AlgoProject

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1 Abstract

This paper will present a greedy algorithm to solve the problem of scheduling classes with constraints of room capacity, time conflict, and teacher conflict. This is a NP-hard problem, but our algorithm can achieve polynomial time complexity and approximate the optimal solution. Using random generated and realistic data, our algorithm can achieve 98% fit percentage of students' preference on average. With real Bryn Mawr registration data in the past, our algorithm can achieve 96.6% fit percentage on average. All the output schedules are valid. During our experimental analysis, we noticed that room capacity is hardly a significant restriction comparing to time conflict between classes when the student body is under 30000. So we recommend registrar consider the high-conflict classes first and try to schedule them in different time slots.

2 Description

We first calculate the popularity of each class by counting how many students want to take the class. Then we create a 2D conflict matrix called $classConflicts$ with row $c_1, c_2, ...c_i$ and column $c_1, c_2, ...c_j$ as all the classes. We calculate the conflict number by recording number of students that want to take both c_i and c_j at $classConflicts[i][j]$. If two classes are taught by the same professor, we set the conflict number to ∞ Then for each unique (i, j) pair, we create a list that stores all the (i, j) pair in the order of decreasing conflict number. For each pair in the list, we consider both classes in order. If the class is not yet scheduled, at each time slot, we consider 1) number of surplus students at the largest available room; and 2) the sum of conflicts with previous class(es) that are scheduled at this time slot. Schedule the class at t_k, where it satisfies the most students who want to take this class and at c_l, where it's the smallest room that can take all the prospective students or otherwise the largest available room.

Once all the classes have been scheduled, we put students into classes according to their preference list as long as those classes don't have conflicts.

3 Pseudo Code

```
Function Scheduling(S,C,R,T,P)
    Initialize a 2D array classConflicts[c][c] where the value of each cell is the
      conflict number of class_{row-number} and class_{column-number}
    for each student s_a do
        for each c_b ∈ s_a's preference list do
            c_b.popularity++
        end
        for each two classes i and j ∈ s_a's preference list do
            classConflicts[i][j]++
            classConflicts[j][i]++
        end
    end
    for each professor do
        a = prof[i].firstClass
        b = prof[i].secondClass
        classConflicts[a][b]++
        classConflicts[b][a]++
    end
    Create a list conflict that store all the unique class pairs
    Sort the list in the order of decreasing conflict number
    Sort the rooms by their capacity
    for each class pair in the conflict list do
        for each class c_k of the two classes do
            if c_k is not scheduled then
                for i = 1 to t do
                    largeRoom = the largest available room at t_i
                    surplus = c_k.popularity − largeRoom.capacity
                    sumOfConflict = ∑ conflicts of all previous scheduled classes
                    finalConflict[i] = max(surplus, sumOfConflict)
                end
                Schedule c_k at t_i where we have minimum finalConflict[i]
                Schedule c_k at the smallest room at t_i that can either fit all students
                  want to take c_k, or the largest available room at t_i
            end
        end
    end
    for each student s_a do
        for each c_b ∈ s_a's preference list do
            if c_b is not filled and s_a is available at c_b's class time then
                register s_a into c_b
            end
        end
    end
```

4 Time Analysis

Creating the 2D array $classConflicts$ requires $O(1)$, because the java array is filled out by default.

The first for loop will loop through all the students, and check each student's 4 preference classes and each unique class pairs, which requires $4s + 6s = 10s$ iterations. Because there is only 4 classes in a student's preference list and the total number of unique class pairs in a student's preference list is $\binom{4}{2} = 6$. Everything inside the loop will be shown that only takes $O(1)$.

We have $\frac{(1+(c-1))\cdot(c-1)}{2} = \frac{c^2-c}{2}$ unique pairs in total, so creating the list of conflicts requires $O(c^2)$. Quick sort is then used to sort the list, which requires $O(c^2logc^2)$. Sorting the classrooms by their capacity using quick sort again takes $O(rlogr)$. To loop through each unique pair of classes and schedule all the classes requires c iterations. The number of time slots in reality is way fewer than the number of classes, so here we will assume it is a small constant. The for loop that iterates through each time slot requires t loops, but the operation inside will be shown to be $O(r)$. Because classes has to put into r rooms and t timeslots, we will simplify $O(rt)$ as $O(c)$. We will show other operations inside the second for loop takes $O(1)$ or $O(c)$ as well. Overall, the second for loop takes $O(c^2)$ to complete.

The last for loop and the loop inside will once again check each student's preference list, which runs $4s$ times and is bounded by $O(s)$.

