

Notes for libEMMI_MGFD

Pengliang Yang
Harbin Institute of Technology, China
E-mail: ypl.2100@gmail.com

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1 Input parameters

- **mode**: mode=0, forward modelling; mode=1, 3D CSEM inversion;
- **freqs**: the frequencies used for CSEM modelling and inversion, a number of frequencies can be given by comma separated values;
- **chsrc**: source channels (i.e. Ex, Ey, Ez, Hx, Hy, Hz)
- **chrec**: receiver channels (i.e. Ex, Ey, Ez, Hx, Hy, Hz)
- **nx,ny,nz**: number of intervals in x, y and z axes for input resistivity model on equispaced FD grid;
- **dx,dy,dz**: grid spacing of input resistivity model on equispaced FD grid;
- **ox,oy,oz**: origin of the 3D coordinates in x, y and z directions;
- **fbathy**: a binary input file of size nx*ny to specify bathymetry information;
- **frho11,frho22,frho33**: binary file of size nx*ny*z to specify resistivities;
- **fsrc**: an ASCII file to specify source locations and orientations;
- **frec**: an ASCII file to specify receiver locations and orientations;
- **fsrcrec**: an ASCII file to specify the connection between source and receivers;
- **niter**: number of iterations for nonlinear optimization;
- **npar**: number of parameters used for inversion, default value=2;
- **bound**: bound=1 uses bounded LBFGS; bound=0 does not apply bound constraint;
- **idxpar**: index of the inversion parameter, default value=1,2 indicating horizontal and vertical resistivities;
- **minpar**: the minimum values for the physical parameters;

- **maxpar**: the maximum values for the physical parameters;
- **gamma1**: strength of 1st order Tikhonov regularization;
- **gamma2**: strength of Total Variational (TV) regularization;

An example job script `run.sh` using the above parameters is listed in the following.

```
#!/bin/bash

export OMP_NUM_THREADS=2
mpirun -n 25 ./bin/main mode=1 \
    freqs=0.25,1,2.75 \
    chsrc=Ex \
    chrec=Ex ,Ey,Hx,Hy \
    nx=100 \
    ny=100 \
    nz=100 \
    dx=200 \
    dy=200 \
    dz=40 \
    ox=-10000 \
    oy=-10000 \
    oz=0 \
    fbathy=fbathy \
    frho11=frho_init \
    frho22=frho_init \
    frho33=frho_init \
    fsrc=sources.txt \
    frecv=receivers.txt \
    fsrcrec=src_rec_table.txt \
    niter=30 \
    npar=2 \
    bound=1 \
    idxpar=1,2 \
    minpar=1.0,1.0 \
    maxpar=100.0,100.0 \
    gamma1=100 \
    gamma2=0
```

2 Source-receiver configuration

The locations and orientations for every source/transmitter and receiver are written in a 6-column table. The following is an example of source table `sources.txt` where `x,y,z` are coordinates, `azimuth,dip` are orientations, `iTx` is the index of the transmitter.

x	y	z	azimuth	dip	iTx
-2196.15234	-8196.15234	903.652222	30.0000000	0	1
401.923828	-6696.15234	870.258484	30.0000000	0	2

3000.00000	-5196.15234	834.029785	30.0000000	0	3
5598.07617	-3696.15234	810.434204	30.0000000	0	4
8196.15234	-2196.15234	809.226013	30.0000000	0	5
-3696.15234	-5598.07617	865.961426	30.0000000	0	6
-1098.07617	-4098.07617	832.172302	30.0000000	0	7
1500.00000	-2598.07617	807.180298	30.0000000	0	8
4098.07617	-1098.07617	802.881104	30.0000000	0	9
6696.15234	401.923828	821.753967	30.0000000	0	10
.....					

The following is an example of receiver table `receivers.txt` where `x,y,z` are coordinates, `azimuth,dip` are orientations, `iRx` is the index of the receiver.

x	y	z	azimuth	dip	iRx
-10000.0000	0.00000000	1000.00000	0	0	1
-9800.00000	0.00000000	1000.00000	0	0	2
-9600.00000	0.00000000	1000.00000	0	0	3
-9400.00000	0.00000000	1000.00000	0	0	4
-9200.00000	0.00000000	1000.00000	0	0	5
-9000.00000	0.00000000	1000.00000	0	0	6
-8800.00000	0.00000000	1000.00000	0	0	7
-8600.00000	0.00000000	1000.00000	0	0	8
-8400.00000	0.00000000	1000.00000	0	0	9
-8200.00000	0.00000000	1000.00000	0	0	10
-8000.00000	0.00000000	1000.00000	0	0	11
-7800.00000	0.00000000	1000.00000	0	0	12
-7600.00000	0.00000000	1000.00000	0	0	13
-7400.00000	0.00000000	1000.00000	0	0	14
-7200.00000	0.00000000	1000.00000	0	0	15
-7000.00000	0.00000000	1000.00000	0	0	16
.....					

