Uncertainty weighted average

When trying to calculated an uncertainty weighted average, we use the Maximum Likelihood estimator.

$$P(\{x_i\}) = \prod f(x_i|\mu,\sigma_i) = \prod \frac{1}{\sqrt{2\pi}\sigma_i}\exp\left(-\frac{1}{2}\frac{(x_i-\mu)^2}{\sigma_i^2}\right)$$

To maximize the above expression, we need to maximize the exponent.

$$\frac{\partial}{\partial\mu}\sum_i\left(-\frac{1}{2}\frac{(x_i-\mu)^2}{\sigma_i^2}\right) = \sum_i \frac{(x_i-\mu)}{\sigma_i^2} = 0$$

Thus,

$$\mu = \frac{\sum_i x_i/\sigma_i^2}{\sum_i 1/\sigma_i^2} = \frac{\sum_i w_i x_i}{\sum_i w_i} \qquad \text{with } w_i = 1/\sigma_i^2$$