The algorithm's overall time complexity will be $O(c^2logc^2 + s)$ if the following operations can be done in $O(1)$ or $O(c)$ or less than $O(c)$:

1. access a student's preference list
2. increment a class's popularity
3. check if two classes are taught by the same professor
4. increment two classes' conflict number
5. access class pair in the conflict list
6. check if a class is scheduled or not
7. find the largest available room at a given time slot
8. access class's popularity
9. access room's capacity
10. calculate $sumOfConflict$
11. populate $finalConflict$ array

12. find the minimum value in $finalConflict$ array
13. schedule a class at a time slot
14. find the smallest room at a given time slot that can fit all students who want to take a class
15. schedule a class at a room
16. check if a class is filled
17. register a student into a class
18. check if a student is available at certain time slot

4.1 Data Structure

The *student* class will contain a unique ID, their preference list, and a list of their final registered classes. We can store all the students in an array *students* by their unique ID. This will allow (1), (17) and (18) to be done in $O(1)$.

The *class* class will contain a unique ID, its popularity, scheduled room and time slot, an array $finalConflict$ of size t, the professor who teaches the class, and an initially empty list *registerationList*. We can store all the classes in an array *classes* by their unique ID. This will allow (2), (3), (6), (8), (13), and (15) to be done in $O(1)$.

The *room* class will contain its capacity and its unique ID. We will create an array of *rooms* and sort it by the capacity of the rooms (sorting is already being considered in the above analysis). The *timeslot* class will contain an array called *roomSchedules*, where the index is the room's rank in *rooms* and the value is the class that is scheduled at this time at certain room. We will store all the time slots in an array *timeSlots*. This will allow (9), (16) to be done in $O(1)$ and (7) to be done in at most $O(r)$. To achieve (14), we need to linearly search each room and access its capacity, then compare it with the popularity of the class, which requires $O(r)$.

The 2D array $classConflicts$ will record the number of conflicts for each pair of different class. The value of $classConflicts[i][j]$ represents the number of conflicts between $class_i$ and $class_j$. This will allow (4) and (5) to be done in $O(1)$. For (10), there is at most r conflict classes at a time slot, therefore calculating the sum of all the conflicts takes $O(r)$. Then, populating final Conflict array, (11), requires $O(rt)$ which can be simplified as $O(c)$. Finding the minimum in this array, (12), takes $O(t) = O(1)$.

5 Proof of Correctness

5.1 Proof of Termination

The pseudocode and time analysis demonstrated that all the loops run a finite number of iterations. We never check the class again once it's scheduled and we never reconsider a student once we tried to put him or her into all 4 preference list classes.

5.2 Proof of Valid Schedule

1. no teacher conflict:

   For any two classes a and b that are taught by the same professor, we set its number of conflicts in the 2D array *classConflicts* to positive infinity. Then, when the algorithm creates the conflict list, these (a, b) pairs will be considered first. Thus, once one of the class, say a, is scheduled at some time slot t_a, $finalConflict[t_a]$ will be the maximum number in the entire array. But the algorithm will always take the minimum $finalConflict[t]$ to schedule b, hence avoiding the situation where two classes taught by the same professor are scheduled at the same time slot.

2. no room conflict (only one class can be scheduled in the room at a given time):

   For each class, we check each time slot once and check the available rooms only once until we schedule the class into one room at a given time. Note that for each time slot we have an array *roomSchedules* to record which room at the time is occupied. When a class is scheduled, we put the class into the *roomSchedules* to indicate the room is not available anymore.

3. all schedulable classes are scheduled (no empty rooms/time slots):

   Suppose there is a class c_d that is schedulable, but has not been assigned to a room or time slot. By the algorithm, it must be that all the time slots and rooms is already scheduled, which contradicts the supposition that the class is still schedulable.

4. no classes are scheduled more than once:

   The class pair in the list is unique, and we only look at each unique pair once during the second for loop. Then we only look at classes that are not scheduled, which means scheduled classes will never be looked at again, making sure no classes are scheduled more than once.

5. no enrolled student has a schedule conflict:

   Suppose there is a student s_e who has a schedule conflict for two classes c_a and c_b that s_e registered in. Also suppose s_e registered in c_a first. According to the algorithm, it must be that c_b is not filled out and s_e is available at c_b's class time. But clearly s_e

is not available at c_b's class time because he or she needs to take c_a. Hence, we have a contradiction.

6 Discussion

This is a greedy algorithm because once we have scheduled a class at a time and in a room or enrolled a student into a class, we never backtrack. It is also conflict based and popularity adjusted, meaning we consider the highly-conflicted classes first and assign the classes to rooms based on their popularity. As we analyzed the problem, we noticed that there are two forms of conflict that make students unable to take the class(es) in his/her preference list: 1) the class that student wants to take is full and 2) the student already has another class in the same time slot (or overlapping time slot, as in our extended implementations).

