

# Notes for libEMMI\_MGFD

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## 1 Input parameters

- **mode**: **mode=0**, forward modelling; **mode=1**, 3D CSEM inversion;
- **freqs**: the frequencies used for CSEM modelling and inversion, a number of frequencies can be given by comma separated values;
- **chsrc**: source channels (i.e. Ex, Ey, Ez, Hx, Hy, Hz)
- **chrec**: receiver channels (i.e. Ex, Ey, Ez, Hx, Hy, Hz)
- **nx,ny,nz**: number of intervals in x, y and z axes for input resistivity model on equispaced FD grid;
- **dx,dy,dz**: grid spacing of input resistivity model on equispaced FD grid;
- **ox,oy,oz**: origin of the 3D coordinates in x, y and z directions;
- **fbathy**: a binary input file of size **nx\*ny** to specify bathymetry information;
- **frho11,frho22,frho33**: binary file of size **nx\*ny\*z** to specify resistivities;
- **fsrc**: an ASCII file to specify source locations and orientations;
- **frec**: an ASCII file to specify receiver locations and orientations;
- **fsrcrec**: an ASCII file to specify the connection between source and receivers;
- **niter**: number of iterations for nonlinear optimization;
- **npar**: number of parameters used for inversion, default value=2;
- **bound**: **bound=1** uses bounded LBFGS; **bound=0** does not apply bound constraint;
- **idxpar**: index of the inversion parameter, default value=1,2 indicating horizontal and vertical resistivities;
- **minpar**: the minimum values for the physical parameters;

- **maxpar**: the maximum values for the physical parameters;
- **gamma1**: strength of 1st order Tikhonov regularization;
- **gamma2**: strength of Total Variational (TV) regularization;

An example job script **run.sh** using the above parameters is listed in the following.

```
#!/bin/bash

export OMP_NUM_THREADS=2
mpirun -n 25 ../bin/main mode=1 \
    freqs=0.25,1,2.75 \
    chsrc=Ex \
    chrec=Ex ,Ey,Hx,Hy \
    nx=100 \
    ny=100 \
    nz=100 \
    dx=200 \
    dy=200 \
    dz=40 \
    ox=-10000 \
    oy=-10000 \
    oz=0 \
    fbathy=fbathy \
    frho11=frho_init \
    frho22=frho_init \
    frho33=frho_init \
    fsrc=sources.txt \
    frec=receivers.txt \
    fsrcrec=src_rec_table.txt \
    niter=30 \
    npar=2 \
    bound=1 \
    idxpar=1,2 \
    minpar=1.0,1.0 \
    maxpar=100.0,100.0 \
    gamma1=100 \
    gamma2=0
```

## 2 Source-receiver configuration

The locations and orientations for every source/transmitter and receiver are written in a 6-column table. The following is an example of source table **sources.txt** where **x,y,z** are coordinates, **azimuth,dip** are orientations, **iTx** is the index of the transmitter.

x	y	z	azimuth	dip	iTx
-2196.15234	-8196.15234	903.652222	30.0000000	0	1
401.923828	-6696.15234	870.258484	30.0000000	0	2

3000.00000	-5196.15234	834.029785	30.0000000	0	3
5598.07617	-3696.15234	810.434204	30.0000000	0	4
8196.15234	-2196.15234	809.226013	30.0000000	0	5
-3696.15234	-5598.07617	865.961426	30.0000000	0	6
-1098.07617	-4098.07617	832.172302	30.0000000	0	7
1500.00000	-2598.07617	807.180298	30.0000000	0	8
4098.07617	-1098.07617	802.881104	30.0000000	0	9
6696.15234	401.923828	821.753967	30.0000000	0	10
.....					

The following is an example of receiver table `receivers.txt` where `x,y,z` are coordinates, `azimuth,dip` are orientations, `iRx` is the index of the receiver.

x	y	z	azimuth	dip	iRx
-10000.00000	0.000000000	1000.00000	0	0	1
-9800.00000	0.000000000	1000.00000	0	0	2
-9600.00000	0.000000000	1000.00000	0	0	3
-9400.00000	0.000000000	1000.00000	0	0	4
-9200.00000	0.000000000	1000.00000	0	0	5
-9000.00000	0.000000000	1000.00000	0	0	6
-8800.00000	0.000000000	1000.00000	0	0	7
-8600.00000	0.000000000	1000.00000	0	0	8
-8400.00000	0.000000000	1000.00000	0	0	9
-8200.00000	0.000000000	1000.00000	0	0	10
-8000.00000	0.000000000	1000.00000	0	0	11
-7800.00000	0.000000000	1000.00000	0	0	12
-7600.00000	0.000000000	1000.00000	0	0	13
-7400.00000	0.000000000	1000.00000	0	0	14
-7200.00000	0.000000000	1000.00000	0	0	15
-7000.00000	0.000000000	1000.00000	0	0	16
.....					

