AI and Human Capital Accumulation: Aggregate and Distributional Implications*

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Abstract

This paper develops a model to analyze the effects of AI advancements on human capital investment and their impact on aggregate and distributional outcomes in the economy. We construct an incomplete markets economy with endogenous asset accumulation and general equilibrium, where households decide on human capital investment and labor supply. Anticipating near-term AI advancements that will alter skill premiums, we analyze the transition dynamics toward a new steady state. Our findings reveal that human capital responses to AI amplify its positive effects on aggregate output and consumption, mitigate the AI-induced rise in precautionary savings, and stabilize the adjustments in wages and asset returns. Furthermore, while AI-driven human capital adjustments increase inequalities in income, earnings, and consumption, they unexpectedly reduce wealth inequality.

Keywords: AI, Job Polarization, Human Capital, Inequality

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1 Introduction

The distinctive nature of AI advancements lies in their ability to perform cognitive, non-routine tasks that previously required significant education and expertise, fundamentally differentiating its impact on the labor market and economy from that of general automation. For example, AI tools in medical diagnostics now assist radiologists in analyzing medical images, potentially reducing demand for entry-level radiologists while simultaneously increasing the productivity of senior professionals. More generally, AI could shift the premium associated with various skills levels, devaluing middle-level skills while increasing the demand for high-level expertise. In anticipation of these changes, households are likely to adjust their human capital investments.

According to the National Center for Education Statistic,[1] college enrollment in the U.S. has been declining since 2010. The National Student Clearinghouse Research Center reports that the undergraduate college enrollment decline has accelerated since the pandemic began, resulting in a loss of almost 6% of total enrollment between fall 2019 to fall 2023, while graduate enrollment has risen by about 5%.[2] These shifts, regardless of their causes, highlight evolving patterns in human capital investment.

This paper develops a model to study the effects of AI advancements on human capital investment and their subsequent impact on aggregate and distributional outcomes of the economy. We posit an economy consisting of three sectors, requiring low, middle and high levels of skill (human capital) with increasing sectoral labor productivity. Households can invest in their human capital to move up to more productive sectors. But if they do not invest, their human capital depreciates and, over time, they will move down to less productive sectors. We model human capital investment at two levels, a low level attainable on the job and a high level requiring full-time commitment, such as pursuing higher education. Households are subject to uninsurable idiosyncratic risk in terms of productivity shocks that affect both labor productivity and effectiveness in human capital investment.

The interaction between human capital investment and labor supply presents a tradeoff at the household level between current wage earning and future wage gains. At aggregate level, the interaction implies that when individuals transition from the middle to the high sector, they may temporarily exit the workforce to upskill, reducing immediate labor supply but improving future labor productivity.

We model AI advancements as increasing the productivity for the low and high sectors but not for the middle sector so that the skill premium of the middle sector decreases and the skill premium of the high sector increases. Allowing for human

[1] https://nces.ed.gov/programs/digest/d22/tables/dt22_303.70.asp

[2] https://public.tableau.com/app/profile/researchcenter/viz/CTEEFall2023dashboard/CTEEFall2023

capital adjustments not only alters AI's economic implications quantitatively, it also makes a qualitative difference.

If the skill distribution is fixed, AI will unambiguously improve the labor productivity of the whole economy. However, allowing human capital to adjust enables workers to upskill or downskill. The response of overall labor productivity could be enhanced, or dampened, or even reverted depending on whether workers move to more or less productive sectors.

Using a two-period model, we show how households' labor supply and human capital investment are affected by their productivity shocks, asset holdings and stocks of human capital. The effects of AI, in this partial equilibrium analysis, are shown to discourage human capital investment for households in the low sector and encourage human capital investment for households in the middle sector, thereby increasing human capital inequality. In addition, AI worsens consumption inequality for households with low levels of human capital and reduces consumption inequality for those with high levels of human capital.

At the economy level, the effects of AI advancements depend on the sectoral distribution of households and the general equilibrium effects via wage and capital return responses. We quantify these effects using a fully-fledged dynamic quantitative model that incorporates an infinite horizon, endogenous asset accumulation, and general equilibrium. The model is calibrated to reflect key features of the U.S. economy, capturing realistic household heterogeneity. The steady state distribution of human capital without AI advancements pins down the sectoral distribution of households. We then introduce fully anticipated AI advancements happening in the near future and study the transition dynamics from the current state of the economy to the eventual new steady state.

We find that aggregate human capital rises sharply even before AI introduction, indicating that a substantial portion of workers, anticipating changes in skill premium, leave the labor force early to accumulate human capital. The economy also experiences AI-induced job polarization, with a notable reallocation of workers from the middle sector to either low or high sectors.

Building on these labor dynamics, our model examines how AI influences both the aggregate and distributional outcomes of the economy, including output, consumption, investment, employment, income inequality, consumption inequality, and wealth inequality. Our focus is on how human capital adjustments reshape AI's effects on each of these outcomes. Specifically, we examine two primary channels through which human capital adjustments operate: the redistribution channel, which reallocates workers across skill sectors, and the general equilibrium channel, which operates through wages and capital return changes.

Our findings reveal that human capital responses to AI amplify its positive effects on aggregate output and consumption, mitigate the AI-induced rise in precautionary savings, and stabilize the adjustments in wages and asset returns. Furthermore, while AI-driven human capital adjustments increase inequalities in income, earnings, and consumption, they unexpectedly reduce wealth inequality. We also show that the redistribution channel is the dominant factor in the effects of human capital adjustments, whereas the general equilibrium channel, via wage and capital return changes, plays a comparatively minor role.

INTRODUCING PRECAUTIONARY SAVING MOTIVE IN THE WAGE POLARIZATION INVESTIGATION Autor *et al.*, (2006)

This paper relates to the literature examining how technological advancements, including AI, have significantly contributed to job polarization. Goos and Manning (2007) show that since 1975, the United Kingdom has experienced job polarization, with increasing employment shares in both high- and low-wage occupations. Autor and Dorn (2013) expanded on this by providing a unified analysis of the growth of low-skill service occupations, highlighting key factors that amplify polarization in the U.S. labor market. Empirical evidence from Goos *et al.*, (2014) further confirms pervasive job polarization across 16 advanced Western European economies. In the U.S., Acemoglu and Restrepo (2020) show that robots can reduce employment and wages, finding robust negative effects of automation on both in various commuting zones.

The introduction of AI and robotics has had adverse effects on labor markets, with significant implications for employment and labor force participation. Lerch (2021) highlights that the increasing use of robots not only displaces workers but also negatively impacts overall labor force participation rates. Similarly, Faber *et al.*, (2022) demonstrate that the detrimental effects of robots on the labor market have resulted in a decline in job opportunities, particularly in sectors where automation is prevalent. These findings suggest that while technological advancements bring productivity gains, they simultaneously reduce employment prospects and participation in the labor market, exacerbating economic challenges for certain groups of workers.

The introduction of AI and robotics also influences human capital accumulation as workers respond to technological disruption. Faced with the employment risks brought about by automation, many exposed workers may invest in additional education as a form of self-insurance, rather than relying on increases in the college wage premium (Atkin, 2016; Beaudry *et al.*, 2016). Empirical evidence supports this response. Di Giacomo and Lerch (2023) find that for every additional robot adopted in U.S. local labor markets between 1993 and 2007, four individuals enrolled in college, particularly in community colleges, indicating a rise in educational investments triggered by automation. Similarly, Dauth *et al.*, (2021) show that within German firms, robot adoption has led to an increase in the share of college-educated workers, as firms prioritize higher-skilled employees over those with apprenticeships.

The response of human capital accumulation to technological disruption could also go to the other extreme. A 2022 report by Higher Education Strategy Associates finds that following decades of growth, dropping student enrollment has become a major trend in higher education in the Global North.[3] In the U.S., the public across the political spectrum has increasingly lost confidence in the economic benefits of a college degree. Pew Research Center reports that about half of Americans say having a college degree is less important today than it was 20 years ago in a survey conducted in 2023.[4] A 2022 study from Public Agenda, a nonpartisan research organization, shows that young Americans without college degrees are most skeptical about the value of higher education.

The rise of AI and automation also plays a significant role in exacerbating general inequality, particularly through its impact on education and wealth distribution. Prettner and Strulik (2020) present a model showing that innovation-driven growth leads to an increasing proportion of college graduates, which in turn drives higher income and wealth inequality. As technology advances, workers with higher educational attainment benefit disproportionately, widening the gap between those with and without advanced skills. Sachs and Kotlikoff (2012) also explore this dynamic, providing a model within an overlapping generations framework that examines the interaction between automation and education. They demonstrate how automation can further entrench inequality by favoring workers with higher levels of education, as those without adequate skills are more likely to be displaced or see their wages stagnate. This interaction between technological change and educational attainment not only amplifies economic inequality but also perpetuates disparities in wealth across generations.

The rest of the paper is organized as follows. Section 2 describes the model environment. Section 3 solves the household's problem using a two-period version of the model. Section 4 solves the fully-fledged quantitative model and calibrates it to fit key features of the U.S. economy, including employment rate, human capital investment, and household heterogeneity. Section 5 incorporates AI into the quantitative model and examines its economic impact on both aggregate and distributional outcomes. Section 6 analyzes how human capital adjustments change the economic impact of AI advancements. Section 7 concludes.

2 Model Environment

Time is discrete and infinite. There is a continuum of households. Each household is endowed with one unit of indivisible labor and faces idiosyncratic productivity

[3] https://higheredstrategy.com/world-higher-education-institutions-students-and-funding/

[4] https://www.pewresearch.org/social-trends/2024/05/23/public-views-on-the-value-of-a-college-degree/

shock, z, that follows an AR(1) process in logs:

$$\ln z' = \rho_z \ln z + \varepsilon_z, \varepsilon_z \overset{\text{iid}}{\sim} N(0, \sigma_z^2) \tag{1}$$

The asset market is incomplete following Aiyagari (1994), and the physical capital, a, is the only asset available to households to insure against this idiosyncratic risk. Households can also invest in human capital, h, which allows them to work in sectors with different human capital requirement.

2.1 Production Technology

The production technology in the economy is a constant-returns-to-scale Cobb-Douglas production function:

$$F(K, L) = K^{1-\alpha} L^{\alpha} \tag{2}$$

K represents the total physical capital accumulated by households, while L denotes the total effective labor supplied by households, aggregated across three sectors: low, middle, and high. The marginal products of capital and effective labor determine the economy-wide wage rate, w, and interest rate, r.

These sectors differ in their technologies for converting labor into effective labor units and in the levels of human capital required for employment. The middle sector employs households with human capital above h_M and converts one unit of labor to one effective labor unit. The high sector, requiring human capital above h_H, converts one unit of labor to $1 + \lambda$ effective units, while the low sector, with no human capital requirement, converts one unit into $1 - \lambda$ effective units. This implies a sectoral labor productivity $x(h)$ that is a step function in human capital:

$$x(h) = \begin{cases} 1 - \lambda & \text{low sector if } h < h_M \\ 1 & \text{middle sector if } h_M < h < h_H \\ 1 + \lambda & \text{high sector if } h > h_H \end{cases} \tag{3}$$

A household i who decides to work thus contributes $z_i x(h_i)$ units of effective labor, where z_i is his idiosyncratic productivity. Denote $n_i \in \{0, 1\}$ as the indicator that takes one if the household works and zero if the household does not. The aggregate labor is

$$L = \int n_i z_i x(h_i) di, \tag{4}$$

assuming perfect substitutability of effective labor across the three sectors.

2.2 *Household's Problem*

Households derive utility from consumption, incur disutility from labor and effort of human capital investment. A household maximizes the expected lifetime utility by optimally choosing consumption, saving, labor supply and human capital investment each period, based on his idiosyncratic productivity shock z_t:

$$\max_{\{c_t, a_{t+1}, n_t, e_t\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t (\ln c_t - \chi_n n_t - \chi_e e_t) \right] \tag{5}$$

where c_t represents consumption, a_{t+1} represents saving, $n_t \in \{0, 1\}$ is labor supply, and e_t is the effort of human capital investment.

If a household decides to work in period t, he will be employed into the appropriate sector according to his human capital h_t and receive labor income $w_t z_t x(h_t)$. The household's budget constraint is

$$c_t + a_{t+1} = n_t(w_t z_t x(h_t)) + (1 + r_t) a_t \tag{6}$$

$$c_t \geq 0 \text{ and } a_{t+1} \geq 0 \tag{7}$$

We prohibit households from borrowing $a_{t+1} \geq 0$ to simplify analysis.[5]

Human capital investment can take three levels of effort: $\{0, e_L, e_H\}$. A non-working household is free to choose any of the three effort levels but a working household cannot devote the highest level of effort e_H, reflecting a trade-off between working and human capital investment. Hence:

$$e_t \in \{0, e_L, (1 - n_t) e_H\}. \tag{8}$$

Its contribution to next-period human capital is subject to the productivity shock:

$$h_{t+1} = z_t e_t + (1 - \delta) h_t \tag{9}$$

where δ is human capital's depreciation rate.

3 Household Decisions in a Two-Period Model

In this section, we solve the household's problem with two periods to gain intuition.

Period-2 decisions Households do not invest in human capital or physical capital in the last period. The only relevant decision is whether to work.

[5] According to Aiyagari (1994), a borrowing constraint is necessarily implied by present value budget balance and nonnegativity of consumption. Since the borrowing limit is not essential to our analysis, we set it to zero for simplicity.