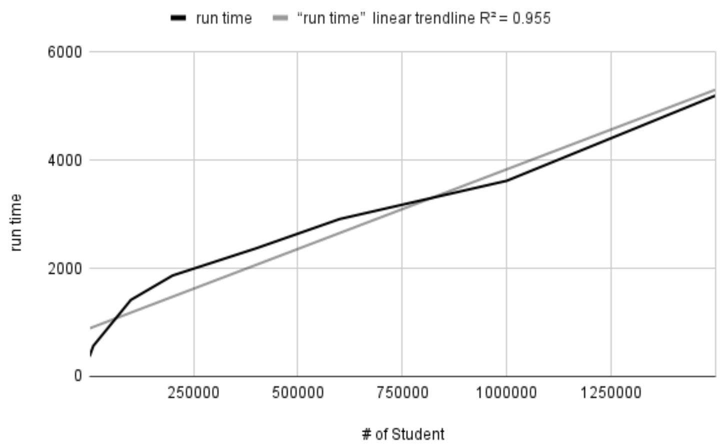
To address these two conflict, we first try to separate the classes that have the same teacher or large number of overlapping students who want to take the classes at the same time to different timeslots. We also make sure the room capacity is well-used, meaning we assign the smallest room for the class that can hold all the students who want to take the class. If no such room exists, we assign the largest room available for the class. Although these two strategies in no way can guarantee the optimal answer, it minimizes the two aforementioned conflicts to some extent.

We implemented the algorithm in Java because it's a object-oriented language. While implementing the algorithm, we had some hard time dealing with MAX_Integer in Java. We used this to represent ∞ in our theoretical formulation. But this went wrong when we tried to sum all the conflict number for certain timeslot - adding number to MAX_Integer would cause overflow and the conflict number would be set back to a very small number. It caused a chain effect that sometimes resulted in the professor-conflict courses still being scheduled at the same time.

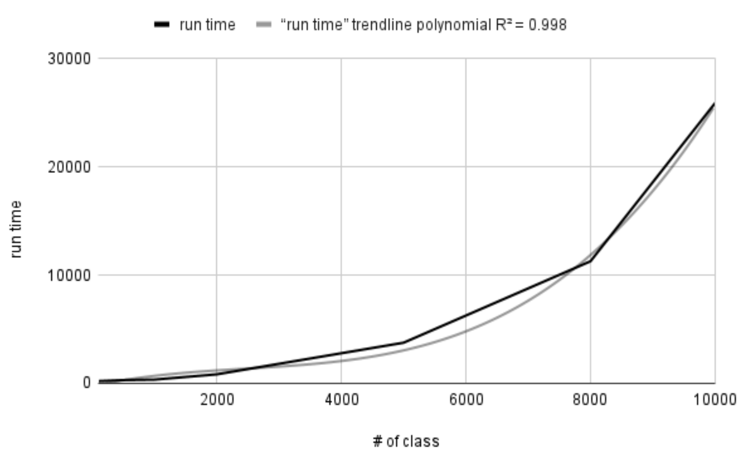
Another trouble we encountered is reading files. While the randomly generated data is indexed from 1, our array starts from 0. So we shift all the id by -1 to accommodate the array indexing. Later on when we use the real Bryn Mawr/Haverford data, the id is in other forms. In retrospect, we should have separated the id and the name of the data as two different instances in the class.

7 Experimental Analysis

7.1 Time Analysis

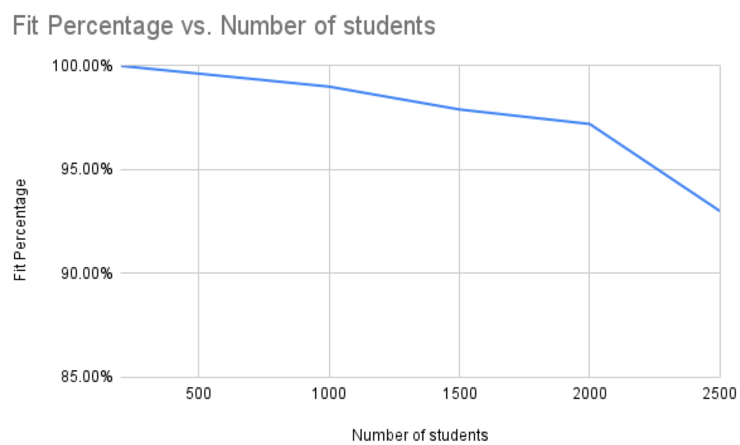


When we keep the number of rooms, classes, and time slots the same, and increase the number of students, we expect the running time would grow linearly as per time analysis suggests. The black line in the graph represents the actual run time of our program and the grey line represents a linear trend line. As the graph suggests, the time complexity is $O(s)$ when other constraints are fixed. The value $R^2 = 0.955$ suggests a very high fit for our analysis.



