

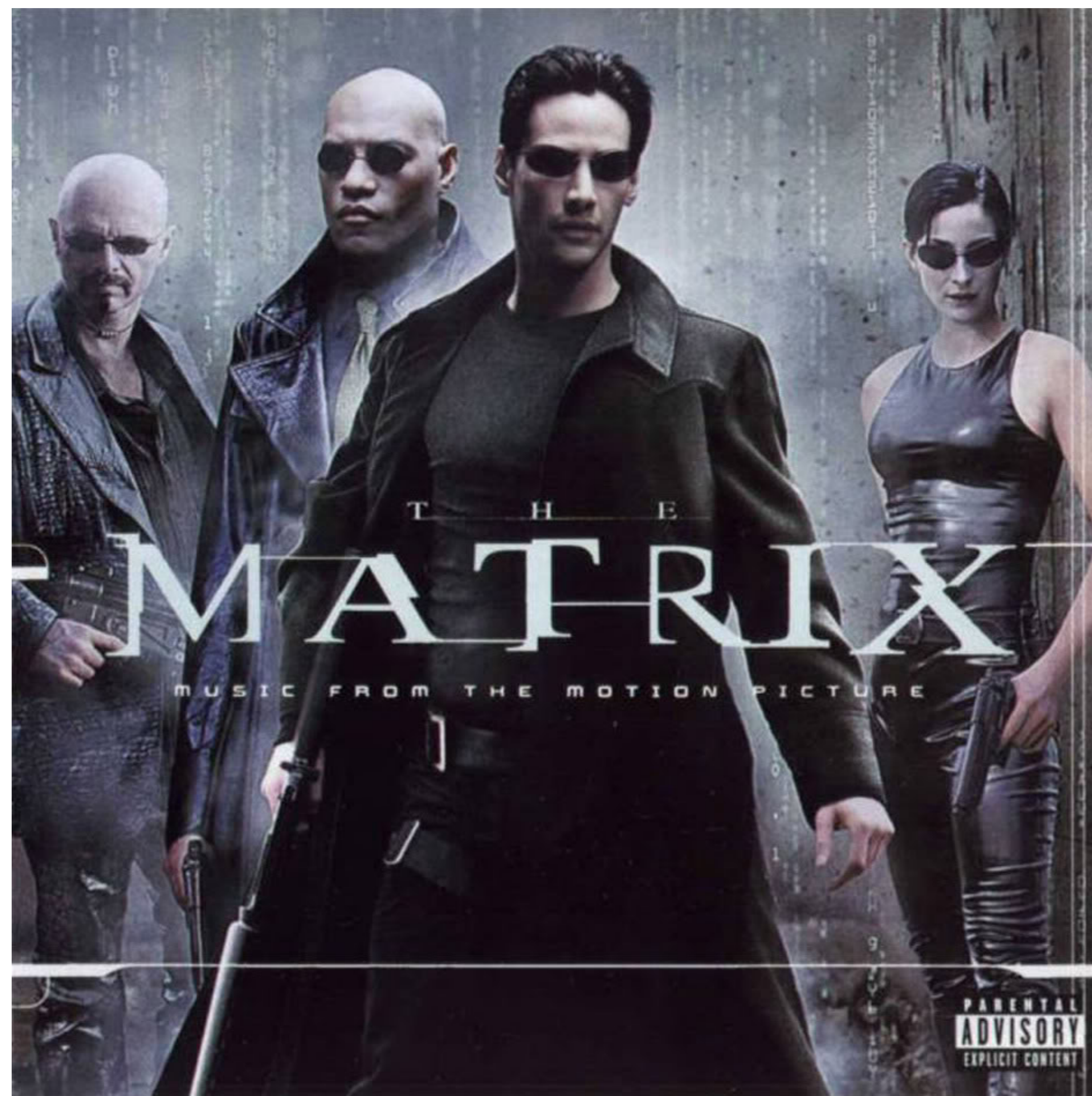
专给程序员设计的线性代数

liuyubobobo

矩阵不是简单的 $m*n$ 个数

什么是矩阵

矩阵 Matrix



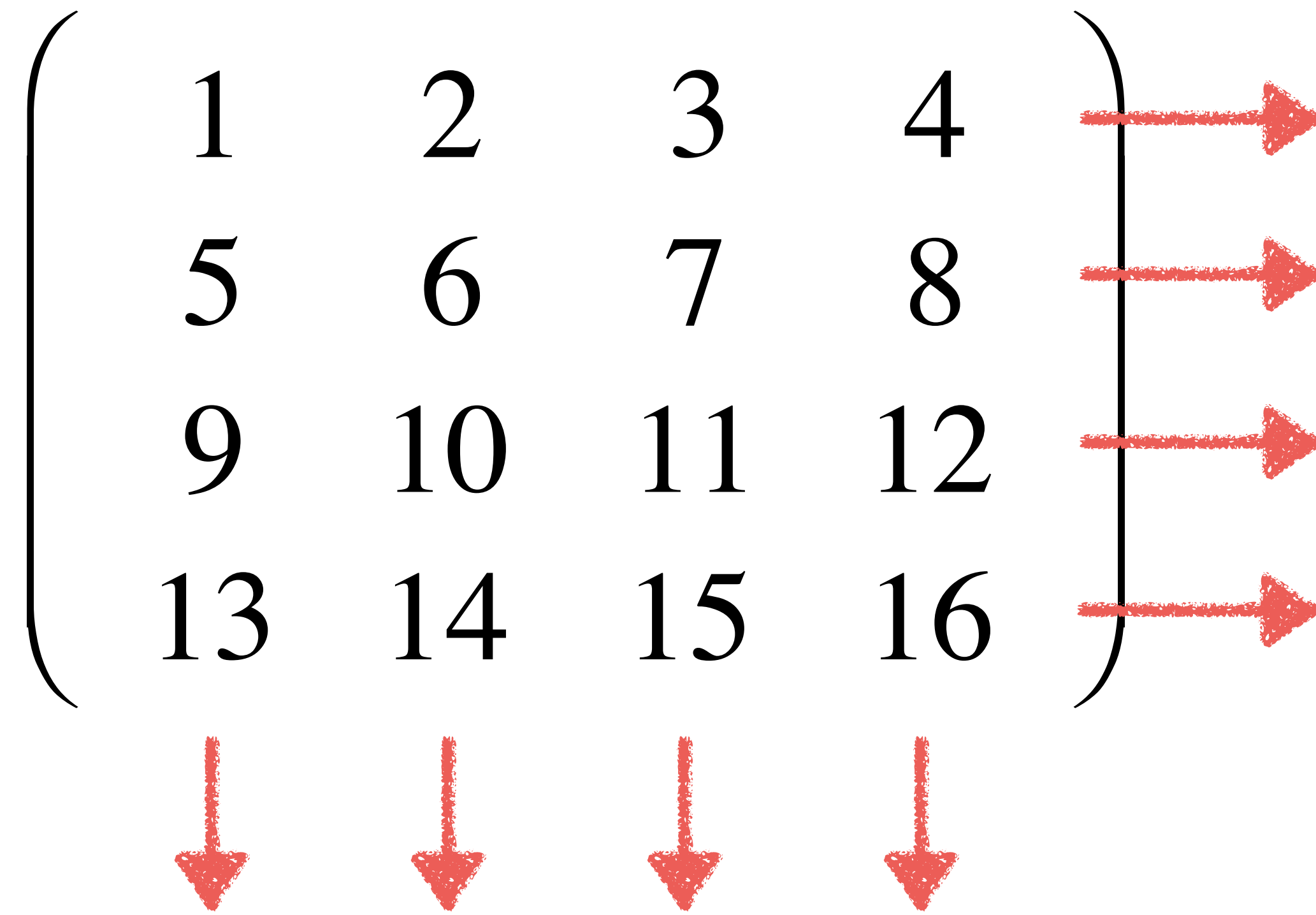
矩阵 Matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

向量是对数的拓展，
一个向量表示一组数

矩阵是对向量的拓展，
一个矩阵表示一组向量

矩阵 Matrix



A 4x4 matrix is shown with elements 1 through 16. Red arrows point from each row to the right, labeled '行向量' (row vector). Red arrows point from each column downwards, labeled '列向量' (column vector).