The connections between sources and receivers (which receivers record data from which source) must be specified by a source-receiver connection table `src_rec_table.txt` according to the index of the source and receivers.

isrc	irec
1	1
1	2
1	3
1	4
1	5
1	6
1	7
1	8
1	9
1	10
...	
2	1
2	2
2	3

```

2          4
2          5
2          6
2          7
2          8
2          9
2         10
...

```

### 3 Output EMF files

The simulated CSEM data are stored in ASCII files. An example EM data file `emf_0001.txt` from source index 0001 includes index of source and receivers (`iTx`, `iRx`), the recording channels of the receiver `chrec`, frequencies in Hz, real and imaginary part of the frequency domain data. They form a 6-column table in the following.

<code>iTx</code>	<code>iRx</code>	<code>chrec</code>	frequency/Hz	Real{E/H}	Imag{E/H}
1	1	Ex	0.25	-8.794095e-15	2.336085e-14
1	2	Ex	0.25	-8.236141e-15	2.632050e-14
1	3	Ex	0.25	-7.451898e-15	2.954083e-14
1	4	Ex	0.25	-6.390576e-15	3.306965e-14
1	5	Ex	0.25	-4.998464e-15	3.694922e-14
1	6	Ex	0.25	-3.228805e-15	4.118009e-14
1	7	Ex	0.25	-1.026235e-15	4.577336e-14
1	8	Ex	0.25	1.681961e-15	5.075206e-14
1	9	Ex	0.25	4.974577e-15	5.613805e-14
1	10	Ex	0.25	8.933074e-15	6.194364e-14
.....					
1	1	Ex	1	-1.045790e-12	9.924710e-13
1	2	Ex	1	-1.119678e-12	1.968149e-12
1	3	Ex	1	-8.904390e-13	3.433207e-12
1	4	Ex	1	-1.103422e-13	6.096019e-12
1	5	Ex	1	1.431065e-12	1.066018e-11
1	6	Ex	1	6.458673e-12	2.313294e-11
1	7	Ex	1	2.208377e-11	5.329251e-11
1	8	Ex	1	1.162063e-10	1.498437e-10
1	9	Ex	1	3.530520e-10	3.849713e-10
1	10	Ex	1	-1.178124e-09	6.435691e-10
.....					

### 4 Convergence information on CSEM inversion

After the 3D inversion, the convergence history of the nonlinear optimization will be stored in an ASCII file named `iterate.txt`.

```

=====
l-BFGS memory length: 5
Maximum number of iterations: 30

```

Convergence tolerance: 1.00e-06  
maximum number of line search: 5  
initial step length: alpha=1

```
=====
iter    fk      fk/f0      ||gk||      alpha      nls      ngrad
  0    1.08e+03    1.00e+00    5.30e+00    1.00e+00     0       0
  1    9.16e+02    8.49e-01    4.50e+00    4.00e+00     2       3
  2    6.48e+02    6.01e-01    5.15e+00    2.50e-01     2       6
  3    5.96e+02    5.52e-01    8.72e+00    1.00e+00     0       7
  4    4.35e+02    4.03e-01    4.40e+00    1.00e+00     0       8
  5    3.35e+02    3.11e-01    3.20e+00    1.00e+00     0       9
  6    2.58e+02    2.39e-01    5.29e+00    5.00e-01     1      11
  7    2.09e+02    1.94e-01    3.69e+00    1.00e+00     0      12
  8    1.71e+02    1.58e-01    2.00e+00    1.00e+00     0      13
  9    1.44e+02    1.34e-01    1.53e+00    1.00e+00     0      14
 10    1.23e+02    1.14e-01    2.09e+00    1.00e+00     0      15
 11    1.16e+02    1.07e-01    2.28e+00    1.00e+00     0      16
 12    8.96e+01    8.31e-02    2.02e+00    1.00e+00     0      17
 13    7.71e+01    7.15e-02    1.77e+00    1.00e+00     0      18
 14    6.27e+01    5.82e-02    8.97e-01    1.00e+00     0      19
 15    5.48e+01    5.08e-02    1.11e+00    1.00e+00     0      20
 16    4.93e+01    4.57e-02    7.89e-01    1.00e+00     0      21
 17    4.45e+01    4.13e-02    6.47e-01    1.00e+00     0      22
 18    4.06e+01    3.76e-02    7.92e-01    1.00e+00     0      23
 19    3.69e+01    3.42e-02    6.41e-01    1.00e+00     0      24
 20    3.42e+01    3.17e-02    7.62e-01    1.00e+00     0      25
 21    3.30e+01    3.06e-02    9.15e-01    1.00e+00     0      26
 22    3.01e+01    2.79e-02    6.26e-01    1.00e+00     0      27
 23    2.80e+01    2.60e-02    6.31e-01    1.00e+00     0      28
 24    2.66e+01    2.47e-02    6.19e-01    1.00e+00     0      29
 25    2.60e+01    2.41e-02    7.20e-01    1.00e+00     0      30
 26    2.58e+01    2.40e-02    1.13e+00    1.00e+00     0      31
 27    2.50e+01    2.31e-02    8.96e-01    2.00e+00     1      38
 28    2.44e+01    2.26e-02    8.16e-01    1.00e+00     0      39
 29    2.35e+01    2.18e-02    4.30e-01    1.00e+00     0      40
```

==>Maximum iteration number reached!

