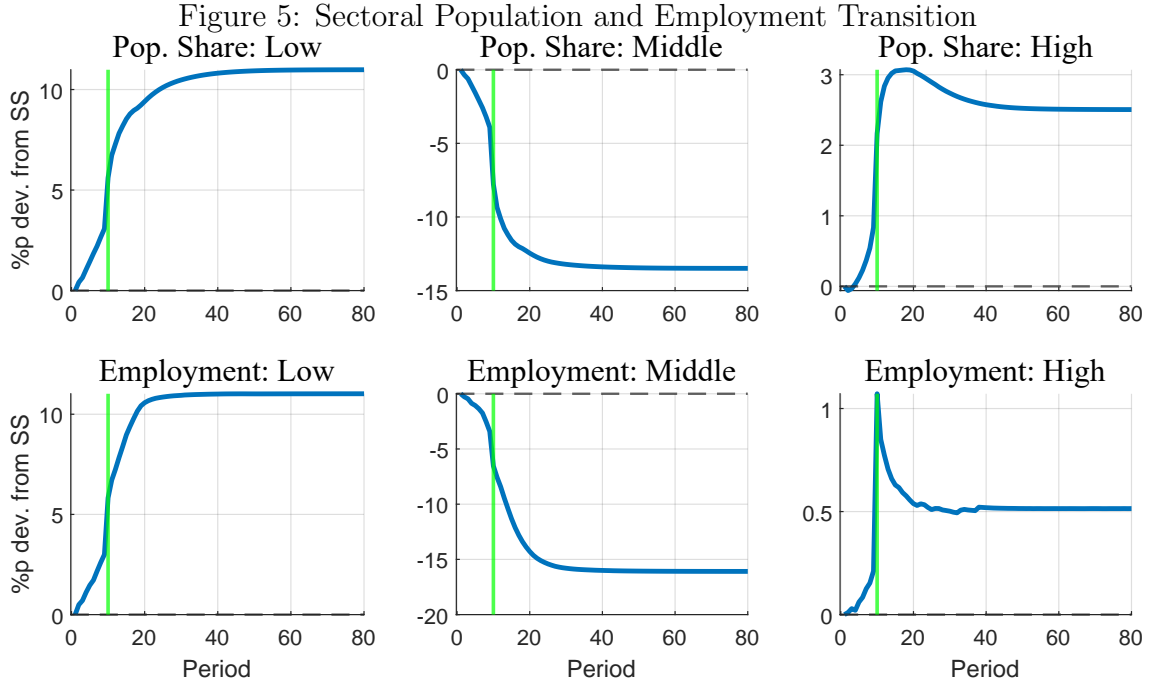
636 This disparity arises from two key factors. First, AI enhances the productivity of  
637 low-sector workers by 7.5% and high-sector workers by 5%. However, this produc-  
638 tivity differential alone does not fully account for the significant asymmetry. The  
639 second factor is the variation in labor supply elasticity across sectors. Compared to  
640 the high sector, the low sector exhibits higher labor supply elasticity, meaning that  
641 the same change in labor earnings triggers larger labor supply responses. This is  
642 because households in the low sector have lower consumption levels, making their  
643 marginal utility of consumption more sensitive to changes in their budget. Con-  
644 sequently, a greater proportion of households in the low sector are at the margin  
645 between employment and non-employment (Chang and Kim, 2006).

## 646 6 The Aggregate and Distributional Effects of AI

647 The aggregate and distributional effects of AI are shaped by both its direct impact on  
648 sectoral productivity and the endogenous response of human capital accumulation.  
649 By altering sectoral productivity, AI changes labor earnings, which in turn influences  
650 labor supply decisions and savings through income effects. Consequently, AI directly  
651 affects the supply of labor and capital, generating aggregate economic responses.  
652 Because AI’s productivity effects are heterogeneous across sectors, its impact is  
653 inherently distributional.

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<sup>14</sup>To emphasize this key difference, our model deliberately abstracts from any direct negative effect of AI on middle-sector workers.



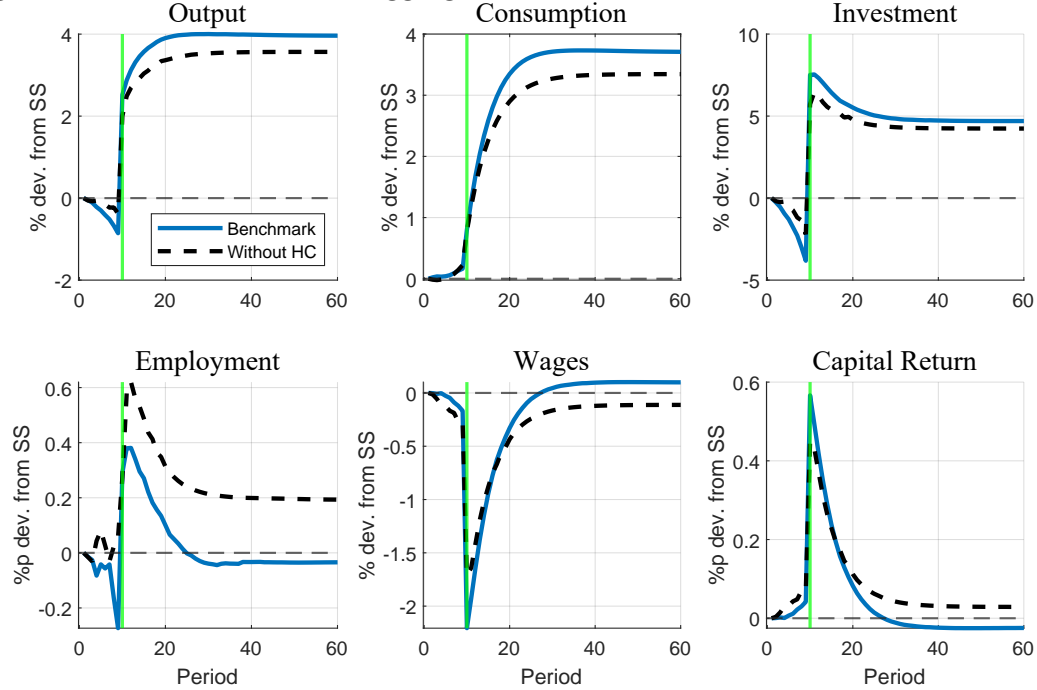
Note: The transition paths within each sector. The x-axis represents years, and the y-axis shows the percentage (or percentage point) deviation from the initial steady state. AI introduction is assumed to occur in period 10. “Pop. Share” denotes the population share within each sector. “Employment” is the percentage of households who are employed in each sector.

654 These sectoral differences also induce human capital adjustments, as households  
655 reallocate across sectors in response to changing incentives. This reallocation not  
656 only shifts the distribution of labor productivity and aggregate productivity, but  
657 also directly shapes distributional outcomes, as households’ relative positions in the  
658 income and asset distributions are altered by their movement across sectors.