The household works $n = 1$ if and only if $z \geq \overline{z}(h, a)$, with $\overline{z}(h, a)$ defined as

$$\ln(w\overline{z}(h, a)x(h) + (1 + r)a) - \chi_n = \ln((1 + r)a) \tag{10}$$

The household faces a trade-off between earning labor income and incurring the disutility of working. Given the sector-specific productivity $x(h)$ specified in (3), the threshold for idiosyncratic productivity, $\overline{z}(h, a)$, takes on three possible values:

$$\overline{z}(h, a) = \begin{cases} \overline{z}(a)\frac{1}{1-\lambda} & \text{if } h < h_M \\ \overline{z}(a) & \text{if } h_M \leq h < h_H \\ \overline{z}(a)\frac{1}{1+\lambda} & \text{if } h > h_H \end{cases} \tag{11}$$

$$\text{where } \overline{z}(a) := \frac{(\exp(\chi_n) - 1)(1 + r)a}{w} \tag{12}$$

Households with higher human capital is more likely to work, whereas households with higher physical capital is less likely to work.

Period-1 decisions In addition to labor supply, period-1 decisions include saving and human capital investment, both of which are forward-looking and affected by the idiosyncratic risk associated with the productivity shock z'. Our model also features a trade-off between human capital investment and labor supply as a working household cannot devote the highest level of effort e_H in human capital investment. Therefore, human capital investment grants households the possibility of a discrete wage hike in the future but may entail a wage loss in the current period.

To see the implication of this trade-off and how it interacts with uninsured idiosyncratic risk, we proceed in two steps. We first derive the period-1 decisions without uncertainty by assuming that z' is known to the household at period 1 and z' is such that the household will work in period 2. We then reintroduce uncertainty in z' and compare the decision rules with the case without uncertainty.

3.1 Period-1 Labor Supply and Human Capital Investment

3.1.1 Consumption and saving without uncertainty

The additive separability of household's utility implies that labor supply n and human capital investment e enters in consumption and saving choices only via the intertemporal budget constraint:

$$c + \frac{c'}{1 + r'} = (1 + r)a + n(wzx(h)) + \frac{w'z'x(h')}{1 + r'}$$
$$\text{with } h' = ze + (1 - \delta)h.$$

The log utility in consumption implies the optimality condition:

$$c' = \beta(1 + r')c. \tag{13}$$

Combining it with the budget constraint, we obtain the optimal consumption as a function of labor supply n and human capital investment e:

$$c(n, e) = \frac{1}{1+\beta}\left[(1+r)a + n(wzx(h)) + \frac{w'z'x\left(h' = ze + (1-\delta)h\right)}{1+r'}\right]. \tag{14}$$

3.1.2 Labor supply and human capital investment

The optimal consumption rules in (14) and (13) allow us to express the household's problem as the maximization of an objective function in labor supply n and human capital investment e:[6]

$$\max_{n,e} (1+\beta)\ln c(n, e) - \chi_n n - \chi_e e \tag{15}$$

This maximization depends critically on the household's current human capital and achievable next-period human capital. Accordingly, we partition households into five ranges of h: $[0, h_M)$, $[h_M, h_M(1-\delta)^{-1})$, $[h_M(1-\delta)^{-1}, h_H)$, $[h_H, h_H(1-\delta)^{-1})$, and $[h_H(1-\delta)^{-1}, h_{\max}]$.

We now derive the decision rules for households $h \in [h_M, h_M(1-\delta)^{-1})$ in detail, as the decision rules for the other four ranges are similar. For households with $h < h_M(1-\delta)^{-1}$, we define two cutoffs in z:

$$\underline{z}_M(h) := \frac{h_M - (1-\delta)h}{e_H}; \overline{z}_M(h) := \frac{h_M - (1-\delta)h}{e_L} \tag{16}$$

These cutoffs divide households into three groups based on their ability to be employed in the middle sector in the next period.

Non-learners are households with $z < \underline{z}_M(h)$. They cannot achieve $h' > h_M$ with either e_L or e_H level of human capital investment today. As a result, they will choose not to invest in human capital, $e = 0$, and their future sectoral productivity will be $x(h') = 1 - \lambda$. These non-learners work $n = 1$ if and only if $z \geq \overline{z}^L_{non}(a)$:

$$\overline{z}^L_{non}(a) = \frac{(\exp(\frac{\chi_n}{1+\beta}) - 1)[(1+r)a + \frac{w'z'(1-\lambda)}{1+r'}]}{w} \tag{17}$$

Slow learners are households with $z \in (\underline{z}_M(h), \overline{z}_M(h))$. These households can reach $h' > h_M$ in the next period only by investing $e = e_H$ today. Their choice is restricted to $e = 0$ or $e = e_H$, since selecting $e = e_L$ incurs a cost without any

[6] This follows since $c' = \beta(1 + r')c$, so $\ln c' = \ln\beta + \ln(1 + r') + \ln c$.

future benefit. Slow learners must trade off between working and human capital investment: choosing $e = e_H$ requires not working today ($n = 0$), while opting to work means forgoing investment in human capital ($n = 1, e = 0$).[7]

Slow learners prefer ($n = 1, e = 0$) to ($n = 0, e = e_H$) if and only if $z \geq \overline{z}^L_{slow}(a)$:

$$\overline{z}^L_{slow}(a) = \frac{(\exp(\frac{\chi_n - \chi_e e_H}{1+\beta}) - 1)[(1+r)a + \frac{w'z'}{1+r'}] + \lambda\frac{w'z'}{1+r'}}{w} \tag{18}$$

Fast learners are households with $z > \overline{z}_M(h)$. They can achieve $h' > h_M$ in the next period if they invest $e = e_L$ today. In this case, there is no need to exert high effort e_H in human capital investment. The fast learners choose among three options: ($n = 1, e = 0$), ($n = 1, e = e_L$), and ($n = 0, e = e_L$).[8]

The decision rule for fast learners are as follows:

$$n(z,h,a), e(z,h,a) = \begin{cases} n = 1, e = 0 & \text{if } z \geq \overline{z}^L_{fast}(a) \\ n = 1, e = e_L & \text{if } \underline{z}^L_{fast}(a) \leq z < \overline{z}^L_{fast}(a) \\ n = 0, e = e_L & \text{if } z < \underline{z}^L_{fast}(a) \end{cases} \tag{19}$$

where

$$\overline{z}^L_{fast}(a) = \frac{\left\{\exp(\frac{\chi_e e_L}{1+\beta})\lambda\left[\exp(\frac{\chi_e e_L}{1+\beta}) - 1\right]^{-1} - 1\right\}\frac{w'z'}{1+r'} - (1+r)a}{w} \tag{20}$$

$$\underline{z}^L_{fast}(a) = \frac{(\exp(\frac{\chi_n}{1+\beta}) - 1)[(1+r)a + \frac{w'z'}{1+r'}]}{w} \tag{21}$$

We set up our model so that $\overline{z}^L_{fast}(a) > \underline{z}^L_{fast}(a)$.[9]

Decision rule diagram: Figure 1 illustrates the decision rule (n, e) as a function of states (z, h, a) for households with $h_M \leq h < h_M\frac{1}{1-\delta}$. The human capital h changes along the horizontal line and the idiosyncratic productivity z changes along the vertical line. The two diagonal lines, $\overline{z}_M(h)$ and $\underline{z}_M(h)$ defined in (16), separate the state space into three areas: the unshaded area represents the non-learners, the lightly-shaded area represents the slow learners, and the darkly-shaded area represents the fast learners. The areas are divided by four dashed horizontal lines associated with cutoffs $\overline{z}^L_{non}(a)$, $\overline{z}^L_{slow}(a)$, $\underline{z}^L_{fast}(a)$, and $\overline{z}^L_{fast}(a)$ that are functions of

[7]The choice between ($n = 0, e = e_H$) and ($n = 0, e = 0$) does not depend on z. For e_H to be relevant, λ must be large enough so that ($n = 0, e = e_H$) is preferred to ($n = 0, e = 0$). See the Appendix for details on the lower bound for λ.

[8]Similar to the case of slow learners, the choice between ($n = 0, e = e_L$) and ($n = 0, e = 0$) does not depend on z. Moreover, since our model is set up so that ($n = 0, e = e_H$) dominates ($n = 0, e = 0$), it implies that ($n = 0, e = e_L$) dominates ($n = 0, e = 0$).

[9]Appendix A.2 provides the parameter restrictions such that the condition for ($n = 0, e = e_H$) to dominate ($n = 0, e = 0$) is sufficient for $\overline{z}^L_{fast}(a) > \underline{z}^L_{fast}(a)$.

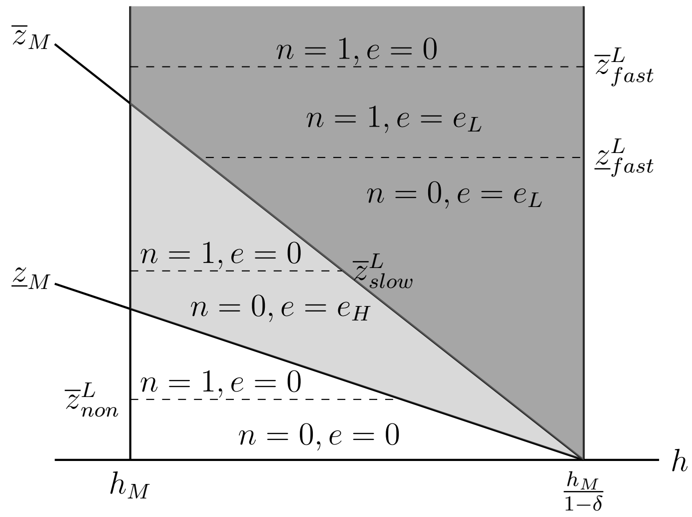


Figure 1: Decision Rule Diagram for $h_M \leq h < h_M(1-\delta)^{-1}$

The human capital h changes along the horizontal line and the idiosyncratic productivity z changes along the vertical line. The two diagonal lines, $\overline{z}_M(h)$ and $\underline{z}_M(h)$, separate the state space into three areas: the unshaded area represents the non-learners, the lightly-shaded area represents the slow learners, and the darkly-shaded area represents the fast learners. The areas are divided by four dashed horizontal lines associated with cutoffs $\overline{z}^L_{non}$, $\overline{z}^L_{slow}$, $\underline{z}^L_{fast}$, and $\overline{z}^L_{fast}$ that are functions of capital holding a.

capital holding a and defined in (17), (18), (21), and (20).

This decision rule diagram is representative for households in other four ranges of human capital. Figure 2 illustrates the regions in which households make positive human capital investments. Striped shading highlights where investment occurs, with dark areas denoting fast learners and light areas representing slow learners.

For households with $h < h_M$, $\overline{z}_M(h)$ and $\underline{z}_M(h)$ continue to be the boundaries that separate non-learners, slow learners and fast learners, but the four cutoffs are $\overline{z}^L_{non}\frac{1}{1-\lambda}$, $\overline{z}^L_{slow}\frac{1}{1-\lambda}$, $\underline{z}^L_{fast}\frac{1}{1-\lambda}$, and $\overline{z}^L_{fast}\frac{1}{1-\lambda}$.

For households with $h_M\frac{1}{1-\delta} \leq h < h_H\frac{1}{1-\delta}$, the boundaries for state space division change to $\overline{z}_H(h)$ and $\underline{z}_H(h)$:

$$\underline{z}_H(h) := \frac{h_H - (1-\delta)h}{e_H}; \qquad \overline{z}_H(h) := \frac{h_H - (1-\delta)h}{e_L} \tag{22}$$

If $h_M\frac{1}{1-\delta} \leq h < h_H$, the four cutoffs that partition the decision regions for households are denoted as $\overline{z}^M_{non}(a)$, $\overline{z}^M_{slow}(a)$, $\underline{z}^M_{fast}(a)$, and $\overline{z}^M_{fast}(a)$ (see Appendix A.1 for the explicit formulae).[10] If $h_H \leq h < h_H\frac{1}{1-\delta}$, the analogous cutoffs are given by $\overline{z}^M_{non}\frac{1}{1+\lambda}$, $\overline{z}^M_{slow}\frac{1}{1+\lambda}$, $\underline{z}^M_{fast}\frac{1}{1+\lambda}$, and $\overline{z}^M_{fast}\frac{1}{1+\lambda}$.

All households with $h \geq h_H\frac{1}{1-\delta}$ are non-learners because their current human capital is enough for employment in the high sector next period even without any

[10]Appendix A.2 provides parameter restrictions for $\overline{z}^M_{fast}(a) > \underline{z}^M_{fast}(a)$.

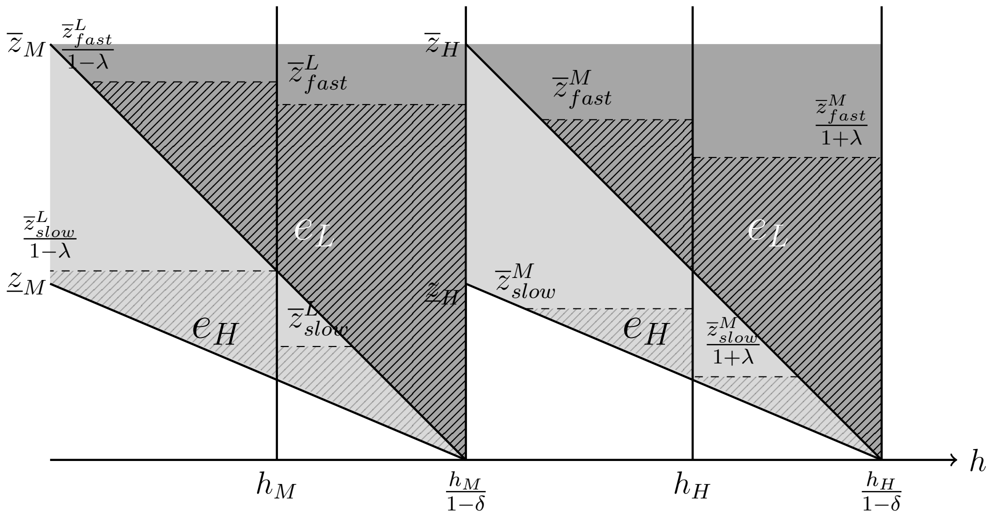


Figure 2: State Space for Human Capital Investment

The darkly-shaded striped areas indicate the state space for human capital investment equal to e_L by the fast learners. The lightly-shaded striped areas indicate the state space for human capital investment equal to e_H by the slow learners.

human capital investment. The only relevant cutoff for them is $\overline{z}^H_{non}(a)\frac{1}{1+\lambda}$ where

$$\overline{z}^H_{non}(a) := \frac{(\exp(\frac{\chi_n}{1+\beta}) - 1)[(1+r)a + \frac{w'z'(1+\lambda)}{1+r'}]}{w} \tag{23}$$

3.2 The Effects of Uninsured Idiosyncratic Risk

We now reintroduce the idiosyncratic risk to households in period 1 by assuming that z' follows a log-normal distribution with mean $\overline{z}'$ and variance σ_z^2.

