

# Report

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## 1 Introduction

This paper introduces the phenomena of loss aversion into the Santa-Fe Institute Artificial Stock Market (SFIASM) (Arthur et al., 1997; LeBaron et al., 1999). We use the agent-based computational approach, SFIASM model, to explore the effect of loss aversion on the financial market dynamics. The aim of this paper is to explore if the investors' loss aversion can explain some of the financial anomalies introduced by Cont (2001): leverage effect and conditional heavy tails in stock returns. In this report, we focus on leverage effect in financial market which refers to most measures of volatility of an asset are negatively correlated with the returns of that asset.

Traditional asset pricing models based on rational expectations and homogeneity have problems explaining the complex and volatile nature of financial markets. The heterogeneity in expectations can lead to market instability and complicate dynamics of prices, which are driven by endogenous market forces (Rekika et al., 2014). In fact, when investors in financial market form their beliefs about future asset return, their expectations are influenced by their behavioral biases such as herding, loss aversion, anchoring and other behavioral biases (Barberis and Thaler, 2003). The phenomena that traditional financial models have trouble to explain are often called “puzzles”. The difficulty in explaining the empirical puzzles using the traditional representative agent structure has driven the development of agent-based models via which we can look deeper into the price dynamics of financial market caused by the investors' behavior. The agent-based model, SFIASM, is developed to facilitate the understanding of some areas that traditional homogeneous models cannot explain very well. SFIASM is an endogenous-expectations market and provides Genetic Algorithm condition/forecast classifier learning (Arthur et al., 1997). By focusing on the dynamics of learning, the SFIASM identifies the learning speed of traders which is able to switch the model to either a regime that is close to the homogeneous rational expectation equilibrium, or to a more complex regime, partially revealing rational expectation equilibrium (LeBaron et al., 1999). Investors in SFIASM update their forecasting rule by learning and then update their expectations of future price and dividend. Updating the forecasting rule does not only affect the changes in the expectation about future but also may influence the cognition of the market. We introduce loss aversion into SFIASM to allow the investors to form their expectations of changes (gains or losses) in their financial wealth based on their selected forecasting rules and information of the market state.

Prospect Theory (PT) is proposed in the seminal paper of Kahneman and Tversky, (1979) which describes loss aversion as the “mental penalty” associated with a given loss is greater than the mental reward from a gain of the same size. According to PT, if investors are loss averse, they may be reluctant to realize losses and may even take increasing risks to escape from a losing position. One of the most cited applications of PT is the “disposition effect” (Shefrin and Statman 1985) which refers to a tendency among investors to hold assets in a loss position too long and to sell assets in gain position too quickly. This provides a viable explanation for “averaging down” investment tactics, whereby investors increase their exposure to the falling stocks, in an attempt to recoup prior losses. Shefrin (2001) terms this phenomenon “escalation bias”. In the model presented in this paper, the investors become passive when they expect losses in the future.

Shimokawa et al. (2007) incorporate two features of PT, the existence of a reference point and differences in the treatment of gains and losses, into an agent-based model, introducing a piece wise exponential utility (CARA)

function depending on gains/losses into the model and show that both an increase in aversion to loss and a decrease in information precision amplify the stock price distortion. Following Shimokawa et al. (2007)’s way to introduce loss aversion into agent-based model, Polach and Kukacka (2016) introduced a PT extension of the Brock and Hommes (1998a) ABS framework and investigate behavior and statistical properties using Monte Carlo simulation. Polach and Kukacka (2016) assessed and identified 3 stylized facts: absence of autocorrelation of returns, fat tails and volatility clustering in their extended model but their simulations showed the overall stability of the market increased if PT investors exist in the market. Shimokawa et al. (2007) and Polach and Kukacka (2016) take the moving average of prices over certain previous periods (they tried 1, 5, 10 and 15) as the reference point and so investors in their models compare the expectation of future price and this moving average to form gains/losses. Although the authors do not use the original LeBaron et al. (1999), Shimokawa et al. (2007) and Polach and Kukacka (2016) propose a relatively straightforward method to implement PT features into agent-based models in which the agents have CARA preferences. SFIASM which is built based the asset pricing model (Grossman 1976, Grossman and Stiglitz 1980) also adopted CARA utility function. We introduce into SFIASM the piece wise utility which depends on gains/losses as in Shimokawa et al. (2007) and Polach and Kukacka (2016). However, we replace the reference point with the status quo scaled up by the risk-free rate (Barberis, Huang and Santos 2001), assuming that the gains and losses refer to changes in the value of the investor’s financial wealth over two successive periods.