In the above example, each columns has clear meaning:

- iter: the iteration index k;
- fk: the misfit at the k-th iteration;
- fk/f0: the normalized misfit at the k-th iteration;
- ||gk||: the norm of the gradient;
- alpha: step length used in line search;
- nls: number of line search at the k-th iteration;
- ngrad: number of gradient evaluations

## 5 The Green's function and the reciprocity

Assume only electrical current  $J_j(x_s, \omega) = \delta(x - x_s)e_j$  where  $e_j$  is the  $j$ -directed unit vector. We have

$$\begin{cases} \nabla \times G_{ij}^{E|E} - i\omega\mu G_{ij}^{H|E} &= 0 \\ \nabla \times G_{ij}^{H|E} - \sigma G_{ij}^{E|E} &= \delta(x - x_s)e_j \end{cases}, \quad (1)$$

which defines two Green's function  $G_{ij}^{E|E}$  and  $G_{ij}^{H|E}$ :  $G_{ij}^{E|E}$  is the  $i$ th electrical ( $E$ ) component of Green's function induced by  $j$ th component of electrical ( $E$ ) source;  $G_{ij}^{H|E}$  is the  $i$ th magnetic ( $H$ ) component of Green's function induced by  $j$ th component of electrical ( $E$ ) source. The representation theorem gives

$$E_i = G_{ij}^{E|E} J_j, H_i = G_{ij}^{H|E} J_j. \quad (2)$$

We can do the same assuming only a magnetic source  $M_j = \delta(x - x_s)e_j$ :  $G_{ij}^{E|H}$  is the  $i$ th electrical ( $E$ ) component of Green's function induced by  $j$ th component of magnetic ( $H$ ) source;  $G_{ij}^{H|H}$  is the  $i$ th magnetic ( $H$ ) component of Green's function induced by  $j$ th component of magnetic ( $H$ ) source.

$$\begin{cases} \nabla \times G_{ij}^{E|H} - i\omega\mu G_{ij}^{H|H} &= \delta(x - x_s)e_j \\ \nabla \times G_{ij}^{H|H} - \sigma G_{ij}^{E|H} &= 0 \end{cases}, \quad (3)$$

which defines another two Green's function  $G_{ij}^{E|H}$  and  $G_{ij}^{H|H}$ . Similar to equation (??), the representation theorem gives

$$E_i = G_{ij}^{E|H} M_j, H_i = G_{ij}^{H|H} M_j. \quad (4)$$

The total electrical and magnetic fields in the coupled system is then the superposition of two contributions:

$$E_i = \sum_j G_{ij}^{E|E} J_j + G_{ij}^{E|H} M_j, \quad H_i = \sum_j G_{ij}^{H|E} J_j + G_{ij}^{H|H} M_j. \quad (5)$$

It is shown that the reciprocity for EM system holds in the following form

$$\begin{cases} G_{ij}^{E|E}(x_s|x_r) = G_{ji}^{E|E}(x_r|x_s), \\ G_{ij}^{H|H}(x_s|x_r) = G_{ji}^{H|H}(x_r|x_s), \\ G_{ij}^{H|E}(x_s|x_r) = -G_{ji}^{E|H}(x_r|x_s). \end{cases} \quad (6)$$

Without magnetic source, we have

$$\underbrace{\begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}}_E = \underbrace{\begin{bmatrix} G_{xx}^{E|E} & G_{xy}^{E|E} & G_{xz}^{E|E} \\ G_{yx}^{E|E} & G_{yy}^{E|E} & G_{yz}^{E|E} \\ G_{zx}^{E|E} & G_{zy}^{E|E} & G_{zz}^{E|E} \end{bmatrix}}_{G^{E|E}} \underbrace{\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix}}_{J_s} = \underbrace{\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}}_H = \underbrace{\begin{bmatrix} G_{xx}^{H|E} & G_{xy}^{H|E} & G_{xz}^{H|E} \\ G_{yx}^{H|E} & G_{yy}^{H|E} & G_{yz}^{H|E} \\ G_{zx}^{H|E} & G_{zy}^{H|E} & G_{zz}^{H|E} \end{bmatrix}}_{G^{H|E}} \underbrace{\begin{bmatrix} J_x \\ J_y \\ J_z \end{bmatrix}}_{J_s}. \quad (7)$$