659 In this section, we examine the importance of endogenous human capital ad-  
660 justment in shaping both the transitional and long-run effects of AI. To do so, we  
661 compare the benchmark economy – where households endogenously adjust their hu-  
662 man capital – with an alternative scenario in which households are held fixed at  
663 their initial steady-state human capital during the AI transition (“No HC model”).  
664 In both cases, households make endogenous decisions about consumption, savings,  
665 and labor supply.

666 By contrasting the transition dynamics across these two economies, we can disen-  
667 tangle the direct and indirect effects of AI. The transition path in the No-HC-model  
668 isolates the direct impact of AI on aggregate and distributional outcomes, as it ab-  
669 stracts from any human capital adjustments. The difference in outcomes between  
670 the benchmark and the No-HC-model then reveals the indirect effects of AI that  
671 operate through households’ adjustments in human capital. This decomposition al-  
672 lows us to assess the relative importance of human capital dynamics in driving both  
673 the aggregate and distributional consequences of AI.

Figure 6: Transition Path of Aggregate Variables: Benchmark vs. No HC Models.



Note: The transition paths of aggregate variables: benchmark vs. No HC models. The x-axis represents years, and the y-axis shows the percentage deviation from the initial steady state. AI introduction is assumed to occur in period 10. The No HC model is an economy in which workers maintain their initial steady-state level of human capital throughout the AI implementation until the new steady state is reached.

## 6.1 Aggregate Implications

Figure 6 shows the transition paths of key macroeconomic variables—output, consumption, investment, and employment—as well as factor prices, including the wage rate and capital return. The blue solid lines depict results from the benchmark model with endogenous human capital adjustment, while the black dashed lines represent the No-HC model in which human capital is held fixed.

### 6.1.1 AI's direct impacts

The No-HC-model isolates the direct effects of AI. In the long run, the introduction of AI leads to higher output, consumption, investment, and employment. However, in anticipation of AI (prior to period 10), output and investment decline, while consumption and employment remain stable.

Before the implementation of AI, sectoral productivity is unchanged; the only difference is households' awareness of future increases in productivity in the low and high sectors beginning in period 10. This anticipation raises households' expected lifetime income, prompting them to save less and consume more ahead of the actual productivity gains. As a result, aggregate capital stock falls, which lowers output and reduces the marginal product of labor while raising the marginal product of capital. Employment remains largely unchanged in this period, as sectoral productivity has not yet shifted.

Following the AI shock, sectoral productivity in the low and high sectors rises,

694 boosting labor income, employment, and output in these sectors. Because produc-  
695 tivity gains are labor-augmenting, the supply of efficient labor units rises sharply,  
696 causing wages to decline and capital returns to increase. Employment and invest-  
697 ment both adjust to dampen these factor price changes. In the new steady state, the  
698 wage rate is slightly below its initial level, while the return to capital is marginally  
699 higher.

### 700 **6.1.2 AI’s indirect impacts via endogenous human capital adjustments**

701 The difference between the No-HC model and the benchmark model captures the  
702 indirect effects of AI operating through endogenous human capital adjustments.  
703 Among all macroeconomic variables, this indirect effect is most pronounced for em-  
704 ployment.

705 In anticipation of AI, employment declines as some households temporarily exit  
706 the labor market to invest in human capital and prepare for the post-AI economy.<sup>15</sup>  
707 During this period, labor productivity remains unchanged, so the decline in em-  
708 ployment directly translates to a reduction in output. Consistent with standard  
709 consumption-smoothing behavior, this reduction is mainly absorbed by lower in-  
710 vestment. Meanwhile, the drop in employment mitigates the direct effects of AI on  
711 both wages and capital returns prior to the AI implementation.

712 After AI is introduced, employment rebounds as sectoral productivity increases.  
713 However, continued human capital investment by middle-sector households keeps  
714 employment lower than in the No-HC model, resulting in an almost neutral long-  
715 run effect of AI on employment. Despite this, output, consumption, and investment  
716 are all higher in the benchmark model because human capital adjustments reallocate  
717 more labor to the low and high sectors, thereby better capturing the productivity  
718 gains from AI.

719 This reallocation also reverses the steady-state comparison of factor prices: en-  
720 dogenous human capital adjustment transforms the negative direct effect of AI on  
721 the wage rate into a positive net effect, and the positive direct effect on capital  
722 returns into a negative net effect.

## 723 *6.2 Distributional Implications*

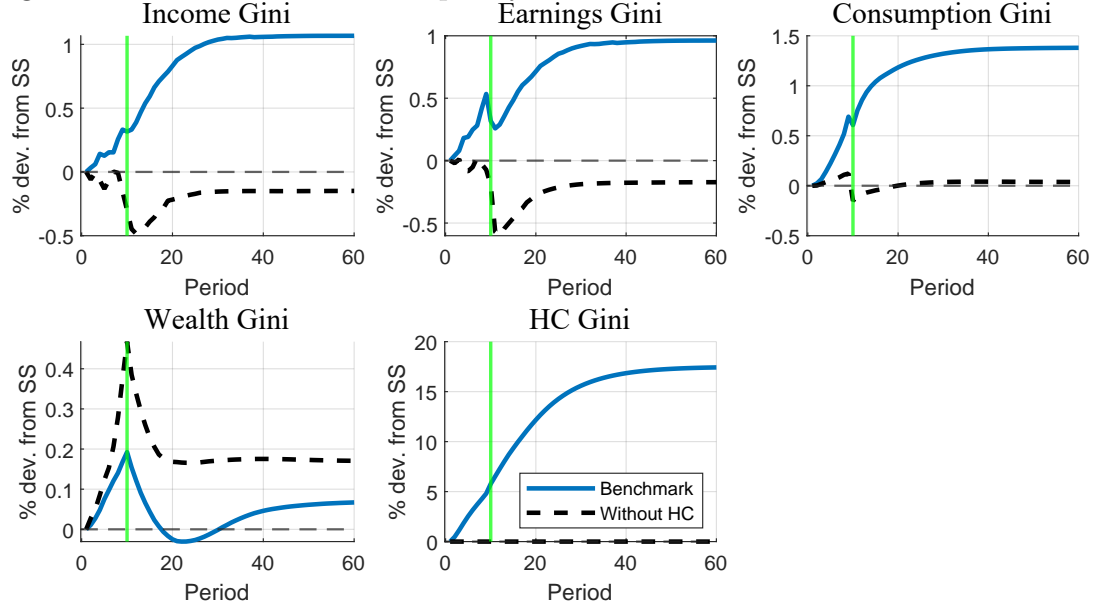
724 The findings above underscore the importance of accounting for human capital ad-  
725 justments when assessing the aggregate impact of AI, as households actively adapt  
726 to a rapidly evolving labor market. When it comes to economic inequality, endoge-  
727 nously adjusting human capital plays an even more significant role.