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

行向量

4 * 4 矩阵

行数为4，列数为4

列向量

矩阵 Matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix}$$

3 * 4 矩阵

行数为3，列数为4

矩阵 Matrix

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{pmatrix}$$

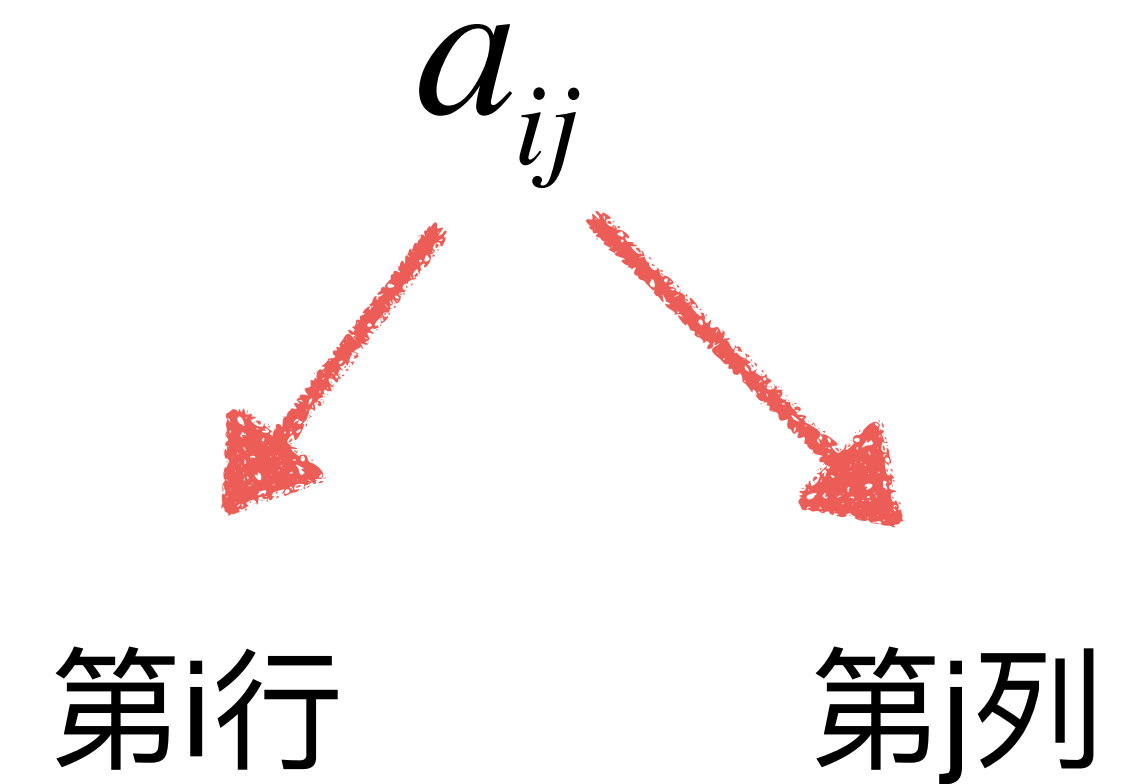
行数 = 列数  方阵

方阵有很多特殊的性质

有很多特殊的矩阵是方阵

矩阵 Matrix

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{pmatrix}$$



和计算机中的二维数组的表示一样！

矩阵 Matrix

	语文	数学	英语	物理	化学	
$A =$	90	76	88	92	90	张三
	88	82	98	95	92	李四
	86	68	70	80	77	王五

实现属于我们自己的矩阵类

实践： 实现属于我们自己的矩阵类

矩阵的基本运算

矩阵的基本运算

回忆向量的基本运算

$$\vec{u} + \vec{v}$$

$$k \cdot \vec{u}$$

矩阵的基本运算

$$A + B$$

$$k \cdot A$$

矩阵的基本运算

矩阵加法:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1c} \\ a_{21} & a_{22} & \dots & a_{2c} \\ \dots & \dots & \dots & \dots \\ a_{r1} & a_{r2} & \dots & a_{rc} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1c} \\ b_{21} & b_{22} & \dots & b_{2c} \\ \dots & \dots & \dots & \dots \\ b_{r1} & b_{r2} & \dots & b_{rc} \end{pmatrix}$$

$A + B$

$$A + B = \begin{pmatrix} a_{11} + b_{11} & a_{12} + b_{12} & \dots & a_{1c} + b_{1c} \\ a_{21} + b_{21} & a_{22} + b_{22} & \dots & a_{2c} + b_{2c} \\ \dots & \dots & \dots & \dots \\ a_{r1} + b_{r1} & a_{r2} + b_{r2} & \dots & a_{rc} + b_{rc} \end{pmatrix}$$

矩阵的基本运算

矩阵加法： $A + B$

	语文	数学	英语	物理	化学	
$A =$	90	76	88	92	90	张三
	88	82	98	95	92	李四
	86	68	70	80	77	王五

上学期成绩

	语文	数学	英语	物理	化学	
$B =$	92	74	96	92	95	张三
	88	92	94	86	78	李四
	82	74	80	88	80	王五

下学期成绩

矩阵的基本运算

矩阵数量乘法:

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1c} \\ a_{21} & a_{22} & \cdots & a_{2c} \\ \cdots & \cdots & \cdots & \cdots \\ a_{r1} & a_{r2} & \cdots & a_{rc} \end{pmatrix}$$

$k \cdot A$

$$k \cdot A = \begin{pmatrix} k \cdot a_{11} & k \cdot a_{12} & \cdots & k \cdot a_{1c} \\ k \cdot a_{21} & k \cdot a_{22} & \cdots & k \cdot a_{2c} \\ \cdots & \cdots & \cdots & \cdots \\ k \cdot a_{r1} & k \cdot a_{r2} & \cdots & k \cdot a_{rc} \end{pmatrix}$$

矩阵的基本运算

矩阵数量乘法： $k \cdot A$

	语文	数学	英语	物理	化学	
$A =$	90	76	88	92	90	张三
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上学期成绩

	语文	数学	英语	物理	化学	
$B =$	92	74	96	92	95	张三
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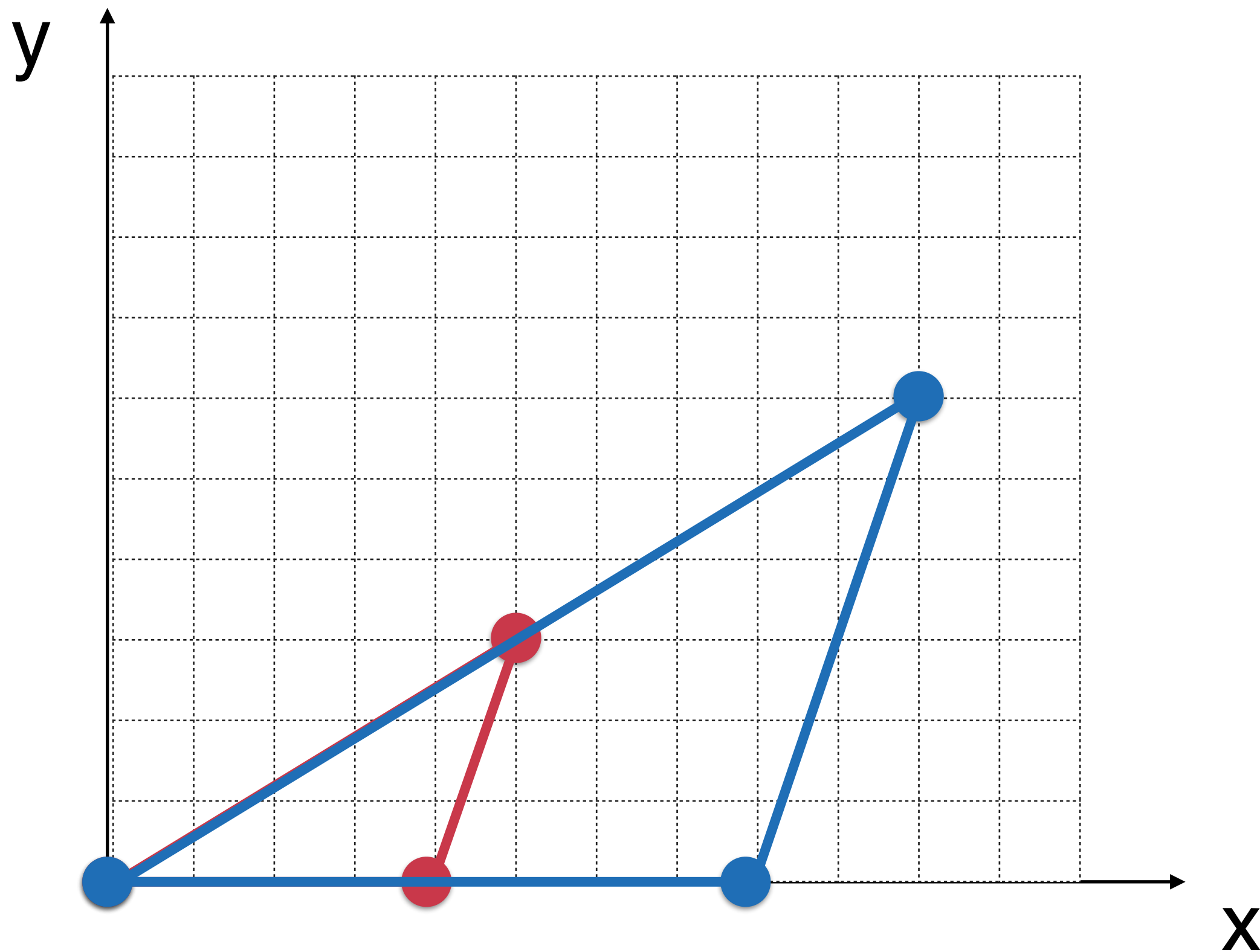
下学期成绩

$$\frac{1}{2} \cdot (A + B)$$

两学期成绩的平均分

矩阵的基本运算

矩阵数量乘法: $k \cdot A$



$$P = \begin{pmatrix} 0 & 0 \\ 4 & 0 \\ 5 & 3 \end{pmatrix}$$

$$2 \cdot P = \begin{pmatrix} 0 & 0 \\ 8 & 0 \\ 10 & 6 \end{pmatrix}$$

矩阵的基本运算性质

矩阵的基本运算

$$A + B \quad k \cdot A$$

矩阵的基本运算性质

$$A + B = B + A$$

$$(A + B) + C = A + (B + C)$$

存在矩阵 O ，满足： $A + O = A$

存在矩阵 $-A$ ，满足： $A + (-A) = O$

$-A$ 唯一； $-A = -1 \cdot A$

矩阵的基本运算性质

矩阵的基本运算性质

$$(ck)A = c(kA)$$

$$k \cdot (A + B) = k \cdot A + k \cdot B$$

$$(c + k) \cdot A = c \cdot A + k \cdot A$$

矩阵的基本运算性质

$$k \cdot (A + B) = k \cdot A + k \cdot B$$

基本证明思路：

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1c} \\ a_{21} & a_{22} & \dots & a_{2c} \\ \dots & \dots & & \dots \\ a_{r1} & a_{r2} & \dots & a_{rc} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1c} \\ b_{21} & b_{22} & \dots & b_{2c} \\ \dots & \dots & & \dots \\ b_{r1} & b_{r2} & \dots & b_{rc} \end{pmatrix}$$

