UNIVERSITY OF CAMBRIDGE

MPhil in Machine Learning and, Machine Intelligence 2018-2019

Module Coursework Feedback

Module Title: Robotics

Module Code: 4M20

Candidate Number: F606F

Coursework Number: 1

I confirm that this piece of work is my own unaided effort and adheres to the Department of Engineering's guidelines on plagiarism. √

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This piece of work has been completed to the following standard *(Please circle as appropriate)*:

	Distinction			Pass			Fail (C+ - marginal fail)		
Overall assessment (circle grade)	**Outstanding**	**A+**	**A**	**A-**	**B+**	**B**	**C+**	**C**	**Unsatisfactory**
Guideline mark (%)	90-100	80-89	75-79	70-74	65-69	60-64	**55-59**	**50-54**	**0-49**
Penalties	**10% of mark for each day, or part day, late (Sunday excluded).**								

The assignment grades are given **for information only**; results are provisional and are subject to confirmation at the Final Examiners Meeting and by the Department of Engineering Degree Committee.

Question 1 (Please open and run the matlab file 'not_fu_a_lot.m')

Let the (x,y) be the position of the end of the foot 2. Given that $L = L_1 = L_2 = L_3 = L_4$, and set a constraint to the sum of all the rotating angles to be π (i.e. $\theta = \theta_1 + \theta_2 + \theta_3 = \pi$).

$$\begin{aligned} \mathrm{x} &= L_2 \sin\theta_1 + L_3 \sin(\theta_1 + \theta_2) + L_4 \sin(\theta_1 + \theta_2 + \theta_3) \\ y &= L_1 + L_2 cos\theta_1 + L_3 cos(\theta_1 + \theta_2) + L_4 cos(\theta_1 + \theta_2 + \theta_3) \end{aligned}$$

The (x,y)-position becomes,

$$\begin{aligned} \frac{x}{L} &= s_{\pi-2-3} + s_{\pi-3} + s_{\pi} = s_{23} + s_3 \\ \frac{y-L}{L} &= c_{\pi-2-3} + c_{\pi-3} + c_{\pi} = -(c_{23} + c_3 + 1) \end{aligned}$$

Combining the equations of x and y positions,

$$\begin{aligned} \frac{x^2 + (y-L)^2}{L^2} &= s_{23}^2 + s_3^2 + c_{23}^2 + c_3^2 + 1 + 2s_{23}s_3 + 2c_{23}c_3 + 2c_3 + 2c_{23} \\ &= 3 + 2(c_2 + c_3 + c_{23}) \end{aligned}$$

Substitute $c_{23} + c_3 = -\left(\frac{y-L}{L} + 1\right)$, so θ_2 is found.

$$\begin{aligned} \frac{x^2 + (y-L)^2}{L^2} &= 3 + 2\left(c_2 - \left(\frac{y-L}{L} + 1\right)\right) \\ \frac{x^2 + (y-L)^2}{L^2} + 2\frac{y-L}{L} &= 1 + 2c_2 \\ c_2 &= \frac{x^2 + (y-L)^2}{2L^2} + \frac{y-L}{L} - \frac{1}{2} \end{aligned}$$

To find θ_3, firstly modify $\frac{x}{L}$ equation,

$$\frac{x}{L} = s_2c_3 + s_3c_2 + s_3 = s_3(1 + c_2) + c_3s_2$$

Then eliminate c_3 by modifying $\frac{y-L}{L}$ equation,

$$\begin{aligned} -\left(\frac{y-L}{L} + 1\right) &= c_{23} + c_3 = c_2c_3 - s_2s_3 + c_3 = c_3(1 + c_2) - s_3s_2 \\ c_3 &= \frac{1}{1 + c_2}\left(s_3s_2 - \left(\frac{y-L}{L} + 1\right)\right) \end{aligned}$$

Substitute it, and θ_3 is found.

$$\frac{x}{L} = s_3(1 + c_2) + \frac{s_2}{1 + c_2}\left(s_3s_2 - \left(\frac{y-L}{L} + 1\right)\right)$$

$$\begin{aligned} \frac{x}{L} + \frac{s_2}{1 + c_2}\left(\frac{y-L}{L} + 1\right) &= s_3\left(1 + c_2 + \frac{s_2^2}{1 + c_2}\right) \\ &= s_3(1 + c_2 + 1 - c_2) = 2s_3 \end{aligned}$$

$$s_3 = \frac{1}{2}\left(\frac{x}{L} + \frac{s_2}{1 + c_2}\left(\frac{y-L}{L} + 1\right)\right)$$

In conclusion, $\theta_1\theta_2\theta_3$ can be found by the following equations.