The connections between sources and receivers (which receivers record data from which source) must be specified by a source-receiver connection table `src_rec_table.txt` according to the index of the source and receivers.

isrc	irec
1	1
1	2
1	3
1	4
1	5
1	6
1	7
1	8
1	9
1	10
...	
2	1
2	2
2	3

```

2      4
2      5
2      6
2      7
2      8
2      9
2      10
...

```

3 Output EMF files

The simulated CSEM data are stored in ASCII files. An example EM data file `emf_0001.txt` from source index 0001 includes index of source and receivers (`iTx`, `iRx`), the recording channels of the receiver `chrec`, frequencies in Hz, real and imaginary part of the frequency domain data. They form a 6-column table in the following.

<code>iTx</code>	<code>iRx</code>	<code>chrec</code>	frequency/Hz	Real{E/H}	Imag{E/H}
1	1	Ex	0.25	-8.794095e-15	2.336085e-14
1	2	Ex	0.25	-8.236141e-15	2.632050e-14
1	3	Ex	0.25	-7.451898e-15	2.954083e-14
1	4	Ex	0.25	-6.390576e-15	3.306965e-14
1	5	Ex	0.25	-4.998464e-15	3.694922e-14
1	6	Ex	0.25	-3.228805e-15	4.118009e-14
1	7	Ex	0.25	-1.026235e-15	4.577336e-14
1	8	Ex	0.25	1.681961e-15	5.075206e-14
1	9	Ex	0.25	4.974577e-15	5.613805e-14
1	10	Ex	0.25	8.933074e-15	6.194364e-14
.....					
1	1	Ex	1	-1.045790e-12	9.924710e-13
1	2	Ex	1	-1.119678e-12	1.968149e-12
1	3	Ex	1	-8.904390e-13	3.433207e-12
1	4	Ex	1	-1.103422e-13	6.096019e-12
1	5	Ex	1	1.431065e-12	1.066018e-11
1	6	Ex	1	6.458673e-12	2.313294e-11
1	7	Ex	1	2.208377e-11	5.329251e-11
1	8	Ex	1	1.162063e-10	1.498437e-10
1	9	Ex	1	3.530520e-10	3.849713e-10
1	10	Ex	1	-1.178124e-09	6.435691e-10
.....					

4 Convergence information on CSEM inversion

After the 3D inversion, the convergence history of the nonlinear optimization will be stored in an ASCII file named `iterate.txt`.

```
=====
1-BFGS memory length: 5
Maximum number of iterations: 30
```

```

Convergence tolerance: 1.00e-06
maximum number of line search: 5
initial step length: alpha=1
=====
iter      fk      fk/f0      ||gk||      alpha      nls      ngrad
  0    1.08e+03  1.00e+00   5.30e+00  1.00e+00    0       0
  1    9.16e+02  8.49e-01   4.50e+00  4.00e+00    2       3
  2    6.48e+02  6.01e-01   5.15e+00  2.50e-01    2       6
  3    5.96e+02  5.52e-01   8.72e+00  1.00e+00    0       7
  4    4.35e+02  4.03e-01   4.40e+00  1.00e+00    0       8
  5    3.35e+02  3.11e-01   3.20e+00  1.00e+00    0       9
  6    2.58e+02  2.39e-01   5.29e+00  5.00e-01    1      11
  7    2.09e+02  1.94e-01   3.69e+00  1.00e+00    0      12
  8    1.71e+02  1.58e-01   2.00e+00  1.00e+00    0      13
  9    1.44e+02  1.34e-01   1.53e+00  1.00e+00    0      14
 10   1.23e+02  1.14e-01   2.09e+00  1.00e+00    0      15
 11   1.16e+02  1.07e-01   2.28e+00  1.00e+00    0      16
 12   8.96e+01  8.31e-02   2.02e+00  1.00e+00    0      17
 13   7.71e+01  7.15e-02   1.77e+00  1.00e+00    0      18
 14   6.27e+01  5.82e-02   8.97e-01  1.00e+00    0      19
 15   5.48e+01  5.08e-02   1.11e+00  1.00e+00    0      20
 16   4.93e+01  4.57e-02   7.89e-01  1.00e+00    0      21
 17   4.45e+01  4.13e-02   6.47e-01  1.00e+00    0      22
 18   4.06e+01  3.76e-02   7.92e-01  1.00e+00    0      23
 19   3.69e+01  3.42e-02   6.41e-01  1.00e+00    0      24
 20   3.42e+01  3.17e-02   7.62e-01  1.00e+00    0      25
 21   3.30e+01  3.06e-02   9.15e-01  1.00e+00    0      26
 22   3.01e+01  2.79e-02   6.26e-01  1.00e+00    0      27
 23   2.80e+01  2.60e-02   6.31e-01  1.00e+00    0      28
 24   2.66e+01  2.47e-02   6.19e-01  1.00e+00    0      29
 25   2.60e+01  2.41e-02   7.20e-01  1.00e+00    0      30
 26   2.58e+01  2.40e-02   1.13e+00  1.00e+00    0      31
 27   2.50e+01  2.31e-02   8.96e-01  2.00e+00    1      38
 28   2.44e+01  2.26e-02   8.16e-01  1.00e+00    0      39
 29   2.35e+01  2.18e-02   4.30e-01  1.00e+00    0      40
==>Maximum iteration number reached!