矩阵的基本运算性质

$$k \cdot (A + B) = k \cdot A + k \cdot B$$

基本证明思路：

$$k \cdot (A + B) = \begin{pmatrix} k(a_{11} + b_{11}) & k(a_{12} + b_{12}) & \dots & k(a_{1c} + b_{1c}) \\ k(a_{21} + b_{21}) & k(a_{22} + b_{22}) & \dots & k(a_{2c} + b_{2c}) \\ \dots & \dots & \dots & \dots \\ k(a_{r1} + b_{r1}) & k(a_{r2} + b_{r2}) & \dots & k(a_{rc} + b_{rc}) \end{pmatrix}$$

$$k \cdot A + k \cdot B = \begin{pmatrix} ka_{11} + kb_{11} & ka_{12} + kb_{12} & \dots & ka_{1c} + kb_{1c} \\ ka_{21} + kb_{21} & ka_{22} + kb_{22} & \dots & ka_{2c} + kb_{2c} \\ \dots & \dots & \dots & \dots \\ ka_{r1} + kb_{r1} & ka_{r2} + kb_{r2} & \dots & ka_{rc} + kb_{rc} \end{pmatrix}$$

矩阵的基本运算性质

$$A + B = B + A$$

$$(A + B) + C = A + (B + C)$$

存在矩阵 O ，满足： $A + O = A$

存在矩阵 $-A$ ，满足： $A + (-A) = O$

$-A$ 唯一； $-A = -1 \cdot A$

$$(ck)A = c(kA)$$

$$(c + k) \cdot A = c \cdot A + k \cdot A$$

$$k \cdot (A + B) = k \cdot A + k \cdot B$$

实现矩阵的基本运算

实践：实现矩阵的基本运算

看待矩阵的另一个视角：系统

矩阵 Matrix

	语文	数学	英语	物理	化学	
$A =$	90	76	88	92	90	张三
	88	82	98	95	92	李四
	86	68	70	80	77	王五

之前，我们的例子中，矩阵式一个数据表格

矩阵还可以表示一个系统

矩阵 Matrix

矩阵还可以表示一个系统

经济系统中，对IT，电子，矿产，房产的投入 x_{it} x_e x_m x_h

$$x_{it} = 100 + 0.2x_e + 0.1x_m + 0.5x_h$$

$$x_e = 50 + 0.5x_{it} + 0.2x_m + 0.1x_h$$

$$x_m = 20 + 0.4x_e + 0.3x_h$$

$$x_h = 666 + 0.2x_{it}$$

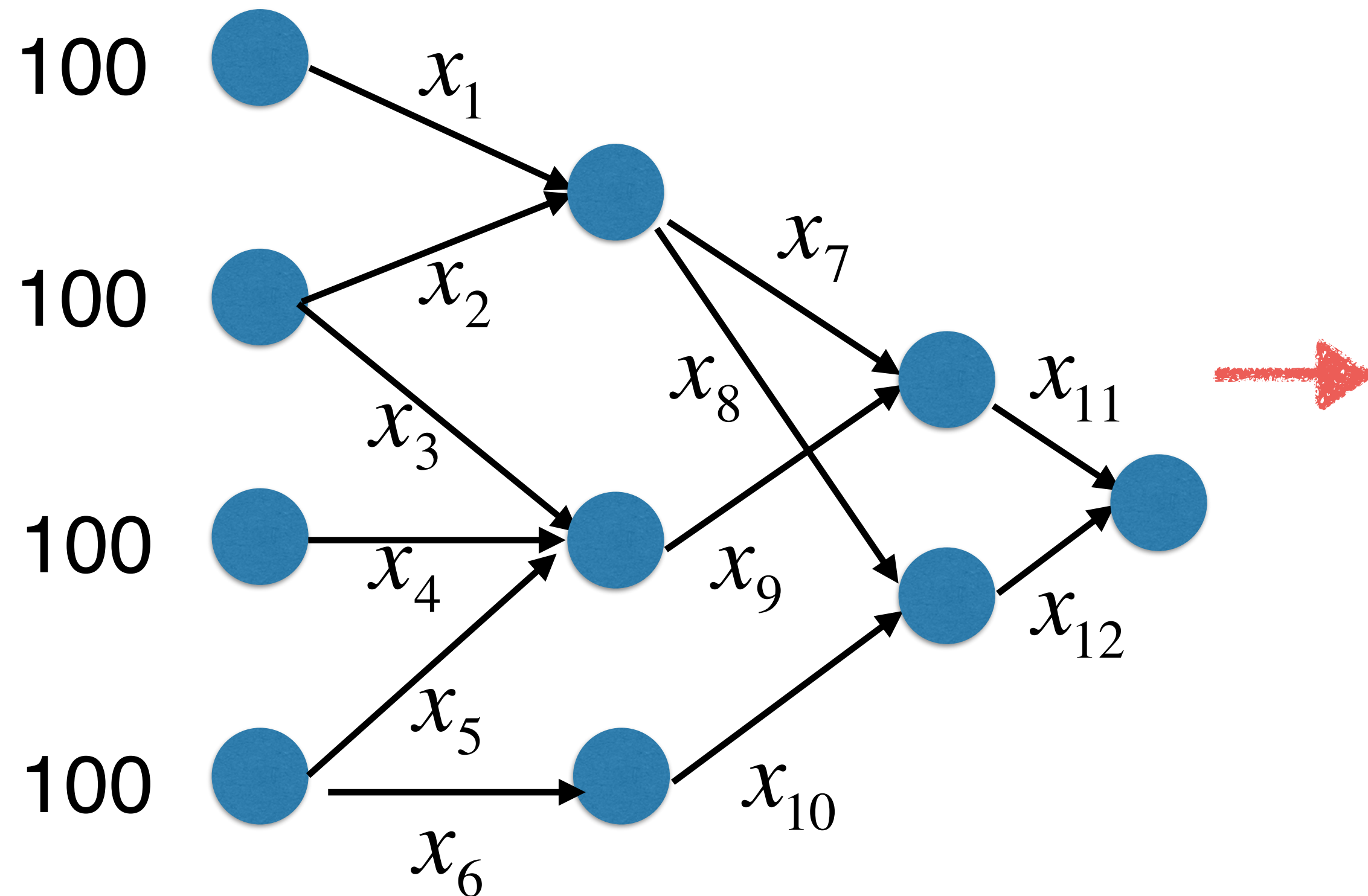
矩阵 Matrix

经济系统中，对IT，电子，矿产，房产的投入 x_{it} x_e x_m x_h

$$\begin{aligned} x_{it} &= 100 + 0.2x_e + 0.1x_m + 0.5x_h \\ x_e &= 50 + 0.5x_{it} + 0.2x_m + 0.1x_h \\ x_m &= 20 + 0.4x_e + 0.3x_h \\ x_h &= 666 + 0.2x_{it} \end{aligned} \quad \begin{array}{l} \rightarrow \left\{ \begin{array}{l} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h = 100 \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h = 50 \\ -0.4x_e - x_m + 0.3x_h = 20 \\ -0.2x_{it} + x_h = 666 \end{array} \right.$$

矩阵 Matrix

网络中 (交通网络, 信息网络...)