$$\begin{aligned} \theta_2 &= cos^{-1}\left(\frac{x^2 + (y-L)^2}{2L^2} + \frac{y-L}{L} - \frac{1}{2}\right) \\ \theta_3 &= sin^{-1}\left(\frac{1}{2}\left(\frac{x}{L} + \frac{s_2}{1 + c_2}\left(\frac{y-L}{L} + 1\right)\right)\right) \end{aligned}$$

$$\theta_1 = \pi - \theta_2 - \theta_3$$

Given that (x,y) = [0.15, 0.05], L = 0.1, all the joint angles are calculated by the matlab. The result is stated as follows in degrees.

$$\theta_1 = 33.8^{\circ}; \theta_2 = 75.5^{\circ}; \theta_3 = 70.7^{\circ}$$

Given that (x,y) = [0.12, 0], the joint angles are shown as follow in degrees.

$$\theta_1 = 36.9^{\circ}; \theta_2 = 106.2^{\circ}; \theta_3 = 36.9^{\circ};$$

Question 2

The centre of mass is defined as follows.

$$M_{COM}\underline{x_{COM}} = \sum_i m_i \underline{x_i}$$

, where $M_{COM} = \sum_i m_i$, the centre of mass equation, m_i the individual mass, $\underline{x_i}$ the position vector of individual mass, $\underline{x_{COM}}$ the location vector of the centre of mass.

Therefore, $M_{COM} = M_1 + M_2 + M_3 + M_4 = 0.1x4 = 0.4kg$.

Consider each position vector of individual mass,

$$x_1 = 0; y_1 = \frac{L}{2}$$

$$x_2 = \frac{L}{2}s_1; y_2 = L\left(1 + \frac{1}{2}c_1\right)$$

$$x_3 = L\left(s_1 + \frac{1}{2}s_{12}\right); y_3 = L\left(1 + c_1 + \frac{1}{2}c_{12}\right)$$

$$x_4 = L\left(s_1 + s_{12} + \frac{1}{2}s_{123}\right); y_4 = L\left(1 + c_1 + c_{12} + \frac{1}{2}c_{123}\right)$$

So, the location of the centre of mass is,

$$\begin{aligned} \mathrm{x_{COM}} &= \frac{M}{4M}\sum_i x_i = \frac{L}{4}\left(\frac{5}{2}s_1 + \frac{3}{2}s_{12} + \frac{1}{2}s_{123}\right) \\ &= \frac{L}{8}(5s_1 + 3s_{12} + s_{123}) \\ \mathrm{y_{COM}} &= \frac{L}{4}\left(\frac{7}{2} + \frac{5}{2}c_1 + \frac{3}{2}c_{12} + \frac{1}{2}c_{123}\right) \\ &= \frac{L}{8}(7 + 5c_1 + 3c_{12} + c_{123}) \end{aligned}$$

Question 3

Since $0 < X_{COM} < L_0$, It sets a lower bound of minimum length of foot 1, from question 2, the projection of the centre of mass on the ground can be used.

$$L_0 > X_{COM} = \frac{L}{8}(5s_1 + 3s_{12} + s_{123})$$

To find the minimum length, extreme case can be set. Assume that $(\theta_1, \theta_2, \theta_3) = (90^{\circ}, 0^{\circ}, 0^{\circ})$,

$$L_0 > \frac{9L}{8} = 0.1125m$$

So the minimum length is 0.1125m.

And the maximum stride length is equal to the maximum distance where the foot 2 can reach with the end touching the ground. The maximum stride length is 2L = 0.2m if the sum of the joint angles are 180 degrees.

Discussion

If foot 1 is too large, it increases the contact surface area with the floor which gives more friction against the locomotion of the robot. It also reduces the energy efficiency since it can slide easier without such a long foot when foot 1 is not fixed.

To move foot 1, it will also be difficult. This is because the robot needs to raise foot 1 higher, so as to land it properly without losing the balance and falling down.

Question 4 (Please open and run the matlab file 'Optimize_code.m')

Background

Assumption:

- No friction exists on the ground and the joints.
- The locomotion only relies on the rotations of links.
- The robot never slides.
- Every time when the front link moves, the rear link is fixed in the ground. Similarly, when the rear link moves, the front link is fixed.

Like question 1, to move the front foot with the fixed point of rear foot, the (x,y) position vector of the front foot is,

$$\begin{pmatrix} x \\ y \end{pmatrix} = L\begin{pmatrix} -(s_3 + s_{23} + s_{123}) \\ 1 + c_3 + c_{23} + c_{123} \end{pmatrix}$$

the joint angles become,

$$\begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix} = \begin{pmatrix} sin^{-1}\frac{1}{2}\left(\frac{-x}{L} + \frac{s_2}{1 + c_2}\frac{y}{L}\right) \\ cos^{-1}\left(\frac{x^2 + y^2}{2L^2} - 1\right) \\ \pi - \theta_1 - \theta_2 \end{pmatrix}$$

This equation helps to find the trajectories of the joint angles for the front link motion.

