EE5321, Spring 2018
Homework Assignment 6: State Reconstruction
Due 4/26/2018

For the upcoming problems, use the following aircraft linear model:

```
a = [   -0.0000    1.0000   -0.0000   -0.0000
        -0.0000   -0.5004   -0.0022    0.0064
       -30.8405  -66.2409    0.0089    0.0199
        -8.8533  224.6563   -0.0360   -0.8097  ];

b = [   -0.0000   0.0000
        -0.0226   0.0212
         0.0010   0.0035
        -0.1601   0.2148 ];

c = [    1.0000   -0.0050    0.0000    0.0000
         0.0000    1.0025    0.0000   -0.0000
         0.1604    0.0084    0.9612    0.2769
         0.0000   -0.0049   -0.0011    0.0040  ];
```

The states are [**theta q u w**], where **u** is the aircraft body x-axis velocity in ft/sec, **w** is the body z-axis velocity in ft/sec, **theta** is the aircraft pitch angle in rad, and **q** is the body pitch rate in rad/sec. The outputs are **[theta q V alpha**], where **V** is the aircraft total velocity in ft/sec, **alpha** is the aircraft angle of attack in rad, **theta** is the aircraft pitch angle in rad, and **q** is the body pitch rate in rad/sec. The input (control) are the two pitch control device deflections in deg, respectively.

Implement this system in Simulink and set the simulation to use a fixed step solver with time step of 0.01 seconds. Add random Gaussian noise to the output with characteristics of zero mean and of variance [0.00001, 0.00001, 0.0001, 0.0001], seed of [1 2 3 4], and sample time of 0.01 seconds. As an initial condition on the state, use x0=[0.05, 0, 0, 0] and use a stop time of 10 seconds.

Problem 1: **Regulator Development (30 points)**

Using the aircraft model described above, perform the following:

a) Show the pole locations, damping, and frequencies of the open loop system.
b) Noting the system is unstable, show a time history of the states. Put the states **u** and **w** on one graph and put **q** and **theta** on another graph. Which states appear to be unstable?
c) Show that the system is both controllable and observable.
d) With Q = eye(4)*10 and R=eye(2), develop a state feedback gain that stabilizes the system.
e) Implement the negative state feedback and regenerate the time histories of the controlled system, again putting **u** and **w** on one graph and **theta** and **q** on another. Also plot the control time history.

Problem 2: **Observer Development (30 points)**

a) Develop an observer using the appropriate sized identity matrices for Q=eye(.)*0.1 [Note – this (.) definition is not to be taken literally – you need to determine the appropriate size for Q and R] and R=eye(.)*5. Implement the observer in Simulink as sketched in class and compare the estimated states with the true states when using the true states as feedback, using the initial value of the state estimate to be the same as the initial value of the true states. Assuming the comparison is good, now use the estimated states in the feedback controller and show the state and control time histories.

b) Now try adjusting the values of Q and R used to compute the observer gain L to make the eigenvalues at least 5 times those of the closed loop eigenvalues (eig(A-BK)), assuming this is possible for this problem. Again use the estimated states in the feedback controller and show the state and control time histories.

Problem 3: **Kalman Filter (40 points)**

a) Into your simulation, implement a Kalman filter using the equations shown in class. Keeping your state feedback controller and observer gain as you had them and using the observer state estimates as the state feedback, set Q=eye(.)*10, R=eye(.), P0=1000*eye(.), and the initial value of the state estimates to be all zero, plot the Kalman filter state estimates and compare them to the true states.

b) Now set R=0.0001*eye(.) and rerun the simulation, plotting the Kalman filter state estimates. Are the estimates better or worse, and is the noise more or less apparent?

c) Finally, assuming the results are both fine, use the Kalman filter state estimates in the controller feedback and plot the state and <u>control time histories for both cases a) and b)</u>. Comment on the comparisons of the observer states against the Kalman filter state estimates. Also comment on the comparison of the control time histories when using the observer or Kalman state estimates as used in the feedback controller.