$$x_1 = 100$$

$$x_2 + x_3 = 100$$

$$x_4 = 100$$

$$x_5 + x_6 = 100$$

$$x_7 + x_8 = x_1 + x_2$$

$$x_9 = x_3 + x_4 + x_5$$

$$x_{10} = x_6$$

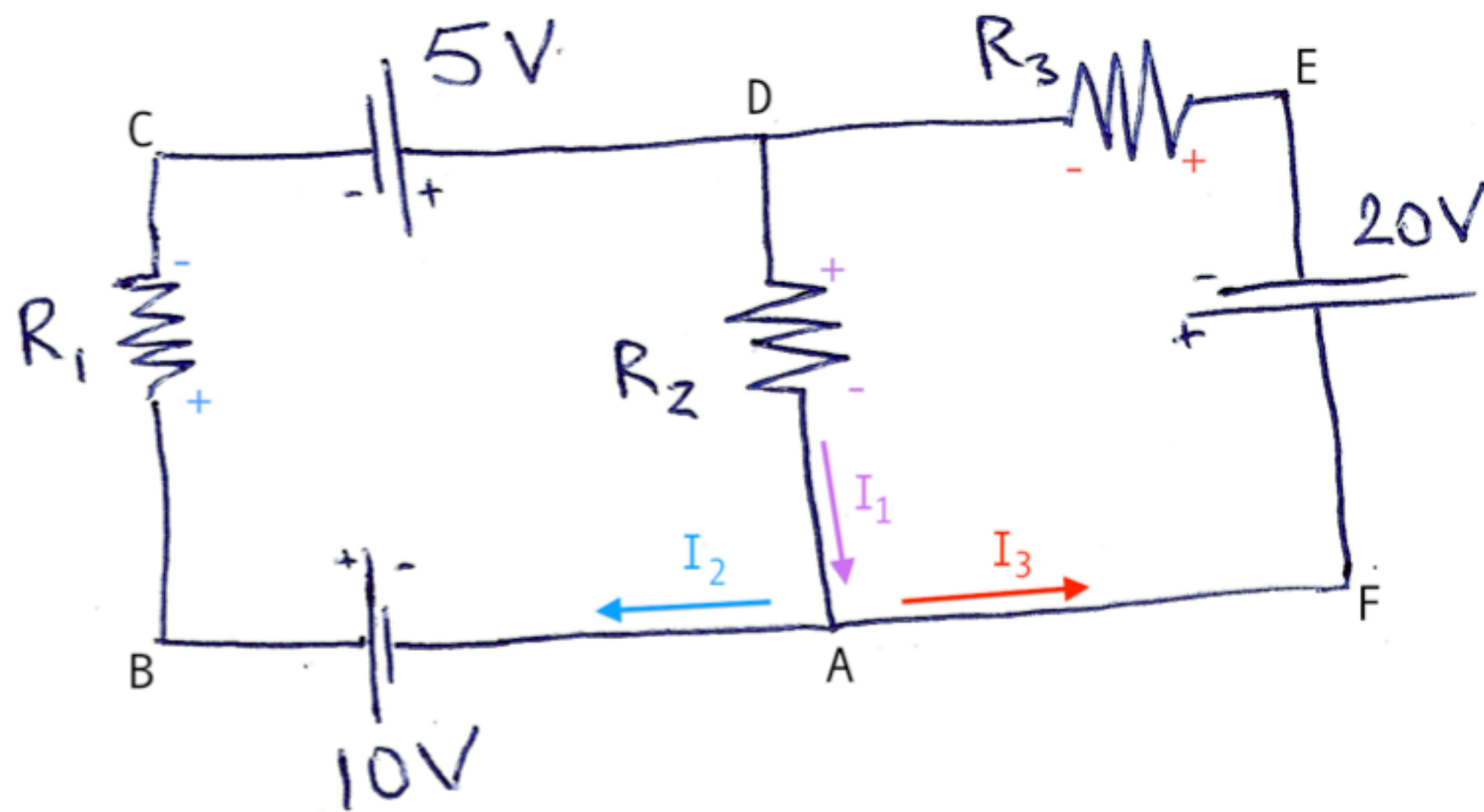
$$x_{11} = x_7 + x_9$$

$$x_{12} = x_8 + x_{10}$$

$$x_{11} + x_{12} = 400$$

矩阵 Matrix

电路系统中



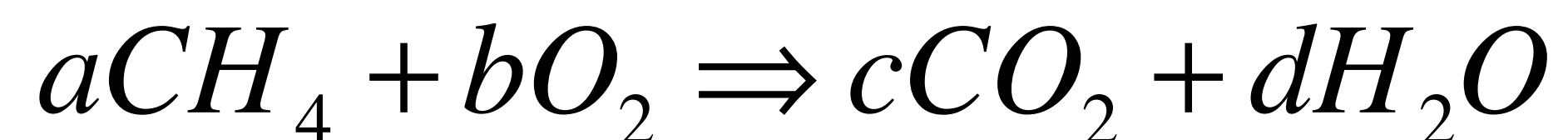
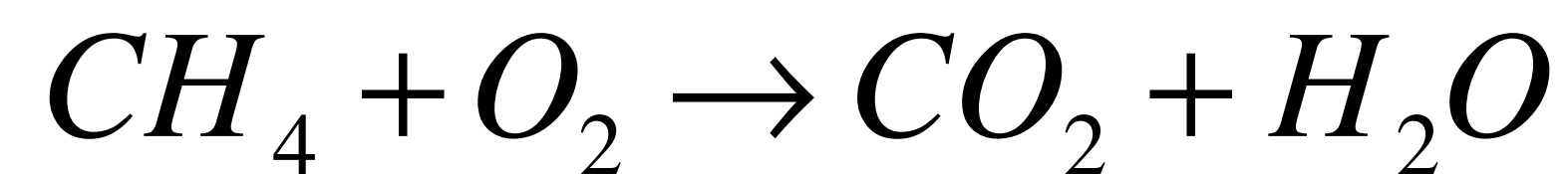
$$10 - R_1 I_2 + 5 - R_2 I_1 = 0$$

$$-20 - R_3 I_3 - R_2 I_1 = 0$$

$$I_1 = I_2 + I_3$$

矩阵 Matrix

化学方程式中



$$a = c$$

$$4a = 2d$$

$$2b = 2c + d$$

矩阵 Matrix

线性方程组在各个领域，有着重要的应用

在线性代数中，称为线性系统

矩阵 Matrix

$$\left\{ \begin{array}{l} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h = 100 \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h = 50 \\ -0.4x_e - x_m + 0.3x_h = 20 \\ -0.2x_{it} + x_h = 666 \end{array} \right.$$

矩阵 Matrix

$$\left\{ \begin{array}{l} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h = 100 \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h = 50 \\ -0.4x_e - x_m + 0.3x_h = 20 \\ -0.2x_{it} + x_h = 666 \end{array} \right.$$

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 100 \\ 50 \\ 20 \\ 666 \end{pmatrix}$$

列向量!

矩阵和向量的乘法

矩阵 Matrix

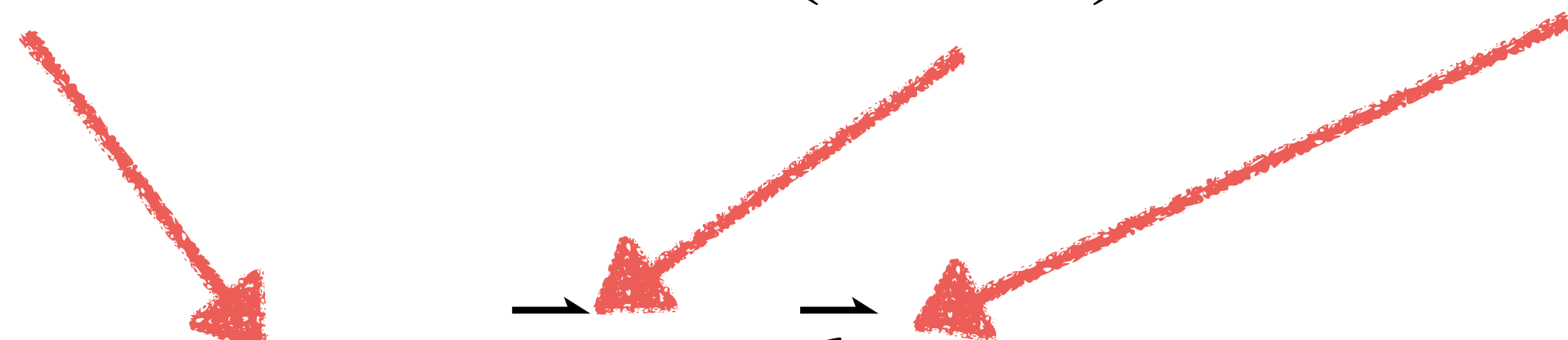
$$\left\{ \begin{array}{l} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h = 100 \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h = 50 \\ -0.4x_e - x_m + 0.3x_h = 20 \\ -0.2x_{it} + x_h = 666 \end{array} \right.$$

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{it} \\ x_e \\ x_m \\ x_h \end{pmatrix} = \begin{pmatrix} 100 \\ 50 \\ 20 \\ 666 \end{pmatrix}$$

列向量!

矩阵和向量相乘

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{it} \\ x_e \\ x_m \\ x_h \end{pmatrix} = \begin{pmatrix} 100 \\ 50 \\ 20 \\ 666 \end{pmatrix}$$


$$A \cdot \vec{x} = \vec{b}$$

矩阵和向量相乘

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{it} \\ x_e \\ x_m \\ x_h \end{pmatrix} = \begin{pmatrix} 100 \\ 50 \\ 20 \\ 666 \end{pmatrix}$$

$$\begin{pmatrix} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h \\ -0.4x_e - x_m + 0.3x_h \\ -0.2x_{it} + x_h \end{pmatrix}$$

矩阵和向量相乘

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{it} \\ x_e \\ x_m \\ x_h \end{pmatrix} = \begin{pmatrix} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h \\ -0.4x_e - x_m + 0.3x_h \\ -0.2x_{it} + x_h \end{pmatrix}$$

矩阵和向量相乘

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{it} \\ x_e \\ x_m \\ x_h \end{pmatrix} = \begin{pmatrix} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h \\ -0.4x_e - x_m + 0.3x_h \\ -0.2x_{it} + x_h \end{pmatrix}$$

矩阵和向量相乘

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{it} \\ x_e \\ x_m \\ x_h \end{pmatrix} = \begin{pmatrix} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h \\ -0.4x_e - x_m + 0.3x_h \\ -0.2x_{it} + x_h \end{pmatrix}$$

矩阵和向量相乘

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{it} \\ x_e \\ x_m \\ x_h \end{pmatrix} = \begin{pmatrix} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h \\ -0.4x_e - x_m + 0.3x_h \\ -0.2x_{it} + x_h \end{pmatrix}$$

矩阵和向量相乘

$$\begin{pmatrix} 1 & -0.2 & 0.1 & 0.5 \\ -0.5 & -1 & 0.2 & 0.1 \\ 0 & -0.4 & -1 & 0.3 \\ -0.2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x_{it} \\ x_e \\ x_m \\ x_h \end{pmatrix} = \begin{pmatrix} x_{it} - 0.2x_e + 0.1x_m + 0.5x_h \\ -0.5x_{it} - x_e + 0.2x_m + 0.1x_h \\ -0.4x_e - x_m + 0.3x_h \\ -0.2x_{it} + x_h \end{pmatrix}$$

矩阵和向量相乘

$$\begin{pmatrix} \text{red bar} \\ \text{red bar} \\ \text{red bar} \\ \text{red bar} \end{pmatrix} \cdot \begin{pmatrix} \text{blue bar} \end{pmatrix} = \begin{pmatrix} \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \end{pmatrix}$$

矩阵和向量相乘

A diagram illustrating matrix-vector multiplication. On the left, a matrix is represented by four horizontal red bars, each enclosed in a large left-facing square bracket. This matrix is multiplied by a vector, represented by a single vertical blue bar enclosed in a large right-facing square bracket. A black dot is placed between the matrix and the vector. To the right of the vector is an equals sign. Further right is the result, which is a vector represented by four horizontal red bars, each followed by a black dot and then a horizontal blue bar. This entire result vector is enclosed in a large right-facing square bracket.