Our previous analysis without uncertainty is a special case with $\sigma_z^2 = 0$. The effects of uninsured idiosyncratic risk can be thought as how households' decisions change when the distribution of z' undergoes a mean-preserving spread in the sense of second-order stochastic dominance.

From a consumption-saving perspective, the uncertain z' is associated with future labor income risk. It is well understood in the literature that idiosyncratic future income risk raises the expected marginal utility of future consumption for households with log utility and makes them save more. In our model, households can also supply more labor to mitigate the effect of idiosyncratic income risk on the marginal utility of consumption.

From the perspective of human capital investment, the uncertain z' is associated with risk in the return to human capital. Conditional on working, households' income increases with z': $c' = (1 + r')a' + w'x(h')z'$. $\ln(c')$ is increasing and concave in z', and a higher $x(h')$ increases the concavity.[11] Consider two levels of h', $\overline{h'} > \underline{h'}$, a mean-preserving spread of z' distribution reduces the expected utility at both levels of h' but the reduction is larger for the higher level $\overline{h'}$. Hence, the expected utility gain of moving from $\underline{h'}$ to $\overline{h'}$ is smaller due to the idiosyncratic risk. Human capital investment is discouraged.

Taking into account endogenous labor supply reinforces the discouragement of human capital investment by the idiosyncratic risk. Recall from Section 3 that households with z' lower than a cutoff do not work. The endogenous labor supply therefore provides insurance against the lower tail risk of the idiosyncratic z'. Moreover, the cutoff in z' is lower for those with higher human capital h'. This makes households with higher h' more exposed to the lower tail risk than those with lower h', further reducing the gain of human capital investment.

Proposition 1. *The uninsured idiosyncratic risk in z' makes households in period 1 save more, work more and invest less in human capital.*

3.3 Period-1 Saving and Human Capital Investment

In this section, we study the impact of endogenous human capital investment on households' saving decisions. Specifically, we compare optimal saving behavior in two scenarios: one in which households can choose to invest in human capital, and an alternative scenario in which human capital is exogenously fixed. To facilitate the comparison, we assume in this section that there is no human capital depreciation.[12]

When the optimal choice of human capital investment is zero, optimal saving is identical in both scenarios. When the optimal human capital investment is either e_L or e_H, we compare the household's optimal saving to the case where human capital investment is exogenously fixed at zero, i.e., $(n = 1, e = 0)$.[13]

To make the human captial relevant, we assume that $n' = 1$ in period 2. The

[11] The marginal effect of z' on $\ln(c')$ is

$$\frac{\partial \ln(c')}{\partial z'} = \frac{w'x(h')}{(1+r')a' + w'x(h')z'} > 0$$

The second derivative is

$$\frac{\partial^2 \ln(c')}{(\partial z')^2} = -\left[\frac{w'x(h')}{(1+r')a' + w'x(h')z'}\right]^2 < 0$$

and is more negative if $x(h')$ is higher.

[12] If depreciation is allowed, the analysis proceeds similarly but involves more comparison paris.

[13] Why not compare to $(n = 0, e = 0)$? Such a comparison is not meaningful when considering $(n = 1, e = e_L)$ because the two scenarios involve different state spaces. To see it, suppose conditions are such that $(n = 1, e = e_L)$ is optimal. If we were to fix $e = 0$ exogenously, the household's lifetime income would fall, and as a result the household would have a greater incentive to work. Thus, it is not possible for the household to deviate from choosing $n = 1$ when human capital is held fixed at $e = 0$. The comparison between $(n = 0, e = 0)$ and $(n = 0, e = e_L$ or $e_H)$ is similar to the comparison between $(n = 1, e = 0)$ to $(n = 1, e = e_L)$, since human capital investment does not affect period-1 labor income in either case.

additive separability of work and human capital investment effort from consumption allows us to consider the optimal saving conditional on a given choice of labor supply and human capital investment.

In particular, the household maximizes expected lifetime utility:

$$\max_{a'} : \ln(c) + \beta \mathbb{E}_{z'}[\ln(c')], \tag{24}$$

subject to the budget constraints

$$c + a' = (1+r)a + n(wzx(h)), \tag{25}$$
$$c' = (1+r')a' + w'z'x(h'), \tag{26}$$
$$\text{with } h' = ze + (1-\delta)h, e \in \{0, e_L, (1-n)e_H\} \tag{27}$$

3.3.1 Effect of on-job-training on saving

We now compare the optimal saving between ($n = 1, e = e_L$) and ($n = 1, e = 0$), where e_L allows households to move to a higher sector in period 2 with higher sectoral productivity $x(h')$.

To simplify the notation while maintaining the key economic forces, we normalize $(1+r) = (1+r') = 1$, $w = w' = 1$, the period-1 productivity shock $z = 1$ and the period-2 productivity shock z' to $\ln z' \sim \mathcal{N}(0, \sigma_z^2)$. The budget constraints become:

$$c + a' = a + x, \quad c' = a' + txz' \tag{28}$$

where $t \geq 1$ represents the effect of human capital investment on period-2 income: $t > 1$ if $e = e_L$; $t = 1$ if $e = 0$.

The optimal saving is determined by the FOC:

$$\frac{1}{a + x - a'} = \beta \mathbb{E}_{z'}\left(\frac{1}{a' + txz'}\right) \tag{29}$$

Denoting the mean and variance of z' as μ and Σ, respectively:

$$\mu \equiv \mathbb{E}[z'] = e^{\sigma_z^2/2}, \quad \Sigma \equiv \text{Var}(z') = e^{\sigma_z^2}(e^{\sigma_z^2} - 1). \tag{30}$$

The second-order approximate solution to the FOC is:

$$a'^*(x, a; t) = \underbrace{\frac{\beta(a+x) - tx\mu}{1+\beta}}_{\text{CE}} + \underbrace{\frac{t^2x^2\Sigma}{\beta(a + x + tx\mu)}}_{\text{Precautionary}} \tag{31}$$

The first term is the *certainty-equivalent* saving, which reflects the consumption smoothing motive, increasing in the period-1 resources $a + x$ and decreasing in the period-2 expected labor income $tx\mu$. The second term is the *precautionary* saving, which is increasing in the variance of period-2 labor income $t^2x^2\Sigma$ and decreasing in the expected total resources $a + x + tx\mu$.

The effect of on-job-training on saving can be decomposed into two components:

$$\frac{\partial a'^{\star}}{\partial t}(x,a;t) = -\frac{x\mu}{1+\beta} + \frac{x^2\Sigma}{\beta}\frac{t\,[2(a+x)+tx\mu]}{(a+x+tx\mu)^2}. \tag{32}$$

The first term being negative captures the *crowd-out* effect on saving via consumption-smoothing motive as on-job-training increases the expected period-2 labor income $tx\mu$. The second positive term captures the *crowd-in* effect via precautionary saving motive as on-job-training exposes households to larger future income risk.

To capture the overall impact of on-job-training on saving, we define:

$$\Delta_{\text{on-job}}(x,a;t) = a'^{\star}(x,a;t) - a'^{\star}(x,a;1) = \int_1^t \frac{\partial a'^{\star}}{\partial u}(x,a;u)du, \tag{33}$$

where $a'^{\star}(x,a;t)$ is the optimal saving when households undertake on-job-training, and $a'^{\star}(x,a;1)$ is the optimal saving when human capital is kept exogenously fixed.

Whether on-job-training increases or decreases saving ultimately depends on the balance between the crowd-out effect (via higher expected future income) and the precautionary crowd-in effect (via heightened future income risk). The next proposition demonstrates that these effects can dominate differently depending on skill, so that the overall impact of on-job-training on saving can differ between low- and high-skilled households.

Proposition 2. *When the idiosyncratic shock is large enough, i.e.,* $\frac{\Sigma}{\mu} > \underline{\sigma}(t)$*, on-job-training crowds out saving for low-skilled households and crowds in saving for high-skilled households: for* $x < x^*(a,t)$*,* $e = e_L$ *lowers saving* $\Delta_{\textit{on-job}}(x,a;t) < 0$*; for* $x > x^*(a,t)$*,* $e = e_L$ *raises saving* $\Delta_{\textit{on-job}}(x,a;t) > 0$.

Proof. See Appendix B. □

3.3.2 Effect of full-time training on saving

We next compare the optimal saving between $(n = 0, e = e_L \text{ or } e_H)$ and $(n = 1, e = 0)$. Note that full-time training requires the households to give up their labor income in period 1, which is not the case for on-job-training. Following the same normalization and notation as in the previous subsection, we can write the budget constraints with full-time training and without training as:

$$e = e_H: \quad c + a' = a, \quad c' = a' + txz' \tag{34}$$

$$e = 0: \quad c + a' = a + x, \quad c' = a' + xz' \tag{35}$$

where $t > 1$ captures the effect of full-time training on period-2 income.

The second-order approximate solution to the optimization problem is:

$$e = e_H: \quad a'^*_{e_H}(x,a;t) = \underbrace{\frac{\beta a - tx\mu}{1+\beta}}_{\text{CE}} + \underbrace{\frac{t^2x^2\Sigma}{\beta(a+tx\mu)}}_{\text{Precautionary}} \tag{36}$$

$$e = 0: \quad a'^*(x,a;1) = \underbrace{\frac{\beta(a+x) - x\mu}{1+\beta}}_{\text{CE}} + \underbrace{\frac{x^2\Sigma}{\beta(a+x+x\mu)}}_{\text{Precautionary}} \tag{37}$$

so that the total effect of full-time training on saving is:

$$\Delta_{\text{full-time}}(x,a;t) = a'^*_{e_H}(x,a;t) - a'^*(x,a;1) \tag{38}$$

$$= \Delta_{\text{on-job}}(x,a;t) - x\frac{\beta}{1+\beta} + \frac{t^2x^2\Sigma}{\beta}\frac{x}{(a+x+tx\mu)(a+tx\mu)} \tag{39}$$

Compared to the effect of on-job-training, represented by $\Delta_{\text{on-job}}(x,a;t)$ defined in (33), full-time training introduces two additional effects on saving. First, it further reduces saving because households forgo their period-1 labor income, as reflected in the second term. Second, it increases precautionary saving, since having lower current resources leaves households less able to self-insure against idiosyncratic risk in period 2, which is captured by the third term. Denote the net additional effect of full-time training on saving as:

$$\Delta_H(x,a;t) \equiv x\Big[-\frac{\beta}{1+\beta} + \frac{\Sigma}{\beta}\frac{t^2x^2}{(a+x+tx\mu)(a+tx\mu)}\Big] \tag{40}$$

so that $\Delta_{\text{full-time}}(x,a;t) = \Delta_{\text{on-job}}(x,a;t) + \Delta_H(x,a;t)$. The next proposition shows that the net additional effect is negative and stronger for higher skilled households.

Proposition 3. *When the idiosyncratic shock is not too large, i.e., $\frac{\Sigma}{\mu} < \overline{\sigma}(t)$, full-time training crowds out more saving than on-job-training, $\Delta_H(x,a;t) < 0$. Moreover, the crowding-out effect is stronger for higher skilled households: $\Delta_H(x,a;t)$ is decreasing in x.*

Proof. See Appendix B. □

3.4 The Effects of an Anticipated Period-2 AI Shock

Suppose that an AI shock is anticipated to occur in period 2 and to increase the labor productivity for the low sector and the high sector but not the middle sector. The effect of AI shock on the sectoral productivity is captured by γ with $0 < \gamma < 1$:

$$x(h') = \begin{cases} 1-\lambda+\gamma\lambda & \text{low sector if } h' < h_M \\ 1 & \text{middle sector if } h_M < h' < h_H \\ 1+\lambda+\gamma\lambda & \text{high sector if } h' > h_H \end{cases} \tag{41}$$

In other words, the AI shock increases average labor productivity, reduces the earnings premium for the middle sector, and enlarges the earnings premium for the high sector relative to the middle sector.

3.4.1 Effects on human capital investment

The AI shock lowers the incentive to work in the middle sector in period 2. Consequently, households with $h < h_M/(1-\delta)$ reduce their human capital investment, while those with $h > h_M/(1-\delta)$ increase it. More specifically, the upper bounds that determine whether households undertake positive human capital investment – denoted by $\overline{z}^L_{slow}$ and $\overline{z}^L_{fast}$ for $h < h_M/(1-\delta)$, and $\overline{z}^M_{slow}$ and $\overline{z}^M_{fast}$ for $h > h_M/(1-\delta)$ – respond in opposite directions to the anticipated shock: the former decrease with γ and the latter increase. This relationship is formalized below.