728 Figure 7 shows the transition paths of Gini coefficients for earnings (labor in-  
729 come), total income (capital and labor income), consumption, wealth (asset hold-

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<sup>15</sup>Empirical studies, such as Lerch (2021) and Faber *et al.*, (2022), support the short-term adverse effects of AI adoption on labor markets.

Figure 7: Transition Path of Inequality Measures: Benchmark vs. No HC Models.



Note: The transition paths of inequality measures: benchmark vs. No HC models. The x-axis represents years, and the y-axis shows the percentage deviation from the initial steady state. AI introduction is assumed to occur in period 10. The No HC model is an economy in which workers maintain their initial steady-state level of human capital throughout the AI implementation until the new steady state is reached.

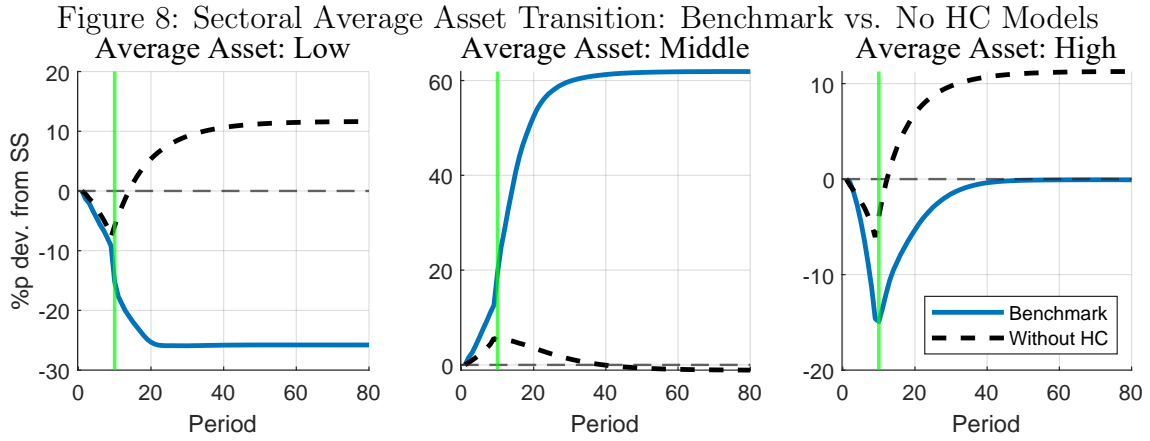
ings), and human capital. The black dashed lines represent results from the No-HC model, capturing the direct impact of AI without human capital adjustment. In contrast, the blue solid lines reflect the benchmark model, where human capital responds endogenously to both anticipated and realized changes in the skill premium induced by AI.

### 6.2.1 Income, earnings, and consumption inequalities

The comparison of transition paths between the No-HC model and the benchmark model reveals that endogenous human capital adjustments fundamentally alter the impact of AI on income, earnings, and consumption inequalities.

**AI's direct impacts:** Without any human capital adjustments, AI's impact on inequalities is primarily driven by productivity gains in the low and high sectors – 7.5% and 5%, respectively. As a result, there is little direct impact on income and earnings Gini coefficients in anticipation of AI before period 10. After AI is implemented, both income and earnings inequality decline: higher labor productivity raises earnings in the low sector, while wage declines in the middle sector compress the distribution. Consumption inequality remains largely unchanged throughout the transition.

**Effects of AI-induced human capital adjustments:** Allowing human capital to adjust endogenously, however, leads to pronounced job polarization, as shown in Section 5.2. Households who would have qualified for middle-sector jobs now tran-



Note: The transition paths within each sector. “Average Asset” is defined as the total assets in a given sector divided by that sector’s population share. The x-axis represents years, and the y-axis shows the percentage (or percentage point) deviation from the initial steady state. AI introduction is assumed to occur in period 10. The No HC model is an economy in which workers maintain their initial steady-state level of human capital throughout the AI implementation until the new steady state is reached.

sition to either the low or high sector. Those moving to the low sector see reduced labor earnings, while those shifting to the high sector enjoy increased earnings. This polarization drives up earnings and income inequality, both before and after AI is implemented. As income disparities widen, consumption inequality also increases.

### 6.2.2 Wealth inequality

In stark contrast to the effects on income and earnings inequality, allowing for endogenous human capital adjustment mitigates the negative direct impact of AI on wealth inequality. While AI’s direct effect would otherwise widen disparities, human capital responses help dampen the increase in wealth inequality, underscoring the stabilizing role of human capital adjustments in the wealth distribution.

As discussed in Section 3.3, the effect of human capital investment on saving is theoretically ambiguous *ex ante*. On the one hand, higher expected future income from on-the-job and full-time training tends to crowd out saving through the standard consumption-smoothing motive. On the other hand, greater exposure to idiosyncratic risk strengthens the precautionary saving motive and can crowd in saving. Propositions 2 and 3 demonstrate that, when idiosyncratic risk is sufficiently large, human capital investment crowds out saving for low-labor-income households but crowds in saving for high-labor-income households. As labor income is positively affected by households productivity, the net effect of human capital investment on saving is positive precisely for the more productive households.

In our quantitative model, this mechanism shows up most clearly in the middle sector. Figure 8 plots the transition of average assets by sector in the benchmark economy and in the counterfactual No HC economy. “Average Asset” is defined as total assets held by households in a given sector divided by that sector’s population share, so it reflects both within-sector saving behavior and the composition of

775 households across sectors.

776 **AI’s direct impacts:** Without any human capital adjustment, AI’s impact on  
777 households’ saving works purely through income effect. In both the low and high  
778 sectors, households reduce their savings in anticipation of AI, expecting higher life-  
779 time labor income. After AI is implemented at period 10, their savings increase  
780 alongside rising labor incomes. In contrast, households in the middle sector, antic-  
781 ipating a negative income effect from AI due to a lower wage rate, increase their  
782 savings prior to period 10. Once AI is introduced and the wage rate recovers,  
783 middle-sector households reduce their savings.

784 **Effects of AI-induced human capital adjustments:** Endogenous human cap-  
785 ital responses introduce an additional channel. Relative to the No-HC model, the  
786 benchmark exhibits a pronounced increase in average assets in the middle sector.<sup>16</sup>  
787 Middle-sector households are relatively productive in our model, and a composi-  
788 tion effect further amplifies their asset accumulation: many less productive middle-  
789 sector households endogenously move down to the low sector, so the remaining  
790 middle-sector population is positively selected on productivity.<sup>17</sup> In addition, as  
791 discussed above, some middle-sector households voluntarily exit employment to in-  
792 vest in human capital full-time. Taken together, these households are the “active  
793 training” and relatively high-productivity workers in our model; thus, as predicted  
794 by Propositions 2 and 3, their human capital investment tends to crowd in saving.  
795 Accordingly, AI-induced human capital adjustment strengthens asset accumulation  
796 in the middle of the distribution and compresses the gap between the middle and  
797 the top. Quantitatively, the increase in wealth inequality in the benchmark economy  
798 with endogenous human capital is therefore markedly smaller than in the No HC  
799 economy, highlighting the stabilizing role of human capital adjustment in the wealth  
800 distribution.