```

In the above example, each columns has clear meaning:

- **iter**: the iteration index k;
- **fk**: the misfit at the k-th iteration;
- **fk/f0**: the normalized misfit at the k-th iteration;
- **||gk||**: the norm of the gradient;
- **alpha**: step length used in line search;
- **nls**: number of line search at the k-th iteration;
- **ngrad**: number of gradient evaluations

5 The Green's function and the reciprocity

Assume only electrical current $J_j(x_s, \omega) = \delta(x - x_s)e_j$ where e_j is the j -directed unit vector. We have

$$\begin{cases} \nabla \times G_{ij}^{E|E} - i\omega\mu G_{ij}^{H|E} &= 0 \\ \nabla \times G_{ij}^{H|E} - \sigma G_{ij}^{E|E} &= \delta(x - x_s)e_j \end{cases}, \quad (1)$$

which defines two Green's function $G_{ij}^{E|E}$ and $G_{ij}^{H|E}$: $G_{ij}^{E|E}$ is the i th electrical (E) component of Green's function induced by j th component of electrical (E) source; $G_{ij}^{H|E}$ is the i th magnetic (H) component of Green's function induced by j th component of electrical (E) source. The representation theorem gives

$$E_i = G_{ij}^{E|E} J_j, H_i = G_{ij}^{H|E} J_j. \quad (2)$$

We can do the same assuming only a magnetic source $M_j = \delta(x - x_s)e_j$: $G_{ij}^{E|H}$ is the i th electrical (E) component of Green's function induced by j th component of magnetic (H) source; $G_{ij}^{H|H}$ is the i th magnetic (H) component of Green's function induced by j th component of magnetic (H) source.

$$\begin{cases} \nabla \times G_{ij}^{E|H} - i\omega\mu G_{ij}^{H|H} &= \delta(x - x_s)e_j \\ \nabla \times G_{ij}^{H|H} - \sigma G_{ij}^{E|H} &= 0 \end{cases}, \quad (3)$$

which defines another two Green's function $G_{ij}^{E|H}$ and $G_{ij}^{H|H}$. Similar to equation (??), the representation theorem gives

$$E_i = G_{ij}^{E|H} M_j, H_i = G_{ij}^{H|H} M_j. \quad (4)$$

The total electrical and magnetic fields in the coupled system is then the superposition of two contributions:

$$E_i = \sum_j G_{ij}^{E|E} J_j + G_{ij}^{E|H} M_j, \quad H_i = \sum_j G_{ij}^{H|E} J_j + G_{ij}^{H|H} M_j. \quad (5)$$

It is shown that the reciprocity for EM system holds in the following form

$$\begin{cases} G_{ij}^{E|E}(x_s|x_r) = G_{ji}^{E|E}(x_r|x_s), \\ G_{ij}^{H|H}(x_s|x_r) = G_{ji}^{H|H}(x_r|x_s), \\ G_{ij}^{H|E}(x_s|x_r) = -G_{ji}^{E|H}(x_r|x_s). \end{cases} \quad (6)$$

Without magnetic source, we have

$$\underbrace{\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}}_E = \underbrace{\begin{bmatrix} G_{xx}^{E|E} & G_{xy}^{E|E} & G_{xz}^{E|E} \\ G_{yx}^{E|E} & G_{yy}^{E|E} & G_{yz}^{E|E} \\ G_{zx}^{E|E} & G_{zy}^{E|E} & G_{zz}^{E|E} \end{bmatrix}}_{G^{E|E}} \underbrace{\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix}}_s, \quad \underbrace{\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}}_H = \underbrace{\begin{bmatrix} G_{xx}^{H|E} & G_{xy}^{H|E} & G_{xz}^{H|E} \\ G_{yx}^{H|E} & G_{yy}^{H|E} & G_{yz}^{H|E} \\ G_{zx}^{H|E} & G_{zy}^{H|E} & G_{zz}^{H|E} \end{bmatrix}}_{G^{H|E}} \underbrace{\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix}}_s. \quad (7)$$

