Line and Circle Detection

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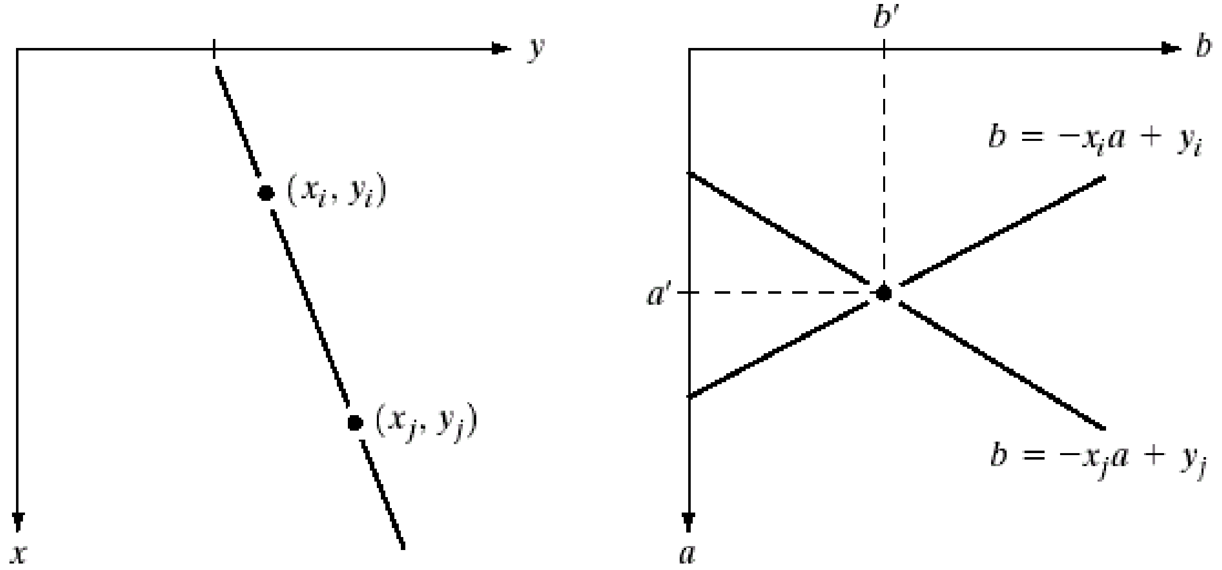
References

- Rafael C. Gonzalez and Richard E. Woods, Digital Image Processing, Third Edition, Pearson Education, 2008:
  - Edge Linking and Boundary Detection: Chapter 10.2.7
- https://en.wikipedia.org/wiki/Hough_transform
- Circle detection, adapted from notes by Harvey Rhody at RIT.

Hough transform for line detection

1. If two edge points (x_i, y_i) and (x_j, y_j) are lying on the same straight line, then they should have the same values of slope and y-intercepts on the xy-plane.



a b

FIGURE 10.17
(a) *xy*-plane.
(b) Parameter space.

2. Concepts and procedures:

   a. For a point (x_i, y_i), we set up a straight-line equation.

$$y_i = ax_i + b \Leftrightarrow b = \left(-x_i\right)a + y_i$$

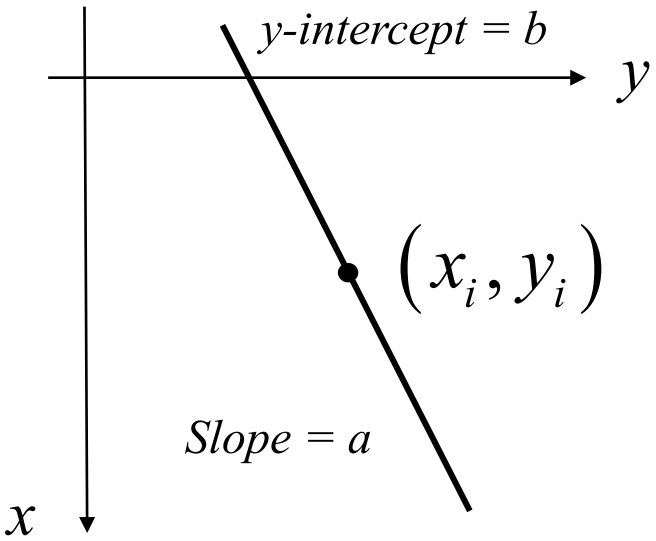
   where a = slope (variable), b = y-intercept (variable), x_i and y_i are known and fixed.