Proposition 4. *An anticipated AI shock decreases human capital investment among households with $h < h_M/(1-\delta)$, but increases it among those with $h > h_M/(1-\delta)$. Specifically, $\overline{z}^L_{slow}$ and $\overline{z}^L_{fast}$ decrease with γ, while $\overline{z}^M_{slow}$ and $\overline{z}^M_{fast}$ increase with γ.*

Proof. See Appendix B. □

3.4.2 Effects on labor supply

via income: The AI shock raises period-2 labor income for households who will work in the low or high sector, leading to a positive income effect that reduces their labor supply in period 1.

via full-time training: Because full-time training and labor supply compete for time, the AI shock affects their tradeoff through its impact on human capital investment incentives. For $h > h_M/(1-\delta)$, where AI makes investing in additional skills more attractive, households are more likely to engage in full-time training and thus reduce period-1 labor supply. In contrast, for $h < h_M/(1-\delta)$, where the AI shock lowers the payoff to investing in skills, households shift away from full-time training and supply more labor in the first period.

3.4.3 Effects on saving

The AI shock increases the sectoral labor productivities for the low and high sectors in period 2, but leaving the middle sector's labor productivity unchanged. Its effect on saving can be analyzed using the households optimal saving problem (24) with varying x' across sectors.

Households who will stay in the same sector We first discuss those households who need no human capital investment or on-job-training to stay in the same sector.

For low-sector and high-sector households, the AI shock increases their period-2 labor income x'. This change of x' is analogous to the effect of on-job-training $\Delta_{\text{on-job}}(x, a; t)$ defined in (33). Proposition 2 shows that $\Delta_{\text{on-job}}(x, a; t)$ has opposite sign for low-skill and high-skill households. Therefore, the AI shock *crowds out low-sector households' saving* and *crowds in high-sector households' saving*. For middel-sector households, the AI shock brings no change to their incomes and saving.

When households need full-time traning to stay in the same sector (middle or high sector), the AI shock affects their incentives to invest e_H. The middle-sector households have weaker incentives for full-time training so that the AI shock makes them save more. The high-sector households have stronger incentives for full-time training and in turn save less in response to the AI shock.

Households who will upskill to a higher sector When households upskill via on-job-training, the low-sector houesholds do not change their saving as the AI shock does not alter their future productivity gain after they upskill. For the middle-sector households, the AI shock improves their future productivity gain from λ to $(1+\gamma)\lambda$. This is equivalent to an increase of t in the on-job-training effect on saving, $\Delta_{\text{on-job}}(x, a; t)$. If the on-job-training effect on saving is positive in absence of the AI shock, the AI shock will increase households' saving. If the on-job-training effect on saving is negative in absence of the AI shock, the AI shock may reduce or increase households' saving depending on the households current sectoral produtivity and γ.

When households upskill via full-time training,

Households who will downskill to a lower sector They must have $e = 0$.

Low sector households: For households who will stay in the low sector in period 2, the AI shock increases their period-2 labor income x' from $1 - \lambda$ to $1 - (1 - \gamma)\lambda$. This change of x' is analogous to the effect of on-job-training $\Delta_{\text{on-job}}(x, a; t)$ defined in (33).

Middle sector households: For households who will stay in the middle sector in period 2, the AI shock brings no change to their incomes and saving.

For households who will work in the high sector in period 2 (via human capital investment)

High sector households:

On-job-training: For the low-skill households, AI increases x but reduces t. Hence, AI reduces the net-crowding-out effect for the low-skill households.

For the middle-skill households, AI does not change x but increases t. Hence, AI enhances the net-crowding-in effect for the middle-skill households.

For the high-skill households, $t = 1$, AI increases x and therefore saving via the conventional channels.

Full-time training:

3.5 Limitations of the two-period model

Up to this point, our analysis has focused on how AI influences household-level decisions regarding human capital investment, labor supply, and saving within the framework of a two-period model. While this provides valuable insights into individual behavioral responses, understanding the broader, economy-wide implications of AI requires moving to a more comprehensive setting – a quantitative model with an infinite time horizon, endogenous asset accumulation, and general equilibrium feedback.

General equilibrium (GE) effects When households adjust their investment in human capital, labor supply, and savings in response to AI, these changes aggregate up to affect the total supply of effective labor and capital in the economy. As these aggregates shift, they exert downward or upward pressure on the wage rate and the interest rate, feeding back into each household's optimization problem. Thus, general equilibrium effects capture the intricate loop by which individual decisions shape, and are shaped by, the macroeconomic environment.

Composition effects Endogenizing human capital investment injects dynamism into how households sort themselves among the three skill sectors. When an AI shock occurs, individuals may choose to retrain, upskill, or even move to lower-skilled work, reshaping the distribution of labor across sectors. This shifting composition changes the relative size of each sector, with significant consequences for both aggregate outcomes and the distributional effects of AI.

4 A Quantitative Model

We now solve the full dynamic model with infinite horizon, endogenous asset accumulation, and general equilibrium. We calibrate the model to reflect key features of the U.S. economy, capturing reasonable household heterogeneity.

4.1 Calibration

We calibrate the model to match the U.S. economy. For several preference parameters, we adopt values commonly used in the literature. Other parameters are calibrated to align with targeted moments. The model operates on an annual time period. Table I summarizes the parameter values used in the benchmark model.

Table I: Parameters for the Calibration

Parameter	Value	Description	Target or Reference
β	0.91795	Time discount factor	Annual interest rate
ρ_z	0.94	Persistence of z shocks	See text
σ_z	0.287	Standard deviation of z shocks	Earnings Gini
$\underline{a}$	0	Borrowing limit	See text
χ_n	2.47	Disutility from working	Employment rate
χ_e	1.48	Disutility from HC effort	See text
$\overline{n}$	1/3	Hours worked	Average hours worked
e_H	1/3	High level of effort	Average hours worked
e_L	1/6	Low level of effort	See text
h_M	0.41	Human capital cutoff for M	See text
h_H	0.96	Human capital cutoff for H	See text
λ	0.2	Skill premium	Income Gini
α	0.36	Capital income share	Standard value
δ	0.1	Capital depreciation rate	Standard value

The time discount factor, β, is calibrated to match an annual interest rate of 4 percent. We set χ_n to replicate an 80 percent employment rate. We calibrate χ_e to match the fact that around 30 percent of the population invests in human capital. The borrowing limit, $\underline{a}$, is set to 0.

We calibrate parameters regarding labor productivity process as follows. We assume that x follows the AR(1) process in logs: $\log z' = \rho_z \log z + \epsilon_z$, where $\epsilon_z \sim N(0, \sigma_z^2)$. The shock process is discretized using the Tauchen (1986) method, resulting in a transition probability matrix with 9 grids. The persistence parameter $\rho_z = 0.94$ is chosen based on estimates from the literature. The standard deviation σ_z, is chosen to match the earnings Gini coefficient of 0.63.

We deviate from the two-period model by assuming that the labor supply is a discrete choice between 0 and $\overline{n} = 1/3$. This change only rescales the two-period model without altering the trade-off facing the households. But such rescaling facilitates the interpretation that households are deciding whether to allocate one-third of their fixed time endowment to work. The high-level human capital accumulation effort, e_H is assumed to equal $\overline{n}$. The low-level effort, e_L is set to half of e_H. The skill premium across sectors, λ, is set at 0.2 to match the income Gini coefficient. Human capital cutoffs, h_M and h_H, are set so that the population shares in low, middle, and high sectors are, respectively, 20, 40, and 40 percent. This population distribution roughly matches the fractions of U.S. workers in 2014 who are employed in routine manual occupations (low sector), routine cognitive and non-routine manual (middle sector), and non-routine cognitive (high sector) (Cortes *et al.*, 2017).

On the production side, we set the capital income share, α, to 0.36, and the depreciation rate, δ, to 0.1.

Table II: Key Moments

Moment	Data	Model
Employment rate	0.80	0.80
Human capital investment ratio	0.29	0.29
Gini coefficient for wealth	0.78	0.76
Gini coefficient for earnings	0.63	0.62
Gini coefficient for income	0.57	0.58

4.2 Key Moments: Data vs. Model

In Table II, we present a comparison of key moments between the model and the empirical data. The model does an excellent job of replicating the 80% employment rate observed in the data. In this context, employment is defined as having positive labor income in the given year, consistent with the common approach used in the literature. According to OECD (1998), the share of the population investing in human capital—those who are actively engaged in skill acquisition or education—is approximately 30%, a figure well matched by the model's predictions. This is an important metric because it reflects the model's capacity to capture the dynamics of human capital formation, which plays a critical role in shaping long-run earnings and income inequality. Additionally, the model accurately captures the distribution of income and earnings, aligning closely with observed data. This suggests that the model effectively incorporates the key mechanisms driving labor market outcomes and the corresponding distributional aspects of earnings. Although the model does not explicitly target the wealth Gini coefficient, it achieves a close match to the data: the empirical wealth Gini is 0.78, while the model produces a value of 0.76. This highlights the model's ability to capture substantial wealth inequality in the economy.

4.3 Steady-state Distribution

Table III presents the steady-state distribution of population, employment, and assets across sectors. The population shares are calibrated to 20%, 40%, and 40% by adjusting the human capital thresholds that define sectors. The shares of employment and assets are endogenously determined by households' labor supply and savings decisions. Notably, the high sector accounts for 46% of total employment—exceeding its population share—indicating that a disproportionate number of households choose to work in that sector. Asset holdings are even more skewed: the high sector holds 68% of total assets, while the low sector holds only 8%.

Figure 3: Steady-state Human Capital Distribution

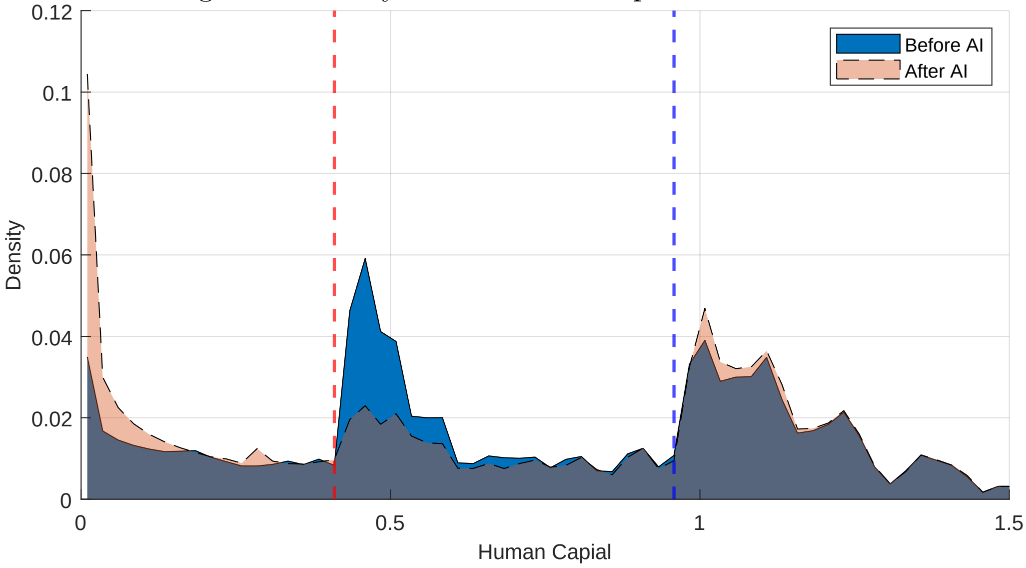


Figure 4: Steady-state Human Capital Investment

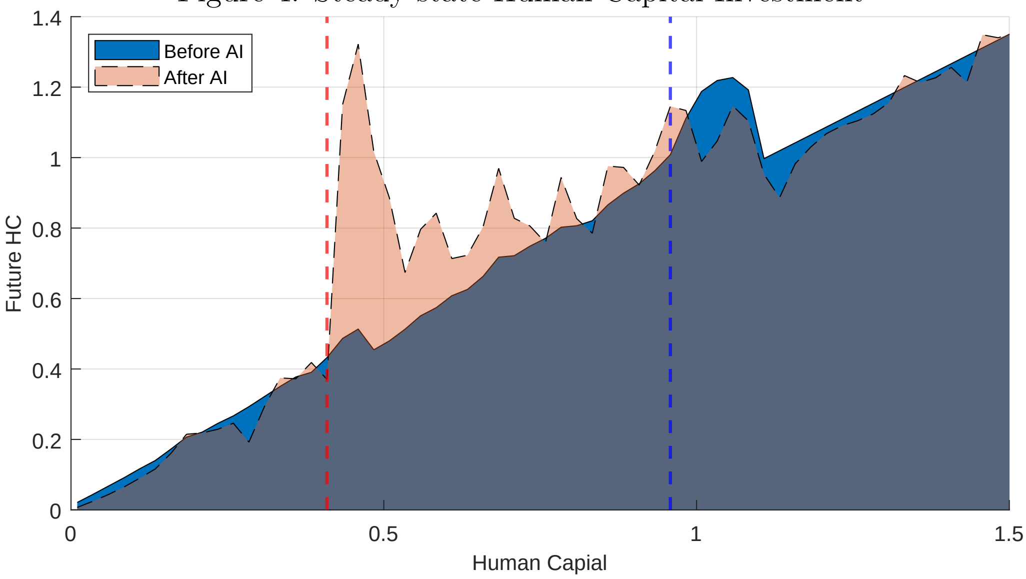


Figure 5: Transition Path for Human Capital Investment

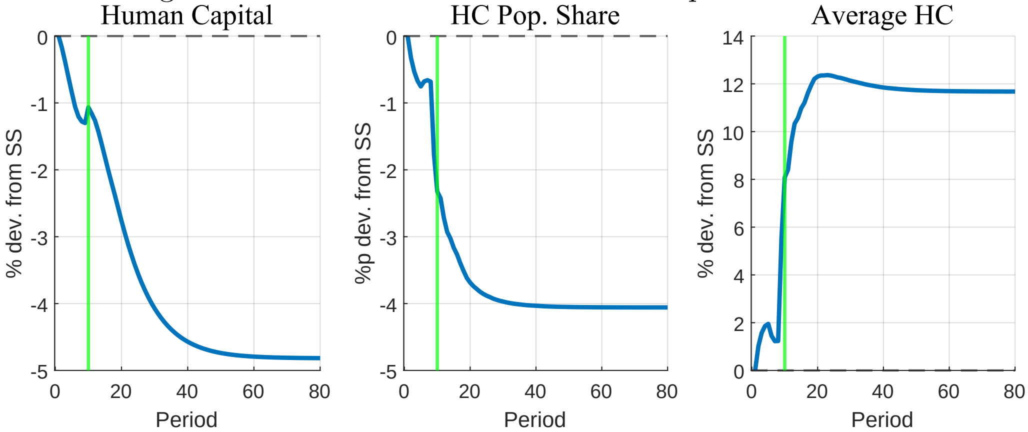


Table III: Distribution of Population, Employment and Assets

Sectors	Pop. Share (%)	Emp. Share (%)	Assets Share (%)
Low	20.76	18.58	8.07
Middle	38.87	35.35	23.92
High	40.35	46.07	68.01

Note: Human capital cutoffs, h_H and h_M, determine the population share across sectors. Employment share and assets share are implied by households labor supply decisions and saving decisions.

