THE UNIVERSITY OF TEXAS AT ARLINGTON, TEXAS
DEPARTMENT OF ELECTRICAL ENGINEERING

EE 5329

Distributed Decision and Control

HW # 2
ASSIGNMENT

by

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Presented to

Dr. Frank Lewis

Jan 30, 2018

<u>EE 5329 Distributed Decision and Control</u>

Spring 2018

Homework Pledge of Honor

On all homeworks in this class - YOU MUST WORK ALONE.

Any cheating or collusion will be severely punished.

It is very easy to compare your software code and determine if you worked together

It does not matter if you change the variable names.

Please sign this form and include it as the first page of all of your submitted homeworks.

....

Typed Name: <u>Soutrik Maiti</u>

Pledge of honor:

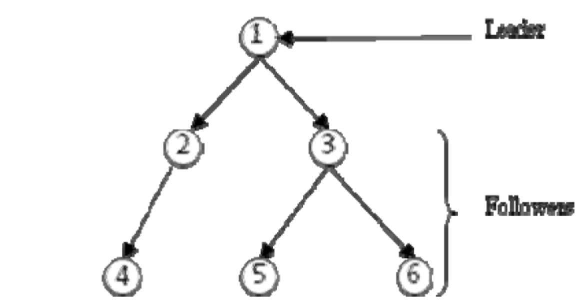
"On my honor I have neither given nor received aid on this homework."

e-Signature: <u>Soutrik Maiti</u>

Graph Laplacian Eigen Values

1.

a) *Adjacent Matrix A and graph Laplacian L for the graph given in problem:*



1. Directed Tree graph. (Formation graph)

Adjacent Matrix:

```
a1 =

         0         0         0         0         0         0
    0.5000         0         0         0         0         0
    0.5000         0         0         0         0         0
         0    0.5000         0         0         0         0
         0         0    0.5000         0         0         0
         0         0    0.5000         0         0         0
```

Laplacian Matrix:

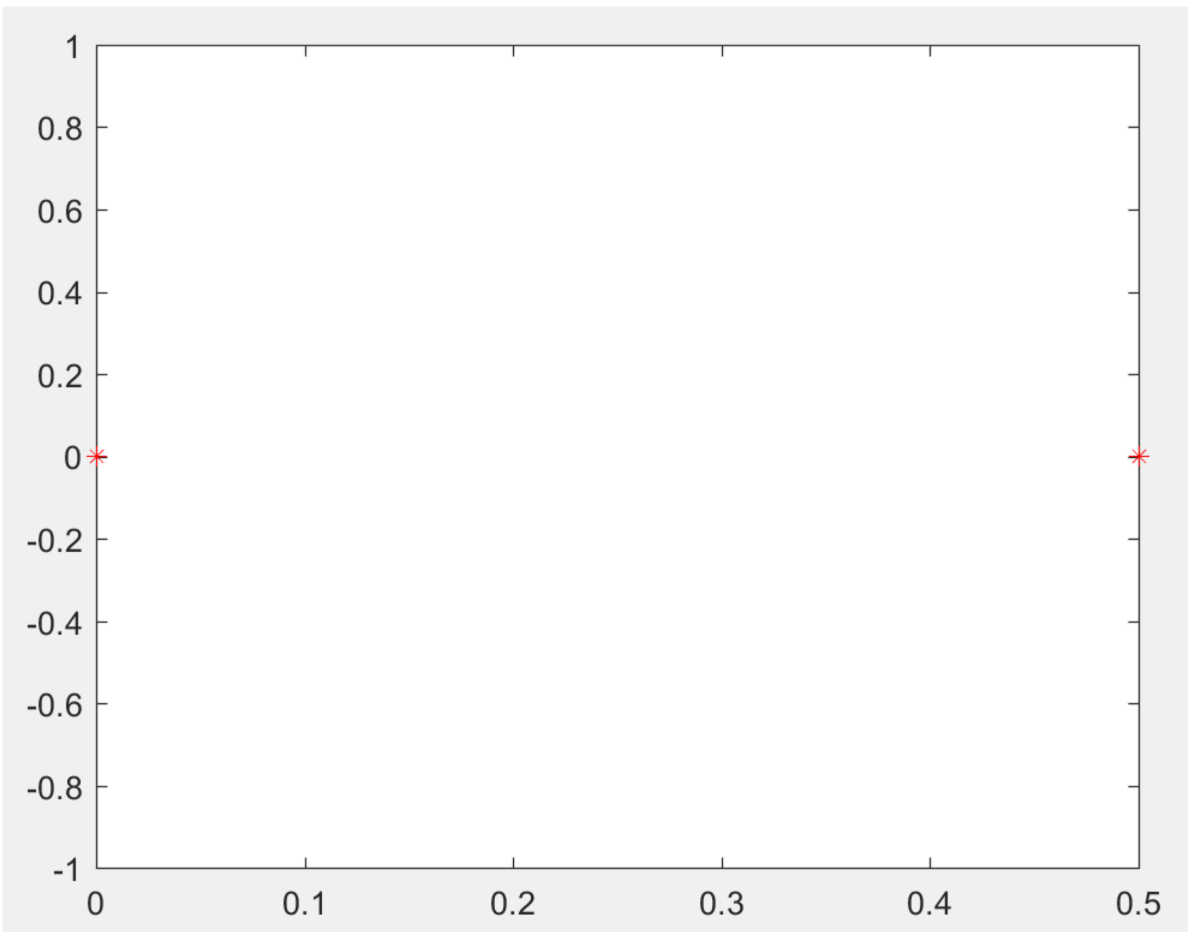
```
l1 =

         0         0         0         0         0         0
   -0.5000    0.5000         0         0         0         0
   -0.5000         0    0.5000         0         0         0
         0   -0.5000         0    0.5000         0         0
         0         0   -0.5000         0    0.5000         0
         0         0   -0.5000         0         0    0.5000
```

