On the Controversy of BF false positive

2014.4.2

Analysing the false positive of Bloom Filter

Defining the temporary function

```
fone[i_, j_, k_, n_, m_] = (-1)^(i - j) j^(k n) i^k / ((m - i) ! j! (i - j) !)
```

$$\frac{(-1)^{i-j}\, i^{k}\, j^{k\,n}}{(i-j)!\; j!\; (-i+m)!}$$

无误差公式

```
fchrist[k_, n_, m_] = m! / (m^(k (n + 1))) (Sum[fone[i, j, k, n, m], {i, 1, m}, {j, 1, i}])
```

$$m^{-k(1+n)}\, m!\sum_{i=1}^{m}\sum_{j=1}^{i}\frac{(-1)^{i-j}\, i^{k}\, j^{k n}}{(i-j)!\, j!\, (-i+m)!}$$

k取最优时公式

```
falsemn[m_, n_] = fchrist[Log[2] / (n (Log[m] - Log[m - 1])), n, m]
```

$$m^{-\frac{(1+n)\,\mathrm{Log}[2]}{n\,(-\mathrm{Log}[-1+m]+\mathrm{Log}[m])}}\, m!\sum_{i=1}^{m}\sum_{j=1}^{i}\left((-1)^{i-j}\, i^{\frac{\mathrm{Log}[2]}{n\,(-\mathrm{Log}[-1+m]+\mathrm{Log}[m])}}\, j^{\frac{\mathrm{Log}[2]}{-\mathrm{Log}[-1+m]+\mathrm{Log}[m]}}\right)\Big/\left((i-j)!\, j!\, (-i+m)!\right)$$

m趋于无穷大

```
Limit[falsemn[m, n], m → ∞]
```

$$\text{Limit}\left[m^{-\frac{(1+n)\,\text{Log}[2]}{n\,(-\text{Log}[-1+m]+\text{Log}[m])}}\, m!\, \sum_{i=1}^{m} \sum_{j=1}^{i} \left((-1)^{i-j}\, i^{\frac{\text{Log}[2]}{n\,(-\text{Log}[-1+m]+\text{Log}[m])}}\, j^{\frac{\text{Log}[2]}{-\text{Log}[-1+m]+\text{Log}[m]}}\right) \Big/ \left((i-j)!\, j!\, (-i+m)!\right),\; m \to \infty\right]$$

代入k=ln2 m/n

```
fchrist[Log[2] m / n, n, m]
```

$$m^{-\frac{m (1+n) \text{Log}[2]}{n}} m! \sum_{i=1}^{m} \sum_{j=1}^{i} \frac{(-1)^{i-j}\, i^{\frac{m \text{Log}[2]}{n}}\, j^{m \text{Log}[2]}}{(i-j)!\, j!\, (-i+m)!}$$

取极限

```
Limit[fchrist[Log[2] m / n, n, m], m → ∞]
```

$$\text{Limit}\left[m^{-\frac{m (1+n) \text{Log}[2]}{n}} m! \sum_{i=1}^{m} \sum_{j=1}^{i} \frac{(-1)^{i-j}\, i^{\frac{m \text{Log}[2]}{n}}\, j^{m \text{Log}[2]}}{(i-j)!\, j!\, (-i+m)!},\ m \to \infty\right]$$