5 AI's Impact on Human Capital Adjustments

We now introduce AI technology into the quantitative model, assuming that it will be implemented in 10 years and that households have full information about its arrival. We examine both the transition dynamics and the differences between the initial and new steady states. This framework allows us to analyze how the economy adjusts in anticipation of, and in response to, the adoption of AI.

The effect of AI on the sectorial productivity is modeled as in (41) with $\gamma = 0.3$. That is, AI boosted the productivity of the low sector workers by 7.5% and the productivity of the high sector workers by 5%, leaving the middle sector intact. It captures the key idea that AI increases average labor productivity (Acemoglu and Restrepo, 2019), but reduces the earning premium for the middle sector, and enlarges the earning premium for the higher sector relative the middle sector.

5.1 *Human Capital Adjustments*

Given the employment distribution in the initial steady state, AI is projected to increase the economy's labor productivity by 4% on average, assuming households do not alter their decisions in response. However, changes in earning premiums incentivize households to adjust their human capital investments.

Steady-state human capital distribution: Figure 3 illustrates how households reallocate across sectors in the new steady state relative to the initial one. The x-axis denotes the level of human capital, while the y-axis indicates the mass of households at each human capital level. The red vertical line marks the cutoff between the low and middle sectors, and the blue vertical line marks the cutoff between the middle and high sectors.

The gray shaded area shows the overlap between the two steady-state distributions. Within each sector, the distribution of households is skewed to the left, reflecting the tendency for human capital investment to be concentrated among those near the sectoral cutoffs. As shown in the decision rule diagram in Figure 2, some households seek to upgrade their skills, while others aim to remain in more skilled sectors. The blue shaded area highlights the mass of households who have exited the middle sector following the AI shock. The pink areas represent the additional mass of households in the new steady-state distribution, concentrated at the lower end of the low sector and the lower end of the high sector.

Steady-state human capital investment: This reallocation pattern reflects shifts in human capital investment incentives driven by AI's impact on the skill premium. Figure 4 plots human capital investment decisions in the initial and new steady states across different human capital levels. Because both the productivity shock (z) and current asset holdings (a) influence human capital investment, the y-axis shows the weighted average of next-period human capital, where the weights reflect the steady-state distribution of households by productivity shock and wealth at each human capital level.

The changes in decision rules before and after the AI shock are highlighted in the blue shaded area, where next-period human capital in the new steady state is lower than in the initial steady state, and in the pick shaded area, where it is higher. The most notable change is that the middle-sector households substantially intensify their human capital investment, aiming to transition into high-sector roles. In contrast, households in the low sector reduce their human capital investment, causing those who might have moved up to the middle sector to remain in the low sector or even drift further down to the very bottom of human capital distribution as shown in Figure 3.

Somewhat surprisingly, most high-sector workers in the new steady state decrease their human capital investment relative to the initial steady state. This is primarily a composition effect: as more households move from the middle-sector to the high sectors, the average asset holdings among high-sector households decline, making intensive human capital investment less affordable [note that this is not supported by the average asset in transition dynamics figure 9].

Transition path Figure 5 reports the transition dynamics of aggregate human capital from the initial to the new steady state. The figure also displays its extensive margin (the share of households making positive human capital investments) and intensive margin (average human capital per household among those who invest).

As households reallocate from the middle sector to the low and high sectors, the net effect is a gradual decline in aggregate human capital along the transition path. This mirrors the steady-state change observed in Figure 3, where the increased mass at the lower end of the low sector outweighs the increase in the high sector.

Additionally, human capital accumulation becomes increasingly concentrated among a smaller share of the population. The proportion of households making positive human capital investments steadily declines, ultimately stabilizing at a level 4% lower than in the initial steady state. Meanwhile, the average human capital among those who invest rises, reaching a level 12% higher than the initial steady state in the long run.[14]

5.2 Job Polarization

An important implication of human capital adjustments to the AI shock is job polarization. Figure 6 illustrate the transition paths of population shares and employment rates in each sector. Notably, the middle sector experiences a significant decline, with its population share decreasing by approximately 13%. Additionally, employment within this sector plummets to a level 16% lower than the initial steady state. In contrast, both the low and high sectors see increases in their population shares and employment rates. These dynamics indicate a reallocation of *workers* from the middle sector to the low and high sectors following the introduction of AI.

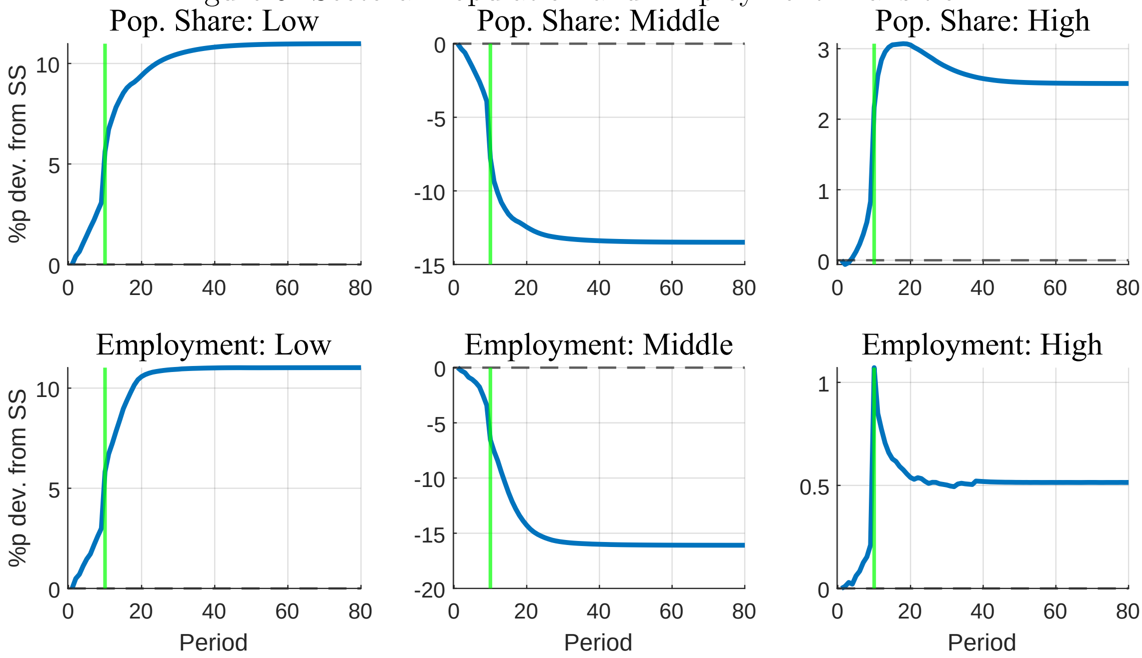
Voluntary job polarization This worker reallocation aligns with the phenomenon of "job polarization"(Goos *et al.*, 2014), where AI and automation technologies disproportionately replace tasks commonly performed by middle-skilled workers. However, our model introduces a complementary mechanism to the conventional understanding of this reallocation. Specifically, households in our model voluntarily exit the middle sector even before AI implementation by adjusting their human capital investments – many middle-sector workers opt for non-employment to invest in skills that will better position them for the post-AI labor market. To emphasize this key difference, our model deliberately abstracts from any direct negative effect of AI on middle-sector workers.

Employment flows more towards the low sector Another intriguing finding in our model is the more pronounced employment effect in the low sector compared to the high sector. In the new steady state, the employment rate in the low sector increases by 12%, whereas in the high sector, it rises by only 0.5%. This asymmetry in employment rate changes suggests an unbalanced reallocation of workers from the middle sector, with a greater flow toward the low sector.

This disparity arises from two key factors. First, AI enhances the productivity of low-sector workers by 7.5% and high-sector workers by 5%. However, this productivity differential alone does not fully account for the significant asymmetry. The second factor is the variation in labor supply elasticity across sectors. Compared to the high sector, the low sector exhibits higher labor supply elasticity, meaning that the same change in labor earnings triggers larger labor supply responses. This is because households in the low sector have lower consumption levels, making their marginal utility of consumption more sensitive to changes in their budget. Consequently, a greater proportion of households in the low sector are at the margin

[14] The only exception to those patterns occurs at period 10 when the positive effects of AI on sectoral productivity are realized.

Figure 6: Sectoral Population and Employment Transition



Note: The transition paths within each sector. The x-axis represents years, and the y-axis shows the percentage (or percentage point) deviation from the initial steady state. AI introduction is assumed to occur in period 10. "Pop. Share" denotes the population share within each sector. "Employment" is the percentage of households who are employed in each sector.

between employment and non-employment (Chang and Kim, 2006).

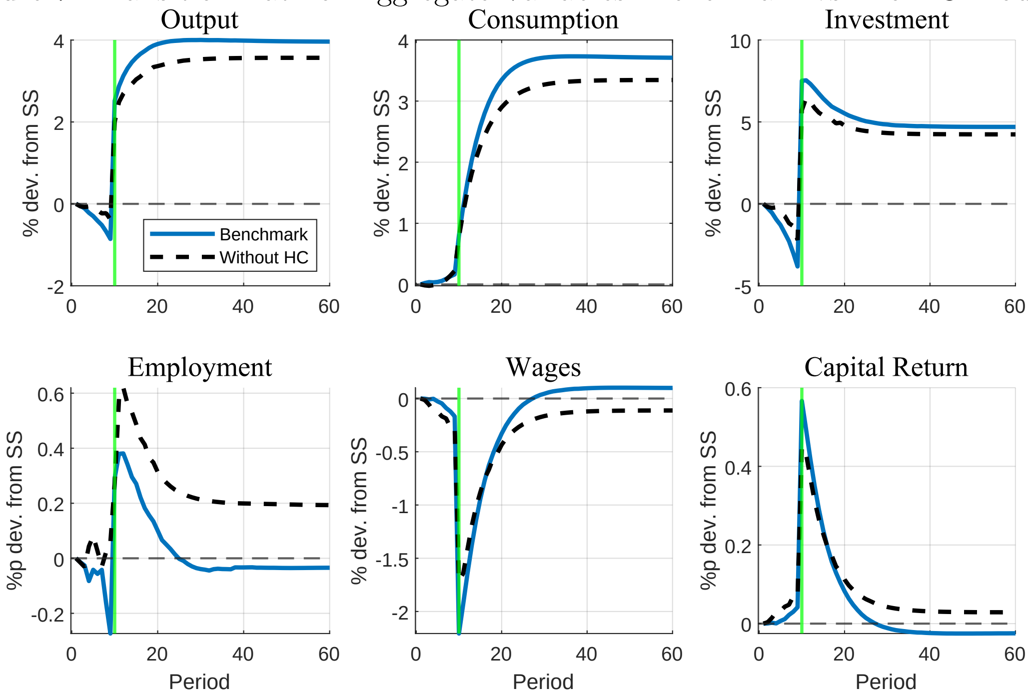
6 The Aggregate and Distributional Effects of AI

The aggregate and distributional effects of AI are shaped by both its direct impact on sectoral productivity and the endogenous response of human capital accumulation. By altering sectoral productivity, AI changes labor earnings, which in turn influences labor supply decisions and savings through income effects. Consequently, AI directly affects the supply of labor and capital, generating aggregate economic responses. Because AI's productivity effects are heterogeneous across sectors, its impact is inherently distributional.

These sectoral differences also induce human capital adjustments, as households reallocate across sectors in response to changing incentives. This reallocation not only shifts the distribution of labor productivity and aggregate productivity, but also directly shapes distributional outcomes, as households' relative positions in the income and asset distributions are altered by their movement across sectors.

In this section, we examine the importance of endogenous human capital adjustment in shaping both the transitional and long-run effects of AI. To do so, we compare the benchmark economy – where households endogenously adjust their human capital – with an alternative scenario in which households are held fixed at their initial steady-state human capital during the AI transition ("No HC model"). In both cases, households make endogenous decisions about consumption, savings,

Figure 7: Transition Path of Aggregate Variables: Benchmark vs. No HC Models.



