Welcome to EL2700: Model predictive control

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The teaching team

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Course administration: EECS Student Services

Outline

- Overview of model predictive control
- Course organization and content
- Linear systems theory

Classical control

Physical systems conveniently described by ordinary differential equations

$$m\ddot{x}(t) = -d\dot{x}(t) - kx(t) + F(t)$$

Letting $F(t)$ be a function of $x(t)$ and $\dot{x}(t)$ allows to alter dynamics.

A design theory based on deep understanding of properties of ODEs.

Particularly powerful when system described by linear ODEs

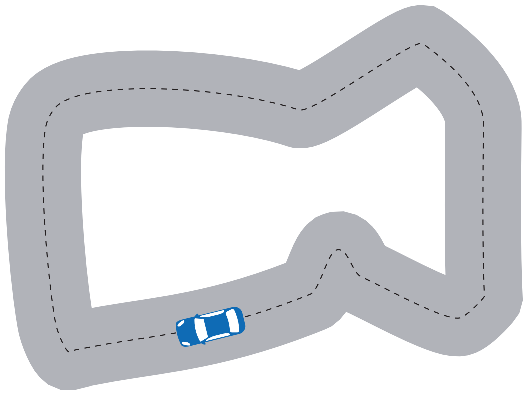
- poles, zeros, transfer functions, frequency responses, ...
- fundamental limitations and inherent trade-offs
- robustness to model uncertainties

Limited practical support for nonlinear and constrained systems.

Model predictive control

Objective:
– minimize lap time.

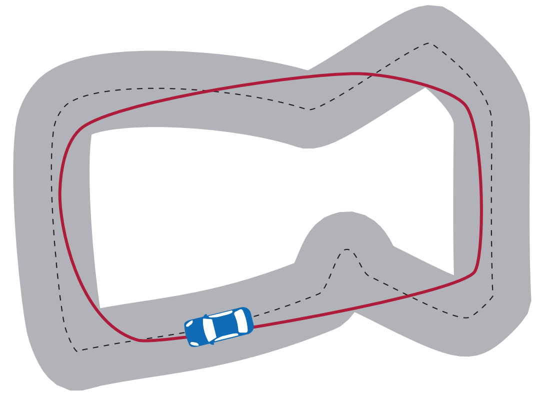
Constraints:
– avoid other cars
– stay on road
– don't skid
– limited acceleration



Intuitive approach: plan based on map, road conditions, car abilities, ...

Model predictive control

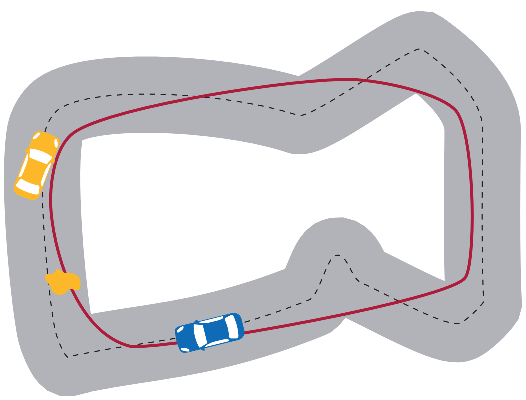
minimize	lap time
subject to	avoid cars
	stay on track
	⋮



Solve optimization problem to find *optimal* path.

Model predictive control

minimize	lap time
subject to	avoid cars
	stay on track
	⋮

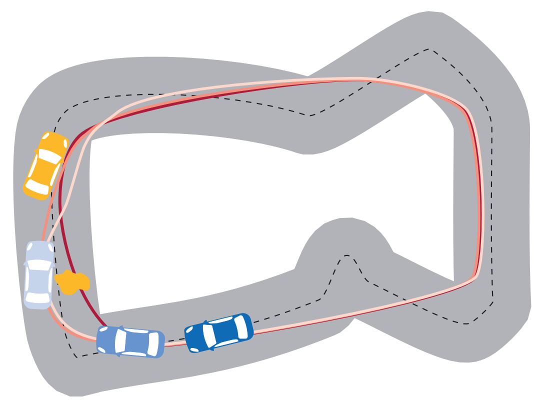


What if something unexpected (oil spill, other cars, ...) appears?

