

微積 I 試験予想問題 解説

@yangniao23

(1)

$$\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 7x} = \frac{3}{7}$$

(2)

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{x - \frac{\pi}{2}}$$

$t = x - \frac{\pi}{2}$ とおくと, $t \rightarrow 0$ だから,

$$\begin{aligned} &= \lim_{t \rightarrow 0} \frac{\cos(t + \frac{\pi}{2})}{t} = \lim_{t \rightarrow 0} \frac{-\sin t}{t} \\ &= -1 \end{aligned}$$

(3)

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sin(\sin^2 x)}{x^2} &= \lim_{x \rightarrow 0} \left(\frac{\sin(\sin^2 x)}{\sin^2 x} \cdot \frac{\sin^2 x}{x^2} \right) \\ &= 1 \end{aligned}$$

(4)

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{2x \tan(3x^2)}{\sin(2x^3)} &= \lim_{x \rightarrow 0} \left(\frac{2x}{\sin(2x^3)} \cdot \frac{\tan(3x^2)}{3x^2} \cdot 3x^2 \right) \\ &= \lim_{x \rightarrow 0} \left(\frac{6x^3}{\sin(2x^3)} \cdot \frac{\sin(3x^2)}{3x^2} \cdot \frac{1}{\cos(3x^2)} \right) = 3 \end{aligned}$$