b) *Plotting the Eigen Values of L on the complex plane :*

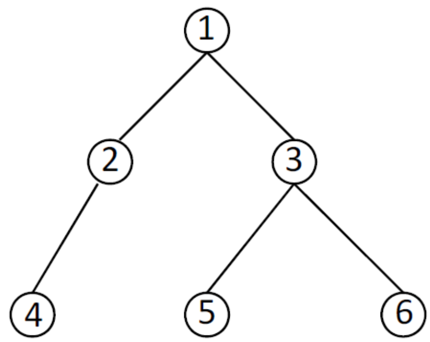
```
ev1 =

    0.5000
    0.5000
    0.5000
    0.5000
    0.5000
         0
```



2.

a) *Adjacent Matrix A and graph Laplacian L for the graphs given in problem:*



2. Undirected Tree graph.

Adjacent Matrix:

```
a2 =

         0    0.5000    0.5000         0         0         0
    0.5000         0         0    0.5000         0         0
    0.5000         0         0         0    0.5000    0.5000
         0    0.5000         0         0         0         0
         0         0    0.5000         0         0         0
         0         0    0.5000         0         0         0
```

Laplacian Matrix:

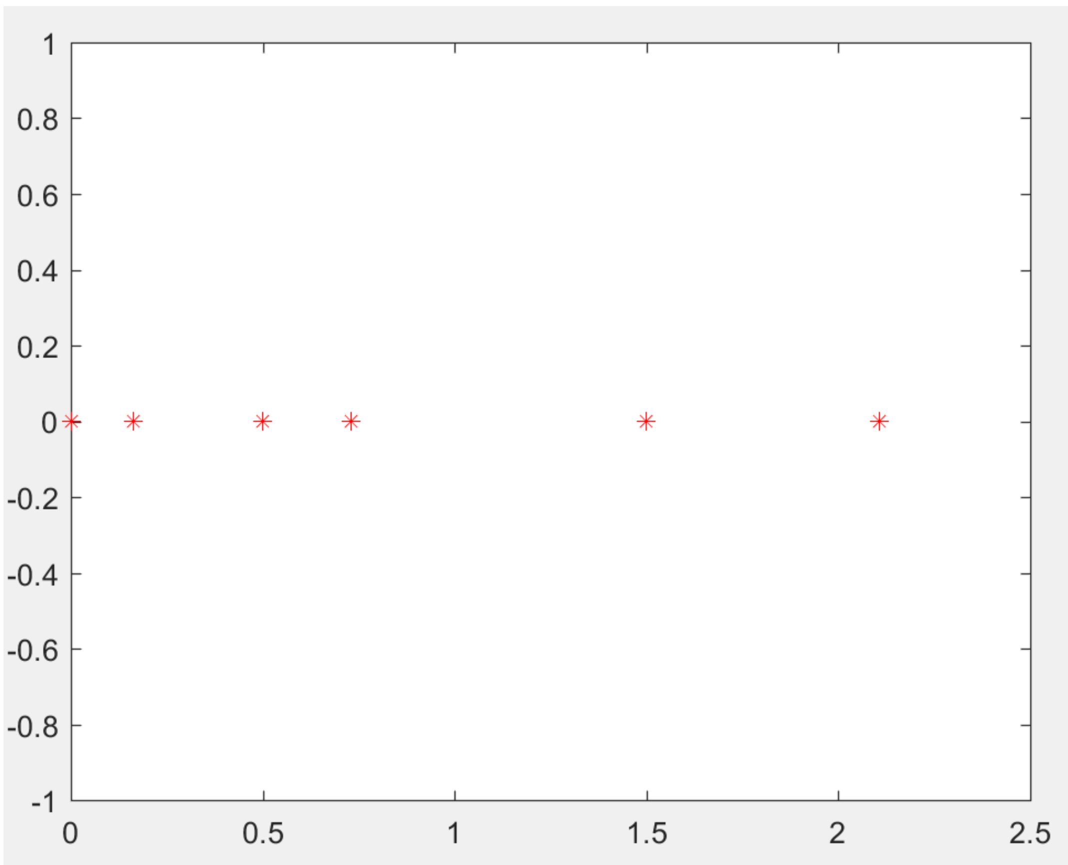
```
l2 =

    1.0000   -0.5000   -0.5000         0         0         0
   -0.5000    1.0000         0   -0.5000         0         0
   -0.5000         0    1.5000         0   -0.5000   -0.5000
         0   -0.5000         0    0.5000         0         0
         0         0   -0.5000         0    0.5000         0
         0         0   -0.5000         0         0    0.5000
```

b) Plotting the Eigen Values of L on the complex plane:

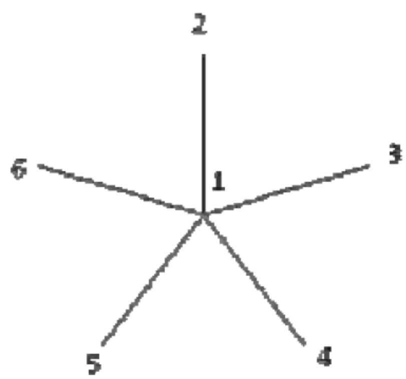
```
ev2 =

    0.0000
    0.1624
    0.5000
    0.7304
    1.5000
    2.1072
```



3.