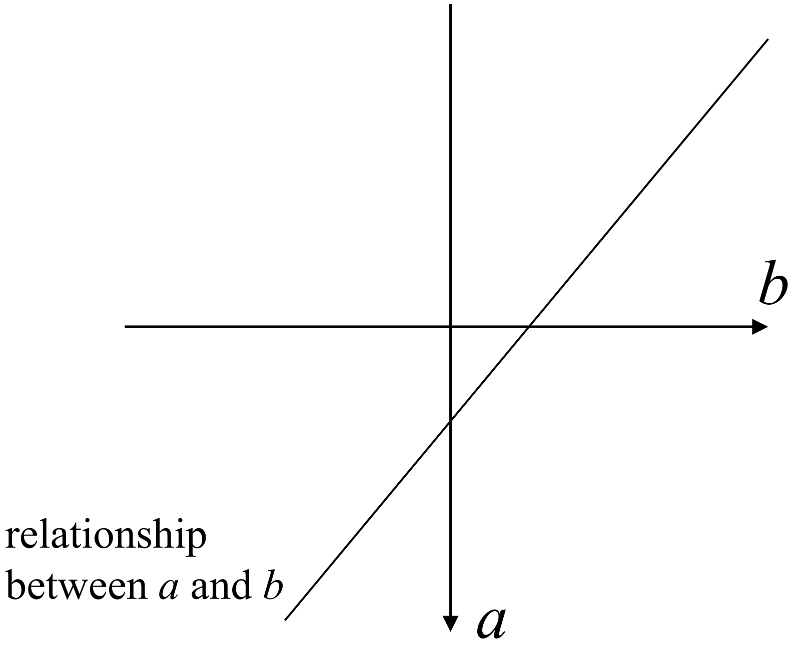
   b. We subdivide a axis into K increments between $[a_{min}, a_{max}]$. For each increment of a, we evaluate the value of b.

   c. A relationship between a and b can be plotted in parameter space.

   d. We partition the parameter space into a number of bins (accumulator cells) and increment the corresponding accumulator bin $A(a,b)$ by 1.

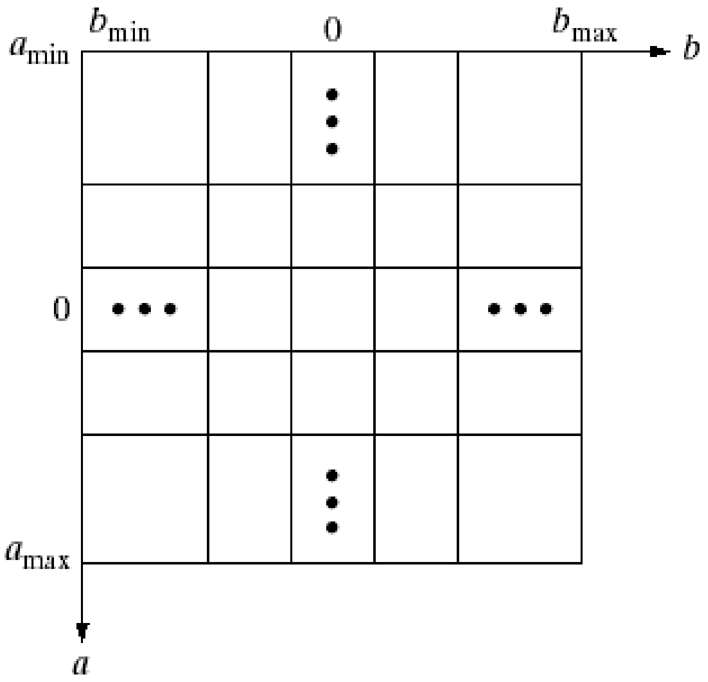
Hough transform for line detection





Hough transform for line detection

FIGURE 10.18 Subdivision of the parameter plane for use in the Hough transform.

