HW4

March 7, 2019

```
In [2]: import numpy as np
        import matplotlib.pyplot as plt
        from scipy import linalg
        from sklearn.preprocessing import StandardScaler
        %matplotlib inline
        FIG_SIZE = (10, 5)
        plt.rcParams['figure.figsize'] = FIG_SIZE
```

1 Problem 1: Decision Trees

Let's first setup our dataset:

```
In [50]: data = np.array([[0,0,1,1,0, -1],
                          [1,1,0,1,0, -1],
                          [0,1,1,1,1, -1],
                          [1,1,1,1,0, -1],
                          [0,1,0,0,0, -1],
                          [1,0,1,1,1,  1],
                          [0,0,1,0,0,  1],
                          [1,0,0,0,0,  1],
                          [1,0,1,1,0,  1],
                          [1,1,1,1,1, -1] ]);
         X = data[:, :-1]
         y = data[:, -1]
```

1.1 Entropy

```
In [51]: import math
         def entropy(y):
             p = np.mean(y==1) # Compute the probability of y = 1
             if p == 0 or p == 1: # If the probability of either class is 1 H(y) = 0
                 return 0
             else:
                 return -(p*math.log2(p) + (1-p)*math.log2(1-p))  # equation for H(y) in terms
         print("H(y) = {} bits".format(entropy(y)))
```

```
H(y) = 0.9709505944546686 bits
```

1.2 Information Gain

```
In [65]: def info_gain(x, y):
             ig = [0.0] * 5 # Setup an output list
             ent = entropy(y) # Compute the original entropy
             for i in range(5):
                 x_i = x[:,i] # Select the i-th feature of each observation
                 p_x_i = np.mean(x_i)# Compute the probability that feature i == 1
                 if p_x_i == 0 or p_x_i == 1:
                     continue
                 # Compute the conditional entropy H(Y|X)
                 cond_ent =p_x_i*entropy(y[x_i==1])+(1-p_x_i)*entropy(y[x_i==0])
                 ig[i] =ent-cond_ent # Store the information gain
             return ig
         print(info_gain(X, y))
```

```
[0.0464393446710154, 0.6099865470109874, 0.0058021490143456145, 0.09127744624168, 0.0058021490
```

So, we should select the 2nd feature.

1.3 Decision Tree

```
In [67]: # Split on feature ...:
         print("Splitting on feature 2:")
         print("Left data:") # data where feature ... is 0
         print(data[X[:,1]==0,:])
         print("Right data:") # data where feature ... is 1
         print(data[X[:,1]==1,:])
```

```
Splitting on feature 2:
Left data:
[[ 0  0  1  1  0 -1]
 [ 1  0  1  1  1  1]
 [ 0  0  1  0  0  1]
 [ 1  0  0  0  0  1]
 [ 1  0  1  1  0  1]]
Right data:
[[ 1  1  0  1  0 -1]
 [ 0  1  1  1  1 -1]
 [ 1  1  1  1  0 -1]
 [ 0  1  0  0  0 -1]
 [ 1  1  1  1  1 -1]]
```

On the right data, we will always predict "-1". On the left data, we'll need to split again. You can see by inspection that the next best feature is the first or you can recompute it as above:

```
In [69]: data_left = data[X[:, 1] == 0, :]
         X_left, y_left = data_left[:, :-1], data_left[:, -1]
         print(info_gain(X_left, y_left))
```

```
[0.3219280948873623, 0.0, 0.07290559532005603, 0.17095059445466865, 0.07290559532005603]
```

```
In [73]: print("Splitting left data on feature 1:")
         print("Left data:") # data_left where feature ... is 0
         print(data_left[X_left[:,0]==0,:])
         print("Right data:") # data_left where feature ... is 1
         print(data_left[X_left[:,0]==1,:])
```

```
Splitting left data on feature 1:
Left data:
[[ 0  0  1  1  0 -1]
 [ 0  0  1  0  0  1]]
Right data:
[[1 0 1 1 1 1]
 [1 0 0 0 0 1]
 [1 0 1 1 0 1]]
```

On the right data, we always predict "+1". On the left data, we can see that only forth data is different and decide if reading or not

So the final rule is:

```
In [74]: # if(long):
         #     discard
         # else:
         #     if(known):
         #         read
         #     else:
         #         if(grade):
         #             discard
         #         else:
         #             read
```

