By Michael S. Yang
2024-03-28

Does this proof strategy work to further reinforce that the non-trivial riemann zeta function zeros all have real part = 1/2?

Function h(s): We have h(s) = (γ(s) / γ'(s)) * ζ(1 - s). This function aims to isolate prime power contributions from the Riemann zeta function (ζ(s)) using its derivative (γ(s) = ζ(s) / ζ(1 - s)).

Assumption: We assume a non-trivial zero (ρ) of ζ(s) exists off the critical line (Re(ρ) ≠ ½).

Function g(s): A function g(s) = (γ(s) / γ'(s)) - (1 / (s - ρ - 1)) is derived from h(s), with ρ shifted by 1. γ'(ρ) ≠ 0: This is an important assumption established earlier. It ensures we don't divide by zero when constructing g(s).

L'Hôpital's Rule: Both the numerator and denominator of g(s) approach zero as s approaches ρ - 1 (due to the properties of γ(s) and the zero of ζ(s) at ρ). Under certain conditions, L'Hôpital's Rule can be applied to evaluate the limit. Non-Zero Residue for g(s): Applying L'Hôpital's Rule (assuming its conditions hold) likely leads to a non-zero limit as s approaches ρ - 1. This implies a non-zero residue for g(s) at that point.

Contradiction with h(s): The subproof previously established that h(s) needs to have a simple pole at s = 1. A simple pole has a non-zero residue. However, the non-zero residue of g(s) at s = ρ - 1 (different from s = 1) contradicts the requirement for h(s) to have a simple pole specifically at s = 1.

This contradiction implies all non-trivial zeros of the Riemann Zeta function have real part = 1/2.