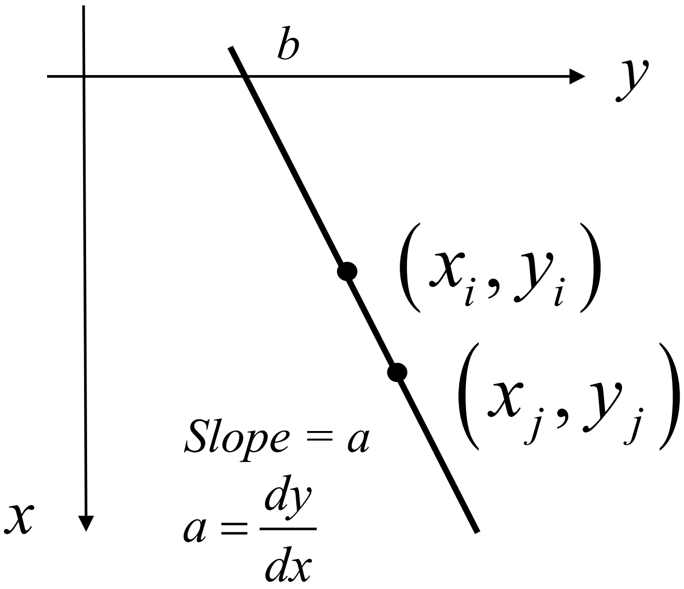


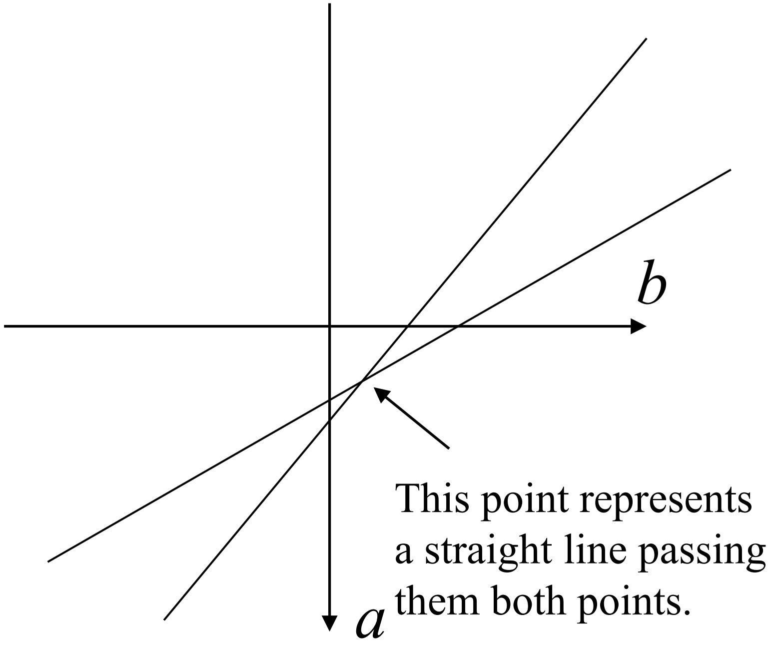
2. Concepts and procedures:
   e. For another point (x_j, y_j), we set up another straight-line equation.

$$y_j = ax_j + b \Leftrightarrow b = \left(-x_j\right)a + y_j$$

   f. Similarly, we subdivide the a axis into K increments between $[a_{min}, a_{max}]$. For each increment of a, we evaluate the value of b. We plot the relationship between a and b in the same parameter space and update bin values.
   g. The accumulator bin $A(a,b)$ having the highest count corresponds to the straight line passing through the points (x_i, y_i) and (x_j, y_j).

Hough transform for line detection



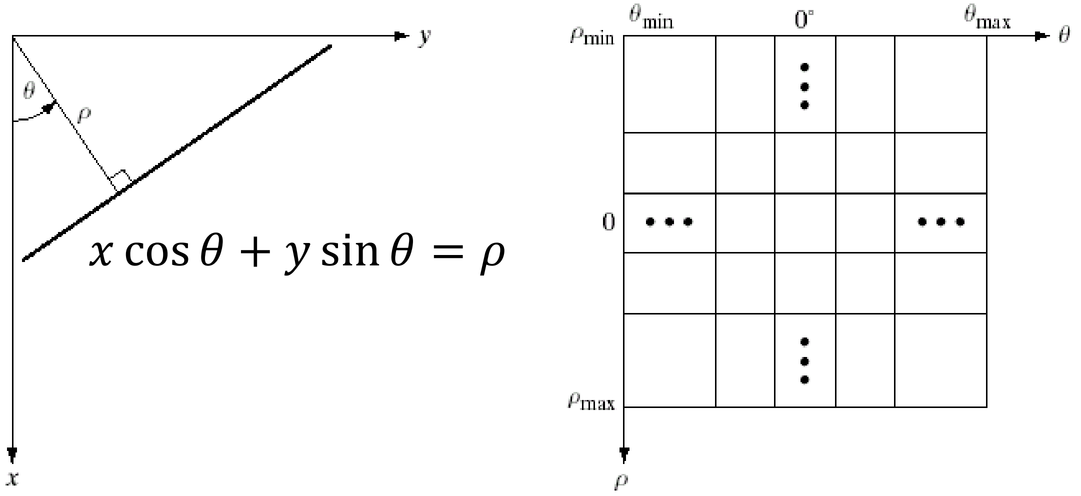


2. Concepts and procedures:
   h. The same procedure can be applied to all points. The accumulator bin $A(a,b)$ having the highest count corresponds to the straight line passing through the largest number of edge points.

3. Problem? Values of a and b run from negative infinity to positive infinity. We need an infinite number of bins!

Hough transform for line detection

4. Solution: use the normal representation of a line.



a b

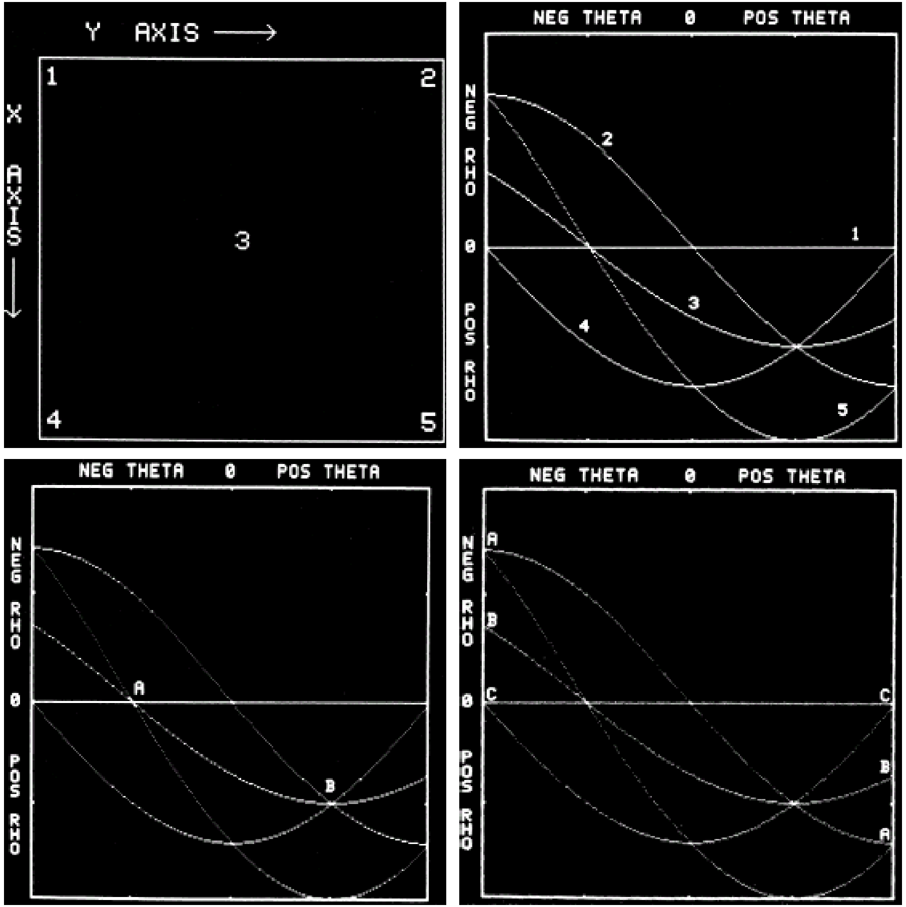
FIGURE 10.19
(a) Normal representation of a line.
(b) Subdivision of the $\rho\theta$-plane into cells.

5. Theta θ runs from –90° to 90°. Rho ρ runs from $-\sqrt{2}D$ to $\sqrt{2}D$, where D is the length/width of the image.

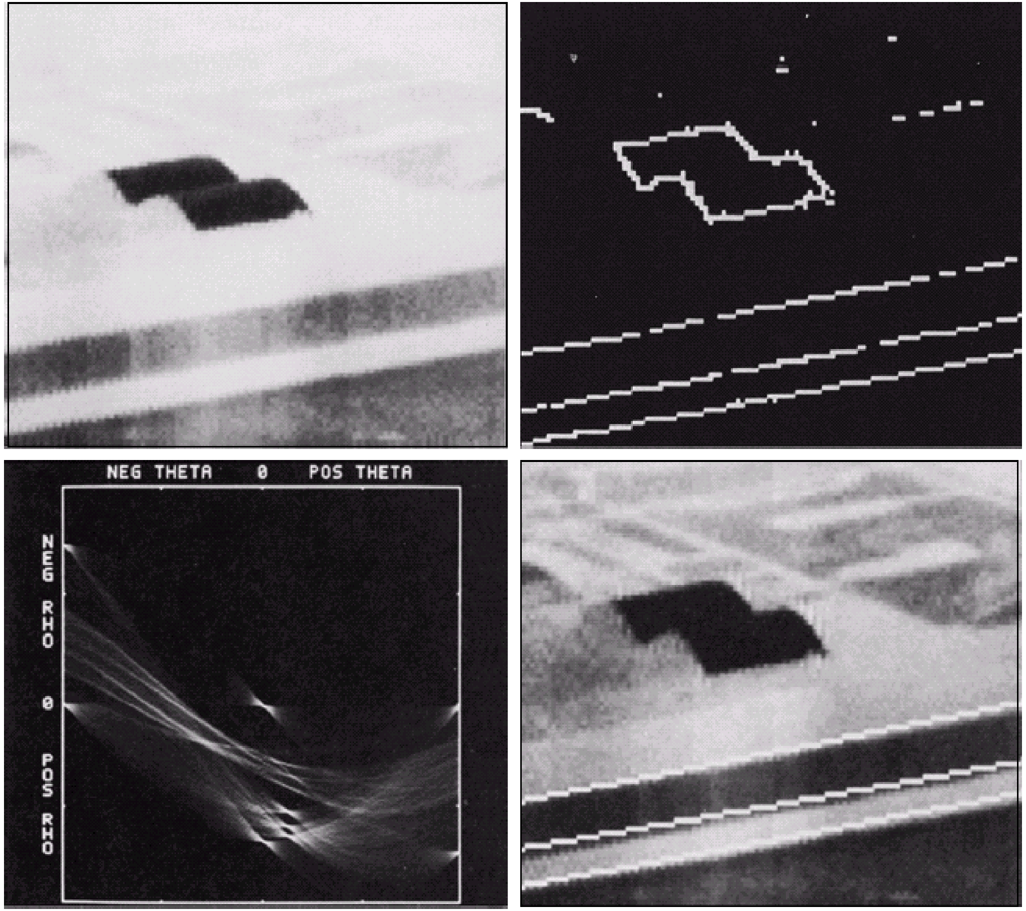
Hough transform for line detection

a	b
c	d

FIGURE 10.20 Illustration of the Hough transform. (Courtesy of Mr. D. R. Cate, Texas Instruments, Inc.)



Hough transform for line detection



a b
c d

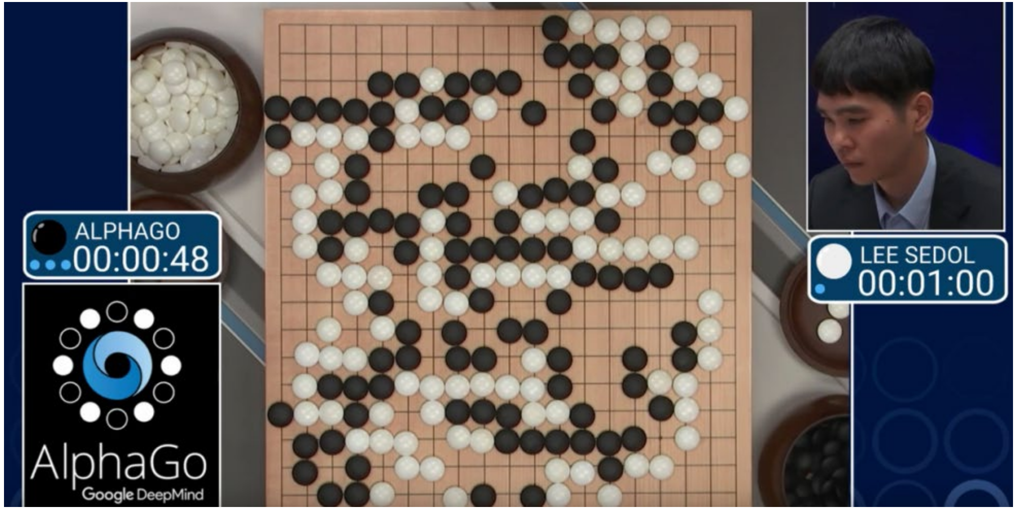
FIGURE 10.21
(a) Infrared image.
(b) Thresholded gradient image.
(c) Hough transform.
(d) Linked pixels.
(Courtesy of Mr. D. R. Cate, Texas Instruments, Inc.)

6. Implementation (straight line detection):
   a. Compute the gradient of an image and threshold it to obtain a binary edge image.
   b. Partition the parameter space, e.g., *a-b* plane or theta-rao plane.
   c. Examine the bin counts, pick the highest count.
   d. Examine the relationship (based on criteria) between pixels.

7. This method can also use circle instead of straight line. How?

$$(x-c_1)^2+(y-c_2)^2=c_3$$

Finding stones in a board of a 19x19 grid……



www.popularmechanics.com

Hough transform for circle detection

Circle Hough Transform (CHT)

The Hough transform can be used to determine the parameters of a circle when a number of points that fall on the perimeter are known. A circle with radius R and center (a, b) can be described with the parametric equations

$$
\begin{aligned}
x &= a + R\cos(\theta) \\
y &= b + R\sin(\theta)
\end{aligned}
$$

When the angle θ sweeps through the full 360 degree range the points (x, y) trace the perimeter of a circle.

If an image contains many points, some of which fall on perimeters of circles, then the job of the search program is to find parameter triplets (a, b, R) to describe each circle. The fact that the parameter space is 3D makes a direct implementation of the Hough technique more expensive in computer memory and time.

https://www.cis.rit.edu/class/simg782/lectures/lecture_10/lec782_05_10.pdf

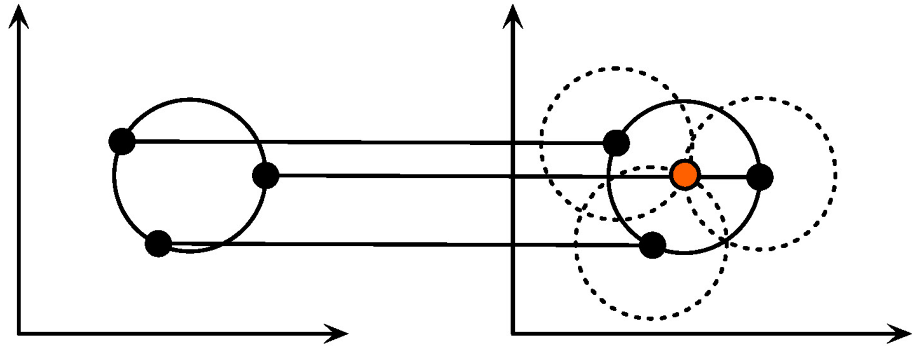
Hough transform for circle detection

Search with fixed R

If the circles in an image are of known radius R, then the search can be reduced to 2D. The objective is to find the (a, b) coordinates of the centers.

$$
\begin{aligned}
x &= a + R\cos(\theta) \\
y &= b + R\sin(\theta)
\end{aligned}
$$

The locus of (a, b) points in the *parameter space* fall on a circle of radius R centered at (x, y). The true center point will be common to all parameter circles, and can be found with a Hough accumulation array.



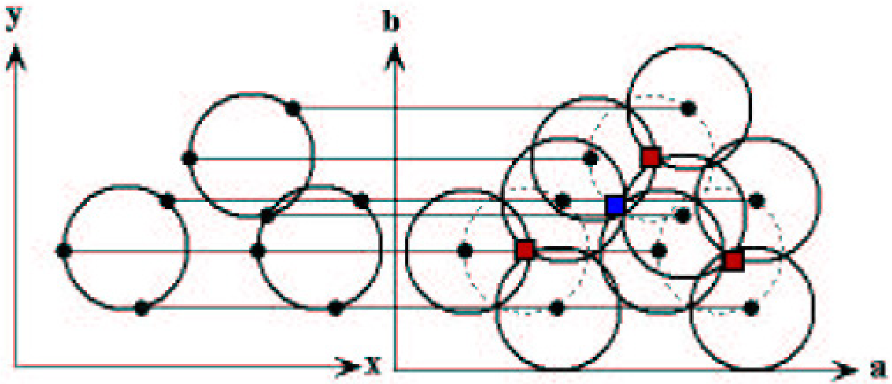
Each point in geometric space (left) generates a circle in parameter space (right). The circles in parameter space intersect at the (a, b) that is the center in geometric space.

https://www.cis.rit.edu/class/simg782/lectures/lecture_10/lec782_05_10.pdf

Hough transform for circle detection

Multiple Circles with known R

Multiple circles with the same radius can be found with the same technique. The centerpoints are represented as red cells in the parameter space drawing. Overlap of circles can cause spurious centers to also be found, such as at the blue cell.
Spurious circles can be removed by matching to circles in the original image.



Each point in geometric space (left) generates a circle in parameter space (right). The circles in parameter space intersect at the (a, b) that is the center in geometric space.

If the radius of the object is unknown, the same procedure can be repeated with different radii.

Then the accumulator bin/cell with the highest count corresponds to the circle that passes through the largest number of edge pixels in the image.

https://www.cis.rit.edu/class/simg782/lectures/lecture_10/lec782_05_10.pdf

Summary

- Line detection using the Hough transform
- Circle detection using the Hough transform