When we input fixed number of rooms, time slots and students, and increase the number of classes, we expect the running time to be $O(c^2 logc^2)$. The black line in the graph represents the actual run time of our program and the grey line represents a cubic polynomial trend line. As the graph suggested, the time complexity of our program is very close to c^3 . $R^2 = 0.998$ suggests our high fit. But since we actually used ArrayList in the actual program, adding classes takes $O(c)$ rather than $O(1)$, which might affect the ultimate run time.

7.2 Solution Quality Analysis



As illustrated in the graph above, with reasonable constraints, our program and algorithm can result in 98% fit percentage on average. As the number of students increase, we reach a lower bound of 93% of fit percentage.

The reason why students can't take certain classes is mainly due to time conflict and not room capacity conflict because when we increase the room capacity in a reasonable range, the fit percentage didn't change much.

8 Real Life Scenarios

When we extend our algorithm to run with the real Bryn Mawr/Haverford registration data in the past few years, there are several things need to be addressed in the code.

1. overlapping time: our previous assumption is that time slots are non-overlapping, but in reality the time is overlapped. We change the program so that if two classes' times is overlapped in any way, students can't take these two classes.
2. identifiers: the real data has a string of numbers to identify a class, student, room,

and professor. We added a step to store these identifiers as the name of the object and assigned another id starting from 0 for each object.

3. location constraints: since from the input list, we have locations followed by each course, we take those locations(rooms) as one of the constraints, i.e., classes could only take place in those locations. Thus, in the class *Courses*, we added an arraylist to store the rooms available for each course to assign, and during scheduling, courses could only be assigned to those locations.

4. number of classes in the preference list: In theoretical data, we assume each student must take 4 courses, but the real input data implies that the number of courses in each student's preference list varies. Some students take only 1 class and some take 5 classes. Thus, in order to calculate the fit percentage, we need to track how many classes students wish to take, rather than using $4 \times s$.

Based on the changes we made above, we try to extend the algorithm to accommodate other scenarios as potential suggestions we can give the registrar.

1. larger room capacity
2. with/without location restriction
3. splitting time slots
4. adding labs without location restriction
5. adding labs with location restriction

Intuitively, increase the room capacity in a reasonable manner could increase the fit percentage because there are students unable to take a class because the room is not large enough. But our data suggests otherwise. When we track the number of time conflicts and the number of capacity conflicts, we noticed that when the student body is small (less than 30000), time conflicts surpass the capacity conflicts. Therefore, for a liberal art college with small student body, increasing the room capacity can't really make a large difference.

We also hypothesized that if there is no location restriction, i.e. STEM classes has to be in Park Science Building, maybe we can have a more flexible schedule, thus resulting in a higher fit percentage. But as our data suggests, with or without the location restriction, we can achieve around 96% fit percentage. So allowing flexibility in class location doesn't help.

Next we split the time slots so that, for example, a class can happen on Monday morning and Tuesday afternoon as long as the total lecture time for the class is 3 hours. We suspected that this way allows for more flexibility in class times; hence less classes would be scheduled at the conflict timeslots. However, we still get a 96% fit percentage on average, which implies that a more flexible schedule does not make a difference.

Next, lab! We noticed some students has the same class in their preference list(actual registration list when using real data) twice, and we suspect that is the lab/discussion component of that class. There are two types of "labs". For CS labs and CHEM labs, for example, those subjects requires special equipment for labs thus they will need special rooms designed. For ECON labs and PSYC labs, since there is no specific lab requirement, labs can be assigned in any room available. We suspect that having labs for some courses may influence student's schedule, thus increasing schedule conflicts. On the other hand, we wonder whether if we assign all labs to certain lab rooms which will not be used as lecture rooms will influence the fit percentage.

We have done two different approaches, one is lab with no location constraints, i.e., we assume that all labs are held in the same classroom as the lectures. The resulting fit percentage is around 96.5%, which means that having labs with no location constraints dose not influence the overall fit percentage. On the other hand, when we assign labs to special rooms, the result fit percentage is around 95.5%, which, while still high, is lower than most of our results. We realized that having a lab with special lab rooms will slightly influence the overall fit percentage.

The reason why there is no significant change in fit percentage for the above implementation, we speculate is due to our original score is already high, and none of the changes can improve the score to a higher level. It is also possible because we use the data as all registered classes with registered locations, there might be some suitable available rooms to use while not in the data, which might help to improve our fit percentage.

Also, we assume courses could always fit in the less conflict timeslots. This could not be achieved if there are location constraints. According to the real data, the number of assigned classrooms for each course is limited, and thus should increase the number of schedule conflicts and decrease the overall fit.

We also speculate that since we did not create special lab timeslots but instead go under our modification with splitted timeslots, we wonder if we create special timeslots for labs as well as having combined timeslots will have some influence on the fit percentage.

Further, we noticed that ESEM, the compulsory writing class for freshman, while having multiple sessions, doesn't reflect on the input data. Dealing with ESEM and college requirements might be something interesting and important for future research.