## 801 7 Conclusion

802 Recent studies on AI suggest that advancements are likely to reduce demand for  
803 junior-level positions in high-skill industries while increasing the need for roles fo-  
804 cused on advanced decision-making and AI oversight. We demonstrate how human  
805 capital investments are expected to adapt in response to these shifts in skill demand,  
806 highlighting the importance of accounting for these human capital responses when  
807 assessing AI’s economic impact.

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<sup>16</sup>In the benchmark model, average assets in the high sector decline due to a composition effect, as relatively low-wealth households move up from the middle sector. In the low sector, average assets also fall, primarily because the scope for precautionary saving is limited, and this effect is reinforced by composition changes.

<sup>17</sup>Note that the share of households moving up is relatively small.

Our work points to several promising directions for future research on the economic impacts of AI. First, if governments implement redistribution policies to address AI-induced inequality, understanding how these policies influence human capital accumulation, and thus their effectiveness, would be valuable. Second, our model assumes households have perfect foresight when making human capital investments. Relaxing this assumption could reveal new insights into the economic trajectory of AI advancements and offer important policy implications.

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## 912 A Household Decision Rule Cutoffs

### 913 A.1 Additional cutoffs formulae for households

$$\bar{z}_{non}^M(a) := \frac{(\exp(\frac{\chi n}{1+\beta}) - 1)[(1+r)a + \frac{w'z'}{1+r'}]}{w} \quad (A.1)$$

$$\bar{z}_{slow}^M(a) := \frac{(\exp(\frac{\chi n - \chi_e e_H}{1+\beta}) - 1)[(1+r)a + \frac{w'z'(1+\lambda)}{1+r'}] + \lambda \frac{w'z'}{1+r'}}{w} \quad (A.2)$$

$$\bar{z}_{fast}^M(a) := \frac{(\exp(\frac{\chi n}{1+\beta}) - 1)[(1+r)a + \frac{w'z'(1+\lambda)}{1+r'}]}{w} \quad (A.3)$$

$$\bar{z}_{fast}^M(a) := \frac{\left\{ \lambda \left[ \exp(\frac{\chi_e e_L}{1+\beta}) - 1 \right]^{-1} - 1 \right\} \frac{w'z'}{1+r'} - (1+r)a}{w} \quad (A.4)$$

$$\bar{z}_{non}^H(a) := \frac{(\exp(\frac{\chi n}{1+\beta}) - 1)[(1+r)a + \frac{w'z'(1+\lambda)}{1+r'}]}{w} \quad (A.5)$$

### 914 A.2 Parameter restrictions for cutoffs ranking

915 To guarantee that  $(n = 0, e = e_H)$  dominates  $(n = 0, e = 0)$ , we need a lower bound  
 916 for  $\lambda$ . The slow learners prefer  $(n = 0, e = e_H)$  if and only if

$$(1 + \beta) \ln c(n = 0, e = e_H) - \chi_e e_H \geq (1 + \beta) \ln c(n = 0, e = 0)$$

917 or equivalently:

$$\lambda \geq \underline{\lambda}_1 := \frac{(1+r)a + \frac{w'z'}{1+r'}}{\frac{w'z'}{1+r'}} \left( 1 - \frac{1}{\exp(\frac{\chi_e e_H}{1+\beta})} \right) \text{ if } h < h_M \frac{1}{1-\delta} \quad (A.6)$$

$$\lambda \geq \underline{\lambda}_3 := \frac{(1+r)a + \frac{w'z'}{1+r'}}{\frac{w'z'}{1+r'}} \left( \exp(\frac{\chi_e e_H}{1+\beta}) - 1 \right) \text{ if } h \geq h_M \frac{1}{1-\delta} \quad (A.7)$$

918 To avoid  $(n = 1, e = e_L)$  from being a dominated choice, we need another lower  
 919 bound for  $\lambda$ . To see it, recall that  $(n = 1, e = 0)$  is better than  $(n = 1, e = e_L)$   
 920 if  $z > \bar{z}_{fast}$ , and  $(n = 1, e = e_L)$  is better than  $(n = 0, e = e_L)$  if  $z > \underline{z}_{fast}$ .  
 921  $(n = 1, e = e_L)$  is therefore the best choice over the interval  $(\underline{z}_{fast}, \bar{z}_{fast})$ . For such an  
 922 interval to exist, it must be the case that when  $z = \underline{z}_{fast}$ ,  $z < \bar{z}_{fast}$ .  $z = \underline{z}_{fast}$  means  
 923 that the fast learners are indifferent between  $(n = 1, e = e_L)$  and  $(n = 0, e = e_L)$  so

924 that

$$(1+r)a + wzx(h) + \frac{w'z'}{1+r'} = \exp\left(\frac{\chi_n}{1+\beta}\right) \left[ (1+r)a + \frac{w'z'}{1+r'} \right] \text{ if } h < h_M \frac{1}{1-\delta} \quad (\text{A.8})$$

$$(1+r)a + wzx(h) + \frac{w'z'(1+\lambda)}{1+r'} = \exp\left(\frac{\chi_n}{1+\beta}\right) \left[ (1+r)a + \frac{w'z'(1+\lambda)}{1+r'} \right] \text{ if } h \geq h_M \frac{1}{1-\delta} \quad (\text{A.9})$$

925 For the fast learners to prefer  $(n=1, e=e_L)$  over  $(n=1, e=0)$ , we need

$$(1+\beta) \ln \frac{c(n=1, e=e_L)}{c(n=1, e=0)} \geq \chi_e e_L \quad (\text{A.10})$$

926 If  $h < h_M \frac{1}{1-\delta}$ , inequality (A.10) is:

$$(1+\beta) \ln \frac{(1+r)a + wzx(h) + \frac{w'z'}{1+r'}}{(1+r)a + wzx(h) + \frac{w'z'(1-\lambda)}{1+r'}} \geq \chi_e e_L$$

927 Evaluating the left-hand-side at  $z = \underline{z}_{fast}$  yields:

$$\lambda \geq \underline{\lambda}_2 := \frac{(1+r)a + \frac{w'z'}{1+r'}}{\frac{w'z'}{1+r'}} \left( 1 - \frac{1}{\exp\left(\frac{\chi_e e_L}{1+\beta}\right)} \right) \exp\left(\frac{\chi_n}{1+\beta}\right) \quad (\text{A.11})$$

928 If  $h > h_M \frac{1}{1-\delta}$ , inequality (A.10) is:

$$(1+\beta) \ln \frac{(1+r)a + wzx(h) + \frac{w'z'(1+\lambda)}{1+r'}}{(1+r)a + wzx(h) + \frac{w'z'}{1+r'}} \geq \chi_e e_L$$

