EE5321, Spring 2018
Extended Homework 1: Fixed Final Time Optimal Control
Due 3/8/2018

Please turn in this signed paper with your exam:
Pledge of honor:
"On my honor I have neither given nor received aid on Problems 1 and 2 in this assignment"
Signature: ____

Problem 1: **Conversion of Hard Constraints to Soft Constraints – Analytic Solution** (35 points)
In the last homework set, you were given the performance index $J = \int_0^8 \frac{1}{2}u^2\, dt$ and the state constraints $\dot{x}_1 = x_2$ and $\dot{x}_2 = u$, with $x_1(0) = 0$ and $x_2(0) = 0$.

Some students asked about the difference between having soft constraints versus hard constraints. So, let's modify the problem where we change the hard constraint requirements $x_1(8) = 100$ and $x_2(8) = 0$ into soft constraints, resulting in a modified performance index $J = \frac{1}{2}W_1(x_1 - 100)^2 + \frac{1}{2}W_2 x_2^2 + \int_0^8 \frac{1}{2}u^2\, dt$. The scale factors W_1 and W_2 give us the ability to weight each constraint independently and influence the amount of control needed to accomplish the objective to a point where the final states might be 'good enough.'

So you can see the change in the solution, solve this problem analytically by forming the Hamiltonian and implementing the necessary conditions for optimality. Leave the scale factors (W_1 and W_2) as variables until you come up with a system of two equations that relate the two final states and the scale factors. Then, set $W_1 = W_2 = 1$ and solve for x_{1f} and x_{2f}. With those values, write the final equations for $x_1(t)$, $x_2(t)$, and $u(t)$.

Problem 2: **Conversion of Hard Constraints to Soft Constraints – Numeric Solution** (35 points)
Knowing what to expect in your final result, implement the state dynamics in Simulink using a fixed step size of 0.1 seconds and use *fminunc* to determine the optimal state and control time histories for the system and constraints described above, breaking the control into 81 parameters that you optimize. You will be experimenting with different values W_1 and W_2.

a) To start, run the optimization with the values set to $W_1 = W_2 = 1$. Plot the state time histories and the control time history along with the optimal cost. How close are the final states you computed analytically in Problem 1 to what you compute in this problem?
b) Now experiment with different values of the weights. Try $W_1 = W_2 = 2, 10,$ and $100.$ Compare these three times histories for the states and associated control. Also list the final cost for the 4 weight cases.
c) Next, try $W_1 = 1, W_2 = 0$ and run the solution. How well does the result compare to the case where there is no constraint on the second state?
d) Lastly, if you set $W_1 = 0, W_2 = 1$, what do you think the optimal solution will be? Give it a try numerically to see what it looks like.

Problem 3: **LQR Experimentation** (30 points)

A linear state-space system of an aircraft was created for a flight condition, below. Your job is to develop a regulator to keep the system at the trim condition, rejecting a specified initial offset/disturbance. The A and B matrices and state and control descriptions are listed below.

a) For the open loop system given, find the eigenvalues and note whether they are positive or negative, meaning the system is unstable or stable. Implement the system in Simulink and run a 10 second time history, taking C = eye(4) and D = zeros(4,2). Set the initial condition to [0.01 0 0 0] and the control inputs to be zero.
b) With R = eye(2) and Q = eye(4) use the MATLAB command ***lqr*** to create a feedback controller. Show the closed loop eigenvalues and note if the system is now stable. Implement the controller in the Simulink simulation and show the state and control time histories.
c) Repeat but with Q=10*eye(4) and again with Q=100*eye(4), showing a table of the closed loop eigenvalues and state and control time histories.
d) Now assume that only the first and third states are the ones about which we're concerned, zero out Q(2,2) and Q(4,4) and have Q(1,1) = Q(3,3) = 1. Add to the table of closed loop eigenvalues and plot the state and control time histories. Note similarities/differences between this run and the case run in part b).
e) Finally, repeat part c) but with the (1,1) and (3,3) values of Q being 1 and Q(2,2) and Q(4,4) being 0. Add to the closed loop eigenvalue table and the plots of state and control time histories.
f) Put Q back to Q = eye(4) and try R = 10*eye(2) and again with R = 0.1*eye(2), plotting the states and control time histories. Compare the R=0.1*eye(2), Q=eye(4) case to where Q=10*eye(4) and R=eye(2). Even though the ratio between them is 10, are they the same (i.e. produce the same time histories)?

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A = [
   -0.0000    1.0000   -0.0000    0.0000;
   -0.0000   -0.5000   -0.0022    0.0015;
  -30.8405  -60.240     0.0089    0.0011;
   -0.0190    0.0092    0.0017    0.0120]

B = [
   -0.0000   -0.0000;
   -0.0226   -0.0212;
    0.0010   -0.0035;
    0.0068   -0.0072]
```

x = [Aircraft pitch angle, theta, rad
Aircraft pitch rate, q, rad/sec
Aircraft x-axis velocity, u, ft/sec
Aircraft z-axis velocity, w, ft/sec]

u = [Left elevon deflection, deg
Right elevon deflection, deg]