a) Adjacent Matrix A and graph Laplacian L for the graphs given in problem:



3. Star $K_{1,5}$

Adjacent Matrix:

a3 =

0	0.5000	0.5000	0.5000	0.5000	0.5000
0.5000	0	0	0	0	0
0.5000	0	0	0	0	0
0.5000	0	0	0	0	0
0.5000	0	0	0	0	0
0.5000	0	0	0	0	0

Laplacian Matrix:

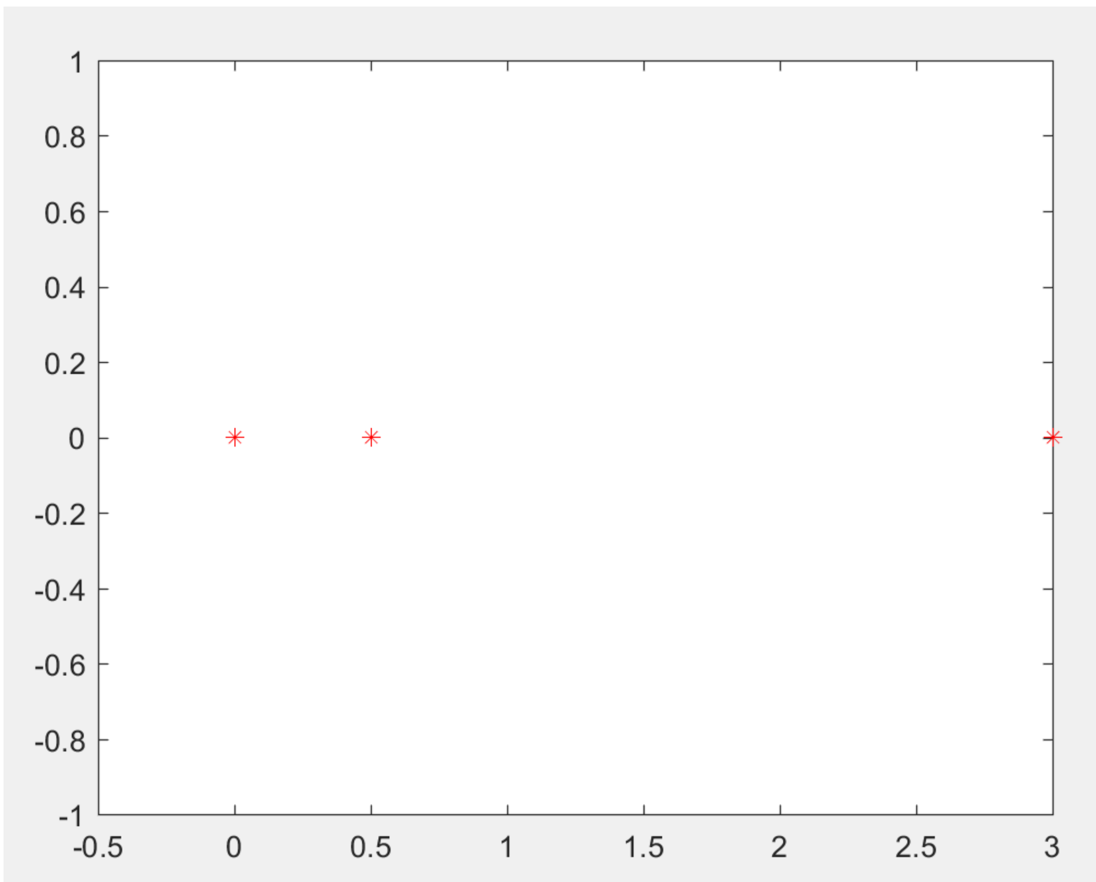
l3 =

2.5000	-0.5000	-0.5000	-0.5000	-0.5000	-0.5000
-0.5000	0.5000	0	0	0	0
-0.5000	0	0.5000	0	0	0
-0.5000	0	0	0.5000	0	0
-0.5000	0	0	0	0.5000	0
-0.5000	0	0	0	0	0.5000

b) Plotting the Eigen Values of L on the complex plane:

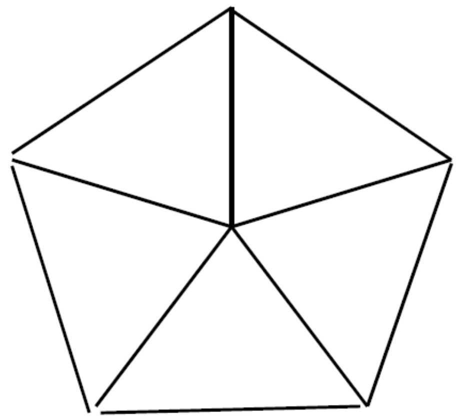
```
ev3 =

   -0.0000
    0.5000
    0.5000
    0.5000
    0.5000
    3.0000
```



4.

a) Adjacent Matrix A and graph Laplacian L for the graphs given in problem:



4. Star with extra edges

Adjacent Matrix:

```
a4 =

         0    0.5000    0.5000    0.5000    0.5000    0.5000
    0.5000         0    0.5000         0         0    0.5000
    0.5000    0.5000         0    0.5000         0         0
    0.5000         0    0.5000         0    0.5000         0
    0.5000         0         0    0.5000         0    0.5000
    0.5000    0.5000         0         0    0.5000         0
```

Laplacian Matrix:

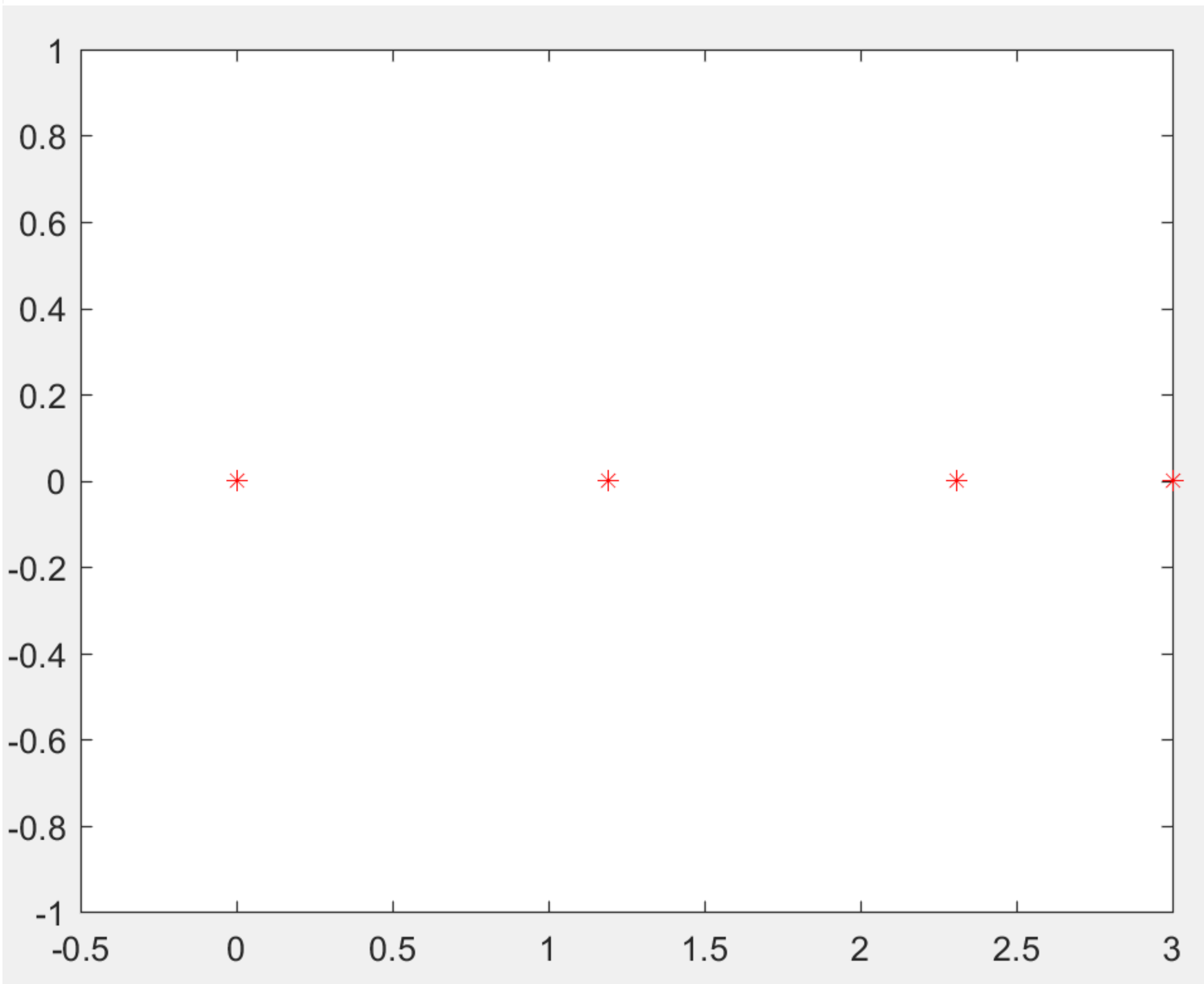
```
l4 =

    2.5000   -0.5000   -0.5000   -0.5000   -0.5000   -0.5000
   -0.5000    1.5000   -0.5000         0         0   -0.5000
   -0.5000   -0.5000    1.5000   -0.5000         0         0
   -0.5000         0   -0.5000    1.5000   -0.5000         0
   -0.5000         0         0   -0.5000    1.5000   -0.5000
   -0.5000   -0.5000         0         0   -0.5000    1.5000
```

b) *Plotting the Eigen Values of L on the complex plane:*

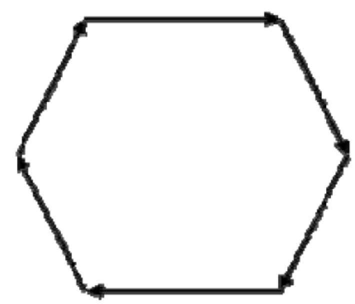
```
ev4 =

  -0.0000
   1.1910
   1.1910
   2.3090
   2.3090
   3.0000
```



5.

a) Adjacent Matrix A and graph Laplacian L for the graphs given in problem:



5. Directed 6-cycle
(6-periodic graph)

Adjacent Matrix:

```
a5 =

         0         0         0         0         0    0.5000
    0.5000         0         0         0         0         0
         0    0.5000         0         0         0         0
         0         0    0.5000         0         0         0
         0         0         0    0.5000         0         0
         0         0         0         0    0.5000         0
```

Laplacian Matrix:

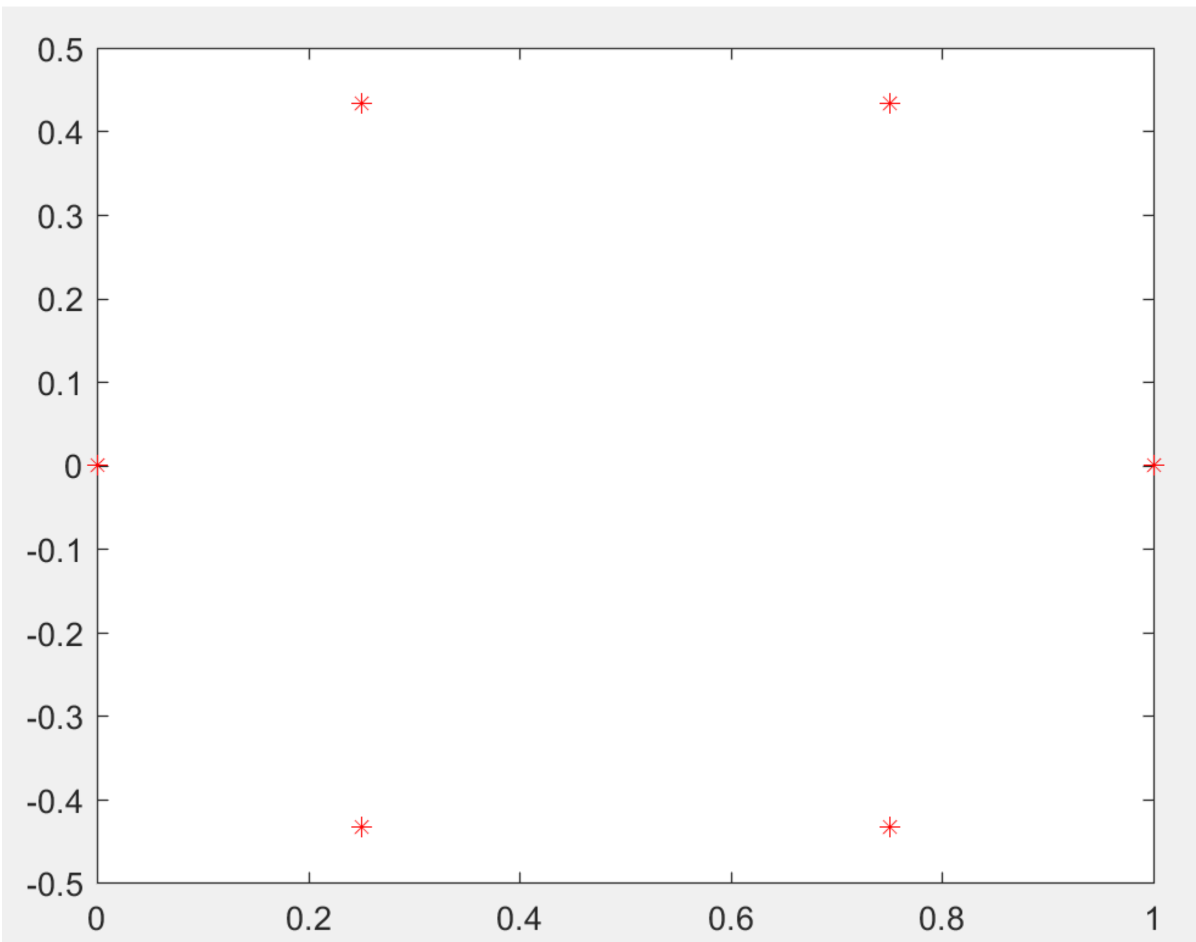
```
l5 =

    0.5000         0         0         0         0   -0.5000
   -0.5000    0.5000         0         0         0         0
         0   -0.5000    0.5000         0         0         0
         0         0   -0.5000    0.5000         0         0
         0         0         0   -0.5000    0.5000         0
         0         0         0         0   -0.5000    0.5000
```

b) Plotting the Eigen Values of L on the complex plane:

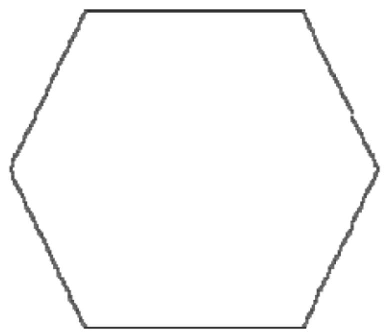
```
ev5 =

  -0.0000 + 0.0000i
   0.2500 + 0.4330i
   0.2500 - 0.4330i
   0.7500 + 0.4330i
   0.7500 - 0.4330i
   1.0000 + 0.0000i
```



6.

a) Adjacent Matrix A and graph Laplacian L for the graphs given in problem:



6. 6-cycle C^6 (2-regular graph).

Adjacent Matrix:

```
a6 =

         0    0.5000         0         0         0    0.5000
    0.5000         0    0.5000         0         0         0
         0    0.5000         0    0.5000         0         0
         0         0    0.5000         0    0.5000         0
         0         0         0    0.5000         0    0.5000
    0.5000         0         0         0    0.5000         0
```

Laplacian Matrix:

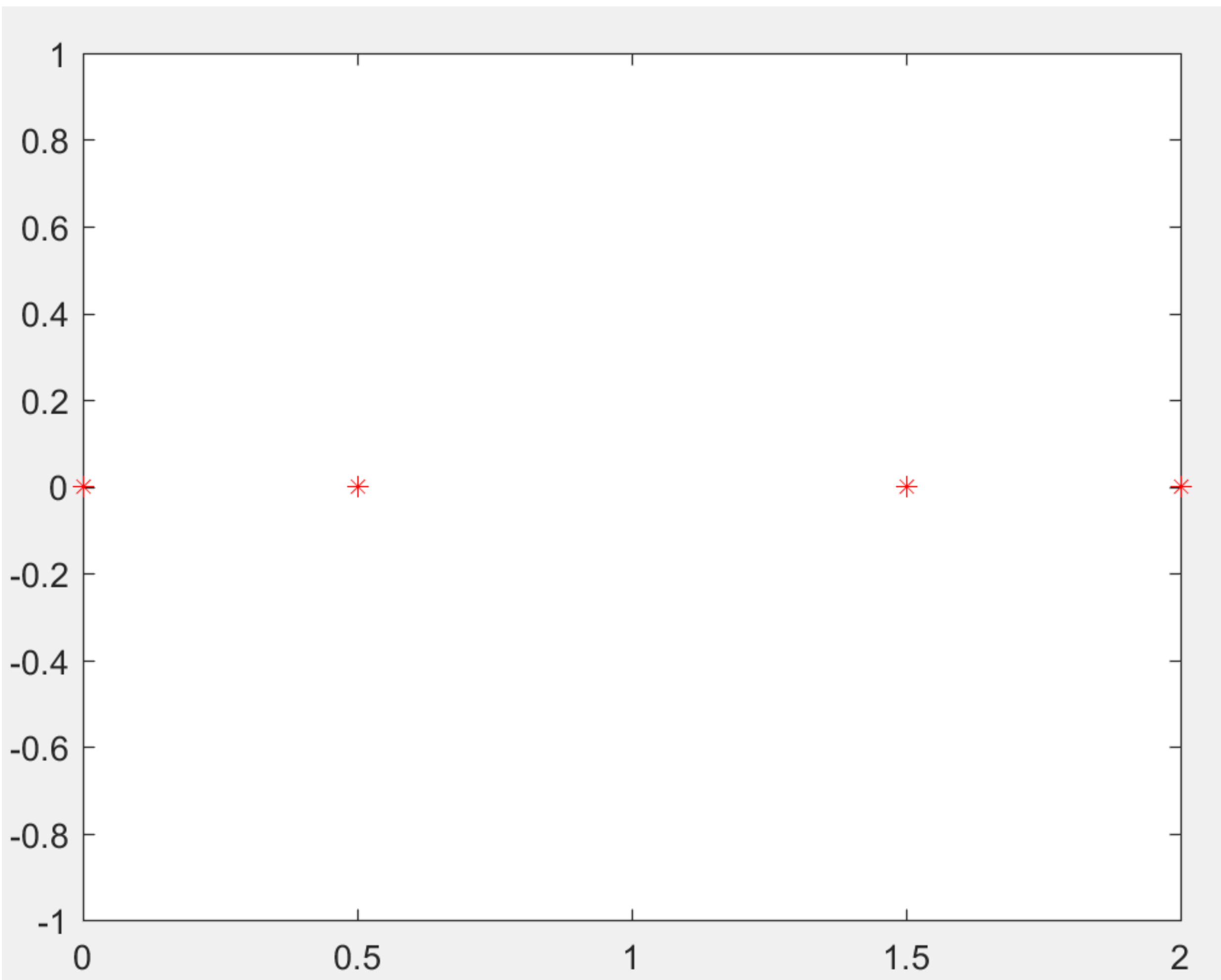
```
l6 =

    1.0000   -0.5000         0         0         0   -0.5000
   -0.5000    1.0000   -0.5000         0         0         0
         0   -0.5000    1.0000   -0.5000         0         0
         0         0   -0.5000    1.0000   -0.5000         0
         0         0         0   -0.5000    1.0000   -0.5000
   -0.5000         0         0         0   -0.5000    1.0000
```

c) *Plotting the Eigen Values of L on the complex plane:*

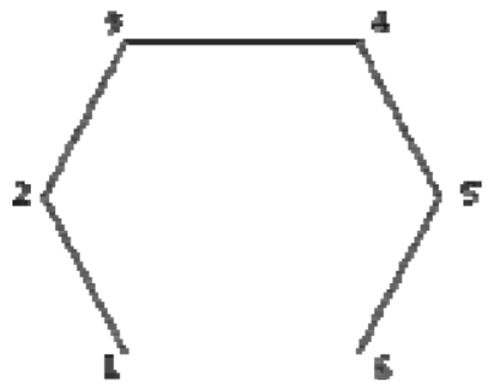
```
ev6 =

   -0.0000
    0.5000
    0.5000
    1.5000
    1.5000
    2.0000
```



7.

a) Adjacent Matrix A and graph Laplacian L for the graphs given in problem:



7. Path P^5.

Adjacent Matrix:

```
a7 =

         0    0.5000         0         0         0         0
    0.5000         0    0.5000         0         0         0
         0    0.5000         0    0.5000         0         0
         0         0    0.5000         0    0.5000         0
         0         0         0    0.5000         0    0.5000
         0         0         0         0    0.5000         0
```

Laplacian Matrix:

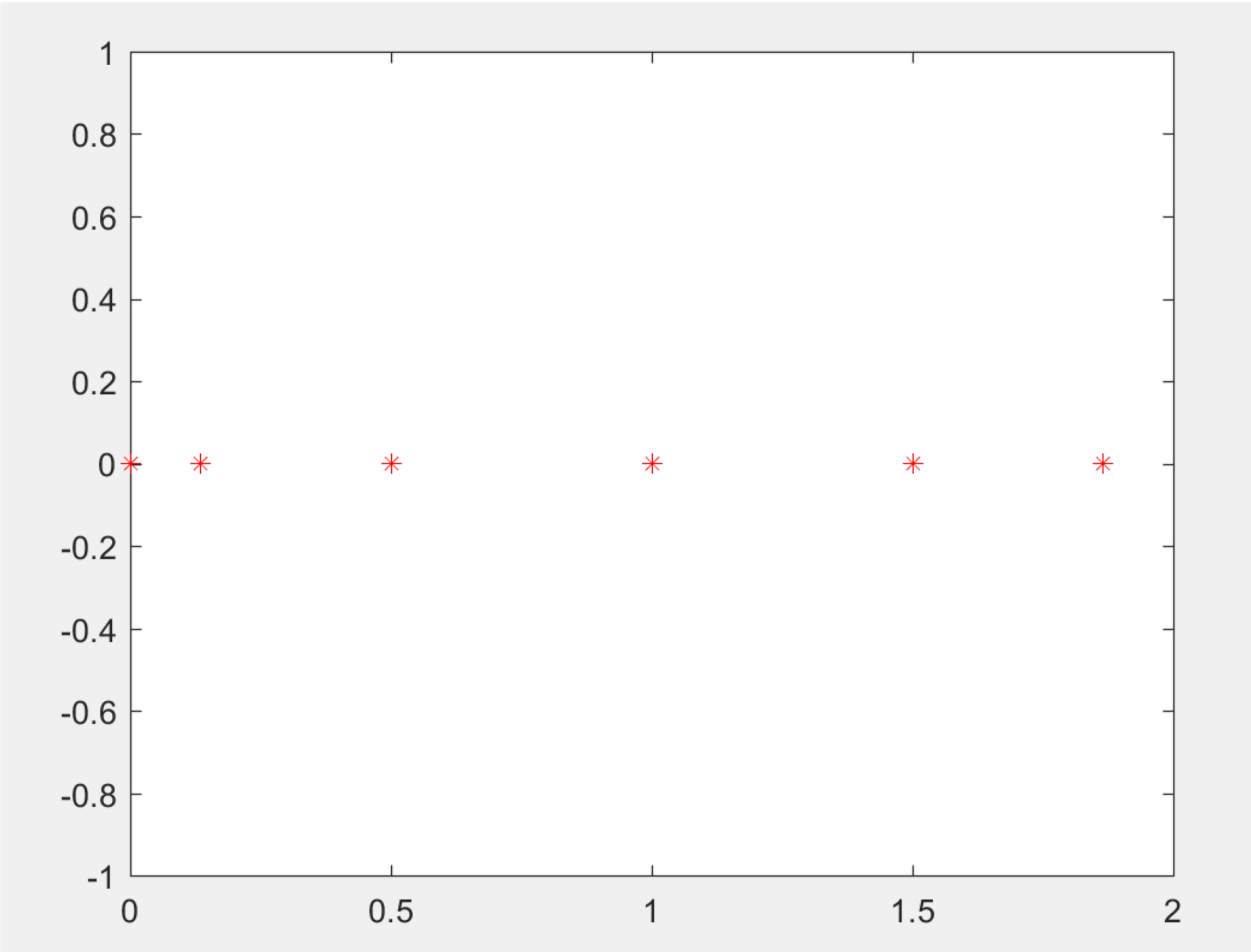
```
l7 =

    0.5000   -0.5000         0         0         0         0
   -0.5000    1.0000   -0.5000         0         0         0
         0   -0.5000    1.0000   -0.5000         0         0
         0         0   -0.5000    1.0000   -0.5000         0
         0         0         0   -0.5000    1.0000   -0.5000
         0         0         0         0   -0.5000    0.5000
```

b) Plotting the Eigen Values of L on the complex plane:

```
ev7 =

    0.0000
    0.1340
    0.5000
    1.0000
    1.5000
    1.8660
```



c)The Fiedler values of the corresponding graphs are:

The Fiedler value represents the rate of convergence of the graph

Graph 1 = 0.5000

Graph 2 = 0.1624

Graph 3 = 0.5000

Graph 4 = 1.1910

Graph 5 = 0.2500 + 0.4330i

Graph 6 = 0.5000

Graph 7 = 0.1340

The Fiedler value for the graph 4(1.1910) is the highest and hence this graph (star with extra edges) would converge the fastest.

d) The Left eigenvector w1 of L for λ1=0 for all the graphs are as follows :

```
w1_L1 =

     1
     0
     0
     0
     0
     0
```

```
w1_L2 =

   -0.4082
   -0.4082
   -0.4082
   -0.4082
   -0.4082
   -0.4082
```

```
w1_L3 =

   -0.4082
   -0.4082
   -0.4082
   -0.4082
   -0.4082
   -0.4082
```

```
w1_L4 =

   -0.4082
   -0.4082
   -0.4082
   -0.4082
   -0.4082
   -0.4082

w1_L5 =

    0.4082
    0.4082
    0.4082
    0.4082
    0.4082
    0.4082

w1_L6 =

    0.4082
    0.4082
    0.4082
    0.4082
    0.4082
    0.4082

w1_L7 =

   -0.4082
   -0.4082
   -0.4082
   -0.4082
   -0.4082
   -0.4082
```

e) The Fiedler Eigen Vector v2 for the corresponding graphs are as follows:

```
v2_L1 =

     0
     0
     0
     1
     0
     0


v2_L2 =

    0.0813
    0.4193
   -0.2831
    0.6211
   -0.4193
   -0.4193


v2_L3 =

         0
   -0.4082
    0.8165
         0
   -0.4082
         0


v2_L4 =

    0.0000
    0.1936
   -0.5128
   -0.5105
    0.1973
    0.6325
```

```
v2_L5 =

  -0.2041 + 0.3536i
  -0.4082 + 0.0000i
  -0.2041 - 0.3536i
   0.2041 - 0.3536i
   0.4082 - 0.0000i
   0.2041 + 0.3536i

v2_L6 =

    0.2887
   -0.2887
   -0.5774
   -0.2887
    0.2887
    0.5774

v2_L7 =

   -0.5577
   -0.4082
   -0.1494
    0.1494
    0.4082
    0.5577
```

