Principle Component Analysis

Spectral Theorem

对称

Let $A \in R^{n,n}$ be symmetric, and $\lambda_i \in R, i = 1, 2, \cdots, n$ be the eigenvalues of A. There exists a set of orthonormal vectors $u_i \in R_n, i = 1, 2, \cdots, n$, such that $Au_i = \lambda_i u_i$. Equivalently, there exists an orthogonal matrix $U = [u_1, \cdots, u_n]$ (i.e., $UU^T = U^T U = I_n$), such that,

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$$A = U \Lambda U^T = \sum_{i=1}^{n} \lambda_i u_i u_i^T, \Lambda = \text{diag}(\lambda_1, \cdots, \lambda_n)$$