929 Evaluating the left-hand-side at  $z = \underline{z}_{fast}$  yields:

$$\lambda \geq \underline{\lambda}_4 := \frac{(1+r)a + \frac{w'z'}{1+r'}}{\frac{w'z'}{1+r'}} \frac{\left( \exp\left(\frac{\chi_e e_L}{1+\beta}\right) - 1 \right) \exp\left(\frac{\chi_n}{1+\beta}\right)}{\exp\left(\frac{\chi_e e_L}{1+\beta}\right) + \exp\left(\frac{\chi_n}{1+\beta}\right) - \exp\left(\frac{\chi_e e_L + \chi_n}{1+\beta}\right)} \quad (\text{A.12})$$

930 We have that  $\underline{\lambda}_1 > \underline{\lambda}_2$  and  $\underline{\lambda}_3 > \underline{\lambda}_4$  if

$$\exp\left(\frac{\chi_e e_H}{1+\beta}\right) > \frac{\exp\left(\frac{\chi_e e_L}{1+\beta}\right)}{\exp\left(\frac{\chi_e e_L}{1+\beta}\right) + \exp\left(\frac{\chi_n}{1+\beta}\right) - \exp\left(\frac{\chi_e e_L + \chi_n}{1+\beta}\right)} \quad (\text{A.13})$$

931 Therefore, the inequality above implies that the conditions (A.6) and (A.7) are  
 932 sufficient for the conditions (A.11) and (A.12). Furthermore,  $\lambda_3 \geq \lambda_1$  so that the  
 933 condition (A.7) is sufficient for the condition (A.6).

934 We can then conclude that the conditions (A.7) and (A.13) are sufficient for  
 935 1) the slower learners always prefers  $(n=0, e=e_H)$  over  $(n=0, e=0)$ , and 2)  
 936  $\bar{z}_{fast} > \underline{z}_{fast}$ , i.e., there exists state space where  $(n=1, e=e_L)$  is optimal.

### 937 *A.3 Other cutoffs ranking for the two-period Model*

938 For the fast learners, their cutoffs rank as follows

$$\frac{\bar{z}_{fast}^L(a)}{1-\lambda} > \bar{z}_{fast}^L(a) > \bar{z}_{fast}^M(a) > \frac{\bar{z}_{fast}^M(a)}{1+\lambda} \quad (\text{A.14})$$

$$\frac{\underline{z}_{fast}^L(a)}{1-\lambda} > \underline{z}_{fast}^M(a) > \underline{z}_{fast}^L(a) > \frac{\underline{z}_{fast}^M(a)}{1+\lambda} \quad (\text{A.15})$$

939 For the slow learners, the rank of their cutoffs is

$$\frac{\bar{z}_{slow}^L(a)}{1-\lambda} > \bar{z}_{slow}^M(a) > \bar{z}_{slow}^L(a) > \frac{\bar{z}_{slow}^M(a)}{1+\lambda} \quad (\text{A.16})$$

940 For the non-learners, the rank of their cutoffs is

$$\frac{\bar{z}_{non}^L(a)}{1-\lambda} > \bar{z}_{non}^M(a) > \frac{\bar{z}_{non}^H(a)}{1+\lambda} > \frac{\bar{z}_{non}^M(a)}{1+\lambda} \quad (\text{A.17})$$

$$\bar{z}_{non}^M(a) > \bar{z}_{non}^L(a) \quad (\text{A.18})$$

## 941 **B Proof of Proposition**

### 942 *B.1 Proof of Proposition 2*

943 The derivative of saving with respect to  $t$  is

$$\frac{\partial a'^*}{\partial t}(x, a; t) = -\frac{x\mu}{1+\beta} + \frac{x^2\Sigma}{\beta} \frac{t[2(x+a) + tx\mu]}{[(x+a) + tx\mu]^2}. \quad (\text{B.1})$$

944 The total effect of on-job-training on saving is

$$\Delta_{\text{on-job}}(x, a; t) = a'^*(x, a; t) - a'^*(x, a; 1) = \int_1^t \frac{\partial a'^*}{\partial u}(x, a; u) du. \quad (\text{B.2})$$

945 Define

$$F(x, a; u) \equiv x \frac{u[2(x+a) + ux\mu]}{[(x+a) + ux\mu]^2}, \quad \bar{F}(x, a; t) \equiv \frac{1}{t-1} \int_1^t F(x, a; u) du.$$

946 Then equation (B.2) can be written as

$$\Delta_{\text{on-job}}(x, a; t) = x(t-1) \left[ \frac{\Sigma}{\beta} \bar{F}(x, a; t) - \frac{\mu}{1+\beta} \right].$$

947 Differentiating  $F(x, a; u)$  with respect to  $x$  gives

$$\frac{\partial F(x, a; u)}{\partial x} = \frac{2u a (a+x)}{(a+(1+u\mu)x)^3} > 0,$$

948 so  $\bar{F}(x, a; t)$  is strictly increasing in  $x$ .

949 The sign of  $\Delta_{\text{on-job}}(x, a; t)$  is governed by

$$S(x, a; t) \equiv \frac{\Sigma}{\beta} \bar{F}(x, a; t) - \frac{\mu}{1 + \beta}.$$

950 Because  $\bar{F}(x, a; t)$  is strictly increasing,  $S(x, a; t)$  increases monotonically with  $x$ .

951 For  $x \rightarrow 0$ ,  $F(x, a; u) \rightarrow 0$  and  $\bar{F}(x, a; t) \rightarrow 0$  so that  $S(x, a; t) \rightarrow -\frac{\mu}{1+\beta} < 0$ ,  
 952 implying  $\Delta_{\text{on-job}}(x, a; t) < 0$  for small  $x$ .

953 For  $x \rightarrow \infty$ ,  $F(x, a; u) \rightarrow \frac{u(2+u\mu)}{(1+u\mu)^2}$  and  $\bar{F}(x, a; t) \rightarrow \bar{F}_\infty(t) \equiv \frac{1}{t-1} \int_1^t \frac{u(2+u\mu)}{(1+u\mu)^2} du$ . If

$$\frac{\Sigma}{\mu} > \sigma^*(t) \equiv \frac{\beta}{1 + \beta} \frac{1}{\bar{F}_\infty(t)} \quad (\text{B.3})$$

954 then  $S(x, a; t) > 0$  for sufficiently large  $x$ , and hence  $\Delta_{\text{on-job}}(x, a; t) > 0$ .

