Appendix: Contract Design for Adaptive Federated Learning

Ni YANG, Yue CUI, Man Hon Cheung

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1 Proof to Lemma 2

Proof.

1. The non-negative weighted sum of convex (concave) functions is convex (concave). [?] $g_1(\boldsymbol{r}) = \sum_{i=1}^{I}\sum_{j=1}^{J} N_{i,j} r_{i,j}\gamma_j$ and $g_2(\boldsymbol{r}) = \sum_{i=1}^{I}\sum_{j=1}^{J} N_{i,j}\theta_i r_{i,j}$ are weighted sum of variables $r_{i,j}$, where $N_{i,j}$, γ_j and θ_i are weights. Thus, $g_1(\boldsymbol{r})$ and $g_2(\boldsymbol{r})$ are both convex and concave.

2. The scalar composition $f(x) = h(g(x))$ is convex if $g(\cdot)$ is concave, $h(\cdot)$ is convex and $\hat{h}$ is non-increasing, where $\hat{h}$ is the extended-value extension of function h, which assigns the value ∞ to points not in **dom** h. Suppose $h(x) = \frac{1}{\sqrt{x}}$, which is a convex and $\hat{h}$ is non-increasing. Thus, $f(\boldsymbol{r}) = h(g_1(\boldsymbol{r}))$ is convex.

3. $C_{com}(\boldsymbol{r})$ is non-negative weighted sum of convex functions $f(\boldsymbol{r})$ and $g_2(\boldsymbol{r})$. Thus, $C_{com}(\boldsymbol{r})$ is convex.

□