Shallow water eqn (explicit time stepping)

① SWE eqn:

$$\gamma = \frac{g\sqrt{u^2+v^2}}{C_z^2 H}$$

$$\begin{cases} \partial_t u + u\partial_x u + v\partial_y u + g\partial_x z = -\gamma u + \tau_x \\ \partial_t v + u\partial_x v + v\partial_y v + g\partial_y z = -\gamma v + \tau_y \\ \partial_t z + \partial_x[(h+z)u] + \partial_y[(h+z)v] = 0 \end{cases}$$

$H(x,y,t) = h(x,y) + z(x,y,t)$

② Time discretization (staggered grid FD)

$$\begin{cases} u^{k+1} = u^k + \Delta t(-u\partial_x u - v\partial_y u - g\partial_x z - \gamma u + \tau_x)^k \\ v^{k+1} = v^k + \Delta t(-u\partial_x v - v\partial_y v - g\partial_y z - \gamma v + \tau_y)^k \\ z^{k+1} = z^k - \Delta t\,\partial_x[(h+z)u]^k - \Delta t\,\partial_y[(h+z)v]^k \end{cases}$$

define $Fw^k = w^k - \Delta t(u\partial_x w + v\partial_y w)^k$, then

$$\begin{cases} u^{k+1} = Fu^k - \Delta t\, g\partial_x z^k - \Delta t(\gamma u - \tau_x)^k \\ v^{k+1} = Fv^k - \Delta t\, g\partial_y z^k - \Delta t(\gamma v - \tau_y)^k \\ z^{k+1} = z^k - \Delta t\,\partial_x[(h+z)u]^k - \Delta t\,\partial_y[(h+z)v]^k \end{cases}$$

$$= z^k - \frac{\Delta t}{\Delta x}\left([(h+z)u]_{i+\frac{1}{2},j} - [(h+z)z]_{i-\frac{1}{2},j}\right)$$

$$- \frac{\Delta t}{\Delta y}\left([(h+z)v]_{i,j+\frac{1}{2}} - [(h+z)v]_{i,j-\frac{1}{2}}\right)$$