955 If idiosyncratic risk is large enough, i.e., condition (B.3) is satisfied, there exists  
 956 a unique threshold  $x^*(t)$  at which the sign flips:

$$\Delta_{\text{on-job}}(x, a; t) < 0 \text{ for } x < x^*(a, t), \quad \Delta_{\text{on-job}}(x, a; t) > 0 \text{ for } x > x^*(a, t).$$

## 957 B.2 Proof of Lemma 1

958 Denote

$$G(x, a; t) \equiv \frac{t^2 x^2}{(a + x + tx\mu)(a + tx\mu)}$$

959 the net additional effect of full-time training on saving can be rewritten as

$$\Delta_H(x, a; t) \equiv x \left[ -\frac{\beta}{1 + \beta} + \frac{\Sigma}{\beta} G(x, a; t) \right]$$

960 Differentiating  $G(x, a; t)$  with respect to  $x$  gives

$$\frac{\partial G(x, a; t)}{\partial x} = \frac{t^2 x a (2tx\mu + 2a + x)}{(a + tx\mu)^2 (a + x + tx\mu)^2} > 0,$$

961 so  $G(x, a; t)$  is strictly increasing in  $x$ .

962 The limits of  $G(x, a; t)$  are:

$$G(x, a; t) \rightarrow 0 \quad (x \rightarrow 0),$$

963

$$G(x, a; t) \rightarrow G_\infty(t) \equiv \frac{t}{\mu(1 + t\mu)} \quad (x \rightarrow \infty),$$

964 Therefore,  $G(x, a; t) < G_\infty(t)$  for any  $x$ .

965 If

$$\frac{\Sigma}{\beta} G_\infty(t) < \frac{\beta}{1 + \beta}, \text{ i.e. } \frac{\Sigma}{\mu} < \hat{\sigma}(t) \equiv \frac{\beta^2}{1 + \beta} \left( \frac{1}{t} + \mu \right). \quad (\text{B.4})$$

966 Then  $\Delta_H(x, a; t) < x[-\frac{\beta}{1+\beta} + \frac{\Sigma}{\beta}G_\infty(t)] < 0$  for any  $x$ . Furthermore, with some  
 967 tedious algebra, we can show that for any  $x$

$$G(x, a; t) + x \frac{\partial G(x, a; t)}{\partial x} < G_\infty(t)$$

968 Hence, the inequality (B.6) also implies that

$$\frac{\partial \Delta_H(x, a; t)}{\partial x} = \frac{\Sigma}{\beta} [G(x, a; t) + x \frac{\partial G(x, a; t)}{\partial x}] - \frac{\beta}{1+\beta} < \frac{\Sigma}{\beta} G_\infty(t) - \frac{\beta}{1+\beta} < 0. \quad (\text{B.5})$$

969 If

$$\frac{\Sigma}{\beta} G_\infty(t) > \frac{\beta}{1+\beta}, \text{ i.e. } \frac{\Sigma}{\mu} > \hat{\sigma}(t) \equiv \frac{\beta^2}{1+\beta} \left( \frac{1}{t} + \mu \right), \quad (\text{B.6})$$

970 since  $G(x, a; t)$  is strictly increasing in  $x$ , there exists a unique  $\hat{x}(a, t)$  such that

$$\Delta_H(x, a; t) = x \left[ -\frac{\beta}{1+\beta} + \frac{\Sigma}{\beta} G(x, a; t) \right] > 0 \Leftrightarrow x > \hat{x}(a, t)$$

971 Moreover,  $\Delta_H(x, a; t) > 0$  implies that

$$\frac{\partial \Delta_H(x, a; t)}{\partial x} > 0.$$

### 972 *B.3 Proof of Proposition 4*

973 The relevant upper bounds of  $z$  for positive human capital investment are functions  
 974 of  $\gamma$  (to the first order approximation):

$$\begin{aligned} \bar{z}_{slow}^L(a; \gamma) &= \bar{z}_{slow}^L(a; \gamma = 0) - \gamma \lambda \frac{w' z'}{w(1+r')} \\ \bar{z}_{fast}^L(a; \gamma) &= \bar{z}_{fast}^L(a; \gamma = 0) - \gamma \lambda \frac{w' z'}{w(1+r')} \frac{\exp(\frac{\chi e e_L}{1+\beta})}{\exp(\frac{\chi e e_L}{1+\beta}) - 1} \\ \bar{z}_{slow}^M(a; \gamma) &= \bar{z}_{slow}^M(a; \gamma = 0) + \gamma \lambda \frac{w' z'}{w(1+r')} \exp\left(\frac{\chi_n - \chi e e_H}{1+\beta}\right) \\ \bar{z}_{fast}^M(a; \gamma) &= \bar{z}_{fast}^M(a; \gamma = 0) + \gamma \lambda \frac{w' z'}{w(1+r')} \frac{1}{\exp(\frac{\chi e e_L}{1+\beta}) - 1} \end{aligned}$$

975 Therefore, an anticipated AI shock,  $\gamma > 0$  makes those with  $h < h_M \frac{1}{1-\delta}$  invest less  
 976 human capital and those with  $h > h_M \frac{1}{1-\delta}$  invest more human capital.

### 977 *B.4 Proof of Proposition 5*

$$\Delta_{\text{on-job}}(x, a; t) = a'^*(x, a; t) - a'^*(x, a; 1) = \int_1^t \frac{\partial a'^*}{\partial u}(x, a; u) du.$$

978 differentiating with respect to  $t$  gives

$$\frac{d\Delta_{\text{on-job}}(x, a; t)}{dt} = \frac{\partial a'^{*}}{\partial t}(x, a; t)$$

979 Since

$$\frac{\partial^2 a'^{*}(x, a; t)}{\partial t^2} = \frac{\partial}{\partial t} \left( -\frac{x\mu}{1+\beta} + \frac{x^2\Sigma}{\beta} \frac{t[2(x+a) + tx\mu]}{[(x+a) + tx\mu]^2} \right) = \frac{2x^2\Sigma(a+x)^2}{\beta(a+x+tx\mu)^3} > 0. \quad (\text{B.7})$$

980 The slope  $\frac{\partial a'^{*}}{\partial t}(x, a; t)$  is strictly increasing in  $t$ . Hence  $\Delta_{\text{on-job}}(x, a; t)$  is convex in  $t$ .

$$\Delta_H(x, a; t) = x \left[ -\frac{\beta}{1+\beta} + \frac{\Sigma}{\beta} G(x, a; t) \right] \text{ with } G(x, a; t) = \frac{t^2 x^2}{(a+x+tx\mu)(a+tx\mu)}$$

981 Differentiating  $G(x, a; t)$  with respect to  $t$  gives

$$\frac{\partial G(x, a; t)}{\partial t} = \frac{tx^2(2a^2 + 2atx\mu + 2ax + \mu tx^2)}{(a+tx\mu)^2(a+x+tx\mu)^2} > 0,$$

982 so  $G(x, a; t)$  is strictly increasing in  $t$ , and so is  $\Delta_H(x, a; t)$ .