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \end{pmatrix} = \begin{pmatrix} a_{11}u_1 + a_{12}u_2 + \dots + a_{1n}u_n \\ a_{21}u_1 + a_{21}u_2 + \dots + a_{2n}u_n \\ \dots \\ a_{m1}u_1 + a_{m2}u_2 + \dots + a_{mn}u_n \end{pmatrix}$$

$$\begin{pmatrix} \text{red bar} \\ \text{red bar} \\ \text{red bar} \\ \text{red bar} \end{pmatrix} \cdot \begin{pmatrix} \text{blue bar} \end{pmatrix} = \begin{pmatrix} \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \end{pmatrix}$$

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \cdot \begin{pmatrix} u_1 \\ u_2 \\ \dots \\ u_n \end{pmatrix} = \begin{pmatrix} a_{11}u_1 + a_{12}u_2 + \dots + a_{1n}u_n \\ a_{21}u_1 + a_{21}u_2 + \dots + a_{2n}u_n \\ \dots \\ a_{m1}u_1 + a_{m2}u_2 + \dots + a_{mn}u_n \end{pmatrix}$$

矩阵A的列数必须和向量u的元素个数一致！

矩阵A的行数没有限制。

$$\begin{pmatrix} \text{red bar} \\ \text{red bar} \\ \text{red bar} \\ \text{red bar} \end{pmatrix} \cdot \begin{pmatrix} \text{blue bar} \end{pmatrix} = \begin{pmatrix} \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \end{pmatrix}$$

$$\begin{pmatrix} \text{---} \vec{r_1} \text{---} \\ \text{---} \vec{r_2} \text{---} \\ \dots \\ \text{---} \vec{r_m} \text{---} \end{pmatrix} \cdot \vec{u} = \begin{pmatrix} \vec{r_1} \cdot \vec{u} \\ \vec{r_2} \cdot \vec{u} \\ \dots \\ \vec{r_m} \cdot \vec{u} \end{pmatrix}$$

行视角

矩阵A的列数必须和向量u的元素个数一致！

矩阵A的行数没有限制。

再看向量的点乘

The diagram illustrates the dot product of a matrix row and a vector. On the left, a matrix A is represented by a large left parenthesis followed by four horizontal red bars, each representing a row, and a large right parenthesis. This is followed by a dot operator \cdot and a vector u , represented by a large left parenthesis followed by a single vertical blue bar and a large right parenthesis. An equals sign $=$ follows. To the right of the equals sign is a large left parenthesis followed by four rows. Each row consists of a horizontal red bar, a dot operator \cdot , and a horizontal blue bar, all enclosed within the large right parenthesis. This represents the resulting vector where each element is the dot product of a row from A and the vector u .

矩阵A的列数必须和向量u的元素个数一致！

矩阵A的行数没有限制。

A的行数为1

再看向量的点乘

The diagram shows the dot product of a row vector and a column vector. On the left, a red horizontal bar representing a row vector is enclosed in large parentheses. This is followed by a black dot representing the dot product operation. Next is a blue vertical bar representing a column vector, also enclosed in large parentheses. An equals sign follows. To the right of the equals sign is a large set of parentheses containing a red horizontal bar (the row vector) followed by a black dot and then a blue horizontal bar (the column vector), representing the element-wise multiplication of the two vectors.

矩阵A的列数必须和向量u的元素个数一致！

矩阵A的行数没有限制。

A的行数为1

再看向量的点乘

The diagram shows the dot product of a matrix and a vector. On the left, a matrix is represented by a red horizontal bar inside large parentheses, followed by a dot operator. Next to it is a vector represented by a blue vertical bar inside large parentheses. An equals sign follows. To the right of the equals sign is a single large parentheses containing a red horizontal bar and a blue horizontal bar separated by a dot operator, representing the resulting scalar value.

矩阵A的列数必须和向量u的元素数一致！

矩阵A的行数没有限制。

A的行数为1

$$\vec{u} \cdot \vec{v} = u_1 \cdot v_1 + u_2 \cdot v_2 + \dots + u_n \cdot v_n$$

矩阵和向量的乘法

$$\begin{pmatrix} \text{red bar} \\ \text{red bar} \\ \text{red bar} \\ \text{red bar} \end{pmatrix} \cdot \begin{pmatrix} \text{blue bar} \end{pmatrix} = \begin{pmatrix} \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \end{pmatrix}$$

$$T \cdot \vec{a} = \vec{b}$$

矩阵T实际上将向量a转换成了向量b!

可以把矩阵理解成向量的函数!

矩阵和矩阵的乘法

矩阵和向量的乘法

$$\begin{pmatrix} \text{red bar} \\ \text{red bar} \\ \text{red bar} \\ \text{red bar} \end{pmatrix} \cdot \begin{pmatrix} \text{blue bar} \end{pmatrix} = \begin{pmatrix} \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \\ \text{red bar} \cdot \text{blue bar} \end{pmatrix}$$

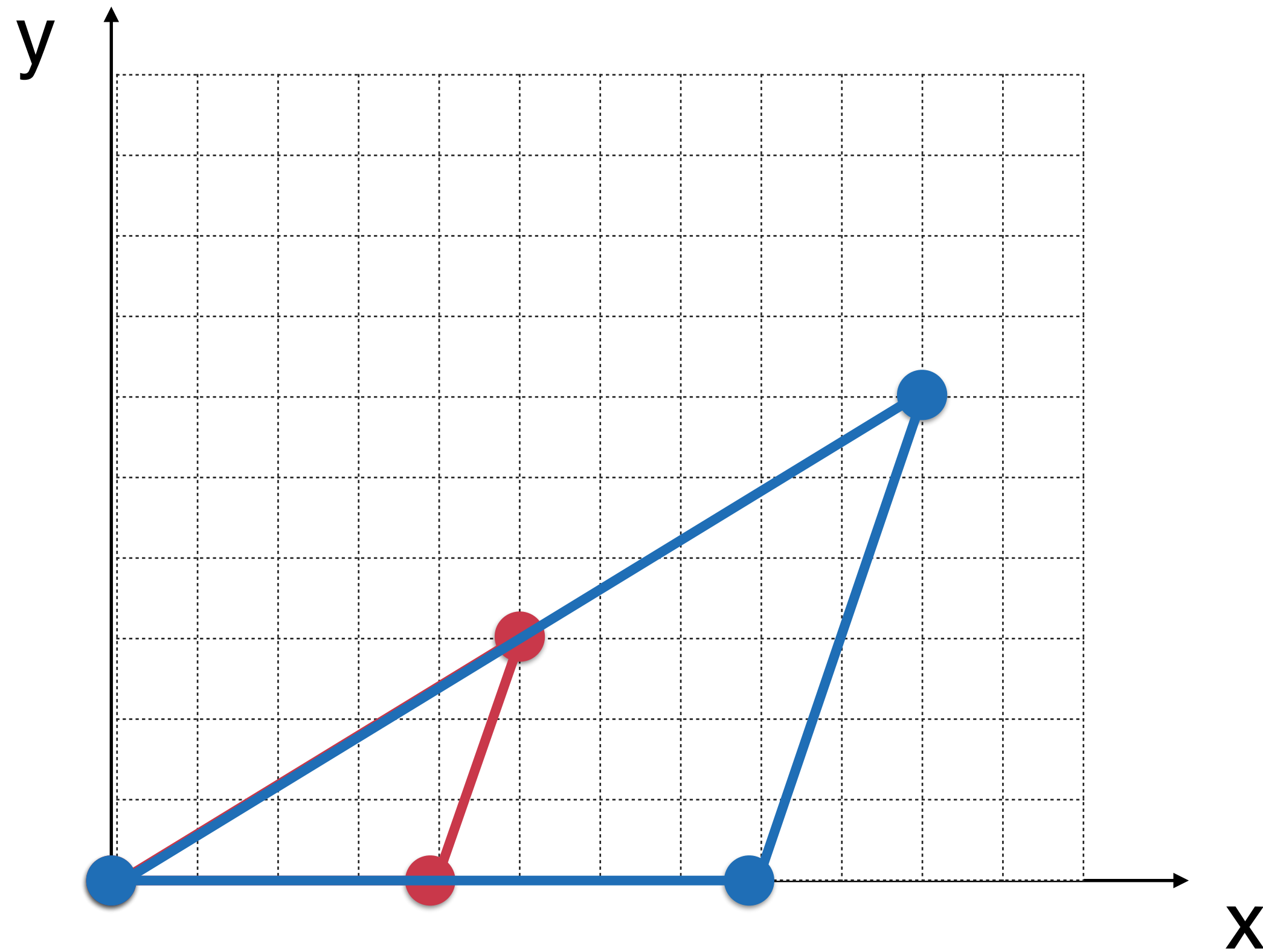
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矩阵在图形变换中的应用