2 Problem 2: EigenFaces

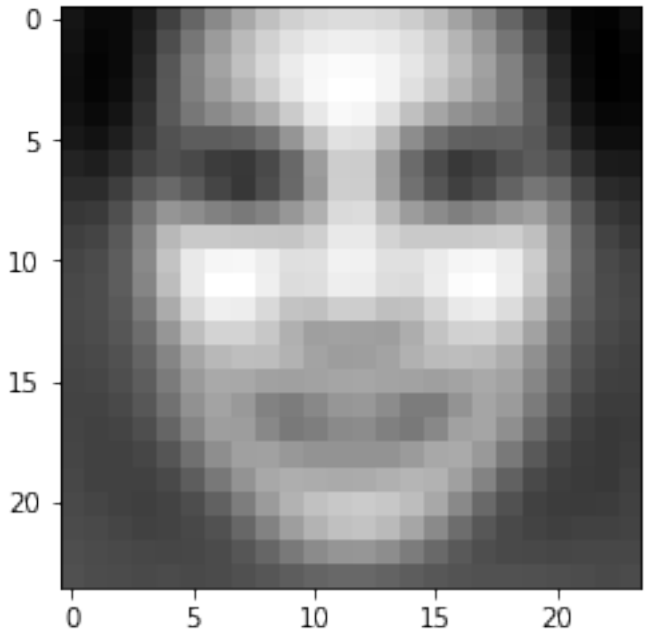
2.1 Part 1

```
In [84]: # Load face dataset
         X = np.genfromtxt("faces.txt",delimiter=None)

         # Remove the mean
         mu = np.mean(X,axis=0) #find mean face
         X0 = X-mu #subtract mean face

         # Plot the mean face
         plt.figure()
         mean_face = np.reshape(mu, (24,24))
         plt.imshow(mean_face.T, cmap="gray")
```

Out[84]: <matplotlib.image.AxesImage at 0x1a1f3a3748>



2.2 Part 2: SVD

```
In [111]: U, S, V = linalg.svd(X0,full_matrices=False)
          W = U.dot(np.diag(S)).dot(V)
          print(W.shape, V.shape)
```

```
(4916, 576) (576, 576)
```

```
In [92]: X.shape
```

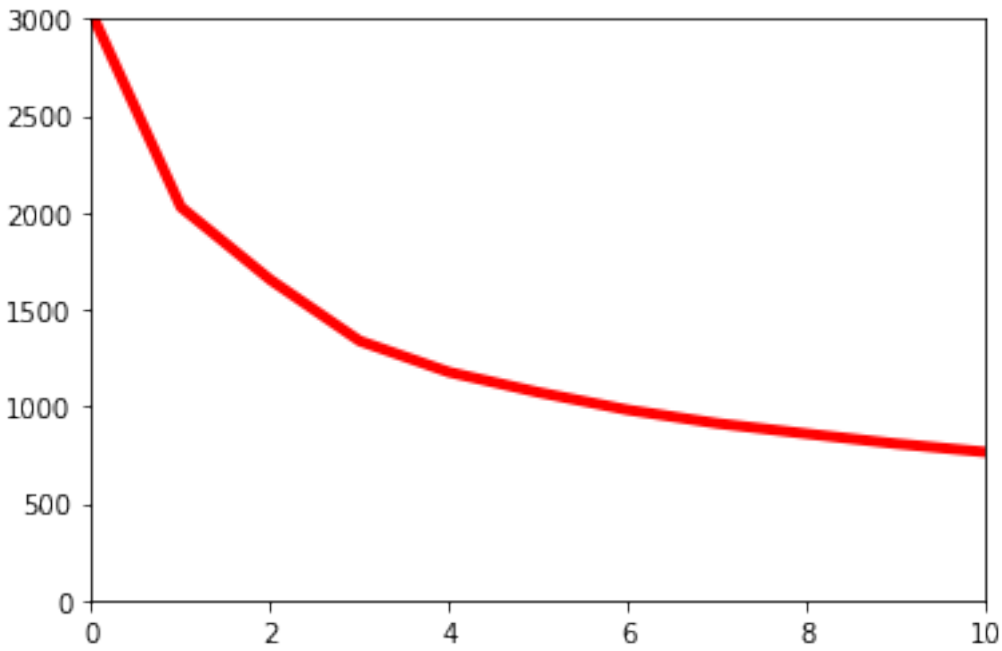
```
Out[92]: (4916, 576)
```

2.3 Part 3

```
In [112]: errors = []
          for K in range(11):
              X0_hat = U[:,0:K].dot(np.diag(S[0:K])).dot(V[0:K,:])
              errors.append(np.mean((W-X0_hat) ** 2))#mse between X0 and X0_hat

          plt.plot(range(11),errors,'r-',linewidth=4)
          plt.axis((0,10,0,3000))
```