983 We now consider the comparison between  $\Delta_{\text{on-job}}(x, a; t)$  and  $\Delta_{\text{on-job}}(x, a; t')$  for  $t' >$   
984  $t$ . Given  $x$  and  $a$ , define

$$f(t) \equiv \frac{\partial a'^{*}}{\partial t}(x, a; t).$$

985 so  $f'(t) > 0$ , i.e.  $f(t)$  is strictly increasing in  $t$ .

986 **Case 1:**  $1 < t < t'$

987 Suppose  $\Delta_{\text{on-job}}(x, a; t) > 0$ . Then

$$\Delta_{\text{on-job}}(x, a; t) = \int_1^t f(u) du > 0.$$

988 Since  $f$  is increasing,

$$f(u) \leq f(t) \quad \text{for all } u \in [1, t],$$

989 which implies

$$\Delta_{\text{on-job}}(x, a; t) = \int_1^t f(u) du \leq (t-1) f(t).$$

990 Because  $t > 1$ , the inequality  $\Delta_{\text{on-job}}(x, a; t) > 0$  forces  $f(t) > 0$ .

991 Now for any  $t' > t$ ,

$$f(u) \geq f(t) > 0 \quad \text{for all } u \in [t, t'],$$



992 and therefore

$$\Delta_{\text{on-job}}(x, a; t') - \Delta_{\text{on-job}}(x, a; t) = \int_t^{t'} f(u) du > 0.$$

993 We then have that:

$$1 < t < t', \Delta_{\text{on-job}}(x, a; t) > 0 \implies \Delta_{\text{on-job}}(x, a; t') > \Delta_{\text{on-job}}(x, a; t) \quad (\text{B.8})$$

994 That is, once  $\Delta_{\text{on-job}}(x, a; t)$  becomes positive for  $t > 1$ , it is strictly increasing in  $t$   
 995 thereafter.

996 **Case 2:**  $t < t' < 1$

997 For  $t < 1$ ,

$$\Delta_{\text{on-job}}(x, a; t) = \int_1^t f(u) du = - \int_t^1 f(u) du.$$

998 Suppose  $\Delta_{\text{on-job}}(x, a; t) > 0$ . Then

$$- \int_t^1 f(u) du > 0 \implies \int_t^1 f(u) du < 0.$$

999 Since  $f$  is increasing

$$f(u) \geq f(t) \quad \text{for all } u \in [t, 1],$$

1000 which implies

$$\int_t^1 f(u) du \geq (1 - t) f(t).$$

1001 Because  $t < 1$ , the inequality  $\Delta_{\text{on-job}}(x, a; t) > 0$  forces  $f(t) < 0$ .

1002 Consider

$$\Delta_{\text{on-job}}(x, a; t') - \Delta_{\text{on-job}}(x, a; t) = \int_t^{t'} f(u) du$$

1003 If  $f(u) < 0$  for all  $u \in [t, t']$ , then  $\int_t^{t'} f(u) du < 0$ .

1004 If there exists some  $t_s \in [t, t']$  such that  $f(t_s) = 0$ , so  $f(u) < 0$  for  $u < t_s$  and  
 1005  $f(u) > 0$  for  $u > t_s$ . Then  $f(u) > 0$  for  $u \in [t', 1]$ . Hence,

$$\int_{t'}^1 f(u) du > 0$$

1006 This implies that

$$\Delta_{\text{on-job}}(x, a; t') = - \int_{t'}^1 f(u) du < 0$$

1007 Together with the inequality  $\Delta_{\text{on-job}}(x, a; t) > 0$ , we have that

$$\Delta_{\text{on-job}}(x, a; t') < \Delta_{\text{on-job}}(x, a; t)$$

1008 We then have that

$$t < t' < 1, \Delta_{\text{on-job}}(x, a; t) > 0 \implies \Delta_{\text{on-job}}(x, a; t') < \Delta_{\text{on-job}}(x, a; t). \quad (\text{B.9})$$

1009 Thus, for  $t < 1$ , whenever  $\Delta_{\text{on-job}}(x, a; t) > 0$ , increasing  $t$  toward 0 makes  $\Delta_{\text{on-job}}$   
1010 strictly decrease.

## 1011 C Computational Procedure for the Quantitative Model

### 1012 C.1 Steady-state Equilibrium

1013 In the steady-state, the measure of households,  $\mu(a, h, z)$ , and the factor prices are  
1014 time-invariant. We find a time-invariant distribution  $\mu$ . We compute the house-  
1015 holds' value functions and the decisions rules, and the time-invariant measure of the  
1016 households. We take the following steps:

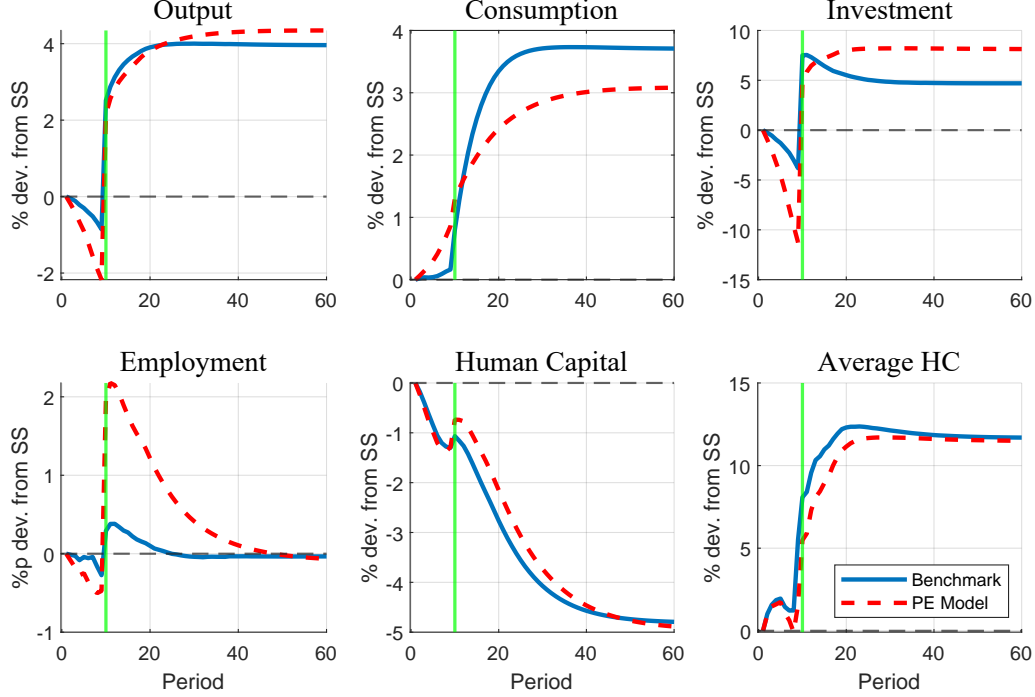
- 1017 1. We choose the number of grid for the risk-free asset,  $a$ , human capital,  $h$ , and  
1018 the idiosyncratic labor productivity,  $z$ . We set  $N_a = 151$ ,  $N_h = 151$ , and  
1019  $N_z = 9$  where  $N$  denotes the number of grid for each variable. To better  
1020 incorporate the saving decisions of households near the borrowing constraint,  
1021 we assign more points to the lower range of the asset and human capital.
- 1022 2. Productivity  $z$  is equally distributed on the range  $[-3\sigma_z/\sqrt{1-\rho_z^2}]$ . As shown  
1023 in the paper, we construct the transition probability matrix  $\pi(z'|z)$  of the  
1024 idiosyncratic labor productivity.
- 1025 3. Given the values of parameters, we find the value functions for each state  
1026  $(a, h, z)$ . We also obtain the decision rules: savings  $a'(a, h, z)$ , and  $h'(a, h, z)$ .  
1027 The computation steps are as follow:
- 1028 4. After obtaining the value functions and the decision rules, we compute the  
1029 time-invariant distribution  $\mu(a, h, z)$ .
- 1030 5. If the variables of interest are close to the targeted values, we have found the  
1031 steady-state. If not, we choose the new parameters and redo the above steps.