Incorporating loss aversion into SFIASM allows for investigation of the impact of loss-averse investors’ behavior on price dynamics. Applying Nelson’s (1990) EGARCH model to test time series generated from SFIASM (LeBaron et al. 1999) and SFIASM with loss aversion, we find that statistically the leverage effect exists in both the two SFIASM models and the leverage effect is larger in SFIASM with loss aversion relative to LeBaron et al. (1999). In our SFIASM with loss aversion, an unanticipated drop in price increases predictable volatility more than an unexpected increase in price of similar magnitude.

## 2 Santa Fe Institute Artificial Stock Market with Loss-averse Investors

In this section, we develop behavioral extension based on PT of SFIASM model (LeBaron et al. 1999).

### 2.1 Overall Structure of the Market

SFIASM consists of a central computational market and a number of artificially intelligent investors. There are  $N$  investors ( $i = 1, 2, \dots, N$ ) interacting with each other via the central market. In all our experimental treatments,  $N = 25$ . Time is broken up into discrete time periods  $t = 1, 2, \dots, T$ .

The investors can either invest in a risk free or in a risky asset. The risk free asset, bond, is in perfect elastic supply and pays a fixed rate of return  $r$ ; the risky asset, stock, with price  $p_t$  (ex-dividend) per share at time  $t$ , pays a stochastic dividend,  $d_t$ . The dividend times series  $d_t$  is itself a stochastic process defined independently of the market and the investors’ actions. Following LeBaron et al. (1999), we use Ornstein-Uhlenbeck process, given by

$$d_{t+1} = \bar{d} + \rho (d_t - \bar{d}) + \epsilon_{t+1} \quad (1)$$

where  $\bar{d}$  is the baseline dividend,  $\rho$  is the persistence parameter, and  $\epsilon_t$  is Gaussian, i.i.d. and  $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ . Following LeBaron et al. (1999), we assume that  $\bar{d} = 10$ ,  $\rho = 0.95$  and  $\sigma_\epsilon = 0.07429$ .

The investors have to decide how much money they want to invest into the stock (which has a fixed total number

of shares - if somebody buys, someone else has to sell), and how much they want to put in the bond. At any time  $t$ , each investor  $i$  holds some number of shares of stock  $x_t^i$  and has some amount of bonds. The investor's wealth  $W_t^i$  dynamics is then

$$W_{t+1}^i = (1 + r) W_t^i + x_t^i (p_{t+1} + d_{t+1} - (1 + r) p_t) \quad (2)$$

In our experimental treatments, for each investor, the initial wealth  $W_0$  is 20,000 units of cash and the initial holding of the stock  $x_0$  is 1 share, following LeBaron et al. (1999).

There exist a fixed amount of shares of the stock available at the beginning of each period  $t$ . Stock shares are assumed to be divisible, i.e., investors can purchase share fractions. In all our experimental treatments, the number of shares is fixed as 25 which was used in LeBaron et al. (1999). The stock trading process is managed by a specialist, the market maker, inside the market. The specialist also has the job of setting,  $p_t$ . Its fundamental problem at each time step is that the number of bids to buy and offers to sell may not match, and yet the total number of shares of stock is fixed. We adopt auction approach to this issue in the sense that the market maker works as an auctioneer. If there are more bids than offers, then the price is raised, so the bids drop and the offers increase, until they match closely. The market clearing price,  $p_t$ , is determined by equating the total demand for the stock to the supply of the stock, i.e.

$$\sum_{i=1}^{N=25} x_t^i = 25 \quad (3)$$

At the start of time period  $t$ , the current dividend  $d_t$  is posted and observed by all investors. Investors then use this information and general information on the state of the market, including the historical dividend sequence  $\{\dots d_{t-2}, d_{t-1}, d_t\}$  and price sequence  $\{\dots p_{t-2}, p_{t-1}\}$ , to form their expectations of the next period's price and dividend.

## 2.2 Loss-averse Investors

Following Shimokawa et al. (2007) and Polach and Kukacka (2016), to introducing loss-averse investors into SFIASM (LeBaron et al. 1999) raises 2 following issues:

- (i) how does the investor measure her gains and losses?
- (ii) how does the investor's utility depend on the gains and losses?