- "optimal" path no longer optimal; need to introduce feedback!

Model predictive control

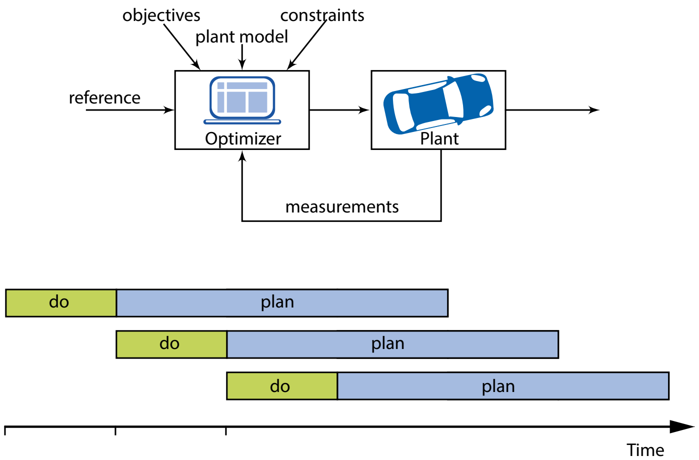
minimize	lap time
subject to	avoid cars
	stay on track
	⋮



Model predictive control:

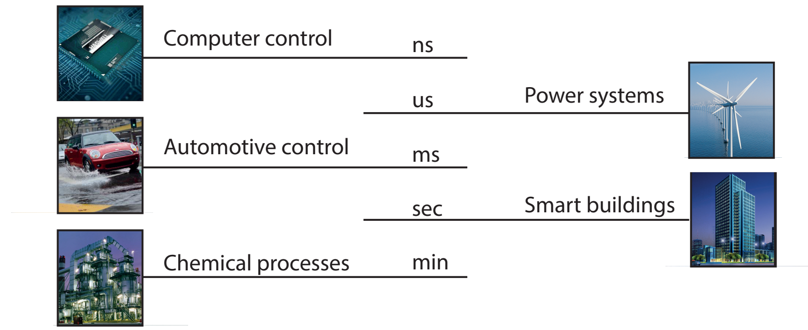
- plan optimal trajectory (series of optimal control actions)
- apply first control action
- observe outcome, repeat planning procedure

Model predictive control



Receding-horizon control introduces feedback!

MPC applications



MPC in industry

Economic assessment of advanced process control, Bauer and Craig (2008)
(answers from 66 industries from five continents.)

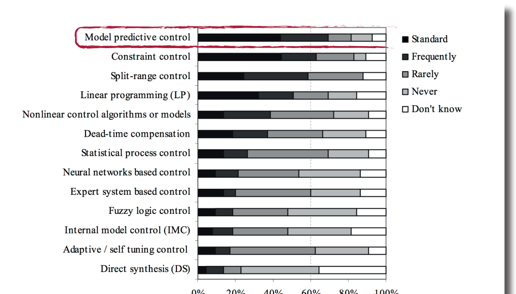
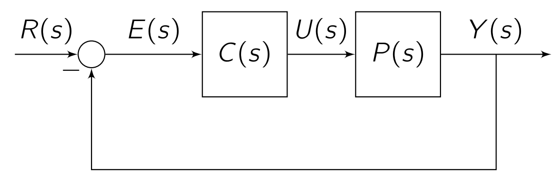


Fig. 5. Industrial use of APC methods: survey results.

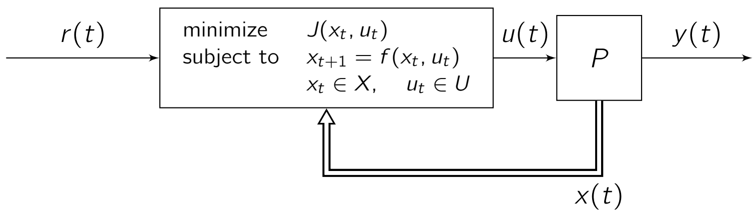
"For us, multivariable control is predictive control."
Tariq Samad, Honeywell (past president of IEEE Control Systems Society), 1997

Classical control theory



- Focus on regulation, i.e. driving system state to zero
- Concerns: disturbance rejection, noise suppression, robustness
- Explicit control laws with simple on-line implementation
- Powerful theory for linear systems (poles, zeroes, frequency responses)

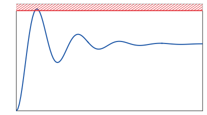
Model predictive control



- Focus on constraints and optimality (in time domain)
- Relies on solving optimization problem on-line, in every sample
- Inherently nonlinear strategy, more difficult to analyze
- Basic idea extends to nonlinear and hybrid systems, tracking

Constraints are everywhere





- Limitations of actuation magnitude and rate-of-change
- Safety limits on states (temperatures, pressures, etc)
- Performance limitations (e.g., maximum overshoot)

Classical control deals with constraints in an ad-hoc manner:

- de-tune controllers to never hit constraints
- design conservative reference trajectories
- use heuristic modifications of linear controllers (e.g. anti-windup)

MPC accounts for constraints explicitly in the design.

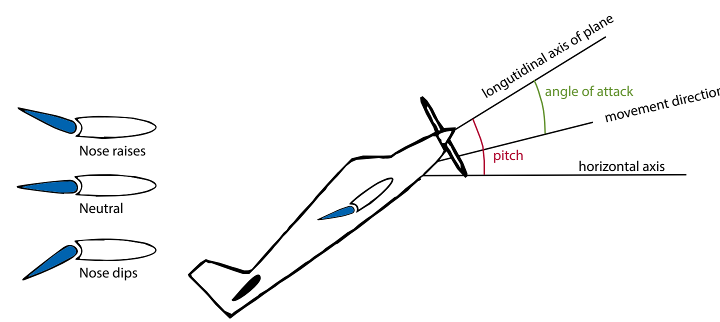
MPC example: Cessna citation aircraft

Linearized model at 5000 m altitude, 128.2m/sec.

$$\dot{x} = \begin{bmatrix} -1.2822 & 0 & 0.98 & 0 \\ 0 & 0 & 1 & 0 \\ -5.4293 & 0 & -1.8366 & 0 \\ -128.2 & 128.2 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} -0.3 \\ 0 \\ -17 \\ 0 \end{bmatrix} u$$

States: x_1: angle of attack, x_2: pitch angle, x_3: pitch rate, x_4: altitude.
Control: u: elevator surface angle.

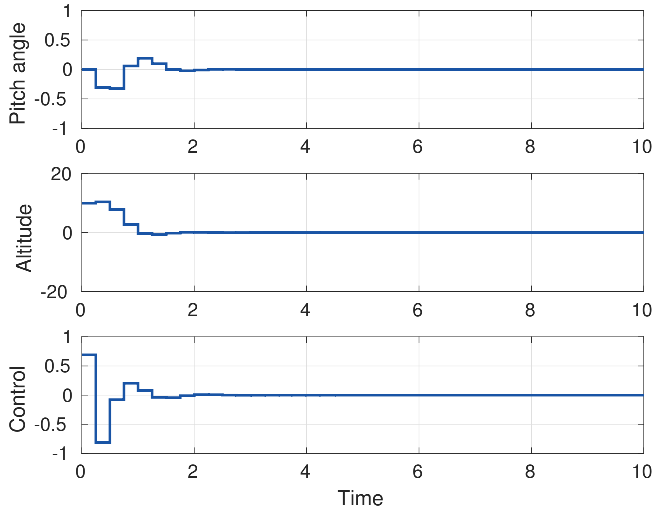
MPC example: terminology and control problem



Objective: manipulate elevators to control pitch and altitude.
Constraints: elevator magnitude $\pm 15°$,
elevator rate-of-change $\pm 30°/s$
pitch angle $\pm 30°$

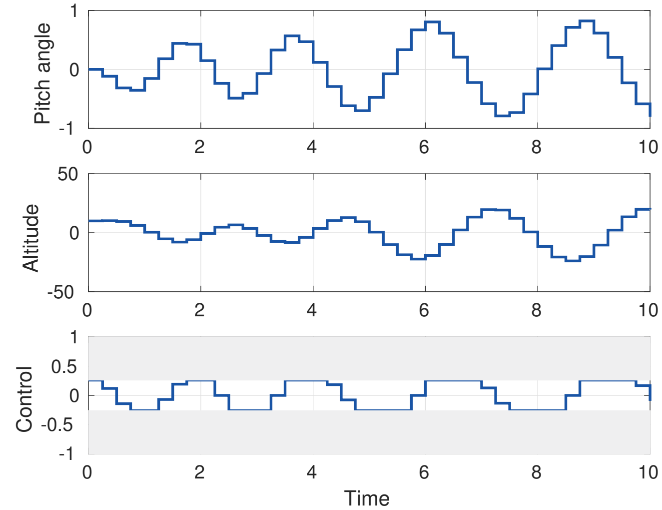
Linear design: altitude change of 10 meters

State feedback looks good on linearized model. . .



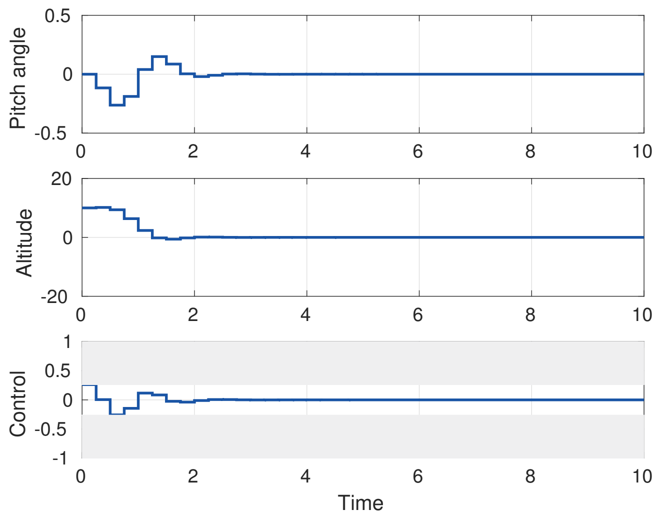
Linear design with constrained acutators

. . . but goes unstable when actuator limitations are inncluded!



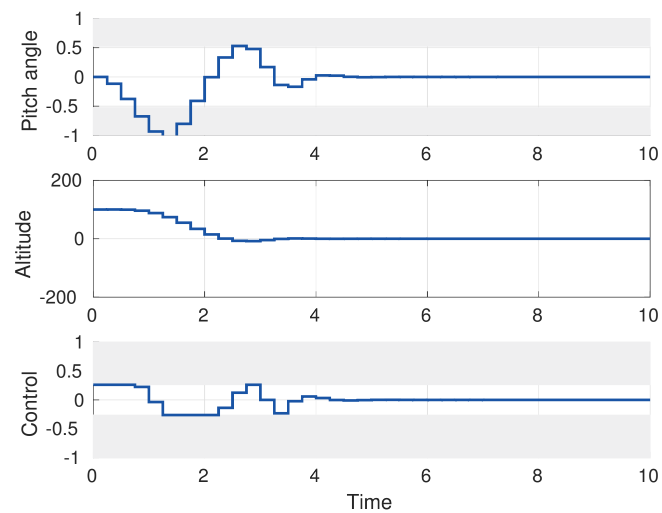
MPC design

Model predictive control accounts directly for actuator constraints.



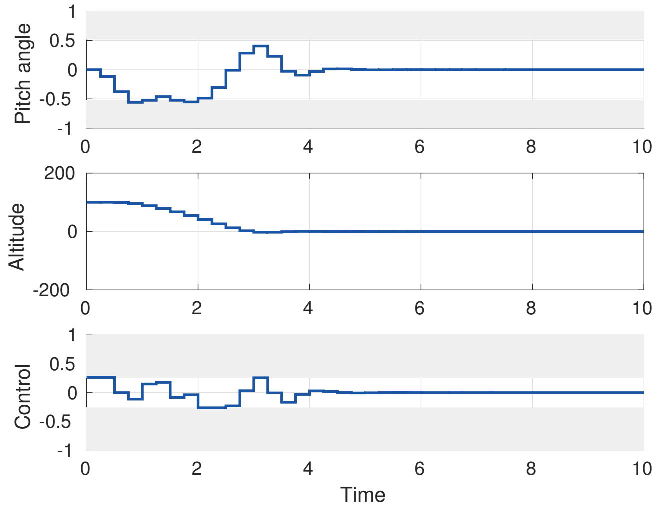
MPC design: altitude change of 100 meters

Violates pitch constraints (since not included in our design yet!)



MPC design: altitude change of 100 meters

State constraints readily included in the design!



Model predictive control

The three key components of MPC:

1. Model-based *prediction* of how control action impacts future states
2. Computation of *optimal control* over finite-horizon
3. Implementation in *receding-horizon* fashion

Model-based prediction

We can use a dynamic model

$$x_{t+1} = f(x_t, u_t)$$

to predict future system states, given initial state x_0

For linear system $x_{t+1} = Ax_t + Bu_t$, predictions are linear in $(x_0, \{u_t\})$

$$x_1 = Ax_0 + Bu_0$$
$$x_2 = Ax_1 + Bu_1 = A^2x_1 + ABu_0 + Bu_1$$
$$\vdots$$
$$x_N = A^N x_0 + \sum_{t=0}^{N-1} A^t B u_{N-1-t}$$

Constraints on x_t translates into constraints on $\{u_0, \dots, u_{t-1}\}$.

Q: how sensitive is this prediction to unmodelled dynamics, disturbances?

Finite-horizon optimal control

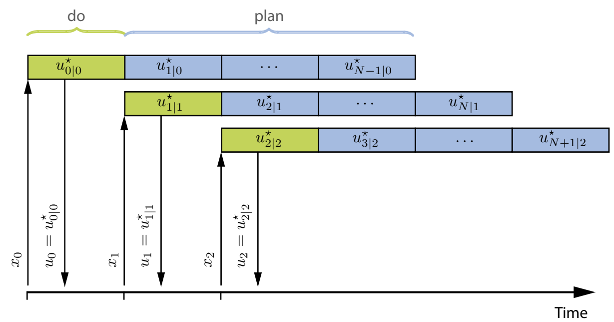
Given system state $x(0)$, find control sequence $\{u_0, u_1, \dots, u_{N-1}\}$ to

$$\begin{array}{lll} \text{minimize} & J = \sum_{t=0}^{N-1} \ell(x_t, u_t) + \ell_f(x_N) & \\ \text{subject to} & x_{t+1} = f(x_t, u_t) & t = 0, 1, \dots, N-1 \\ & x_t \in X, \;\; u_t \in U & t = 0, 1, \dots, N-1 \\ & x_N \in X_f, \quad x_0 = x(0) & \end{array}$$

Q: when can we solve an optimization problem quickly and reliably?

- essentially, when objective and constraint sets are *convex*
- important problem classes: linear and quadratic programming

Receding-horizon implementation



Q: what are the effects of feedback introduced by receding horizon control?

- likely to improve robustness to unmodelled disturbances
- but could potentially render closed-loop system unstable

Course content

- Linear systems in discrete-time
- Optimal control with quadratic cost functions
- Model predictive control (MPC)
- Stability and invariance of constrained systems
- Optimal estimation and output feedback control
- Disturbance modeling and disturbance compensation
- Nonlinear MPC, reference tracking, implementation aspects

What you can expect from EL2700

- learn interesting and widely useful theory
  - linear systems, dynamic optimization, Lyapunov stability, invariance, . . .
- gain deep understanding of linear quadratic and model predictive control
- enhance your engineering and control design skills
- an opportunity to challenge yourself and have fun!

. . . and what we expect from you

Help to make this an interesting and rewarding learning experience!

- participate in class
- use office hours and the teaching staff
- take the opportunity to learn!

Course organization

14 lectures:	theory and design methodology
7 exercises:	mathematical tools and analytical abilities
1 design project:	apply theory and methodology studied in class
1 lab session :	insight and understanding, validation on real system
1 paper presentation:	in-depth understanding of one advanced topic
Office hours	Wednesdays 15-16, Malvinas väg 10, floor 6.

Course materials

Course book:	Lecture notes (free in electronic form)
Recommended:	"Predictive control with constraints" J. Maciejowski, Prentice Hall
	Additional free resources posted on home page
Lecture slides:	posted on Canvas every Monday
Exercises:	posted on Canvas
Lab instructions:	posted on Canvas

If you do not have access to Canvas, please contact Student Services!