If  $J_s|_{x=x_s} = (1, 0, 0)^T$ , we have the 1st column of the matrix  $G^{E|E}$  and  $G^{H|E}$  extracted from vector fields  $E$  and  $H$

$$\begin{bmatrix} E_x(x_r) \\ E_y(x_r) \\ E_z(x_r) \end{bmatrix} = \begin{bmatrix} G_{xx}^{E|E}(x_r|x_s) \\ G_{yx}^{E|E}(x_r|x_s) \\ G_{zx}^{E|E}(x_r|x_s) \end{bmatrix} = \begin{bmatrix} G_{xx}^{E|E}(x_s|x_r) \\ G_{xy}^{E|E}(x_s|x_r) \\ G_{xz}^{E|E}(x_s|x_r) \end{bmatrix}, \quad (8a)$$

$$\begin{bmatrix} H_x(x_r) \\ H_y(x_r) \\ H_z(x_r) \end{bmatrix} = \begin{bmatrix} G_{xx}^{H|E}(x_r|x_s) \\ G_{yx}^{H|E}(x_r|x_s) \\ G_{zx}^{H|E}(x_r|x_s) \end{bmatrix} = - \begin{bmatrix} G_{xx}^{E|H}(x_s|x_r) \\ G_{xy}^{E|H}(x_s|x_r) \\ G_{xz}^{E|H}(x_s|x_r) \end{bmatrix}, \quad (8b)$$

where the last equality comes from the reciprocity in (??). It implies that by switching the source and receiver position, we can reproduce the  $E_x$ ,  $E_y$  and  $E_z$  response from the  $E_x$ -channel of the receiver at source position by repeating the modeling using the electrical sources at receiver location, i.e.,  $J_s|_{x=x_r} = (1, 0, 0)^T$ ,  $J_s|_{x=x_r} = (0, 1, 0)^T$  and  $J_s|_{x=x_r} = (0, 0, 1)^T$ . Similarly, we should reproduce  $-H_x$ ,  $-H_y$  and  $-H_z$  response from the  $E_x$ -channel of the receiver at source position by repeating the modeling using the magnetic sources at receiver location, i.e.,  $M_s|_{x=x_r} = (1, 0, 0)^T$ ,  $M_s|_{x=x_r} = (0, 1, 0)^T$  and  $M_s|_{x=x_r} = (0, 0, 1)^T$ .

If  $J_s|_{x=x_s} = (0, 1, 0)^T$ , we have the 2nd column of the matrix  $G^{E|E}$  and  $G^{H|E}$  extracted from vector fields  $E$  and  $H$

$$\begin{bmatrix} E_x(x_r) \\ E_y(x_r) \\ E_z(x_r) \end{bmatrix} = \begin{bmatrix} G_{xy}^{E|E}(x_r|x_s) \\ G_{yy}^{E|E}(x_r|x_s) \\ G_{zy}^{E|E}(x_r|x_s) \end{bmatrix} = \begin{bmatrix} G_{yx}^{E|E}(x_s|x_r) \\ G_{yy}^{E|E}(x_s|x_r) \\ G_{yz}^{E|E}(x_s|x_r) \end{bmatrix}, \quad (9a)$$

$$\begin{bmatrix} H_x(x_r) \\ H_y(x_r) \\ H_z(x_r) \end{bmatrix} = \begin{bmatrix} G_{xy}^{H|E}(x_r|x_s) \\ G_{yy}^{H|E}(x_r|x_s) \\ G_{zy}^{H|E}(x_r|x_s) \end{bmatrix} = - \begin{bmatrix} G_{yx}^{E|H}(x_s|x_r) \\ G_{yy}^{E|H}(x_s|x_r) \\ G_{yz}^{E|H}(x_s|x_r) \end{bmatrix}. \quad (9b)$$

By switching the source and receiver position, we obtain the  $E_x$ ,  $E_y$  and  $E_z$  response from the  $E_y$ -channel of the receiver at source position by repeating the modeling placing the sources at receiver location, i.e.,  $J_s|_{x=x_r} = (1, 0, 0)^T$ ,  $J_s|_{x=x_r} = (0, 1, 0)^T$  and  $J_s|_{x=x_r} = (0, 0, 1)^T$ . Also, we obtain  $-H_x$ ,  $-H_y$  and  $-H_z$  response from the  $E_y$ -channel of the receiver at source position by repeating the modeling placing the magnetic sources at receiver location, i.e.,  $M_s|_{x=x_r} = (1, 0, 0)^T$ ,  $M_s|_{x=x_r} = (0, 1, 0)^T$  and  $M_s|_{x=x_r} = (0, 0, 1)^T$ .