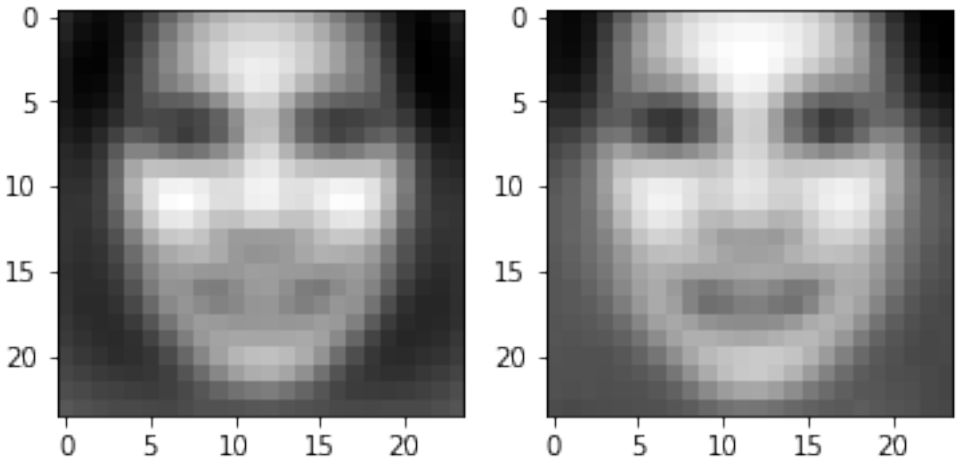
```
Out[112]: (0, 10, 0, 3000)
```



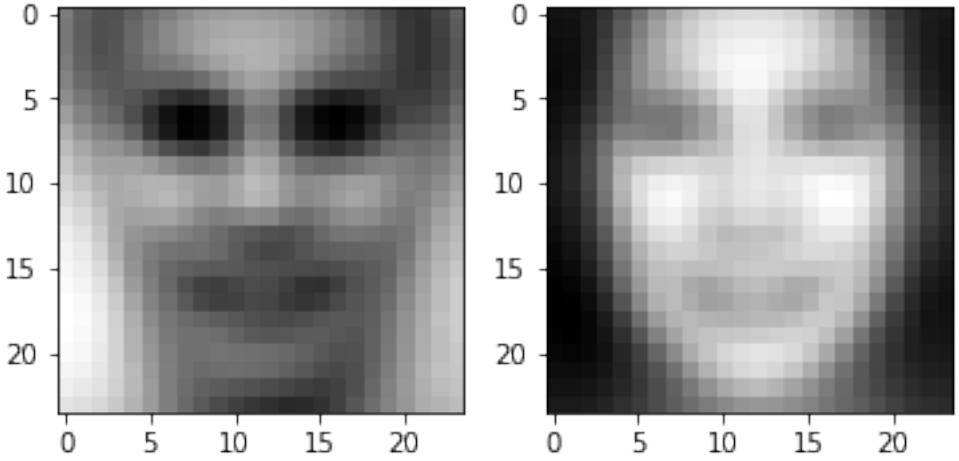
2.4 Part 4

```
In [113]: # Let's look at how the faces vary along each principal component
          W = U.dot(np.diag(S))
          for k in range(3):
              alpha = 2*np.median( np.abs( W[:,k] )); #scaler
              im1 = np.reshape(mu + alpha*V[k,:],(24,24)) #add PC to the mean
              im2 = np.reshape(mu - alpha*V[k,:] ,(24,24)) #subtract PC to the mean
              plt.figure();
              f,(ax1,ax2) = plt.subplots(1,2);
              ax1.imshow(im1.T, cmap="gray");
              ax2.imshow(im2.T, cmap="gray");
```