Note: The transition paths of aggregate variables: benchmark vs. No HC models. The x-axis represents years, and the y-axis shows the percentage deviation from the initial steady state. AI introduction is assumed to occur in period 10. The No HC model is an economy in which workers maintain their initial steady-state level of human capital throughout the AI implementation until the new steady state is reached.

and labor supply.

By contrasting the transition dynamics across these two economies, we can disentangle the direct and indirect effects of AI. The transition path in the No-HC-model isolates the direct impact of AI on aggregate and distributional outcomes, as it abstracts from any human capital adjustments. The difference in outcomes between the benchmark and the No-HC-model then reveals the indirect effects of AI that operate through households' adjustments in human capital. This decomposition allows us to assess the relative importance of human capital dynamics in driving both the aggregate and distributional consequences of AI.

6.1 Aggregate Implications

Figure 7 shows the transition paths of key macroeconomic variables—output, consumption, investment, and employment—as well as factor prices, including the wage rate and capital return. The blue solid lines depict results from the benchmark model with endogenous human capital adjustment, while the black dashed lines represent the No-HC model in which human capital is held fixed.

6.1.1 AI's direct impacts

The No-HC-model isolates the direct effects of AI. In the long run, the introduction of AI leads to higher output, consumption, investment, and employment. However, in anticipation of AI (prior to period 10), output and investment decline, while consumption and employment remain stable.

Before the implementation of AI, sectoral productivity is unchanged; the only difference is households' awareness of future increases in productivity in the low and high sectors beginning in period 10. This anticipation raises households' expected lifetime income, prompting them to save less and consume more ahead of the actual productivity gains. As a result, aggregate capital stock falls, which lowers output and reduces the marginal product of labor while raising the marginal product of capital. Employment remains largely unchanged in this period, as sectoral productivity has not yet shifted.

Following the AI shock, sectoral productivity in the low and high sectors rises, boosting labor income, employment, and output in these sectors. Because productivity gains are labor-augmenting, the supply of efficient labor units rises sharply, causing wages to decline and capital returns to increase. Employment and investment both adjust to dampen these factor price changes. In the new steady state, the wage rate is slightly below its initial level, while the return to capital is marginally higher.

6.1.2 AI's indirect impacts via endogenous human capital adjustments

The difference between the No-HC model and the benchmark model captures the indirect effects of AI operating through endogenous human capital adjustments. Among all macroeconomic variables, this indirect effect is most pronounced for employment.

In anticipation of AI, employment declines as some households temporarily exit the labor market to invest in human capital and prepare for the post-AI economy.[15] During this period, labor productivity remains unchanged, so the decline in employment directly translates to a reduction in output. Consistent with standard consumption-smoothing behavior, this reduction is mainly absorbed by lower investment. Meanwhile, the drop in employment mitigates the direct effects of AI on both wages and capital returns prior to the AI implementation.

After AI is introduced, employment rebounds as sectoral productivity increases. However, continued human capital investment by middle-sector households keeps employment lower than in the No-HC model, resulting in an almost neutral long-run effect of AI on employment. Despite this, output, consumption, and investment are all higher in the benchmark model because human capital adjustments reallocate more labor to the low and high sectors, thereby better capturing the productivity gains from AI.

This reallocation also reverses the steady-state comparison of factor prices: endogenous human capital adjustment transforms the negative direct effect of AI on

[15] Empirical studies, such as Lerch (2021) and Faber *et al.*, (2022), support the short-term adverse effects of AI adoption on labor markets.

the wage rate into a positive net effect, and the positive direct effect on capital returns into a negative net effect.

6.2 Distributional Implications

The findings above underscore the importance of accounting for human capital adjustments when assessing the aggregate impact of AI, as households actively adapt to a rapidly evolving labor market. When it comes to economic inequality, endogenously adjusting human capital plays an even more significant role.

Figure 8 shows the transition paths of Gini coefficients for earnings (labor income), total income (capital and labor income), consumption, wealth (asset holdings), and human capital. The black dashed lines represent results from the No-HC model, capturing the direct impact of AI without human capital adjustment. In contrast, the blue solid lines reflect the benchmark model, where human capital responds endogenously to both anticipated and realized changes in the skill premium induced by AI.

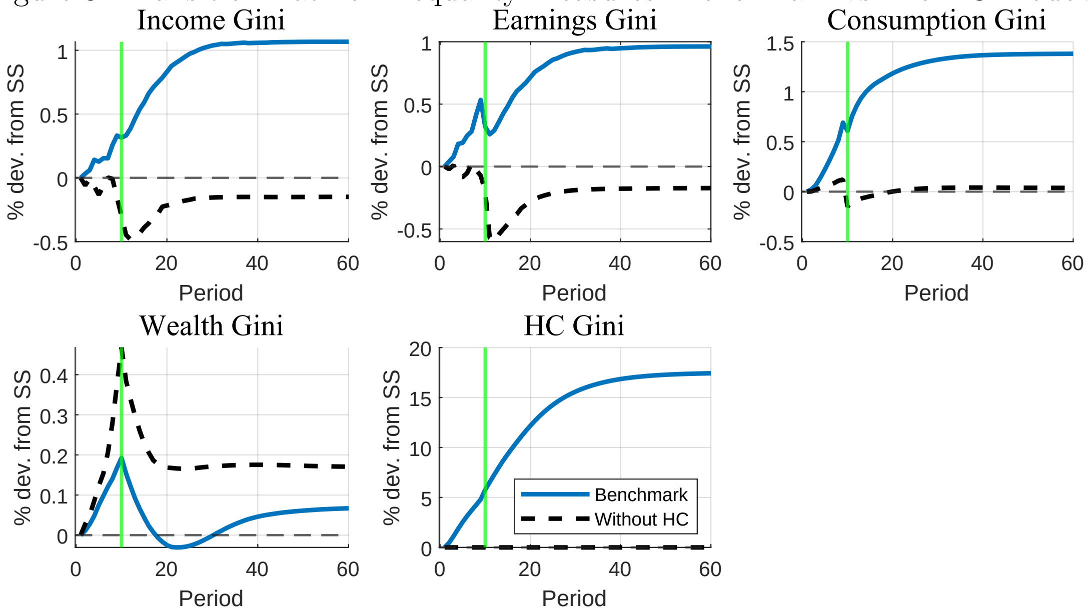
6.2.1 Income, earnings, and consumption inequalities

The comparison of transition paths between the No-HC model and the benchmark model reveals that endogenous human capital adjustments fundamentally alter the impact of AI on income, earnings, and consumption inequalities.

AI's direct impacts: Without any human capital adjustments, AI's impact on inequalities is primarily driven by productivity gains in the low and high sectors – 7.5% and 5%, respectively. As a result, there is little direct impact on income and earnings Gini coefficients in anticipation of AI before period 10. After AI is implemented, both income and earnings inequality decline: higher labor productivity raises earnings in the low sector, while wage declines in the middle sector compress the distribution. Consumption inequality remains largely unchanged throughout the transition.

Effects of AI-induced human capital adjustments: Allowing human capital to adjust endogenously, however, leads to pronounced job polarization, as shown in Section 5.2. Households who would have qualified for middle-sector jobs now transition to either the low or high sector. Those moving to the low sector see reduced labor earnings, while those shifting to the high sector enjoy increased earnings. This polarization drives up earnings and income inequality, both before and after AI is implemented. As income disparities widen, consumption inequality also increases.

Figure 8: Transition Path of Inequality Measures: Benchmark vs. No HC Models.



Note: The transition paths of inequality measures: benchmark vs. No HC models. The x-axis represents years, and the y-axis shows the percentage deviation from the initial steady state. AI introduction is assumed to occur in period 10. The No HC model is an economy in which workers maintain their initial steady-state level of human capital throughout the AI implementation until the new steady state is reached.

6.2.2 Wealth inequality

In stark contrast to the effects on income and earnings inequality, allowing for endogenous human capital adjustment actually mitigates the negative direct impact of AI on wealth inequality. While AI's direct effect would otherwise widen disparities, human capital responses help dampen the increase in wealth inequality, underscoring the stabilizing role of human capital adjustments in the wealth distribution.

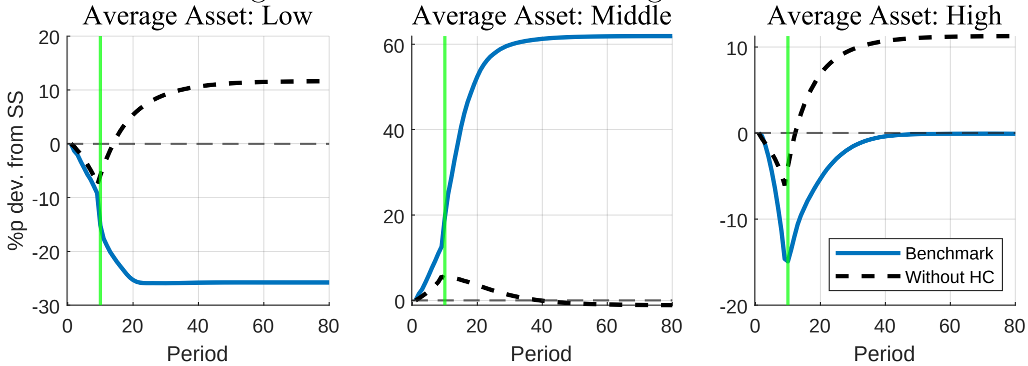
To disentangle the direct and indirect effects of AI on wealth inequality, Figure 9 presents the sectoral transition paths for asset holdings, while Figure 10 compares steady-state asset investment decisions across different human capital levels.

[Add a figure that compares the steady-state asset investment in the No-HC-model (a counterpart of Figure 10).]

AI's direct impacts: We first focus on the black dashed lines in Figure 9. Without households reallocation across sectors, total assets and average asset holdings follow similar patterns. In both the low and high sectors, households reduce their savings in anticipation of AI, expecting higher lifetime labor income. After AI is implemented at period 10, their savings increase alongside rising labor incomes. In contrast, households in the middle sector, anticipating a negative income effect from AI due to a lower wage rate, increase their savings prior to period 10. Once AI is introduced and the wage rate recovers, middle-sector households reduce their savings.

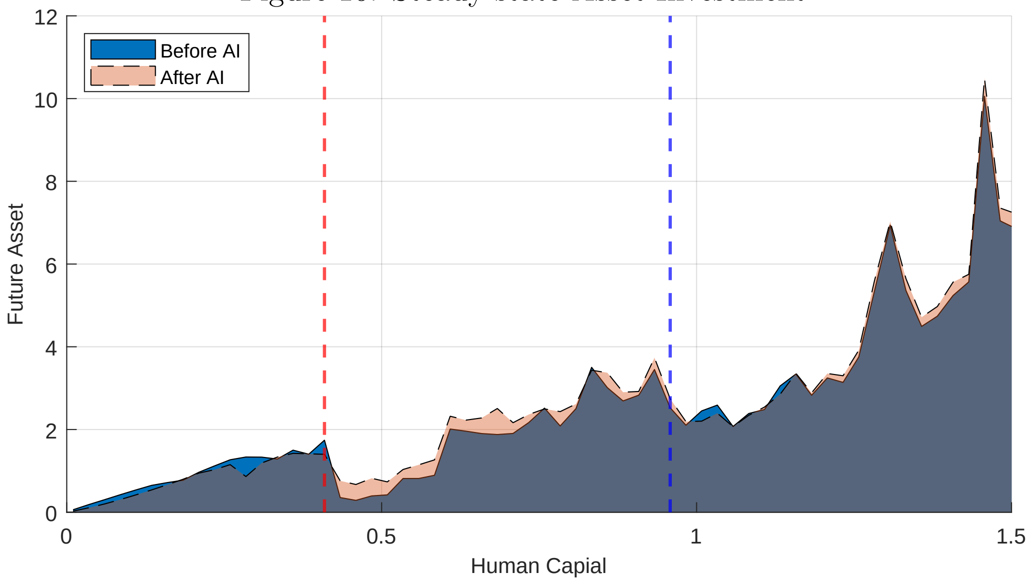
These shifts in sectoral saving patterns sharply increase wealth inequality before period 10, as low-sector households – typically the least wealthy – reduce their asset

Figure 9: Sectoral Asset-holding Transition



Note: The transition paths of average capital within each sector. The x-axis represents years, and the y-axis shows the percentage deviation from the initial steady state. AI introduction is assumed to occur in period 10. "Average Capital" denotes the physical assets per household in each sector.

Figure 10: Steady-state Asset Investment



holdings. After AI is implemented and saving rates in the low sector recover, the wealth Gini coefficient declines from its peak and stabilizes at a level about 0.2% higher than its initial steady state.

Effects of AI-induced human capital adjustments: Average asset holding isolates us from movements in the population share along the transition path.

1. Selection effect is dominant: From middle to low: low productivity and middle-sector level wealth. Due to higher wealth level than the low-sector, the influx should have increased the arrearage asset holding of the low sector, but because they are low productivity households and they experience a reduction of sectoral productivity. [But we still should have seen an increase in Average asset before period 10???]

From middle to high: high productivity and middle-sector level wealth. Due to lower wealth level than the high-sector, the influx of middle-sector households reduces the average asset holding of the high sector. But since they are high-productivity households, their saving rate increases.

2. Precautionary saving motive changes: For the low sector, the reduction of skill premium in the benchmark model implies a reduction in idiosyncratic risk, so households reduce saving. For the high sector, the opposite is true. In the No-HC-model, changes in skill premium does not affect idiosyncratic risk since households cannot change sector.