To find the torque required for each of three motors, the following equation will be used.

$$\underline{\tau} = \underline{\underline{J}}^{\mathrm{T}}\underline{F}$$

$$\begin{pmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{pmatrix} = \begin{pmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} & \frac{\partial x}{\partial \theta_3} \\ \frac{\partial y}{\partial \theta_1} & \frac{\partial y}{\partial \theta_2} & \frac{\partial y}{\partial \theta_3} \end{pmatrix}^T \begin{pmatrix} F_x \\ F_y \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\partial x}{\partial \theta_1} & \frac{\partial y}{\partial \theta_1} \\ \frac{\partial x}{\partial \theta_2} & \frac{\partial y}{\partial \theta_2} \\ \frac{\partial x}{\partial \theta_3} & \frac{\partial y}{\partial \theta_3} \end{pmatrix} \begin{pmatrix} F_x \\ F_y \end{pmatrix}$$

Consider the robot is always touching the floor, there must be a reaction force paired up with its weight. (i.e. $F_x = 0, F_y = -M_{COM}g$). Therefore, the 1st column of the transpose Jacobian matrix does not matter since the x-component of the force is zero, the torque vector becomes,

$$\begin{pmatrix}\tau_1\\ \tau_2\\ \tau 3\end{pmatrix} = F_y \begin{pmatrix}\frac{\partial y}{\partial \theta_1}\\ \frac{\partial y}{\partial \theta_2}\\ \frac{\partial y}{\partial \theta_3}\end{pmatrix} = F_y \frac{\partial y}{\partial \underline{\theta}}$$

For the rear foot,

$$\frac{\partial y}{\partial \underline{\theta}} = \begin{pmatrix}-s_1 - s_{12} - s_{123}\\ -s_{12} - s_{123}\\ -s_{123}\end{pmatrix}$$

For the front foot,

$$\frac{\partial y}{\partial \underline{\theta}} = \begin{pmatrix}-s_{123}\\ -s_{23} - s_{123}\\ -s_3 - s_{23} - s_{123}\end{pmatrix}$$

So, for every motion of the robot, the joint angles change, then the torque for each motor will be stored into an array. To find the minimum torque required for each motor, matlab max function will be used to find the maximum value in the array.

Since the torque is found, the angular acceleration can also be computed to find out the duration of each action by only considering the motion of one joint. This calculation assumes that all joint angles move evenly together (i.e. they begin to move and stop at the same time).

$$\tau = \mathrm{I}\,\alpha$$

, where $I = \frac{1}{3}ML^2$ the rotational inertia of the rod link, α the angular acceleration, τ the torque of the motor, M the mass of the rod and L the length of the rod.

To achieve 'trapezoidal velocity profile', assume that the time of this profile ends at $t = t_f$, and the object accelerates from $\mathrm{t} = 0$ to t_c. Assume that the angular acceleration is constant, the duration required to reach certain degrees is,

$$t_f = \sqrt{\frac{\Delta\theta}{\alpha a(1-a)}}$$

, where $\Delta\theta$ the joint angle rotation, $a = \frac{t_c}{t_f}$ and t_f the time spent to achieve the rotation. In the simulation, a is set to 0.001.

Since all the joint angles move evenly together, one calculation is only need for each motion. For example, when moving foot 1, the duration is equal to the time required to move θ_3. Similarly, when moving foot 2, the duration is the time required to move θ_1. This helps to simplify the calculation.

So, duration required to accomplish the task is the sum of duration of each individual motion.

Assume that no friction exists in the motors or on the ground, the energy efficiency should be the ratio between the energy put in forward motion and the total energy created by the motors.

The total energy can be computed by the following equation,

$$\mathrm{E} = \int \tau \mathrm{d}\theta \approx \sum_{\mathrm{i}} \tau_{\mathrm{i}} \delta\theta_{\mathrm{i}}$$

Using the torque arrays above, they will be dot-product with the vector decremental angular movement. Thus, the total energy generated by the motors are computed.

The energy of the forward motion should be the difference between the total energy and the work done by gravity,

$$E_f = \Delta E = \int \tau d\theta - \int F_g dy \approx \sum(\tau\delta\theta - F_g\delta y)$$

The work done by gravity equals to the y-axis motion of the centre of mass times the gravitational acceleration and the centre of mass.

$$F_g = M_{COM}gh$$

, where h the y-axis motion distance of the centre of mass, M_{COM} the centre of mass and g the gravitational acceleration.