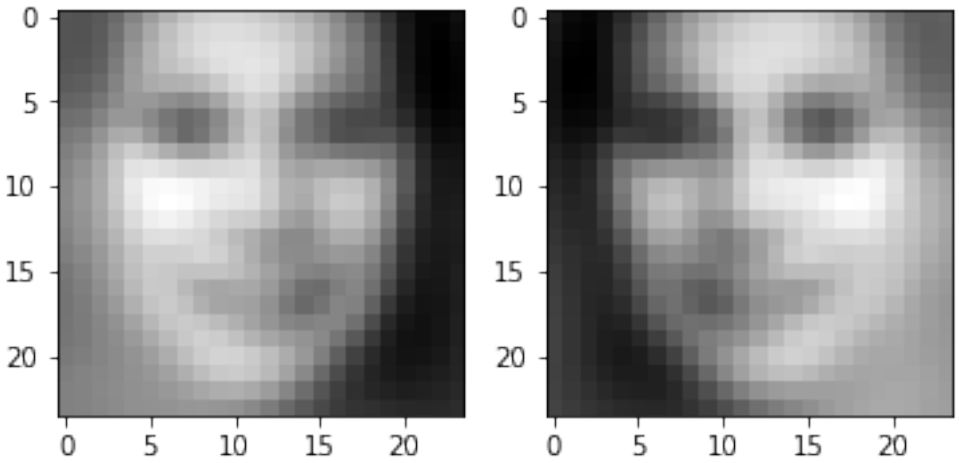
```
<Figure size 432x288 with 0 Axes>
```




```
<Figure size 432x288 with 0 Axes>
```



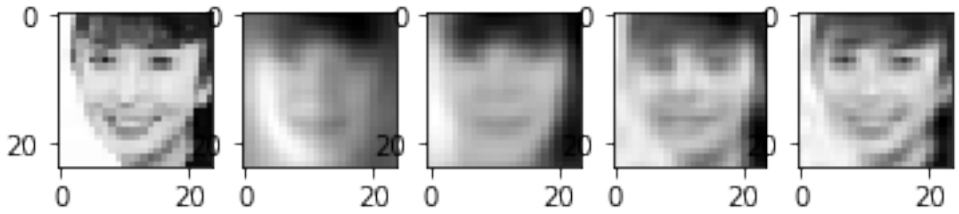

```
<Figure size 432x288 with 0 Axes>
```



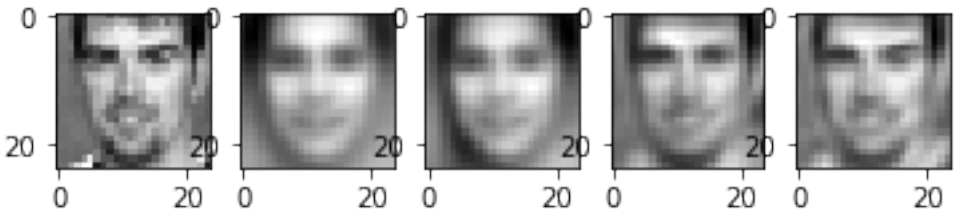
2.5 Part 5:

```
In [115]: # Reconstruct two faces using a few components
          for i in [24,35]:
              im = X[i,:];
              im = np.reshape(im, (24,24));
              plt.figure()
              f,ax = plt.subplots(1,5);
              ax[0].imshow(im.T, cmap="gray"); #show original image
              for j,k in enumerate([5,10,50,100]):
                  im = mu+W[i:i+1,0:k].dot(V[0:k,:]) #reconstruct using k principal components
                  im = np.reshape(im,(24,24))# and reshape im to 24x24
                  ax[j+1].imshow(im.T, cmap="gray");
```

```
<Figure size 432x288 with 0 Axes>
```