If $J_s|_{x=x_s} = (1, 0, 0)^T$, we have the 1st column of the matrix $G^{E|E}$ and $G^{H|E}$ extracted from vector fields E and H

$$\begin{bmatrix} E_x(x_r) \\ E_y(x_r) \\ E_z(x_r) \end{bmatrix} = \begin{bmatrix} G_{xx}^{E|E}(x_r|x_s) \\ G_{yx}^{E|E}(x_r|x_s) \\ G_{zx}^{E|E}(x_r|x_s) \end{bmatrix} = \begin{bmatrix} G_{xx}^{E|E}(x_s|x_r) \\ G_{xy}^{E|E}(x_s|x_r) \\ G_{xz}^{E|E}(x_s|x_r) \end{bmatrix}, \quad (8a)$$

$$\begin{bmatrix} H_x(x_r) \\ H_y(x_r) \\ H_z(x_r) \end{bmatrix} = \begin{bmatrix} G_{xx}^{H|E}(x_r|x_s) \\ G_{yx}^{H|E}(x_r|x_s) \\ G_{zx}^{H|E}(x_r|x_s) \end{bmatrix} = - \begin{bmatrix} G_{xx}^{E|H}(x_s|x_r) \\ G_{xy}^{E|H}(x_s|x_r) \\ G_{xz}^{E|H}(x_s|x_r) \end{bmatrix}, \quad (8b)$$

where the last equality comes from the reciprocity in (??). It implies that by switching the source and receiver position, we can reproduce the E_x , E_y and E_z response from the E_x -channel of the receiver at source position by repeating the modeling using the electrical sources at receiver location, i.e., $J_s|_{x=x_r} = (1, 0, 0)^T$, $J_s|_{x=x_s} = (0, 1, 0)^T$ and $J_s|_{x=x_r} = (0, 0, 1)^T$. Similarly, we should reproduce $-H_x$, $-H_y$ and $-H_z$ response from the E_x -channel of the receiver at source position by repeating the modeling using the magnetic sources at receiver location, i.e., $M_s|_{x=x_r} = (1, 0, 0)^T$, $M_s|_{x=x_s} = (0, 1, 0)^T$ and $M_s|_{x=x_r} = (0, 0, 1)^T$.

If $J_s|_{x=x_s} = (0, 1, 0)^T$, we have the 2nd column of the matrix $G^{E|E}$ and $G^{H|E}$ extracted from vector fields E and H

$$\begin{bmatrix} E_x(x_r) \\ E_y(x_r) \\ E_z(x_r) \end{bmatrix} = \begin{bmatrix} G_{xy}^{E|E}(x_r|x_s) \\ G_{yy}^{E|E}(x_r|x_s) \\ G_{zy}^{E|E}(x_r|x_s) \end{bmatrix} = \begin{bmatrix} G_{yx}^{E|E}(x_s|x_r) \\ G_{yy}^{E|E}(x_s|x_r) \\ G_{yz}^{E|E}(x_s|x_r) \end{bmatrix}, \quad (9a)$$

$$\begin{bmatrix} H_x(x_r) \\ H_y(x_r) \\ H_z(x_r) \end{bmatrix} = \begin{bmatrix} G_{xy}^{H|E}(x_r|x_s) \\ G_{yy}^{H|E}(x_r|x_s) \\ G_{zy}^{H|E}(x_r|x_s) \end{bmatrix} = - \begin{bmatrix} G_{yx}^{E|H}(x_s|x_r) \\ G_{yy}^{E|H}(x_s|x_r) \\ G_{yz}^{E|H}(x_s|x_r) \end{bmatrix}. \quad (9b)$$

By switching the source and receiver position, we obtain the E_x , E_y and E_z response from the E_y -channel of the receiver at source position by repeating the modeling placing the sources at receiver location, i.e., $J_s|_{x=x_r} = (1, 0, 0)^T$, $J_s|_{x=x_s} = (0, 1, 0)^T$ and $J_s|_{x=x_r} = (0, 0, 1)^T$. Also, we obtain $-H_x$, $-H_y$ and $-H_z$ response from the E_y -channel of the receiver at source position by repeating the modeling placing the magnetic sources at receiver location, i.e., $M_s|_{x=x_r} = (1, 0, 0)^T$, $M_s|_{x=x_s} = (0, 1, 0)^T$ and $M_s|_{x=x_r} = (0, 0, 1)^T$.