

Does this proof strategy work to further reinforce that the non-trivial Riemann zeta function zeros all have real part = $1/2$?

Function $h(s)$: We have $h(s) = (\gamma(s) / \gamma'(s)) * \zeta(1 - s)$. This function aims to isolate prime power contributions from the Riemann zeta function ($\zeta(s)$) using its derivative ($\gamma(s) = \zeta(s) / \zeta(1 - s)$).

Assumption: We assume a non-trivial zero (ρ) of $\zeta(s)$ exists off the critical line ($\text{Re}(\rho) \neq 1/2$).

Function $g(s)$: A function $g(s) = (\gamma(s) / \gamma'(s)) - (1 / (s - \rho - 1))$ is derived from $h(s)$, with ρ shifted by 1. $\gamma'(\rho) \neq 0$: This is an important assumption established earlier. It ensures we don't divide by zero when constructing $g(s)$.

L'Hôpital's Rule: Both the numerator and denominator of $g(s)$ approach zero as s approaches $\rho - 1$ (due to the properties of $\gamma(s)$ and the zero of $\zeta(s)$ at ρ). Under certain conditions, L'Hôpital's Rule can be applied to evaluate the limit. Non-Zero Residue for $g(s)$: Applying L'Hôpital's Rule (assuming its conditions hold) likely leads to a non-zero limit as s approaches $\rho - 1$. This implies a non-zero residue for $g(s)$ at that point.

Contradiction with $h(s)$: The subproof previously established that $h(s)$ needs to have a simple pole at $s = 1$. A simple pole has a non-zero residue. However, the non-zero residue of $g(s)$ at $s = \rho - 1$ (different from $s = 1$) contradicts the requirement for $h(s)$ to have a simple pole specifically at $s = 1$.

This contradiction implies all non-trivial zeros of the Riemann Zeta function have real part = $1/2$.