Then the Energy Efficiency is,

$$\text{Efficiency} = \frac{\Delta E}{E}$$

Results

The front leg is originally placed at x=0, and the rear leg separation is around 0.2m initially behind it. Then the rear leg moves firstly by raising 0.05m and proceeding 0.15m. Now the legs separation is 0.05m. The 1st motion mentioned above has the following trajectory of joint angles,

$$\begin{pmatrix}\theta_1\\\theta_2\\\theta_3\end{pmatrix} = \begin{pmatrix}90^\circ\\0^\circ\\90^\circ\end{pmatrix} \to \begin{pmatrix}20.5^\circ\\95.4^\circ\\64.1^\circ\end{pmatrix} \to \begin{pmatrix}14.5^\circ\\151.0^\circ\\14.5^\circ\end{pmatrix}$$

The front leg also moves by raising 0.05m and proceeding 0.15m. Now the legs separation gets back to 0.2 m again. The 2nd motion then has the following joint angles trajectory.

$$\begin{pmatrix}\theta_1\\\theta_2\\\theta_3\end{pmatrix} = \begin{pmatrix}14.5^\circ\\151.0^\circ\\14.5^\circ\end{pmatrix} \to \begin{pmatrix}64.1^\circ\\95.4^\circ\\20.5^\circ\end{pmatrix} \to \begin{pmatrix}90^\circ\\0^\circ\\90^\circ\end{pmatrix}$$

So the rear leg walks on the following x-spots: (0.20, 0.05, -0.10, -0.25, -0.39). And the front leg walks on the following x-spots: (0.00, -0.15, -0.30, -0.45). Always the rear first, then the front follows, and so on.

To overcome the step, the front leg then goes from (x,y) = (-0.45, 0.00) to (-0.59, 0.0005) m by the following joint angles.

$$\begin{pmatrix}\theta_1\\\theta_2\\\theta_3\end{pmatrix} = \begin{pmatrix}14.5^\circ\\151.0^\circ\\14.5^\circ\end{pmatrix} \to \begin{pmatrix}63.3^\circ\\98.8^\circ\\17.9^\circ\end{pmatrix} \to \begin{pmatrix}72.0^\circ\\36.4^\circ\\71.6^\circ\end{pmatrix}$$

The rear leg then follows by moving from (x,y) = (-0.39, 0.00) to (-0.53, 0.0005) m with the following joint angles.

$$\begin{pmatrix}\theta_1\\\theta_2\\\theta_3\end{pmatrix} = \begin{pmatrix}72.0^\circ\\36.4^\circ\\71.6^\circ\end{pmatrix} \to \begin{pmatrix}18.0^\circ\\99.0^\circ\\63.0^\circ\end{pmatrix} \to \begin{pmatrix}14.5^\circ\\151.0^\circ\\14.5^\circ\end{pmatrix}$$

On the platform at the level of 0.0005m, the front leg goes first by the path (-0.59, -0.74, -0.89, -1.04), the rear follows by the path (-0.53, -0.69, -0.84, -0.99).

The minimum torques are,

$$\begin{pmatrix}\tau_1\\\tau_2\\\tau_3\end{pmatrix} = \begin{pmatrix}0.784\\0.392\\0.785\end{pmatrix}\text{Nm}$$

The duration required is 16.24s.

And the energy efficiency is 85.85%.

Discussion

To find how the trajectory affects the minimum torques, duration and energy efficiency, we can compare the results from Table 1.

Table 1

Trajectories parameters			Performance Metrices				
Legs maximum separation (m)	Maximum stride length (m)	Maximum stride height (m)	Torques (Nm)			Duration (s)	Energy Efficiency (%)
			1	2	3		
0.15	0.12	0.05	0.659	0.345	0.667	19.41	77.2
0.15	0.12	0.1	0.663	0.392	0.667	22.72	65.6
0.17	0.12	0.05	0.74	0.37	0.75	15.03	79.1
0.15	0.10	0.05	0.74	0.37	0.75	20.13	76.0

The highlighted blocks are with the changed values.

When the step height is increased, the minimum torques will not vary a lot. But it takes more time to travel, and the energy efficiency goes down since more energy is lost as potential energy.

When the legs maximum separation increases, the minimum torques required increase since the robot needs more torques to support a wider leg separation. However, it takes shorter duration to travel, and enhance the energy efficiency.

When the step width is decreased, the torques also increase. Since the robot takes smaller steps, it takes longer time to travel to the goal. The energy efficiency then decreases.

The torque is found to be quite affected by the posture of robot when standing on the ground. The more the leg separation is, more torque is required.