Allowing for endogenous human capital adjustment results in time-varying population shares across sectors along the transition path, which drives the divergence between sectoral total and average asset holdings. In both the low and high sectors, although the average household's asset holding declines substantially, the total asset holding in the low sector remains relatively stable, and in the high sector even increases, due to the influx of households from the middle sector. Conversely, while the average household in the middle sector saves more, the total asset holding in the middle sector declines as its population share shrinks. These offsetting effects between sectoral average asset holdings and shifting population shares help dampen fluctuations in the wealth Gini coefficient along the transition path, compared to the No-HC model (see Figure 8).

I cannot explain why the wealth gini in the benchmark model is lower than in the No-HC-model, since from the total asset graphs, benchmark model has more total assets in the higher sector in new steady state. So we have to turn to the comparison of asset holding decision rule.

Steady-state change in asset investment: To explain the contrasting sectoral changes in average asset holdings between the benchmark model and the No-HC-model in the new steady state, Figure 10 shows how next-period asset holdings change from the initial to the new steady state at each human capital level in the benchmark model, while Figure XXX presents the corresponding results for the No-HC-model. As in Figure 4, the y-axis displays the weighted average of next-period asset holdings, with weights reflecting the steady-state distribution of households by productivity shocks (z) and wealth (a) at each human capital level. Pink shaded areas indicate an increase in next-period asset holdings, while blue shaded areas indicate a decrease.

Note that in the benchmark model, the pink shaded areas are mostly located in the middle sector. This is due to a "selection effect" since the households who stays in the middle sector in the new steady after the AI shock are those with higher productivity than those in the initial steady state. It is because those with lower productivity would have already flow in the low sector. As productivity is positively correlated with wealth, households remaining in the middle sector in the new steady state tends to have more wealth, which boosts their saving. I cannot explain why the high-sector average asset-holding remains unchanged in the new steady state whereas the asset investment figure shows that the next-period asset holding is reduced in the high sector.

Reduction in saving in the low sector, because of the influx of low-productivity households from the middle sector? High sector, it is a mix so that average asset holding remains the same as the initial steady state. in the benchmark, in the initial steady state, the middle sector's idiosyncratic productivity on average is lower than the high sector households (that is the why they stay in the middle sector that has requires lower human capital investment. Therefore, those moving to the high sector has on average lower z and lower a. That explains why there is a reduction of asset investment in the low end to high sector in the new steady state as the result of more mover from the middle sector. Income effects are still present for the higher end of high sector, which acts as a counterforce to the reduction of average asset holding in the low end.

7 Conclusion

Recent studies on AI suggest that advancements are likely to reduce demand for junior-level positions in high-skill industries while increasing the need for roles focused on advanced decision-making and AI oversight. We demonstrate how human capital investments are expected to adapt in response to these shifts in skill demand, highlighting the importance of accounting for these human capital responses when assessing AI's economic impact.

Our work points to several promising directions for future research on the economic impacts of AI. First, while general equilibrium effects—such as wage and capital return adjustments—have a limited role in our model, further research could examine how these effects might vary under different economic conditions or policy environments. Second, if governments implement redistribution policies to address

AI-induced inequality, understanding how these policies influence human capital accumulation, and thus their effectiveness, would be valuable. Finally, our model assumes households have perfect foresight when making human capital investments. Relaxing this assumption could reveal new insights into the economic trajectory of AI advancements and offer important policy implications.

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A Household Decision Rule Cutoffs

A.1 Cutoffs formulae for households with $h_M\frac{1}{1-\delta} \leq h < h_H\frac{1}{1-\delta}$

$$\overline{z}^M_{non}(a) := \frac{(\exp(\frac{\chi_n}{1+\beta}) - 1)[(1+r)a + \frac{w'z'}{1+r'}]}{w} \tag{A.1}$$

$$\overline{z}^M_{slow}(a) := \frac{(\exp(\frac{\chi_n - \chi_e e_H}{1+\beta}) - 1)[(1+r)a + \frac{w'z'(1+\lambda)}{1+r'}] + \lambda\frac{w'z'}{1+r'}}{w} \tag{A.2}$$

$$\underline{z}^M_{fast}(a) := \frac{(\exp(\frac{\chi_n}{1+\beta}) - 1)[(1+r)a + \frac{w'z'(1+\lambda)}{1+r'}]}{w} \tag{A.3}$$

$$\overline{z}^M_{fast}(a) := \frac{\left\{\lambda\left[\exp(\frac{\chi_e e_L}{1+\beta}) - 1\right]^{-1} - 1\right\}\frac{w'z'}{1+r'} - (1+r)a}{w} \tag{A.4}$$

A.2 Parameter restrictions for cutoffs ranking

To guarantee that $(n=0, e=e_H)$ dominates $(n=0, e=0)$, we need a lower bound for λ. The slow learners prefer $(n=0, e=e_H)$ if and only if

$$(1+\beta)\ln c(n=0, e=e_H) - \chi_e e_H \geq (1+\beta)\ln c(n=0, e=0)$$

or equivalently:

$$\lambda \geq \underline{\lambda}_1 := \frac{(1+r)a + \frac{w'z'}{1+r'}}{\frac{w'z'}{1+r'}} \left(1 - \frac{1}{\exp(\frac{\chi_e e_H}{1+\beta})}\right) \text{ if } h < h_M \frac{1}{1-\delta} \tag{A.5}$$

$$\lambda \geq \underline{\lambda}_3 := \frac{(1+r)a + \frac{w'z'}{1+r'}}{\frac{w'z'}{1+r'}} \left(\exp(\frac{\chi_e e_H}{1+\beta}) - 1\right) \text{ if } h \geq h_M \frac{1}{1-\delta} \tag{A.6}$$

To avoid $(n = 1, e = e_L)$ from being a dominated choice, we need another lower bound for λ. To see it, recall that $(n = 1, e = 0)$ is better than $(n = 1, e = e_L)$ if $z > \overline{z}_{fast}$, and $(n = 1, e = e_L)$ is better than $(n = 0, e = e_L)$ if $z > \underline{z}_{fast}$. $(n = 1, e = e_L)$ is therefore the best choice over the interval $(\underline{z}_{fast}, \overline{z}_{fast})$. For such an interval to exist, it must be the case that when $z = \underline{z}_{fast}$, $z < \overline{z}_{fast}$. $z = \underline{z}_{fast}$ means that the fast learners are indifferent between $(n = 1, e = e_L)$ and $(n = 0, e = e_L)$ so that

$$(1+r)a + wzx(h) + \frac{w'z'}{1+r'} = \exp(\frac{\chi_n}{1+\beta}) \left[(1+r)a + \frac{w'z'}{1+r'}\right] \text{ if } h < h_M \frac{1}{1-\delta} \tag{A.7}$$

$$(1+r)a + wzx(h) + \frac{w'z'(1+\lambda)}{1+r'} = \exp(\frac{\chi_n}{1+\beta}) \left[(1+r)a + \frac{w'z'(1+\lambda)}{1+r'}\right] \text{ if } h \geq h_M \frac{1}{1-\delta} \tag{A.8}$$

For the fast learners to prefer $(n = 1, e = e_L)$ over $(n = 1, e = 0)$, we need

$$(1+\beta) \ln \frac{c(n = 1, e = e_L)}{c(n = 1, e = 0)} \geq \chi_e e_L \tag{A.9}$$

If $h < h_M \frac{1}{1-\delta}$, inequality (A.9) is:

$$(1+\beta) \ln \frac{(1+r)a + wzx(h) + \frac{w'z'}{1+r'}}{(1+r)a + wzx(h) + \frac{w'z'(1-\lambda)}{1+r'}} \geq \chi_e e_L$$

Evaluating the left-hand-side at $z = \underline{z}_{fast}$ yields:

$$\lambda \geq \underline{\lambda}_2 := \frac{(1+r)a + \frac{w'z'}{1+r'}}{\frac{w'z'}{1+r'}} \left(1 - \frac{1}{\exp(\frac{\chi_e e_L}{1+\beta})}\right) \exp(\frac{\chi_n}{1+\beta}) \tag{A.10}$$

If $h > h_M \frac{1}{1-\delta}$, inequality (A.9) is:

$$(1+\beta) \ln \frac{(1+r)a + wzx(h) + \frac{w'z'(1+\lambda)}{1+r'}}{(1+r)a + wzx(h) + \frac{w'z'}{1+r'}} \geq \chi_e e_L$$

Evaluating the left-hand-side at $z = \underline{z}_{fast}$ yields:

$$\lambda \geq \underline{\lambda}_4 := \frac{(1+r)a + \frac{w'z'}{1+r'}}{\frac{w'z'}{1+r'}} \frac{\left(\exp(\frac{\chi_e e_L}{1+\beta}) - 1\right)\exp(\frac{\chi_n}{1+\beta})}{\exp(\frac{\chi_e e_L}{1+\beta}) + \exp(\frac{\chi_n}{1+\beta}) - \exp(\frac{\chi_e e_L + \chi_n}{1+\beta})} \tag{A.11}$$

We have that $\underline{\lambda}_1 > \underline{\lambda}_2$ and $\underline{\lambda}_3 > \underline{\lambda}_4$ if

$$\exp(\frac{\chi_e e_H}{1+\beta}) > \frac{\exp(\frac{\chi_e e_L}{1+\beta})}{\exp(\frac{\chi_e e_L}{1+\beta}) + \exp(\frac{\chi_n}{1+\beta}) - \exp(\frac{\chi_e e_L + \chi_n}{1+\beta})} \tag{A.12}$$

Therefore, the inequality above implies that the conditions (A.5) and (A.6) are sufficient for the conditions (A.10) and (A.11). Furthermore, $\lambda_3 \geq \lambda_1$ so that the condition (A.6) is sufficient for the condition (A.5).

We can then conclude that the conditions (A.6) and (A.12) are sufficient for 1) the slower learners always prefers $(n = 0, e = e_H)$ over $(n = 0, e = 0)$, and 2) $\overline{z}_{fast} > \underline{z}_{fast}$, i.e., there exists state space where $(n = 1, e = e_L)$ is optimal.

A.3 Other cutoffs ranking for the two-period Model

For the fast learners, their cutoffs rank as follows

$$\frac{\overline{z}^L_{fast}(a)}{1-\lambda} > \overline{z}^L_{fast}(a) > \overline{z}^M_{fast}(a) > \frac{\overline{z}^M_{fast}(a)}{1+\lambda} \tag{A.13}$$

$$\frac{\underline{z}^L_{fast}(a)}{1-\lambda} > \underline{z}^M_{fast}(a) > \underline{z}^L_{fast}(a) > \frac{\underline{z}^M_{fast}(a)}{1+\lambda} \tag{A.14}$$

For the slow learners, the rank of their cutoffs is

$$\frac{\overline{z}^L_{slow}(a)}{1-\lambda} > \overline{z}^M_{slow}(a) > \overline{z}^L_{slow}(a) > \frac{\overline{z}^M_{slow}(a)}{1+\lambda} \tag{A.15}$$

For the non-learners, the rank of their cutoffs is

$$\frac{\overline{z}^L_{non}(a)}{1-\lambda} > \overline{z}^M_{non}(a) > \frac{\overline{z}^H_{non}(a)}{1+\lambda} > \frac{\overline{z}^M_{non}(a)}{1+\lambda} \tag{A.16}$$

$$\overline{z}^M_{non}(a) > \overline{z}^L_{non}(a) \tag{A.17}$$

B Proof of Proposition

B.1 Proof of Proposition 2

The derivative of saving with respect to t is

$$\frac{\partial a'^{\star}}{\partial t}(x, a; t) = -\frac{x\mu}{1+\beta} + \frac{x^2\Sigma}{\beta} \frac{t\left[2(x+a) + tx\mu\right]}{\left[(x+a) + tx\mu\right]^2}. \tag{B.1}$$

The total effect of on-job-training on saving is

$$\Delta_{\text{on-job}}(x,a;t) = a'^{\star}(x,a;t) - a'^{\star}(x,a;1) = \int_1^t \frac{\partial a'^{\star}}{\partial u}(x,a;u)du. \tag{B.2}$$

Define

$$F(x,a;u) \equiv x\,\frac{u[2(x+a)+ux\mu]}{[(x+a)+ux\mu]^2}, \quad \overline{F}(x,a;t) \equiv \frac{1}{t-1}\int_1^t F(x,a;u)du.$$

Then equation (B.2) can be written as

$$\Delta_{\text{on-job}}(x,a;t) = x(t-1)\Big[\frac{\Sigma}{\beta}\overline{F}(x,a;t) - \frac{\mu}{1+\beta}\Big].$$

Differentiating $F(x,a;u)$ with respect to x gives

$$\frac{\partial F(x,a;u)}{\partial x} = \frac{2u\,a\,(a+x)}{(a+(1+u\mu)x)^3} > 0,$$

so $\overline{F}(x,a;t)$ is strictly increasing in x.

The sign of $\Delta_{\text{on-job}}(x,a;t)$ is governed by

$$S(x,a;t) \equiv \frac{\Sigma}{\beta}\overline{F}(x,a;t) - \frac{\mu}{1+\beta}.$$

Because $\overline{F}(x,a;t)$ is strictly increasing, $S(x,a;t)$ increases monotonically with x.

For $x \to 0$, $F(x,a;u) \to 0$ and $\overline{F}(x,a;t) \to 0$ so that $S(x,a;t) \to -\frac{\mu}{1+\beta} < 0$, implying $\Delta_{\text{on-job}}(x,a;t) < 0$ for small x.