MATLAB CODE:

```
clear all

clc

close all

%A matrix for first problem

a1= [0 0 0 0 0 0;

   0.5 0 0 0 0 0;

   0.5 0 0 0 0 0;
```

```
    0 0.5 0 0 0 0;
    0 0 0.5 0 0 0;
    0 0 0.5 0 0 0]
%A matrix for second problem
a2= [0 0.5 0.5 0 0 0;
    0.5 0 0 0.5 0 0;
    0.5 0 0 0 0.5 0.5;
    0 0.5 0 0 0 0;
    0 0 0.5 0 0 0;
    0 0 0.5 0 0 0]
%A matrix for third problem
a3= [0 0.5 0.5 0.5 0.5 0.5;
    0.5 0 0 0 0 0;
    0.5 0 0 0 0 0;
    0.5 0 0 0 0 0;
    0.5 0 0 0 0 0;
    0.5 0 0 0 0 0]
%A matrix for fourth problem
a4= [0 0.5 0.5 0.5 0.5 0.5;
    0.5 0 0.5 0 0 0.5;
    0.5 0.5 0 0.5 0 0;
    0.5 0 0.5 0 0.5 0;
    0.5 0 0 0.5 0 0.5;
    0.5 0.5 0 0 0.5 0]
%A matrix for fifth problem
a5= [0 0 0 0 0 0.5;
    0.5 0 0 0 0 0;
    0 0.5 0 0 0 0;
    0 0 0.5 0 0 0;
```

```
    0 0 0 0.5 0 0;
    0 0 0 0 0.5 0]
%A matrix for sixth problem
a6= [0 0.5 0 0 0 0.5;
    0.5 0 0.5 0 0 0;
    0 0.5 0 0.5 0 0;
    0 0 0.5 0 0.5 0;
    0 0 0 0.5 0 0.5;
    0.5 0 0 0 0.5 0]
%A matrix for seventh problem
a7= [0 0.5 0 0 0 0;
    0.5 0 0.5 0 0 0;
    0 0.5 0 0.5 0 0;
    0 0 0.5 0 0.5 0;
    0 0 0 0.5 0 0.5;
    0 0 0 0 0.5 0]


d1= diag([sum(a1(1,:));sum(a1(2,:));sum(a1(3,:));sum(a1(4,:));sum(a1(5,:));sum(a1(6,:))]);
d2= diag([sum(a2(1,:));sum(a2(2,:));sum(a2(3,:));sum(a2(4,:));sum(a2(5,:));sum(a2(6,:))]);
d3= diag([sum(a3(1,:));sum(a3(2,:));sum(a3(3,:));sum(a3(4,:));sum(a3(5,:));sum(a3(6,:))]);
d4= diag([sum(a4(1,:));sum(a4(2,:));sum(a4(3,:));sum(a4(4,:));sum(a4(5,:));sum(a4(6,:))]);
d5= diag([sum(a5(1,:));sum(a5(2,:));sum(a5(3,:));sum(a5(4,:));sum(a5(5,:));sum(a5(6,:))]);
d6= diag([sum(a6(1,:));sum(a6(2,:));sum(a6(3,:));sum(a6(4,:));sum(a6(5,:));sum(a6(6,:))]);
d7= diag([sum(a7(1,:));sum(a7(2,:));sum(a7(3,:));sum(a7(4,:));sum(a7(5,:));sum(a7(6,:))]);

l1= d1-a1
l2= d2-a2
l3= d3-a3
```

```
l4= d4-a4
l5= d5-a5
l6= d6-a6
l7= d7-a7

[V1,D1,W1]= eig(l1);
o=ones(6,1);
ev1=D1*o
r1=real(ev1);
i1=imag(ev1);
figure
plot(r1,i1,'r*')
hold on

[V2,D2,W2]= eig(l2);
ev2=D2*o
r2=real(ev2);
i2=imag(ev2);
figure
plot(r2,i2,'r*')
hold on

[V3,D3,W3]= eig(l3);
ev3=D3*o
r3=real(ev3);
i3=imag(ev3);
figure
plot(r3,i3,'r*')
hold on
```

```
[V4,D4,W4]= eig(l4);
ev4=D4*o
r4=real(ev4);
i4=imag(ev4);
figure
plot(r4,i4,'r*')
hold on

[V5,D5,W5]= eig(l5);
ev5=D5*o
r5=real(ev5);
i5=imag(ev5);
figure
plot(r5,i5,'r*')
hold on

[V6,D6,W6]= eig(l6);
ev6=D6*o
r6=real(ev6);
i6=imag(ev6);
figure
plot(r6,i6,'r*')
hold on

[V7,D7,W7]= eig(l7);
ev7=D7*o
r7=real(ev7);
i7=imag(ev7);
```

```
figure

plot(r7,i7,'r*')

hold on


% The left eigen vectors of the graphs corresponding to lamda = 0

w1_L1=W1(:,6)

w1_L2=W2(:,1)

w1_L3=W3(:,1)

w1_L4=W4(:,1)

w1_L5=W5(:,1)

w1_L6=W6(:,1)

w1_L7=W7(:,1)


%The Fiedler value v2 of the graphs are as follows:

v2_L1=V1(:,2)

v2_L2=V2(:,2)

v2_L3=V3(:,2)

v2_L4=V4(:,2)

v2_L5=V5(:,2)

v2_L6=V6(:,2)

v2_L7=V7(:,2)
```