让每个点的横坐标扩大1.5倍，纵坐标扩大2倍

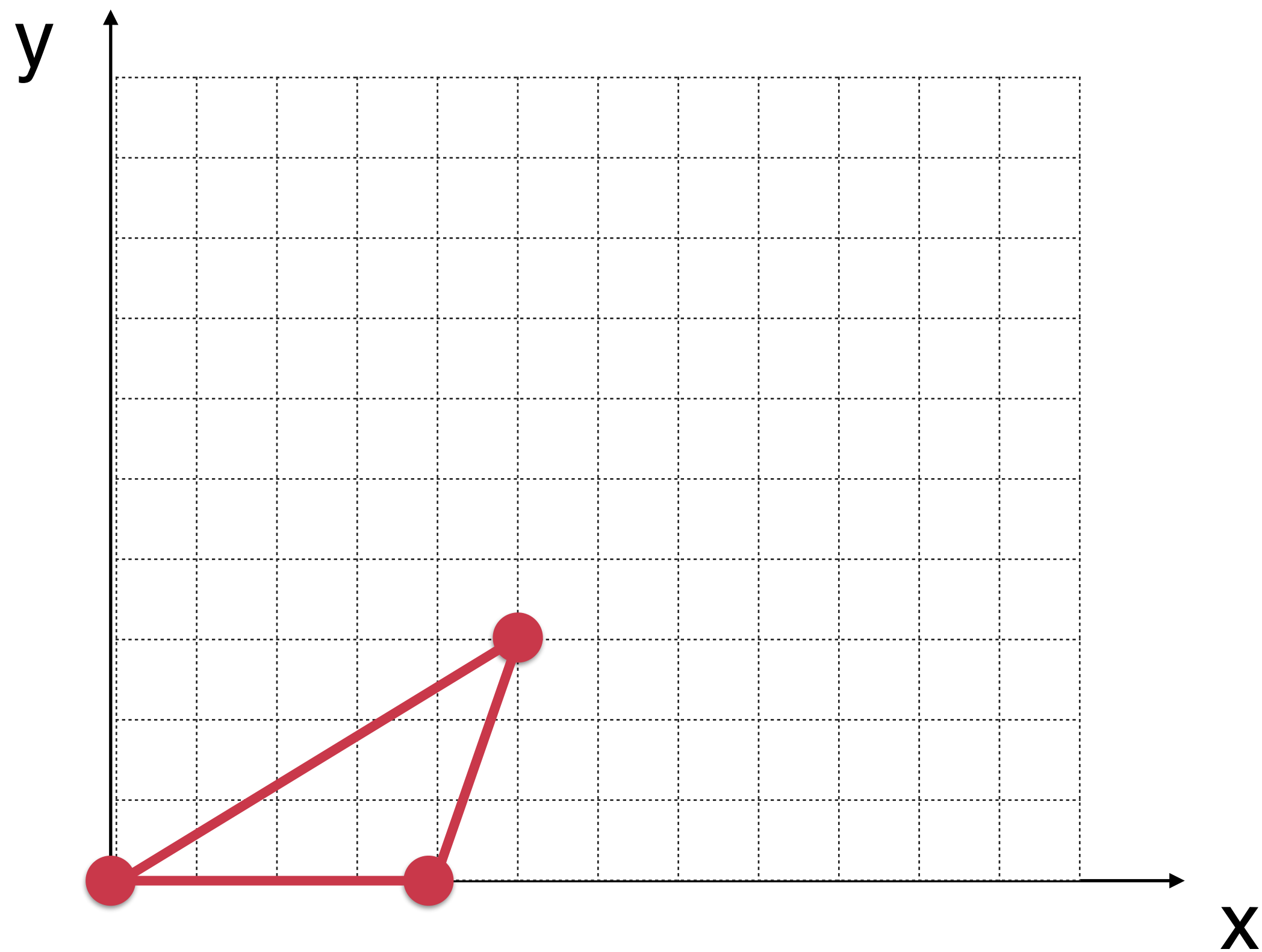


$$P = \begin{pmatrix} 0 & 0 \\ 4 & 0 \\ 5 & 3 \end{pmatrix}$$

$$2 \cdot P = \begin{pmatrix} 0 & 0 \\ 8 & 0 \\ 10 & 6 \end{pmatrix}$$

矩阵在图形变换中的应用

让每个点的横坐标扩大1.5倍，纵坐标扩大2倍



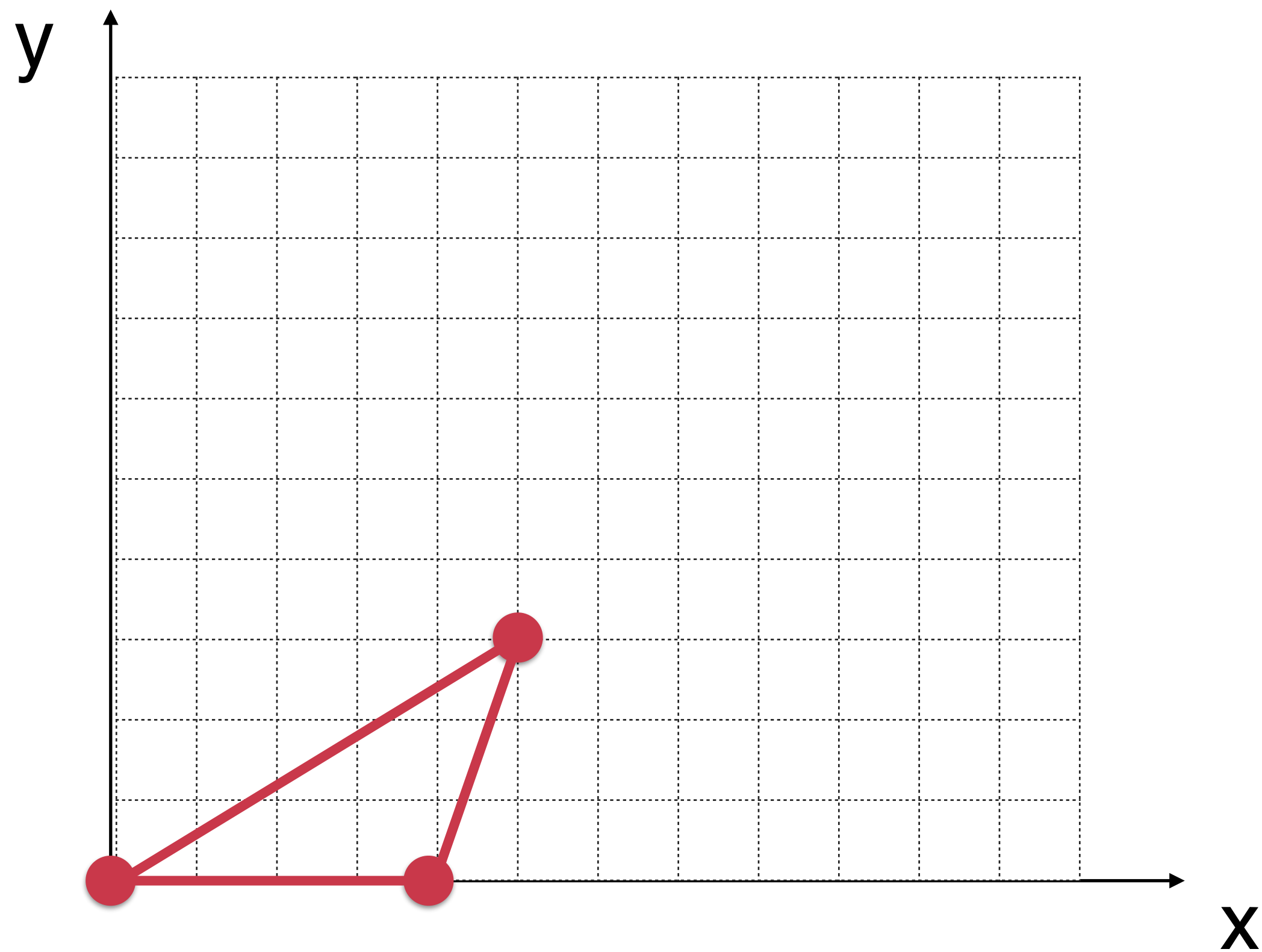
$$T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1.5x \\ 2y \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax + by \\ cx + dy \end{pmatrix}$$

$$T = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix}$$

矩阵在图形变换中的应用

让每个点的横坐标扩大1.5倍，纵坐标扩大2倍



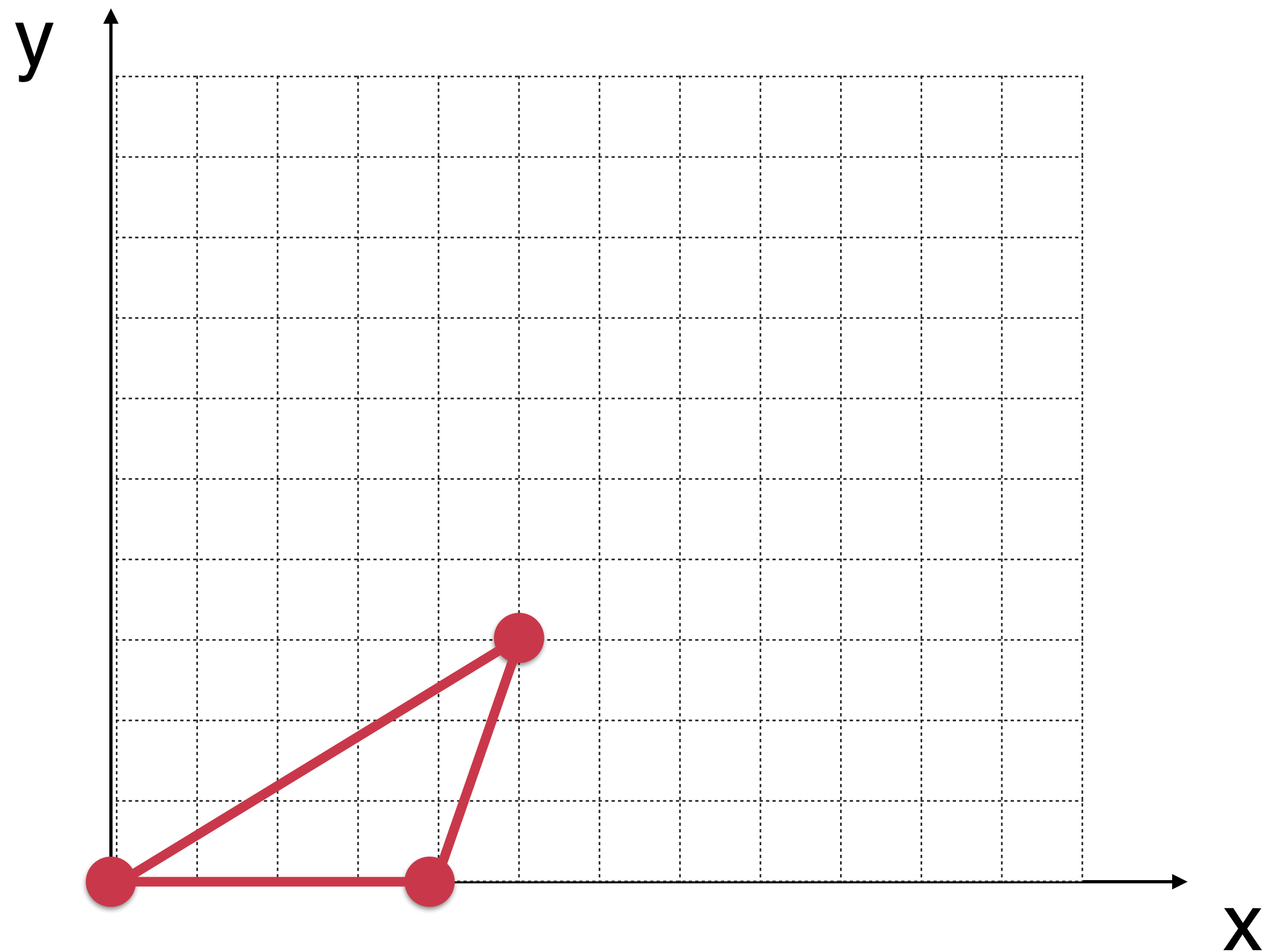
$$T \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1.5x \\ 2y \end{pmatrix}$$

$$T = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1.5x \\ 2y \end{pmatrix}$$