For $x \to \infty$, $F(x,a;u) \to \frac{u(2+u\mu)}{(1+u\mu)^2}$ and $\overline{F}(x,a;t) \to \overline{F}_\infty(t) \equiv \frac{1}{t-1}\int_1^t \frac{u(2+u\mu)}{(1+u\mu)^2}du$. If

$$\frac{\Sigma}{\mu} > \underline{\sigma}(t) \equiv \frac{\beta}{1+\beta}\frac{1}{\overline{F}_\infty(t)} \tag{B.3}$$

then $S(x,a;t) > 0$ for sufficiently large x, and hence $\Delta_{\text{on-job}}(x,a;t) > 0$.

If idiosyncratic risk is large enough, i.e., condition (B.3) is satisfied, there exists a unique threshold $x^*(t)$ at which the sign flips:

$$\Delta_{\text{on-job}}(x,a;t) < 0 \text{ for } x < x^*(a,t), \qquad \Delta_{\text{on-job}}(x,a;t) > 0 \text{ for } x > x^*(a,t).$$

B.2 Proof of another proposition

$a'^*(x,a;t)$ is convex in t:

$$\frac{\partial^2 a'^*(x,a;t)}{\partial t^2} = \frac{\partial}{\partial t}\left(-\frac{x\mu}{1+\beta} + \frac{x^2\Sigma}{\beta}\,\frac{t\,[2(x+a)+tx\mu]}{\big[(x+a)+tx\mu\big]^2}\right) = \frac{2x^2\Sigma(a+x)^2}{\beta(a+x+tx\mu)^3} > 0. \tag{B.4}$$

Increasing t by Δt changes the total effect of on-job-training on saving by

$$\int_t^{t+\Delta t} \frac{\partial a'^{\star}}{\partial u}(x, a; u)du. \tag{B.5}$$

B.3 Proof of Proposition 3

Denote

$$G(x, a; t) \equiv \frac{t^2x^2}{(a + x + tx\mu)(a + tx\mu)}$$

the net additional effect of full-time training on saving can be rewritten as

$$\Delta_H(x, a; t) \equiv x\big[-\frac{\beta}{1+\beta} + \frac{\Sigma}{\beta}G(x, a; t)\big]$$

Differentiating $G(x, a; t)$ with respect to x gives

$$\frac{\partial G(x, a; t)}{\partial x} = \frac{t^2xa(2tx\mu + 2a + x)}{(a + tx\mu)^2(a + x + tx\mu)^2} > 0,$$

so $G(x, a; t)$ is strictly increasing in x.

The limits of $G(x, a; t)$ are:

$$G(x, a; t) \to 0 \quad (x \to 0),$$

$$G(x, a; t) \to G_\infty(t) \equiv \frac{t}{\mu(1 + t\mu)} \quad (x \to \infty),$$

Therefore, $G(x, a; t) < G_\infty(t)$ for any x.

If

$$\frac{\Sigma}{\beta}G_\infty(t) < \frac{\beta}{1+\beta}, \text{ i.e. } \frac{\Sigma}{\mu} < \overline{\sigma}(t) \equiv \frac{\beta^2}{1+\beta}(\frac{1}{t} + \mu). \tag{B.6}$$

Then $\Delta_H(x, a; t) < x[-\frac{\beta}{1+\beta} + \frac{\Sigma}{\beta}G_\infty(t)] < 0$ for any x.

Furthermore, with some tedious algebra, we can show that for any x

$$G(x, a; t) + x\frac{\partial G(x, a; t)}{\partial x} < G_\infty(t)$$

Hence, the inequality (B.6) also implies that

$$\frac{\partial \Delta_H(x, a; t)}{\partial x} = \frac{\Sigma}{\beta}[G(x, a; t) + x\frac{\partial G(x, a; t)}{\partial x}] - \frac{\beta}{1+\beta} < \frac{\Sigma}{\beta}G_\infty(t) - \frac{\beta}{1+\beta} < 0. \tag{B.7}$$

B.4 Proof of Proposition 4

The relevant upper bounds of z for positive human capital investment are functions of γ (to the first order approximation):

$$
\begin{aligned}
\overline{z}^{L}_{slow}(a;\gamma) &= \overline{z}^{L}_{slow}(a;\gamma=0) - \gamma\lambda\frac{w'z'}{w(1+r')}\\
\overline{z}^{L}_{fast}(a;\gamma) &= \overline{z}^{L}_{fast}(a;\gamma=0) - \gamma\lambda\frac{w'z'}{w(1+r')}\frac{\exp(\frac{\chi_e e_L}{1+\beta})}{\exp(\frac{\chi_e e_L}{1+\beta})-1}\\
\overline{z}^{M}_{slow}(a;\gamma) &= \overline{z}^{M}_{slow}(a;\gamma=0) + \gamma\lambda\frac{w'z'}{w(1+r')}\exp(\frac{\chi_n-\chi_e e_H}{1+\beta})\\
\overline{z}^{M}_{fast}(a;\gamma) &= \overline{z}^{M}_{fast}(a;\gamma=0) + \gamma\lambda\frac{w'z'}{w(1+r')}\frac{1}{\exp(\frac{\chi_e e_L}{1+\beta})-1}
\end{aligned}
$$

Therefore, an anticipated AI shock, $\gamma > 0$ makes those with $h < h_M\frac{1}{1-\delta}$ invest less human capital and those with $h > h_M\frac{1}{1-\delta}$ invest more human capital.

C Computational Procedure for the Quantitative Model

C.1 Steady-state Equilibrium

In the steady-state, the measure of households, $\mu(a,h,x)$, and the factor prices are time-invariant. We find a time-invariant distribution μ. We compute the households' value functions and the decisions rules, and the time-invariant measure of the households. We take the following steps:

1. We choose the number of grid for the risk-free asset, a, human capital, h, and the idiosyncratic labor productivity, x. We set $N_a = 151$, $N_h = 151$, and $N_x = 9$ where N denotes the number of grid for each variable. To better incorporate the saving decisions of households near the borrowing constraint, we assign more points to the lower range of the asset and human capital.

2. Productivity x is equally distributed on the range $[-3\sigma_x/\sqrt{1-\rho_x^2}]$. As shown in the paper, we construct the transition probability matrix $\pi(x'|x)$ of the idiosyncratic labor productivity.

3. Given the values of parameters, we find the value functions for each state (a,h,x). We also obtain the decision rules: savings $a'(a,h,x)$, and $h'(a,h,x)$. The computation steps are as follow:

4. After obtaining the value functions and the decision rules, we compute the time-invariant distribution $\mu(a,h,x)$.

5. If the variables of interest are close to the targeted values, we have found the steady-state. If not, we choose the new parameters and redo the above steps.

C.2 Transition Dynamics

We incorporate the transition path from the status quo to the new steady state. We describe the steps below.

1. We obtain the initial steady state and the new steady state.

2. We assume that the economy arrives at the new steady state at time T. We set the T to 100. The unit of time is a year.

3. We initialize the capital-labor ratio $\{K_t/L_t\}_{t=2}^{T-1}$ and obtain the associated factor prices $\{r_t, w_t\}_{t=2}^{T-1}$.

4. As we know the value functions at time T, we can obtain the value functions and the decision rules in the transition path from $t = T - 1$ to 1.

5. We compute the measures $\{\mu_t\}_{t=2}^{T}$ with the measures at the initial steady state and the decision rules in the transition path.

6. We obtain the aggregate variables in the transition path with the decision rules and the distribution measures.

7. We compare the assumed paths of capital and the effective labor with the updated ones. If the absolute difference between them in each period is close enough, we obtain the converged transition path. Otherwise, we assume new capital-labor ratio and go back to 3.

D Investigating the GE channel of AI's impact

Redistribution versus general equilibrium effects: The effects of human capital adjustments on AI's aggregate impacts operate through two primary channels: the *redistribution channel*, which reallocates households across skill sectors, and the *general equilibrium (GE) channel*, which operates through changes in wages and capital returns. We now assess the relative importance of these channels in shaping economic outcomes.

Figure **??** compares the transition dynamics between scenarios with and without human capital adjustments, while holding wages and capital returns fixed at their initial steady-state levels to eliminate GE effects. We refer to the former as the PE Model" and the latter as the "No-HC PE Model." The difference between the solid blue line and the dashed red line isolates the effect of redistribution channel. Comparing this difference to the gap between the benchmark model and the No HC model in Figure 7 enables us to evaluate the importance of the redistribution channel relative to the GE channel. Two key observations emerge.

First, the *redistribution channel* alone accounts for all the *qualitative effects* of human capital adjustments on AI's aggregate impacts. Redistribution of human

Figure 11: Caption

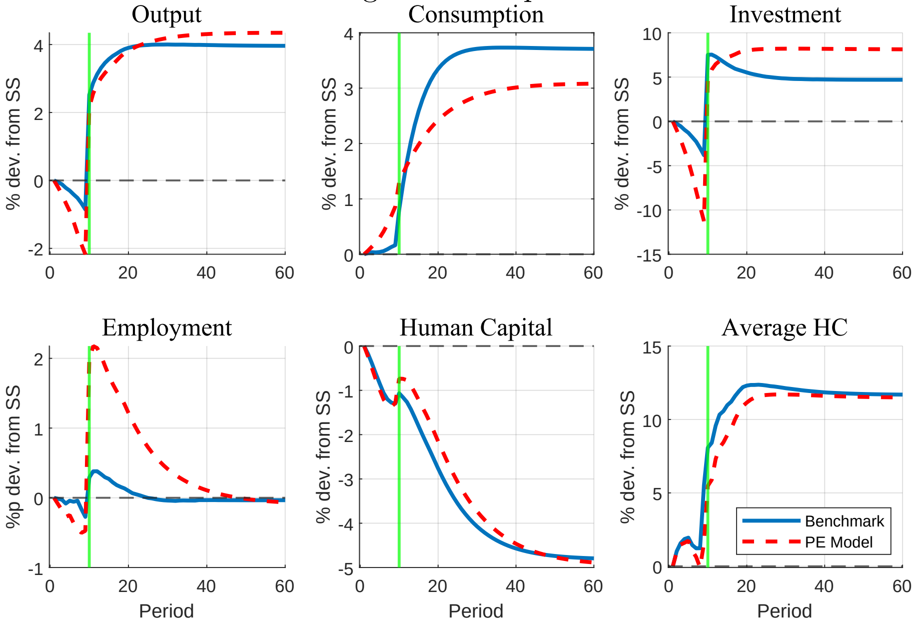
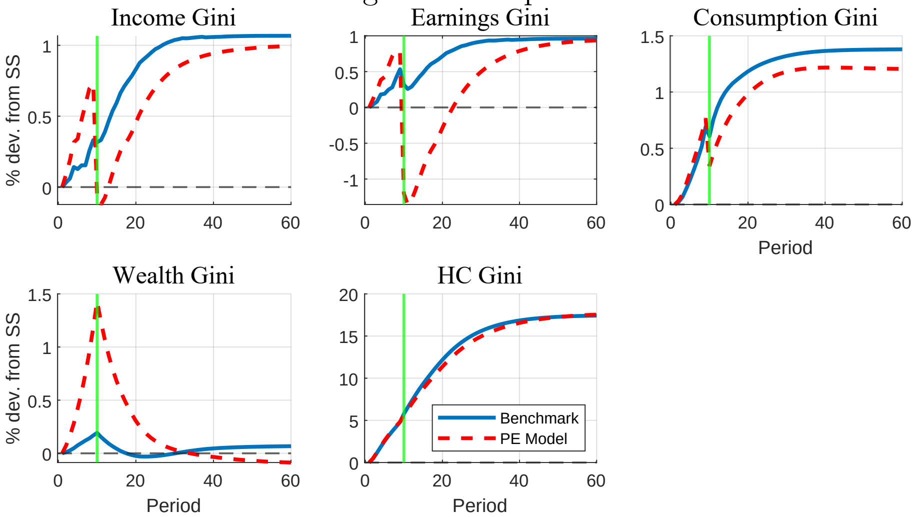


Figure 12: Caption



capital increases consumption, even before AI implementation, as more households shift to the high sector. It also reduces investment by mitigating precautionary savings and lowers employment as middle-sector workers leave the labor market to invest in human capital. In the long run, redistribution amplifies AI's positive impact on output by reallocating more workers to sectors that benefit most from AI advancements.

Second, the *GE channel* primarily affects the *quantitative magnitude* of human capital adjustments' impact on AI's aggregate outcomes. When the GE channel is included, the differences in output, consumption, and employment between models with and without human capital adjustments are smaller compared to when the GE channel is excluded. In contrast, and somewhat unexpectedly, the difference in investment is larger when the GE channel is included. This indicates that allowing capital returns to adjust amplifies the impact of human capital accumulation on how household savings respond to AI.

When the *GE channel* is active (Figure **??**), AI reduces the wealth Gini, but the *redistribution channel* moderates this effect. However, when the *GE channel* is disabled (Figure **??**), AI increases wealth inequality in the long run without the *redistribution channel* from human capital adjustment. In contrast, with the *redistribution channel* active, AI reduces wealth inequality.

These observations lead to two key conclusions:

First, the *redistribution channel* alone introduces a qualitative shift in AI's long-run impact on the wealth Gini (as shown in Figure **??**).

Second, the *GE channel*, when combined with human capital adjustment, qualitatively alters the effect of anticipating AI on the wealth Gini (as shown by comparing the blue lines in Figures **??** and **??**).

Policy implications: The impact of human capital adjustments on AI's distributional outcomes, along with the roles of the *redistribution channel* and *GE channel*, provides valuable insights for policy discussions on how to address the challenges posed by AI shocks.

In particular, government interventions aimed at stabilizing wages in response to AI-induced economic shocks may unintentionally worsen wealth inequality. Our analysis indicates that if wages are prevented from adjusting to reflect productivity differences, this distorts households' incentives to adjust their human capital and precautionary savings—both of which play a critical role in mitigating wealth inequality.