### 1032 C.2 Transition Dynamics

1033 We incorporate the transition path from the status quo to the new steady state. We  
1034 describe the steps below.

- 1035 1. We obtain the initial steady state and the new steady state.
- 1036 2. We assume that the economy arrives at the new steady state at time  $T$ . We  
1037 set the  $T$  to 100. The unit of time is a year.

Figure 9: Transition Path of Aggregate Variables: Benchmark vs. PE Models



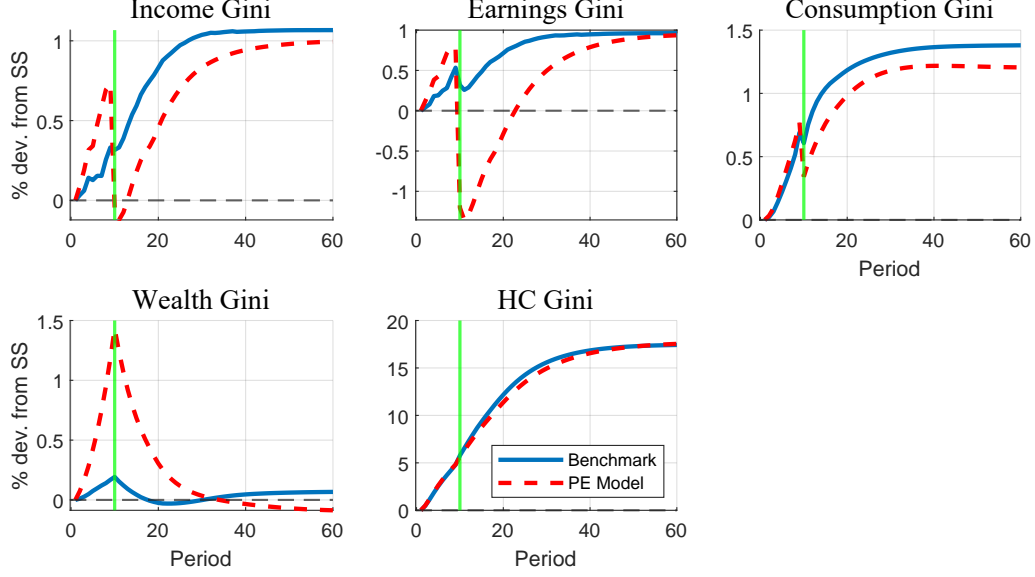
Note: The transition paths of aggregate variables: benchmark vs. PE models. The x-axis represents years, and the y-axis shows the percentage deviation from the initial steady state. AI introduction is assumed to occur in period 10. The PE model is an economy in which factor prices are held fixed at their initial steady-state values until the new steady state is reached.

- 1038 3. We initialize the capital-labor ratio  $\{K_t/L_t\}_{t=2}^{T-1}$  and obtain the associated  
1039 factor prices  $\{r_t, w_t\}_{t=2}^{T-1}$ .
- 1040 4. As we know the value functions at time  $T$ , we can obtain the value functions  
1041 and the decision rules in the transition path from  $t = T - 1$  to 1.
- 1042 5. We compute the measures  $\{\mu_t\}_{t=2}^T$  with the measures at the initial steady state  
1043 and the decision rules in the transition path.
- 1044 6. We obtain the aggregate variables in the transition path with the decision rules  
1045 and the distribution measures.
- 1046 7. We compare the assumed paths of capital and the effective labor with the  
1047 updated ones. If the absolute difference between them in each period is close  
1048 enough, we obtain the converged transition path. Otherwise, we assume new  
1049 capital-labor ratio and go back to 3.

## 1050 D Investigating the GE channel of AI's impact

1051 Figures 9 and 10 compare the transition dynamics in the benchmark general-equilibrium  
1052 model with those in a partial-equilibrium (PE) version of the model, where individ-  
1053 ual behavior responds to AI adoption but factor prices are held fixed at their initial  
1054 steady-state values. The green vertical line marks the date of AI adoption.

Figure 10: Transition Path of Inequality Measures: Benchmark vs. PE Models



Note: The transition paths of inequality measures: benchmark vs. PE models. The x-axis represents years, and the y-axis shows the percentage deviation from the initial steady state. AI introduction is assumed to occur in period 10. The PE model is an economy in which factor prices are held fixed at their initial steady-state values until the new steady state is reached.

On the aggregate side (Figure 9), both models deliver a long-run expansion in output, consumption, and investment after AI adoption. In the PE model, GDP responses are quite similar across the two models, but the composition of that response differs. In the PE model, consumption rises by less, while investment rises by more in the long run. The reason is that, in the benchmark, the long-run return on capital becomes negative (as shown in Figure 6), whereas in the PE model there is no such price effect. As a result, households in the PE environment have stronger incentives to save and invest, tilting the response toward investment rather than consumption. Even though aggregate human-capital dynamics do not differ much across the two environments, employment behaves very differently around the adoption date. In the PE model, employment rises sharply when AI is introduced because wages do not fall as employment increases.

Turning to inequality dynamics (Figure 10), the long-run behavior is similar across the two environments, but the impact responses differ markedly. As noted above, employment rises more on impact in the PE model even though output responses are similar. This implies that the additional employment mainly comes from low-productivity households. Consequently, the Gini coefficients for income, earnings, and consumption fall more on impact in the PE model but then move toward levels similar to those in the benchmark once job polarization and skill reallocation take hold. The human-capital Gini shows virtually no difference between the two models. By contrast, the wealth Gini exhibits very different transition dynamics. In the PE model, it displays a pronounced but short-lived spike early in the transition because poor households save less, as wages do not fall there in response to AI adoption, unlike in the benchmark economy. In the long run, however, the wealth

1079 Gini converges to a level similar to the benchmark, mainly because middle-sector  
1080 households gradually increase their savings, as discussed in the main text.