矩阵在图形变换中的应用

让每个点的横坐标扩大1.5倍，纵坐标扩大2倍



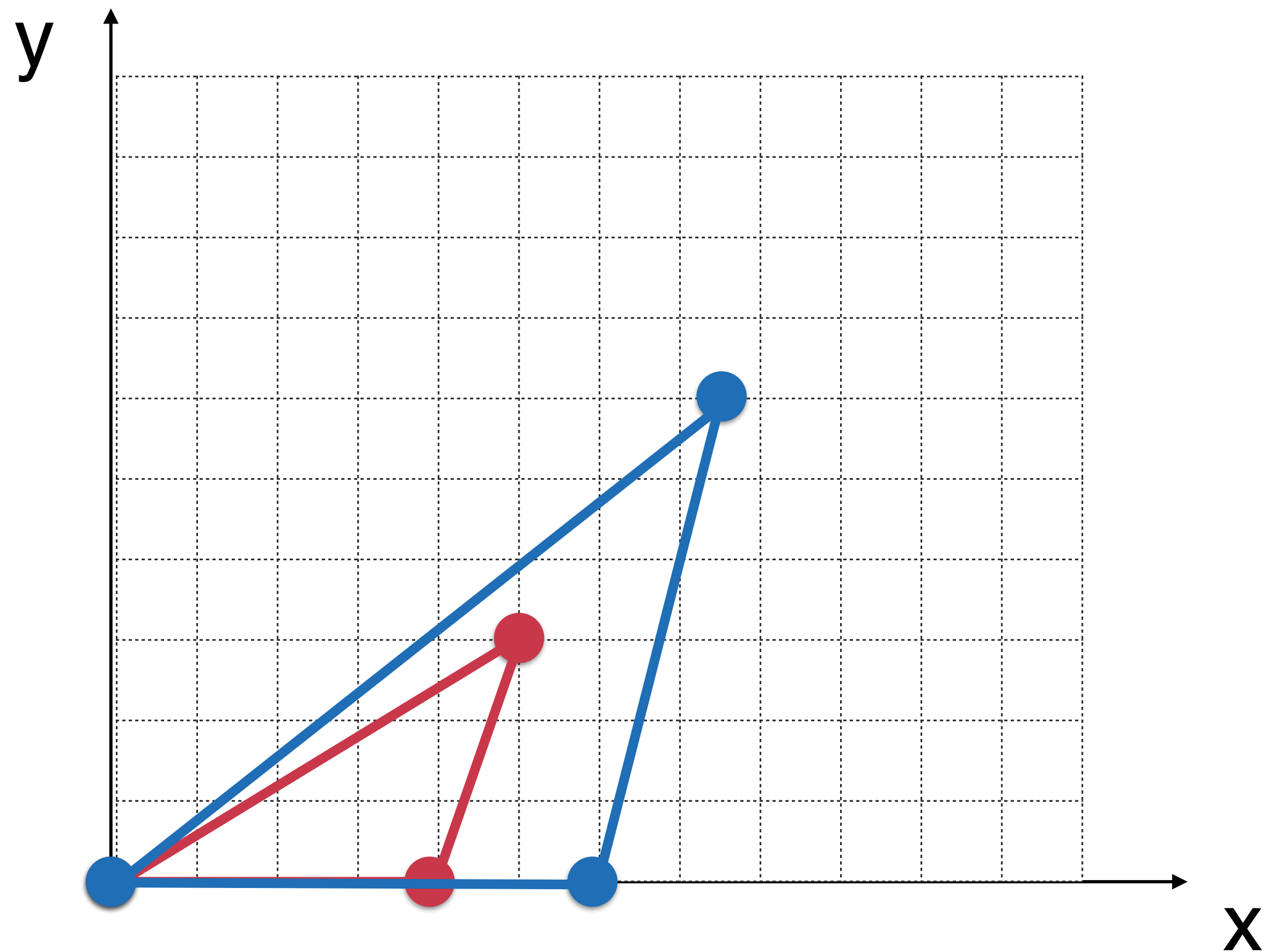
$$T = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix}$$

$$P = \begin{pmatrix} 0 & 4 & 5 \\ 0 & 0 & 3 \end{pmatrix}$$

\downarrow \downarrow \downarrow
 p_1 p_2 p_3

矩阵在图形变换中的应用

让每个点的横坐标扩大1.5倍，纵坐标扩大2倍

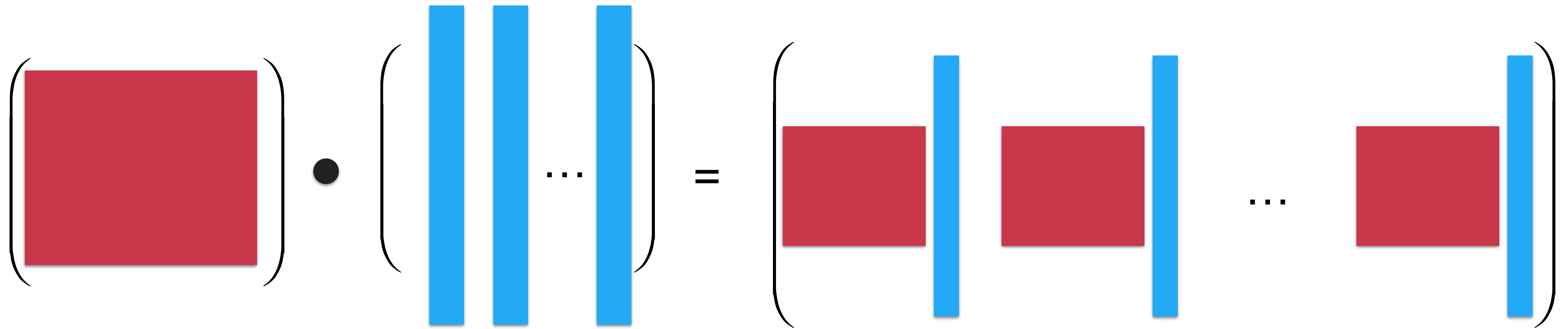


$$T = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix} \quad P = \begin{pmatrix} 0 & 4 & 5 \\ 0 & 0 & 3 \end{pmatrix}$$

$$T \cdot P = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 0 & 4 & 5 \\ 0 & 0 & 3 \end{pmatrix} \\ = \begin{pmatrix} 0 & 6 & 7.5 \\ 0 & 0 & 6 \end{pmatrix}$$

矩阵乘法

$$T \cdot P = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 0 & 4 & 5 \\ 0 & 0 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 6 & 7.5 \\ 0 & 0 & 6 \end{pmatrix}$$



矩阵A的列数必须和矩阵B的行数一致！

矩阵乘法

$$A \cdot B = A \cdot \begin{pmatrix} | & | & \dots & | \\ \overrightarrow{c_1} & \overrightarrow{c_2} & \dots & \overrightarrow{c_n} \\ | & | & & | \end{pmatrix} = \begin{pmatrix} | & | & \dots & | \\ A \cdot \overrightarrow{c_1} & A \cdot \overrightarrow{c_2} & \dots & A \cdot \overrightarrow{c_n} \\ | & | & & | \end{pmatrix}$$

$$\begin{pmatrix} - & \overrightarrow{r_1} & - \\ - & \overrightarrow{r_2} & - \\ & \dots & \\ - & \overrightarrow{r_m} & - \end{pmatrix} \cdot \begin{pmatrix} | & | & \dots & | \\ \overrightarrow{c_1} & \overrightarrow{c_2} & \dots & \overrightarrow{c_n} \\ | & | & & | \end{pmatrix} = \begin{pmatrix} \overrightarrow{r_1} \cdot \overrightarrow{c_1} & \overrightarrow{r_1} \cdot \overrightarrow{c_2} & \dots & \overrightarrow{r_1} \cdot \overrightarrow{c_n} \\ \overrightarrow{r_2} \cdot \overrightarrow{c_1} & \overrightarrow{r_2} \cdot \overrightarrow{c_2} & \dots & \overrightarrow{r_2} \cdot \overrightarrow{c_n} \\ \dots & \dots & & \dots \\ \overrightarrow{r_m} \cdot \overrightarrow{c_1} & \overrightarrow{r_m} \cdot \overrightarrow{c_2} & \dots & \overrightarrow{r_m} \cdot \overrightarrow{c_n} \end{pmatrix}$$