### 2.2.1 Measuring Gains and Losses

The gains and losses in our model refer to changes in the value of the investor's financial wealth, even though this is only one component of his overall wealth. We suppose that the investor cares only about fluctuations in the value of the risky asset. Each agent's wealth dynamics follows the equation (2). Gains and losses are defined to some neutral reference point which usually corresponds to the current asset position (Kahneman and Tversky, 1979). We take the reference point for investor  $i$  to be her current wealth scaled up by the risk-free rate,

$$W_t^{i0} = W_t^i (1 + r) \quad (4)$$

Investor  $i$ 's realized Gains/Losses over periods  $t$  and  $t + 1$  is given by

$$\Delta W_{t,t+1}^i = W_{t+1}^i - W_t^{i0} = x_t^i (p_{t+1} + d_{t+1} - (1 + r) p_t) \quad (5)$$

If  $p_{t+1} + d_{t+1} - (1 + r)p_t > 0$ , then the investor  $i$  get gains,  $\Delta W_{t,t+1}^+$ . If  $p_{t+1} + d_{t+1} - (1 + r)p_t < 0$ , then the investor  $i$  get losses,  $\Delta W_{t,t+1}^-$ . The idea here is that with a riskless return of  $r$ , the investor  $i$  is prone to being disappointed if her stock market return is less than  $r$ .

Investor  $i$ 's expected Gains/Losses over periods  $t$  and  $t + 1$  is given by

$$E_{i,t} [\Delta W_{t,t+1}^i] = x_t^i (E_{i,t} [p_{t+1} + d_{t+1}] - (1 + r)p_t) \quad (6)$$

At time period  $t$ , the loss-averse investor  $i$  forecasts gains/losses in period  $t + 1$ . If  $E_{i,t} [p_{t+1} + d_{t+1}] - (1 + r)p_t \geq 0$ , then the investor  $i$  expects gains at period  $t$ . If  $E_{i,t} [p_{t+1} + d_{t+1}] - (1 + r)p_t < 0$ , then the investor  $i$  expects losses at period  $t$ .

### 2.2.2 Utility depending on Expectation of Gains and Loss

We formulate the loss-averse investor  $i$ 's utility function based on PT in accordance with the characteristic of decision making that people tend to estimate losses larger than losses. Thus, we use the utility function involving loss aversion feature according to Shimokawa et al. (2007) and Polach and Kukacka (2016) as follows:

$$U(W_{t+1}^i) = \begin{cases} -\exp(-\gamma W_{t+1}^i), & \text{if } E_{i,t} [p_{t+1} + d_{t+1}] - (1 + r)p_t \geq 0 \\ -\exp(-\gamma \lambda W_{t+1}^i), & \text{if } E_{i,t} [p_{t+1} + d_{t+1}] - (1 + r)p_t < 0 \end{cases} \quad (7)$$

where  $\lambda = 4$  is the loss aversion parameter and  $\gamma = 0.5$  is the risk aversion parameter.

Investor  $i$ 's predictions at time  $t$  of next period's price and dividend are normally distributed, with conditional mean and variance,  $E_{i,t} [p_{t+1} + d_{t+1}]$  and  $\sigma_{i,t,p+d}^2$ . Under the piece-wised CARA utility function (7) and normal distribution for predictions, the optimal share demand that investor  $i$  desires to hold is determined as follows:

$$x_{i,t}^* = \begin{cases} \frac{E_{i,t} [p_{t+1} + d_{t+1}] - (1 + r)p_t}{\gamma \sigma_{i,t,p+d}^2}, & \text{if } E_{i,t} [p_{t+1} + d_{t+1}] - (1 + r)p_t \geq 0 \\ \frac{E_{i,t} [p_{t+1} + d_{t+1}] - (1 + r)p_t}{\gamma \lambda \sigma_{i,t,p+d}^2}, & \text{if } E_{i,t} [p_{t+1} + d_{t+1}] - (1 + r)p_t < 0 \end{cases} \quad (8)$$

## 2.3 Formation of Expectations

Investors form their expectations about the future price, dividend and gains/losses individually. Each investor at any time possesses a multiplicity of linear forecasting rules and uses those that are both best suited to the current state of the market and have recently proved most reliable. Investors then learn by discovering which of their rules prove best and by occasionally developing new rules via the genetic algorithm, that attempts to develop useful pieces of good rules into even better rules.