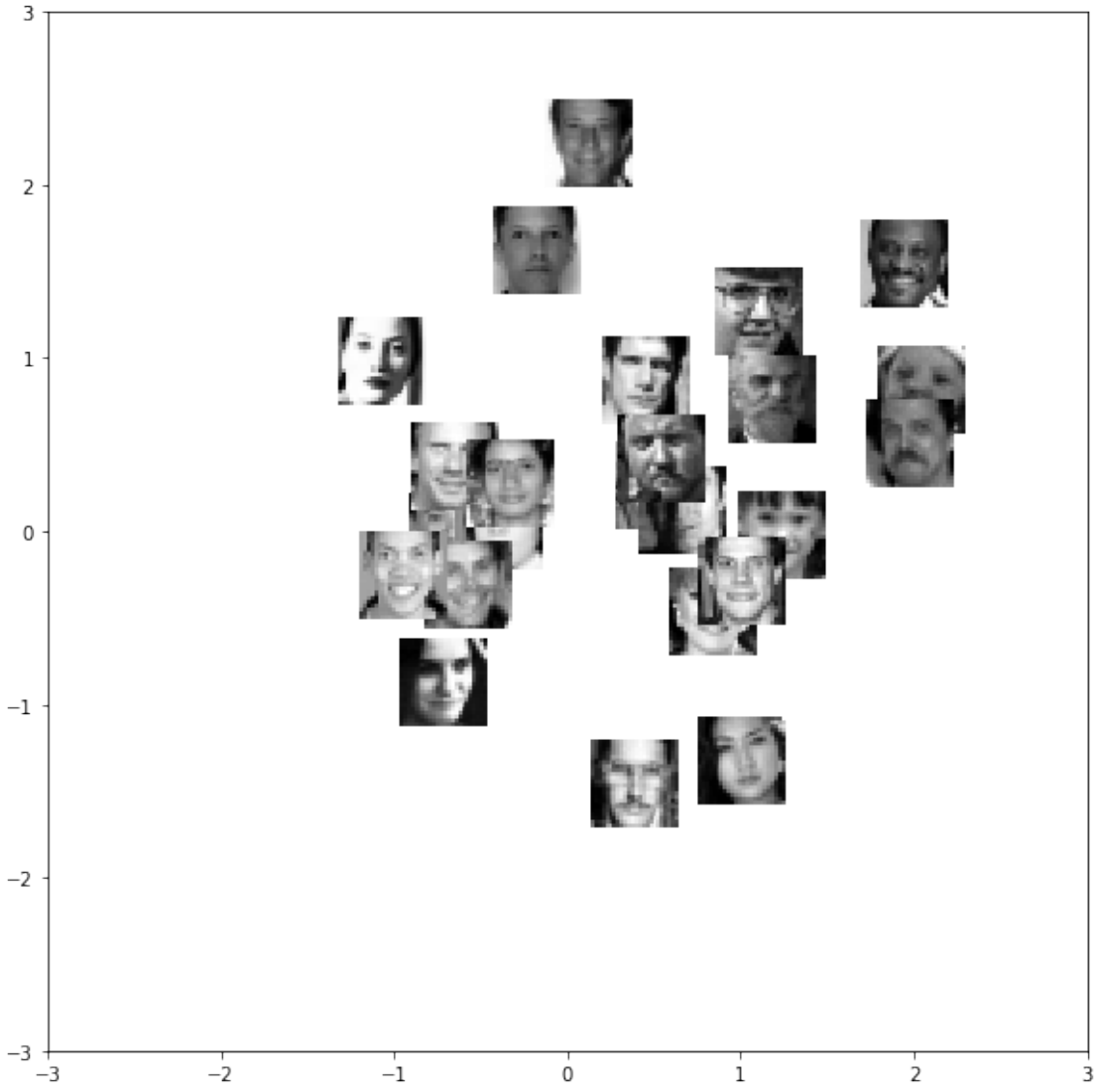



```
<Figure size 432x288 with 0 Axes>
```



2.6 Part 6

```
In [130]: idx = np.random.choice(576,25)# sample 25 images without replacement using np.random
          coord = StandardScaler().fit_transform(W[:, :2])
          plt.figure(figsize = (10, 10))
          for i in idx:
              loc = (coord[i,0],coord[i,0]+0.5, coord[i,1],coord[i,1]+0.5)# compute where to p
              img = np.reshape( X[i,:], (24,24) )# reshape to square
              plt.imshow( img.T , cmap="gray", extent=loc ) # draw each image
              plt.axis( (-3,3,-3,3) ) # set axis to a reasonable scale
```



3 Problem 3: Statement of Collaboration

For doing Homework 4, I use the discussion file as the guide and follow the instruction step by step. Also, I checked the piazza question posts for doing problem 2.4 and 2.5. I went to a study group with Wanjing Zhang and Jiaxiang Wang to discuss the details.

In [ ]: