Image Segmentation by Global Thresholding

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References

- Rafael C. Gonzalez and Richard E. Woods, Digital Image Processing, Third Edition, Pearson Education, 2008:
  - Thresholding: Chapter 10.3

- https://en.wikipedia.org/wiki/Thresholding_(image_processing)

- https://en.wikipedia.org/wiki/Expectation%E2%80%93maximization_algorithm

Image Segmentation

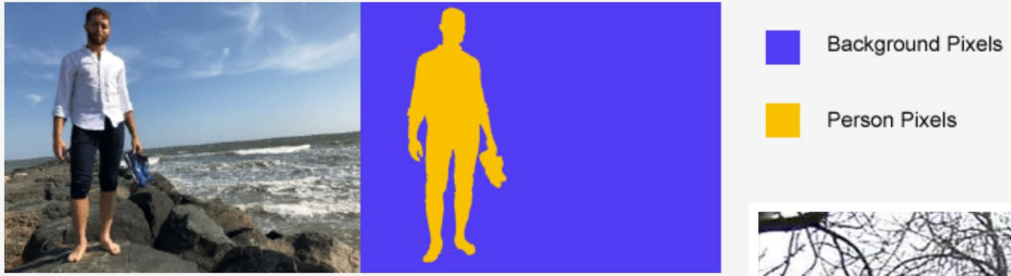
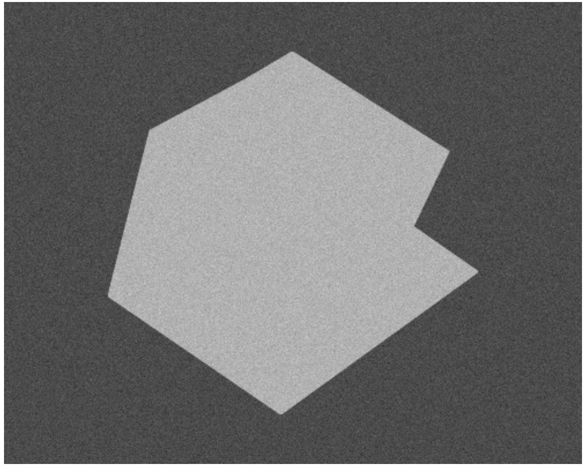


Figure 1: Image and labeled pixels.

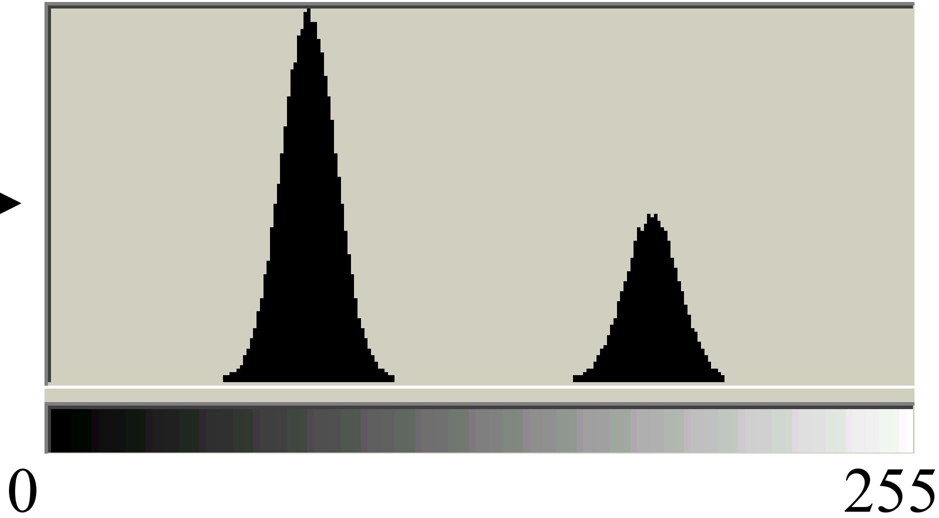
Figure 4: Semantic segmentation for an automated driving application.