Each investor possesses an individual set of  $M = 100$  forecasting rules (or predictors). Investors build forecasts using what are called "if condition-then forecast" rules. Each forecasting rule generates an expectation of the sum of next period's stock price and dividend together with an updated estimate of the rule's "forecast variance". Each of these rules contains both a market condition that may at times be fulfilled by the current state of the market and a forecasting formula for the next period's price and dividend. A forecast is derived according to "if (condition fulfilled), then (use forecasting formula to derive forecast)". The condition part determines when each particular predictor is activated, as explained below. Only activated predictors produce forecasts, using their forecast part.

Following LeBaron et al. (1999), the current market state is described by a 12-bit array MarketState. The condition part of each predictor is implemented with a classifier system, in which the condition part is represented by a ternary string of the symbols  $\{0, 1, 2\}$ . A 1-value signals the occurrence of the described state; 0 signals non-occurrence; and 2 signals don't care. A condition array matches the current market state if all its 0's and 1's match the corresponding bits for the market state with the 2's matching either a 1 or a 0. Fewer 2's contained in the condition part of a rule indicates a more-particularized rule. For example, if the second bit corresponds to "the opportunity cost of holding a share of the stock is greater than half of its dividend", a forecasting rule with condition part (212222222222) will be activated as long as this market state occurs. The set of states in our model includes both fundamental and technical information. The fundamental information will be based on price-dividend ratios, and the technical information will use moving average types of trading rules. Each element corresponds to whether the conditions in Table 1 are true or false.

Table 1

Bit	Market state descriptors
1	$\frac{pr}{d} > \frac{1}{4}$
2	$\frac{pr}{d} > \frac{1}{2}$
3	$\frac{pr}{d} > \frac{3}{4}$
4	$\frac{pr}{d} > \frac{7}{8}$
5	$\frac{pr}{d} > 1$
6	$\frac{pr}{d} > \frac{9}{8}$
7	$p > 5\text{-period MA}$
8	$p > 10\text{-period MA}$
9	$p > 100\text{-period MA}$
10	$p > 500\text{-period MA}$
11	On: 1
12	Off: 0

We need to clarify what is price-dividend ratios. Palmer, Arthur, Holland and LeBaron (1999) defines this ratio as  $\frac{p_t r}{d_t}$  because this quantity greater (or less) than 1 indicates that  $p_t$  is greater than (or less) the stock's previous-period fundamental value. As we define the current wealth scaled up by  $r$  as the reference point, the loss-averse investors in our model choose  $\frac{p_{t-1} r}{d_t}$  as the fundamental measure of the market state to compare the returns of bond and stock.

The forecast part, is built as linear functions of current prices and dividends,

$$E_{ij,t}(p_{t+1} + d_{t+1}) = a_{ij,t}(p_t + d_t) + b_{ij,t} \quad (9)$$

where  $E_{ij,t}$  denotes the expected (predicted) value for investor  $i$ 's  $j$ th predictor, and  $a_{ij,t}$  and  $b_{ij,t}$  are the coefficients that constitute the forecast part of this predictor.

The conditional part and the linear forecast parameters  $a_{ij}$  and  $b_{ij}$  of the initial forecasting rules are randomly generated. The conditional variance estimates  $\hat{\sigma}_{i,j}^2$  for all  $i$  and  $j$  are initially set equal to 4. One period later, the accuracy of all selected rules is checked by comparing their predictions  $E_{ij,t}(p_{t+1} + d_{t+1})$  with the actual realization of  $p_{t+1} + d_{t+1}$ . A rule's forecast accuracy is the inverse of its weighted average of squared forecast error, which is

determined as

$$v_{ij,t}^2 = \left(1 - \frac{1}{\theta}\right) v_{ij,t-1}^2 + \frac{1}{\theta} [(p_t + d_t) - E_{ij,t-1}(p_t + d_t)]^2 \quad (10)$$

The value  $v_{ij,t}^2$  then becomes its conditional variance estimates  $\hat{\sigma}_{ij,t}^2$ . For all experiments in this paper, the parameter related to the time horizon,  $\theta$ , is fixed at 75 to leave the evolutionary frequency as the unique control variable that affects the learning quality. As in LeBaron et al. (1999), a value of 75 is chosen for  $\theta$ .

At each point in time, from the set of 100 individual forecasting rules that each investor possesses, more than one normally matches the specified market condition. Each activated rule will be assigned a probability of being used based on its fitness, which is calculated as

$$f_{ij,t} = -v_{ij,t}^2 - C \times \text{specificity} \quad (11)$$

where  $C = 0.05$  is Bit Cost and specificity is the number of conditions in a rule that are not ignored. A rule with a high fitness receives a higher probability to be selected than a rule with a low fitness. Then, the conditional variance estimate  $\hat{\sigma}_{ij,t}^2$  of the selected rule becomes  $\hat{\sigma}_{i,t,p+d}^2$  in Equation (8).

In our code, the investor  $i$ 's  $j$ th predictor at time  $t$  is defined as, 12-bit descriptor + vector( $a_{ij,t}, b_{ij,t}, v_{ij,t}^2$ ). The investors use the current forecasting rule to form the expectation of gains/losses in next period, and we use a binary number 0 or 1 as the indicator of gains(0) and losses(1). We talk about in detail how to form the expectation of gains/losses using forecasting rule in section 2.4.

## 2.4 Time Line and Genetic Algorithm

### 2.4.1 Time Line of Activities

The following time line is for an arbitrary period  $t$  greater than 1.

- At the beginning of period  $t$ , a new dividend  $d_t$  earned by each outstanding share of stock is announced observed by all investors.
- Each investor  $i$  then proceeds to use the newly posted dividend information, together with general information on the state of the market (including any past observed dividends and prices), to determine which of his forecasting rules is activated in the current period.
- From the active rule set, agents select the rule with the highest fitness.
- The price formation process is then initiated by the auctioneer:
  - 1) The auctioneer announces the first trial price  $p_{\text{trial1}}$  equal to last period's price  $p_{t-1}$
  - 2) Based on this trial price, investors form their expectation about next period's price, dividend and gains/losses, determine their optimal demand for stock, and then submit their offers and bids to the auctioneer. Note: different trial price may lead to different expectation of gains/losses.
  - 3) If the bids and offers cannot be matched, the auctioneer determines a trial price according to the pricing function as follows,

$$p_{\text{trial}} = p_{\text{trial}} + \alpha \left( \sum_{i=1}^{N=25} x_{i,t} - 25 \right) \quad (12)$$

where  $\alpha$  is a scalar which is sensitive to different excess demand or supply. We chose different  $\alpha$  according to specific interval of excess demand or supply. Knowing  $p_{\text{trial}}$ , the investor  $i$  forms expectation of gains/losses as

$$E_{ij,t}(p_{t+1} + d_{t+1}) - p_{trial}(1 + r) = a_{ij,t}(p_{trial} + d_t) + b_{ij,t} - p_{trial}(1 + r) \quad (13)$$

And the whole process starts all over. This iterative process ends when the offers are balanced by the submitted bids or after 1000 unsuccessful trial rounds. In the latter case, one side of the market will be proportionally rationed. The last trial price is announced to be the stock price for period  $t$  and all trades between investors are executed at that price.

- After the current market clearing price  $p_t$  is realized, the dividend and the stock price in period  $t$  are known to the investors, they are now able to update the forecast performance of all rules which were activated in the last period and actually produced a forecast about this period's price and dividend.
- Subsequent to this fitness reassessment, with some fixed probability  $p_{learning}$  each trader updates her entire forecasting rule set by means of a genetic algorithm involving elitism, recombination, and mutation operations.
- At the start of the next period  $t + 1$ , a new exogenously determined dividend  $d_{t+1}$  is posted, and the process repeats.

### 2.4.2 Genetic Algorithm

Genetic algorithm is a method for moving from one population of “chromosomes” to a new population by using a kind of “natural selection” together with the genetics-inspired operators of crossover, mutation, and inversion (Mitchell Melanie, 1998). Genetic Algorithms, originated by Holland (1975), are commonly used to generate high-quality solutions to optimization and search problems.

Some of a investor's predictors may give good predictions when they are activated, while others may not. A genetic algorithm is used to adjust and evolve a better set of predictors. For each investor at each period we run the genetic algorithm with probability  $p_{learning}$ . The genetic algorithm eliminates some of the worst predictors (those than have the highest variance) and generates some new ones to replace them. We replace 20 out of 100 predictors following LeBaron et al. (1999).

**Elimination and Elitism:** For all rules, the worst-performing (least fit) rules are eliminated. In all experimental treatments, this replacement rate is set at 20 percent.

**Replacement by Offspring:** The eliminated rules are replaced with new “offspring” forecasting rules that are formed as variants of “parent” forecasting rules selected from among the retained (elite) rules. With probability  $P$  (where  $P$  depends on the experimental treatment,  $P = 0.1$  for fast learning regime and  $P = 0.3$  for slow learning regime), the offspring forecasting rule is generated purely by crossover operations. With probability  $1 - P$ , the offspring forecasting rule is generated purely by mutation operations.

### 3 Experiments

#### 3.1 Experimental Design

In LeBaron et al. (1999), agents who learned, on average, once every 1000 time periods were described as slow-learning agents, and those who learned, on average, once every 250 time periods were described as fast-learning agents. In this paper, we follow LeBaron et al. (1999)'s definitions of slow and fast learning. To investigate the price dynamics caused by loss aversion in SFI-ASM, we use the results of LeBaron et al (1999) as the baseline. To make our results comparable with LeBaron et al (1999), the CARA market of LeBaron et al (1999) and our loss-averse market have the same initial population, initialization period (500 steps) before trading begins and dividend process and the CARA and loss-averse investors trade in their own markets respectively. The common parameters for the experiments are summarized in Table 2. Moreover, LeBaron et al (1999) solved for a homogeneous linear rational expectation equilibrium (REE) conjecturing a linear function mapping the current state into a price (REE price),

$$p_t = f d_t + e \quad (14)$$

where  $f = \frac{\rho}{1+r-\rho}$ ,  $e = \frac{\bar{d}(f+1)(1-\rho)-\gamma\sigma_{p+d}^2}{r}$ .

In the first experiment, investors have loss aversion and fast learning. To avoid the initial conditions problem, 25 independent runs are implemented, and each run lasts for 270,000 market periods ( $T = 270,000$ ). (In this report, I present results from only 1 run of simulated data.)

Table 2: Value of common parameters

Parameter	Simulation value
Degree of risk aversion $\gamma$	0.5
Degree of loss aversion $\lambda$	4
Baseline dividend $\bar{d}$	10
Autoregressive persistence $\rho$	0.95
Risk-free interest rate $r$	0.1
Initial Conditional variance $\sigma_{p+d}^2$	4
Initial wealth per investor $W_0$	20,000
Initial share per investor $x_0$	1
Number of forecasting rules one possesses $M$	100
Number of bits in a descriptor $J$	12
Crossover and mutation rate $P_c, P_m$	0.1, 0.9
Number of investors $N$	25
Number of stock shares $N$	25
$a$ range	[0.7, 1.2]
$b$ range	[10.0, 19.0]



### 3.2 Time Series Features

To generate stock price time series, the market was run for 270,000 time periods to allow sufficient learning, and for early transient to die out. Then time series were recorded for 20,000 time periods after period 250,000. Figure 1 displays prices generated from CARA and LA markets (CARA price and LA price) from period 255,000 to 255,200. Figure 2 displays prices for 1000 period after period 255,000.

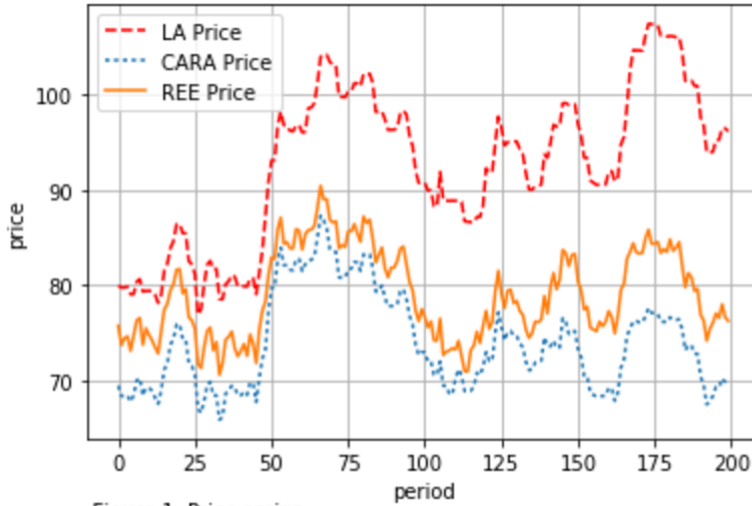


Figure 1. Price series.  
Starting period=255000, solid line=REE price, dotted line=CARA price, dash line=LA price

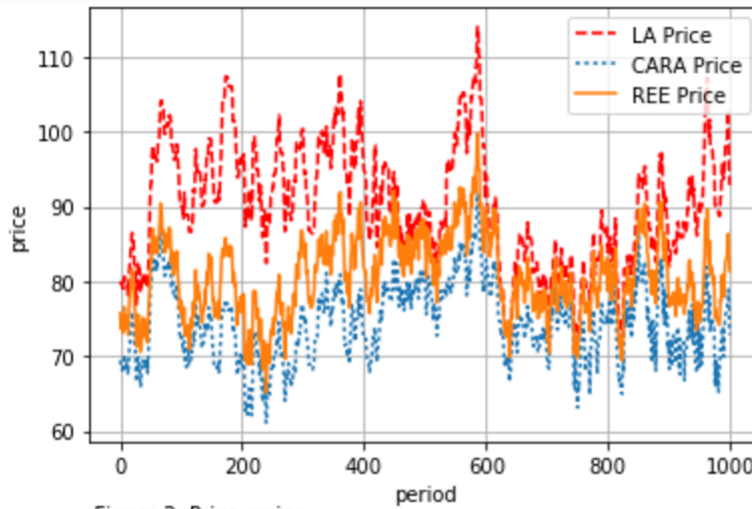


Figure 2. Price series.  
Starting period=255000, solid line=REE price, dotted line=CARA price, dash line=LA price

As shown in the two graphs above, the REE price is almost always in the middle of the LA and the CARA prices. Since there is stochastic learning going on and different gains/losses expectations in LA market, the LA price would always have an extra amount of variability relative to the REE benchmark and our experimental baseline CARA. From figure 1 or 2, we see both the LA and CARA prices sharply fall down at some period around period 70 or 590 and seemingly take up more fluctuations in some periods after this crash than in some periods before the crash. It seems to be that “bad news” (decrease in price) increases the fluctuation more than “good news” (increase in price) in these two markets, which is called leverage effect. And from figure 1, we observe that the overall magnitude of the fluctuations in LA prices is larger than CARA prices. A question rises that whether the leverage effect is more significant in LA market relative to CARA market. In next section, we use a statistical estimation to test whether the leverage effects exist in the two markets and which is larger if exist.

### 3.3 Test for Leverage Effect

Statistically, the leverage effect occurs when an unanticipated drop in price increases predictable volatility more than an unexpected increase in price of similar magnitude. One method proposed by Engle and Victor (1993) to capture such asymmetric effects is Nelson's (1990) EGARCH model. As the investors are myopic one-period decision makers in our model, we adopt EGARCH(1,1) model (lag 1 forecast) as follows:

$$\log(h_t) = \omega + \beta \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}} + \alpha \left[ \frac{|\epsilon_{t-1}|}{\sqrt{h_{t-1}}} - \sqrt{\frac{2}{\pi}} \right] + \zeta \log(h_{t-1}) \quad (15)$$

where  $h_t$  is the conditional variance of price and  $\epsilon_t$  (so-called news) is white noise in the first-order autoregression model of price. If  $\alpha > 0$ , Nelson's model implies that a deviation of  $\frac{|\epsilon_{t-1}|}{\sqrt{h_{t-1}}}$  from its expected value. If  $\beta = 0$ , then an unexpected increase in price has the same effect on volatility as an unexpected decrease in price of the same magnitude. If  $-1 < \beta < 0$ , then an unexpected increase in price has less effect on volatility than an unexpected decrease in price. If  $\beta < -1$ , then an unexpected increase in price actually reduces the volatility while an unexpected decrease in price increases the volatility. We test price time series of 5,000 periods (from 250,000 to 255,000) generated from LA and CARA markets using EGARCH(1,1) model, and table 3 shows the results of the test.

