4M20 Robotics (2017) Comments for Coursework 1
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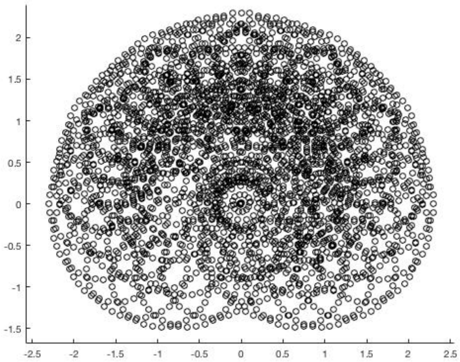
Q1
The reachable range of the end-effector should be roughly as shown in the figure below.



Q2
There are three points to consider for this question. (1) Arm cannot collide with the object, (2) Arm cannot collide with its own links, (3) Arm needs to path through singularity postures. Given these constraints, the shortest path should be composed of two via-points of the end effector that should be connected by straight-lines. The first point is the top right corner of the object. Second, we need to find a point of end effector somewhere above the object, around which the arm can rotate without colliding to the object and ground, while the shortest path to Point B. The coordinates of these two points should be [1.05, 0.5] and [0.54, 1.05], and a total travel distance of the end effector should be 3.18m.

Q3
Here we need to optimize 6 parameters, i.e. L1, L2, L3, as well as θ1, θ2, θ3, in terms of the total sum of angular motions (not the shortest distance of end effector, according to the problem statement). We have a set of boundary conditions, i.e. XA, XB, XO, the ground, and collisions to its own body, but these are not sufficient to analytically find an optimum solution. To find an exact optimum solution, one needs an exhaustive search. A good approximation, however could be what follows. From Q2, one of the critical parameter is L3, which should be short to move the entire arm shortest. Therefore a good approximation is L3 to be the length just enough to reach Point A (i.e. L3=0.56m). This gives another via-point [0.65, 1.06]. To reach all these end-effector positions, we can derive the remaining design parameter as L1=0.75m, L2=0.49m.

Q4
Lagrangian method should give you the equations as follows:

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\dot{\mathbf{q}}, \mathbf{q})\dot{\mathbf{q}} + \mathbf{G}(\mathbf{q}) = \mathbf{T}$$

$$\mathbf{M} = \begin{bmatrix} I_1 + I_2 + I_3 + (\frac{1}{4}m_1 + m_2 + m_3)L_1^2 + (\frac{1}{4}m_2 + m_3)L_2^2 + (m_2 + 2m_3)L_1L_2C_2 & I_2 + I_3 + (\frac{1}{4}m_2 + m_3)L_2^2 + (\frac{1}{2}m_2 + m_3)L_1L_2C_2 \\ -I_2 - I_3 - (\frac{1}{4}m_2 + m_3)L_2^2 - (\frac{1}{2}m_2 + m_3)L_1L_2C_2 & I_2 + I_3 + (\frac{1}{4}m_2 + m_3)L_2^2 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} -(m_2 + 2m_3)L_1L_2S_2\dot{\theta}_2 & (\frac{1}{2}m_2 + m_3)L_1L_2S_2\dot{\theta}_2 \\ (m_2 + 2m_3)L_1L_2S_2\dot{\theta}_1 & -(\frac{1}{2}m_2 + m_3)L_1L_2S_2\dot{\theta}_1 \end{bmatrix}$$

$$\mathbf{G} = \begin{bmatrix} -\frac{1}{2}m_1gL_1S_1 - m_2gL_1S_1 + \frac{1}{2}L_2S_{2-1} - m_3gL_1S_1 + m_3gL_2S_{2-1} \\ -\frac{1}{2}m_2gL_2S_{2-1} - m_3gL_2S_{2-1} \end{bmatrix}$$

Q5.

From Q4, the stationary torque required can be calculated as 1.52Nm and 15.6Nm for joint 1 and 2 respectively. The peak torque is dependent on the joint trajectories it follows because velocity and acceleration terms are dependent on the trajectories. Assuming slow motion with negligible velocity and acceleration, the peak torque should be 19.4Nm and 15.6Nm.

Q6

Theoretical solutions can be derived from the equations of motion in Q4, in which one can find L1 and L2 without using any energy (passively moving from A to B), though it requires some specific initial conditions (e.g. some initial velocities). And such solution can be found also in the longer L1 and L2 which makes the motion faster. In practice, however, the upper bound should be found by the limitation of weight and inertia values (which we assume to be constant in this question).