矩阵乘法

$$\begin{pmatrix} - & \overrightarrow{r_1} & - \\ - & \overrightarrow{r_2} & - \\ & \dots & \\ - & \overrightarrow{r_m} & - \end{pmatrix} \cdot \begin{pmatrix} | & | & & | \\ \overleftarrow{c_1} & \overleftarrow{c_2} & \dots & \overleftarrow{c_n} \\ | & | & & | \end{pmatrix} = \begin{pmatrix} \overrightarrow{r_1} \cdot \overleftarrow{c_1} & \overrightarrow{r_1} \cdot \overleftarrow{c_2} & \dots & \overrightarrow{r_1} \cdot \overleftarrow{c_n} \\ \overrightarrow{r_2} \cdot \overleftarrow{c_1} & \overrightarrow{r_2} \cdot \overleftarrow{c_2} & \dots & \overrightarrow{r_2} \cdot \overleftarrow{c_n} \\ \dots & \dots & & \dots \\ \overrightarrow{r_m} \cdot \overleftarrow{c_1} & \overrightarrow{r_m} \cdot \overleftarrow{c_2} & \dots & \overrightarrow{r_m} \cdot \overleftarrow{c_n} \end{pmatrix}$$

矩阵A的列数必须和矩阵B的行数一致！

A是m*k的矩阵； B是k*n的矩阵， 则结果矩阵为m*n的矩阵

矩阵乘法

A是m*k的矩阵； B是k*n的矩阵， 则结果矩阵为m*n的矩阵

矩阵乘法不遵守交换律！ $A \cdot B \neq B \cdot A$

很有可能根本不能相乘！

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

即使可以相乘， 结果也不一样

$$\begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

实现矩阵的乘法

实践：实现矩阵和向量的乘法

实践：实现矩阵和矩阵的乘法

矩阵乘法的更多性质和矩阵的幂

矩阵乘法的性质

矩阵乘法不遵守交换律！ $A \cdot B \neq B \cdot A$

矩阵乘法遵守： $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

$$A \cdot (B + C) = A \cdot B + A \cdot C$$

$$(B + C) \cdot A = B \cdot A + C \cdot A$$

对任意 $r \times c$ 的矩阵 A ，存在 $c \times x$ 的矩阵 O ，满足： $A \cdot O_{cx} = O_{rx}$

对任意 $r \times c$ 的矩阵 A ，存在 $x \times r$ 的矩阵 O ，满足： $O_{xr} \cdot A = O_{xc}$

矩阵乘法的性质

证明思路: $(A \cdot B) \cdot C = A \cdot (B \cdot C)$

假设A, B, C是任意矩阵:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1k} \\ a_{21} & a_{22} & \dots & a_{2k} \\ \dots & \dots & & \dots \\ a_{m1} & a_{m2} & \dots & a_{mk} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & & \dots \\ b_{k1} & b_{k2} & \dots & b_{kn} \end{pmatrix} \quad C = \begin{pmatrix} c_{11} & c_{12} & \dots & c_{1l} \\ c_{21} & c_{22} & \dots & c_{2l} \\ \dots & \dots & & \dots \\ c_{n1} & c_{n2} & \dots & c_{nl} \end{pmatrix}$$

矩阵乘法的性质

矩阵的幂：
$$A^k = \underbrace{A \cdot A \cdot \dots \cdot A}_k$$

只有方阵才可以进行矩阵的幂运算！

$$A^0 ? \quad A^{-1} ? \quad A^{-2} ?$$

下一章见分晓：)

矩阵乘法的性质

$$(A + B)^2 \neq A^2 + 2AB + B^2$$

$$(A + B)^2 = (A + B) \cdot (A + B)$$

$$= A \cdot (A + B) + B \cdot (A + B)$$

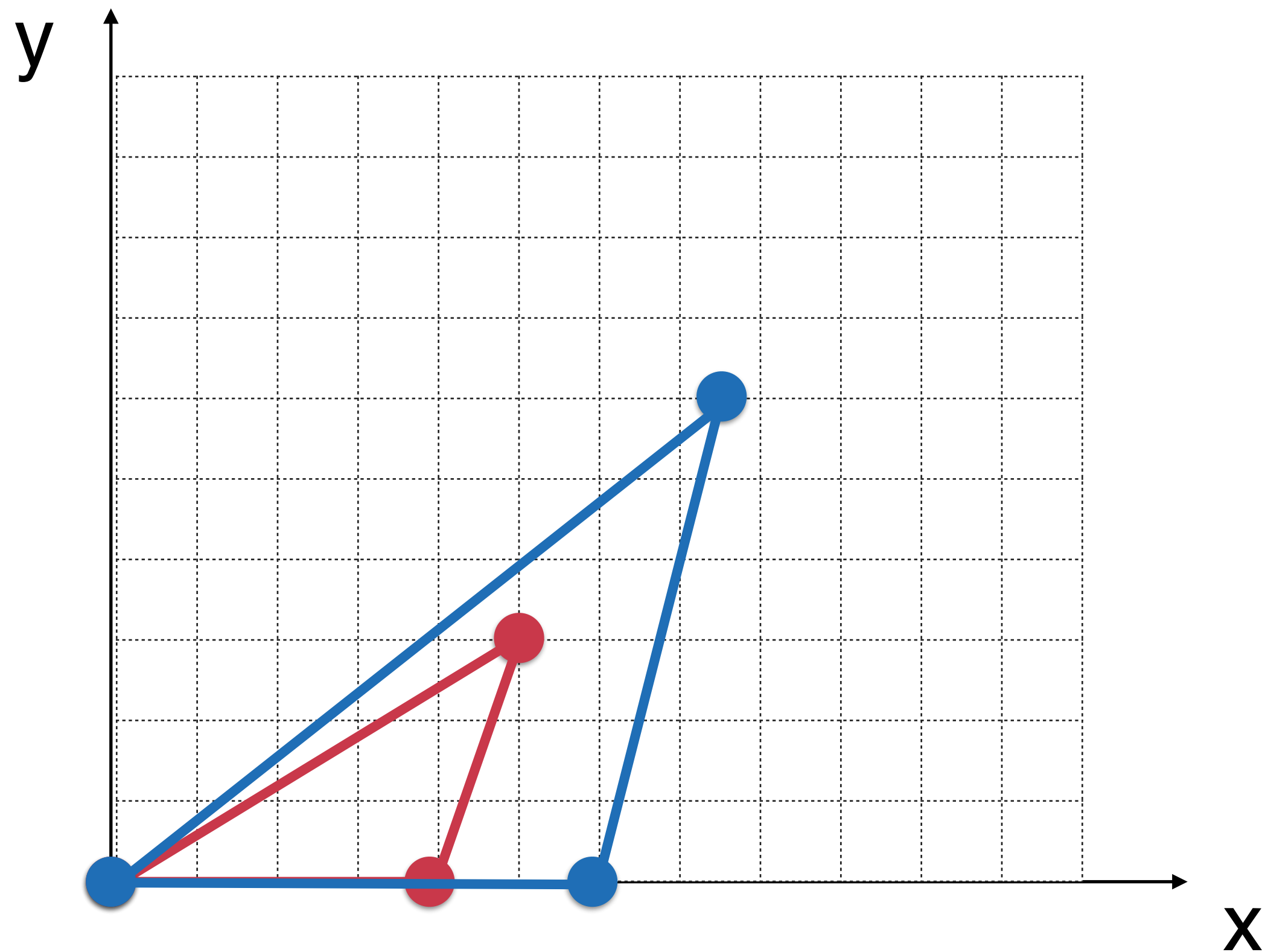
$$= A \cdot A + A \cdot B + B \cdot A + B \cdot B$$

$$= A^2 + A \cdot B + B \cdot A + B^2 \neq A^2 + 2AB + B^2$$

矩阵的转置

矩阵的转置

让每个点的横坐标扩大1.5倍，纵坐标扩大2倍



$$T = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix} \quad P = \begin{pmatrix} 0 & 4 & 5 \\ 0 & 0 & 3 \end{pmatrix}$$

$$T \cdot P = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} 0 & 4 & 5 \\ 0 & 0 & 3 \end{pmatrix} \\ = \begin{pmatrix} 0 & 6 & 7.5 \\ 0 & 0 & 6 \end{pmatrix}$$

矩阵的转置

$$T = \begin{pmatrix} 1.5 & 0 \\ 0 & 2 \end{pmatrix}$$

矩阵的转置：行变成列；列变成行

$$P = \begin{pmatrix} 0 & 0 \\ 4 & 0 \\ 5 & 3 \end{pmatrix}$$

$$P^T = \begin{pmatrix} 0 & 4 & 5 \\ 0 & 0 & 3 \end{pmatrix}$$

$$A = (a_{ij})$$

$$A^T = (a_{ji})$$

矩阵的转置

回忆： 行向量和列向量 $(3,4)$ $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

由于横版印刷原因，使用符号： $(3,4)^T$

矩阵转置的性质

$$(A^T)^T = A$$

$$(A + B)^T = A^T + B^T$$

$$(k \cdot A)^T = k \cdot A^T$$

矩阵转置的性质

证明思路: $(A + B)^T = A^T + B^T$

假设A, B是任意矩阵:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \dots & \dots & & \dots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{pmatrix}$$

矩阵转置的性质

$$(A^T)^T = A$$

$$(A + B)^T = A^T + B^T$$

$$(k \cdot A)^T = k \cdot A^T$$

$$(A \cdot B)^T = B^T \cdot A^T$$

矩阵转置的性质

证明思路: $(A \cdot B)^T = B^T \cdot A^T$

A是m*k的矩阵, B是k*n的矩阵

AB是m*n的矩阵, AB的转置是n*m的矩阵

A的转置是k*m的矩阵, B的转置是n*k的矩阵

(B的转置)(A的转置)是n*m的矩阵

矩阵不是简单的 $m*n$ 个数

实现矩阵的转置和numpy中的矩阵

实践： 实现矩阵的转置和numpy中的矩阵

矩阵不是简单的 $m*n$ 个数

其他

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专给程序员设计的线性代数

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