Table 3

Market	Coefficient	Value	Std. Err.	Prob.
CARA	$\alpha$	0.0304	0.0089	0.001
	$\beta$	-0.0384	0.0066	0.0
LA	$\alpha$	0.1740	0.0051	0.0
	$\beta$	-0.0601	0.0065	0.0

The negative  $\beta$  in Table 3 implies that unanticipated price decreases increase the volatilities in the two markets more than unanticipated price increases. This is an indication for a leverage effect in these two markets. The absolute value of  $\beta_{LA}$  is nearly twice as large as  $\beta_{CARA}$ , which indicates the leverage effect in LA market is larger compared with CARA market. To quantitatively compare leverage effects in these two markets, we draw the news impact curves (Engle and Victor 1993) of the EGARCH(1,1) model above, which are displayed in Figure 3.

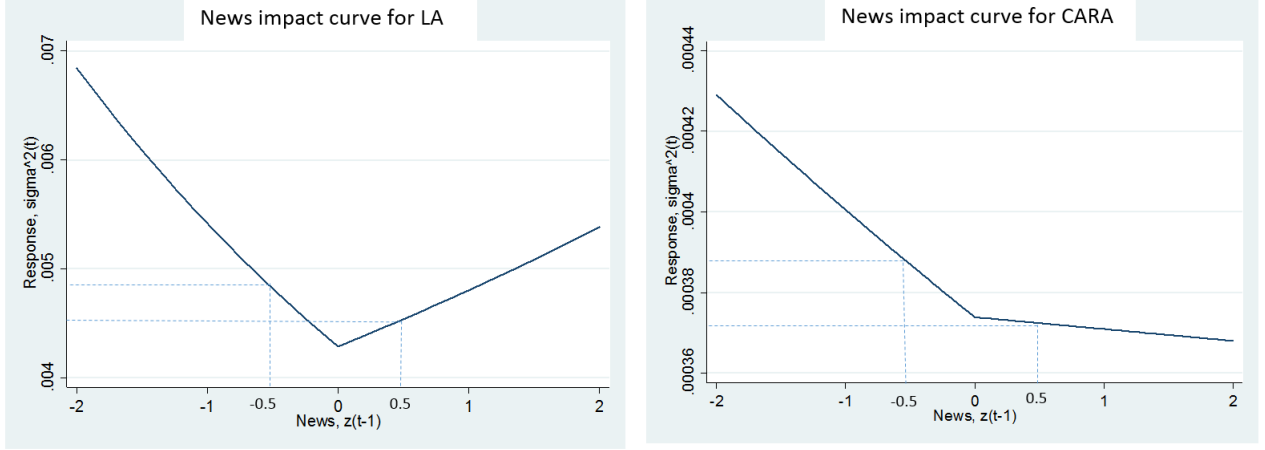


Figure 3: News Impact Curves, the response in the conditional variance to an innovation in the standardized error term.

The vertical axis represents the response in the conditional variance,  $\sigma_t^2$  (SATA standard notation) which is  $h_t$ , and the horizontal axis shows the an innovation in the standardized error term,  $z_{t-1}$  (SATA standard notation) which is  $\epsilon_{t-1}$ . If we take a news with magnitude of 0.5, then the responses in the conditional variance to good news (0.5) and bad news (-0.5) in LA market are about 0.048 and 0.046 correspondingly, while the responses in the conditional variance to such good and bad news in CARA market are around 0.000385 and 0.000375. According to the news impact curves in Figure 3, the leverage effect of a news with magnitude of 0.5 is about  $0.048 - 0.046 = 0.002$  in LA market, and the leverage effect of a news with magnitude of 0.5 is about  $0.000385 - 0.000375 = 0.00001$ . We did not measure all possible leverage effects of all possible news shown in the curves above. As the magnitudes of the response in LA and CARA markets are measured by 0.001 and 0.0001 respectively, the leverage effect is much larger in LA market relative to CARA market if the news is not close to 0.

## 4 Discussion

According the result in section 3.3, it indicates that loss aversion may explain the why leverage effect exists in the financial market. But this conclusion is only based on the simulation of 1 run of SFIASM. We need to get simulation data from more runs to avoid the initialization bias.

We store the data of evolution of the forecasting rule for each investor. We also store the data of fundamental and technical measures of the market state for each investor. If we relate changes in their strategies to the price dynamics, we may more specifically explain how the loss aversion affect the price though the investors' behavior.

We could compare our results with the real world data to check our assessments of stylized facts.

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