Source: https://uk.mathworks.com/solutions/image-video-processing/semantic-segmentation.html

Global Thresholding



Original image



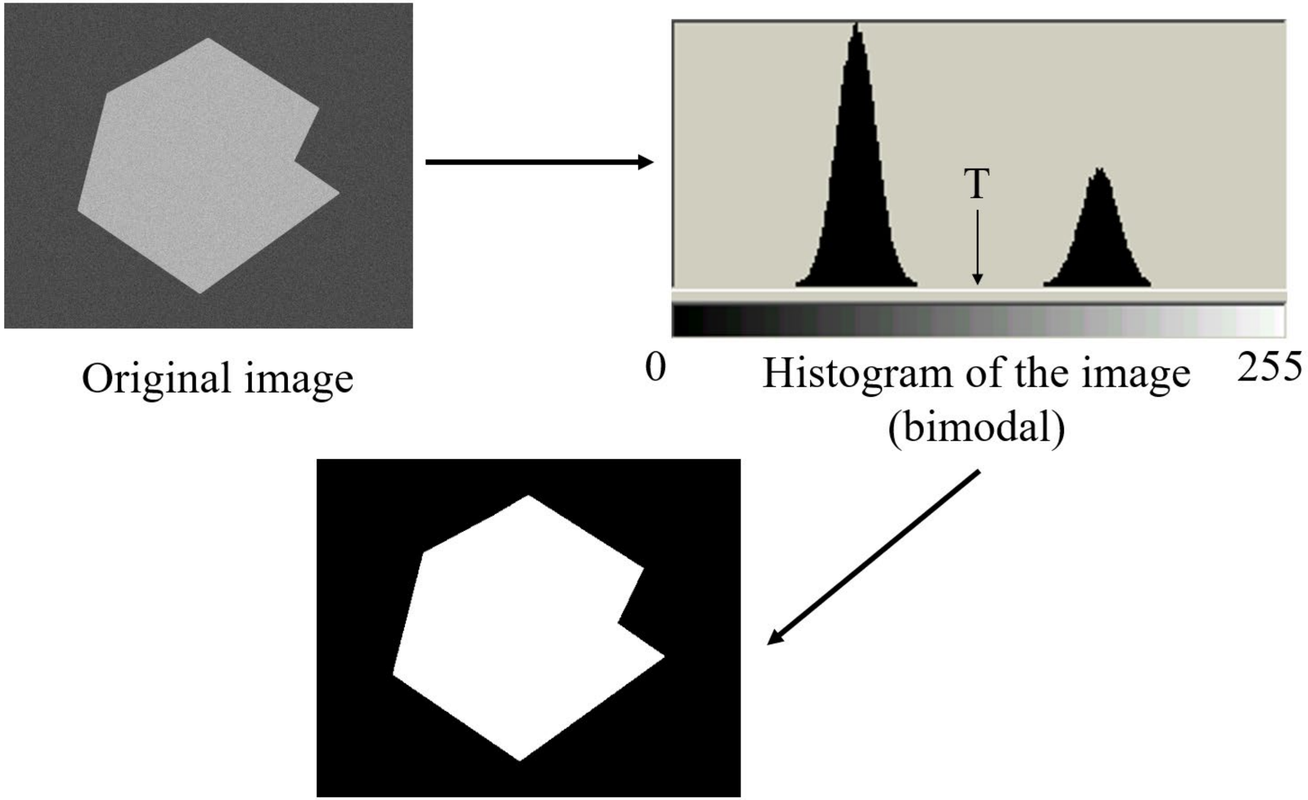
Histogram of an image

Global Thresholding

- For an image f (x, y) having a light object and dark background, the histogram consists of two dominant modes (two groups on intensity values).

- The object is extracted by selecting a threshold T that separates these modes.

Global Thresholding



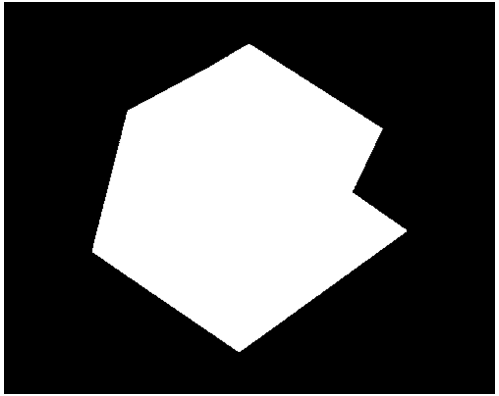
The thresholded (binary) image

Global Thresholding

- Each pixel is then labeled according to a pre-defined pixel labeling rule, 2-class labelling, which is given by

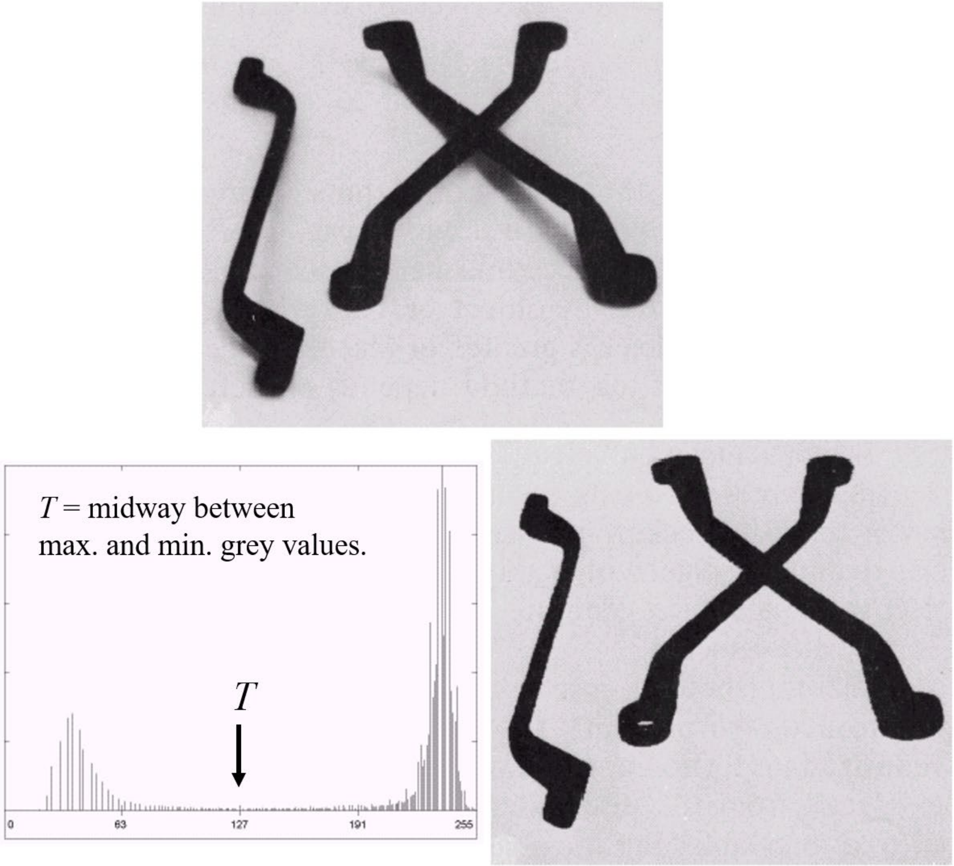
$$g(x,y)=\begin{cases}1 & \text{if } f(x,y)\geq T\\ 0 & \text{if } f(x,y)<T\end{cases}$$

- If $T = 127$,



A thresholded (binary) image

Example

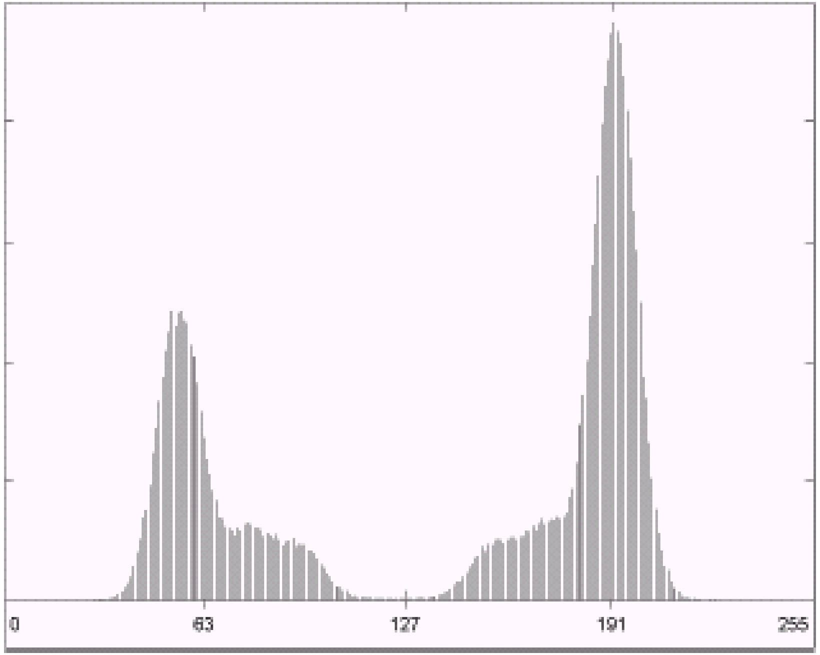


a
b c

FIGURE 10.28 (a) Original image. (b) Image histogram. (c) Result of global thresholding with *T* midway between the maximum and minimum gray levels.

Gaussian Mixture Model (GMM)

- Assume that the image histogram is a mixture of Gaussian distributions.



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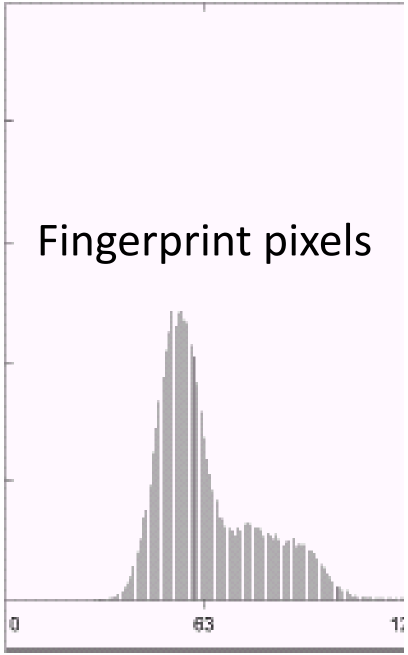

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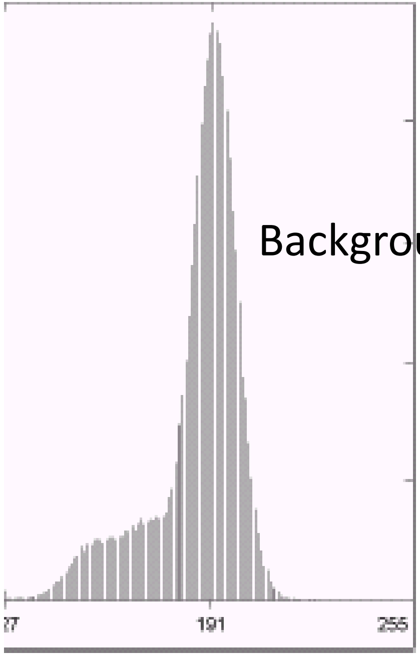

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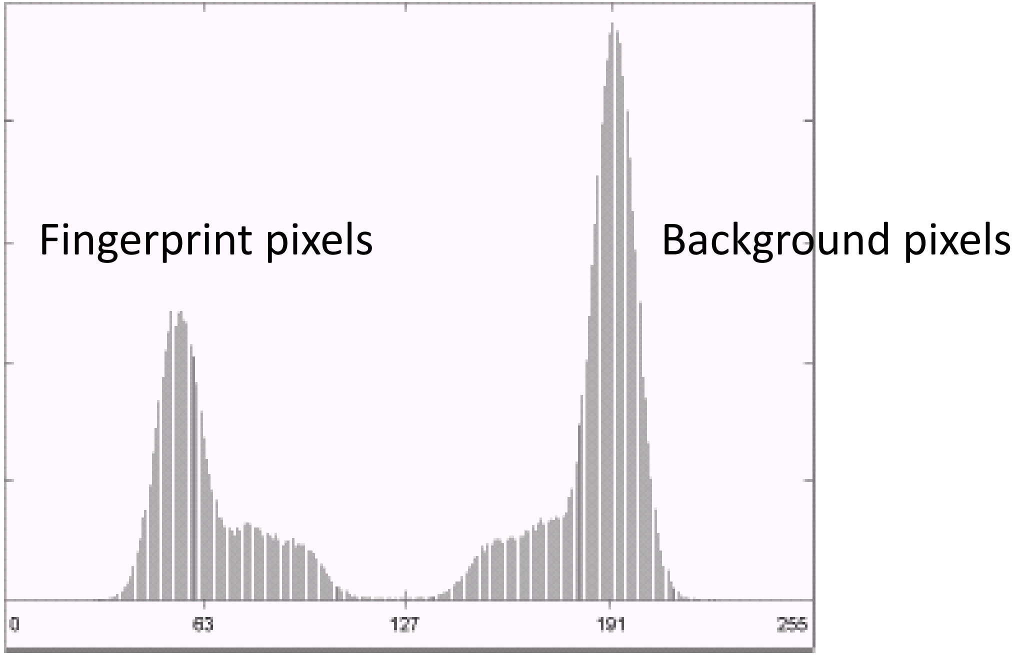
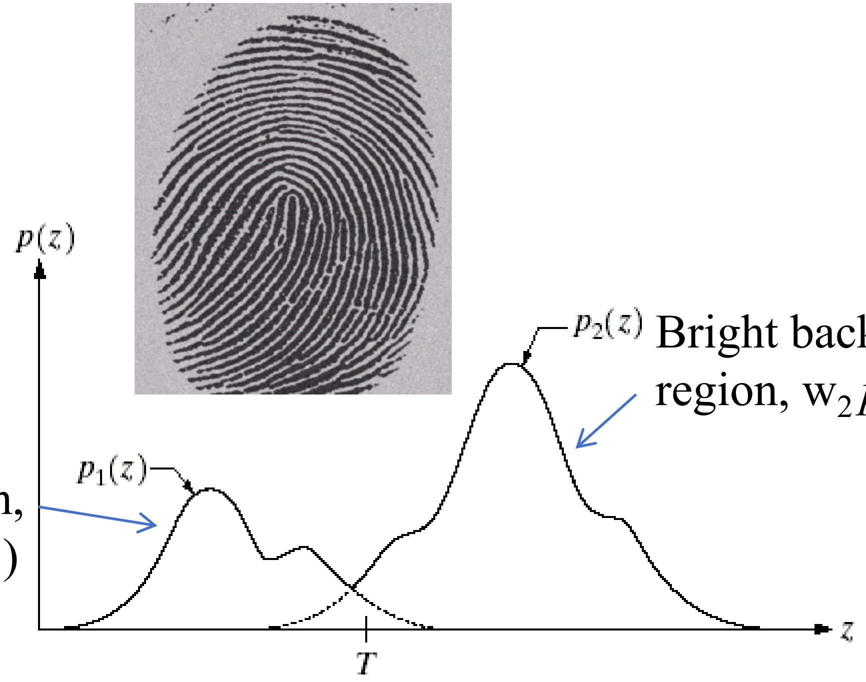


Gaussian Mixture Model (GMM)

- Assume that the image histogram is a mixture of Gaussian distributions.

FIGURE 10.32 Gray-level probability density functions of two regions in an image.



Gaussian Mixture Model (GMM)

- If there are two under lying Gaussian distributions, $p(z|1)$ and $p(z|2)$ represent the Gaussian distributions or other distributions.
- z represents intensity value.
- The mixture model is defined as

$$p(z) = w_1 p(z|1) + w_2 p(z|2)$$

where $w_1 + w_2 = 1$, and $0 \le w_1, w_2 \le 1$

Gaussian Mixture Model (GMM)

- The probability density function (PDF) is formed from a linear combination of M basis functions (e.g., Gaussian distribution).
- Mixture distribution (or PDF) is given by

$$p(z) = \sum_{j=1}^{M} p(z \mid j)\, p(j)$$

where M represents the number of basis functions

$p(z \mid j)$ represents basis function/likelihood

$p(j) = w_j$ represents mixing parameter/prior probability

Gaussian Mixture Model (GMM)

Constraints

(1) $$\sum_{j=1}^{M} p(j) = 1$$

(2) $$0 \le p(j) \le 1$$

(3) $$\int p(z \mid j)\,dx = 1$$

$$\text{e.g., } p(z \mid j) = \frac{1}{\sqrt{2\pi\sigma_j^2}} \cdot \exp\left[\frac{-(z-\mu_j)^2}{2\sigma_j^2}\right]$$

Gaussian distribution

Posterior probability

$$p(j \mid z) = \frac{p(z \mid j)\, p(j)}{p(z)}$$

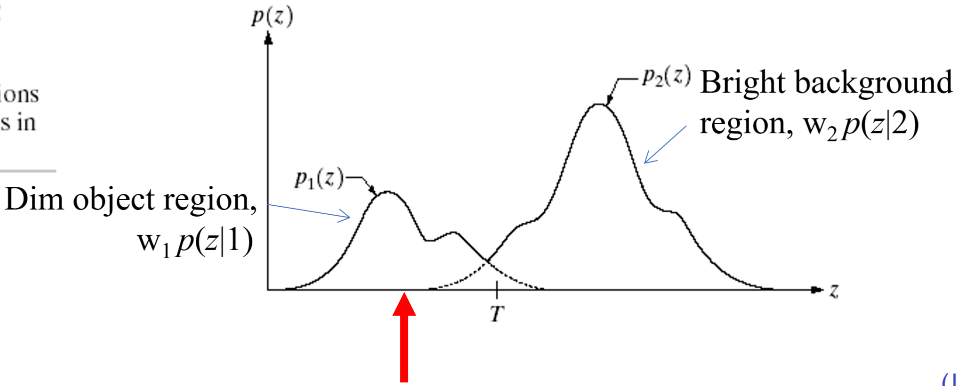
$$\sum_{j=1}^{M} p(j \mid z) = 1$$

Gaussian Mixture Model (GMM)

Fingerprint pixel

$$p(1|z) > p(2|z)$$

FIGURE 10.32 Gray-level probability density functions of two regions in an image.

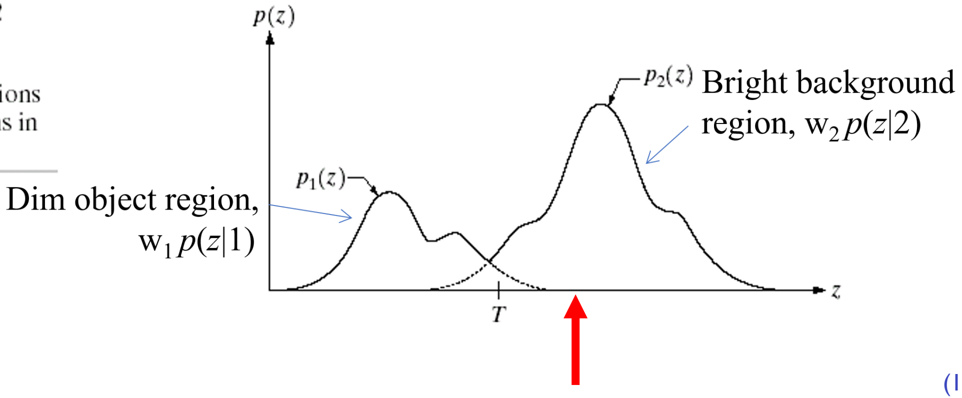


Gaussian Mixture Model (GMM)

Background pixel

$$p(1|z) < p(2|z)$$

FIGURE 10.32 Gray-level probability density functions of two regions in an image.

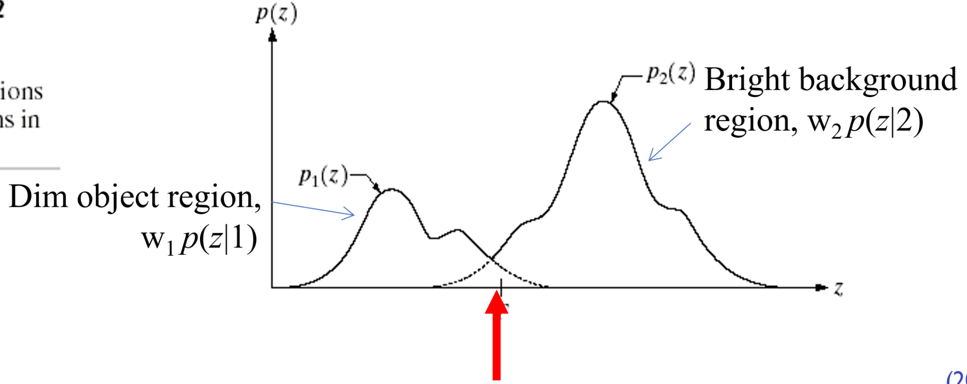


Gaussian Mixture Model (GMM)

?? pixel

$$p(1|z) < p(2|z) \; or \; p(1|z) > \; p(2|z)$$

FIGURE 10.32 Gray-level probability density functions of two regions in an image.



Expectation-Maximization (EM) Method

- The EM method is an iterative scheme for finding the values of the parameters in a mixture model. The concept is to maximize the expected negative log-likelihood function of an observed image.

$$E = -\ln L$$

Initial values of μ_j, σ_j and $p(j)$

Estimate the expected $p^{old}(j \mid z)$

Estimate the values of μ_j^{new}, σ_j^{new} and $p^{new}(j)$ based on the principle of maximum likelihood

Expectation-Maximization (EM) Method

- Update equations are shown below. Let N be the number of pixels in an image. As such, pixel index n = 1, …, N. N = total number of pixels.
- z^n represents pixel intensity at the n^{th} pixel.
- Iteration continues until convergence is reached.

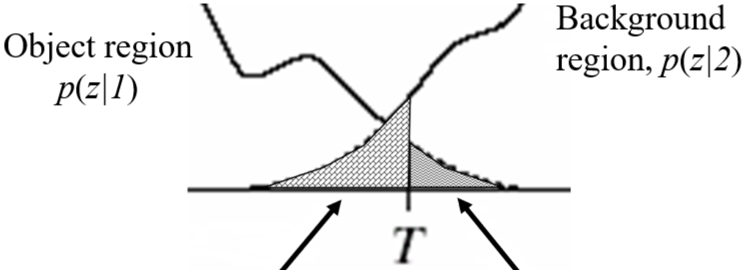
$$\mu_j^{new} = \frac{\sum_n p^{old}\left(j \mid z^n\right) z^n}{\sum_n p^{old}\left(j \mid z^n\right)}$$

$$(\sigma_j^{new})^2 = \frac{\sum_n p^{old}\left(j \mid z^n\right)\left(z^n - \mu_j^{new}\right)^2}{\sum_n p^{old}\left(j \mid z^n\right)}$$

$$p\left(j\right)^{new} = \frac{1}{N}\sum_n p^{old}\left(j \mid z^n\right)$$

Global Thresholding

- Classification/segmentation error is defined as



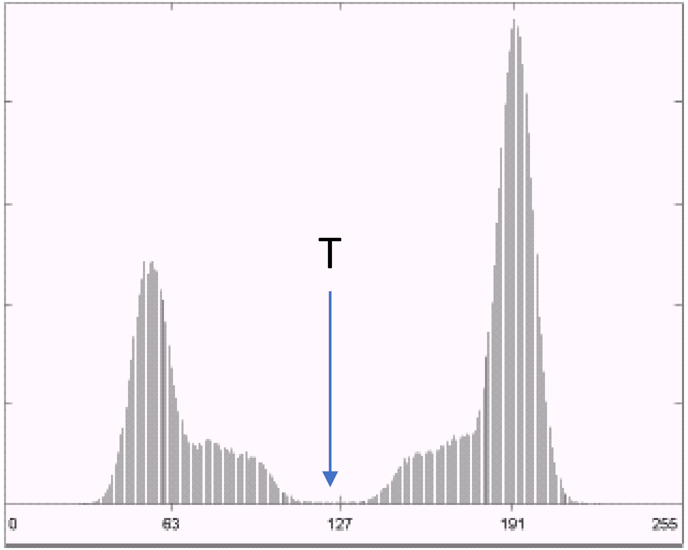
$$E(T) = \int_{-\infty}^{T} w_2 p(z \mid 2)\,dz + \int_{T}^{\infty} w_1 p(z \mid 1)\,dz$$

Misclassifying background pixels into object pixels. | Misclassifying object pixels into background pixels.

- By minimizing E(T) with respect to T, we get

when $w_1 p(T \mid 1) = w_2 p(T \mid 2)$
$E(T)$ is minimum.

Global Thresholding



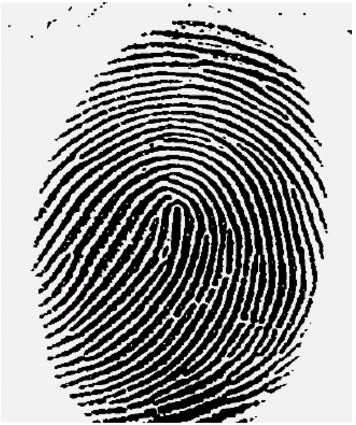


FIGURE 10.29 (a) Original image. (b) Image histogram. (c) Result of segmentation with the threshold estimated by iteration. (Original courtesy of the National Institute of Standards and Technology.)

Summary

- Global thresholding concept
- Gaussian mixture model (GMM)
- Expectation-Maximization (EM) method