COMP30026_Assignment_01_904904

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COMP30026 Models of Computation
Assignment 1

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September 13, 2019

Challenge 1

$$
\begin{aligned}
F &= \forall x\ (\neg P(x,x)) \\
G &= \forall x\ \forall y\ \forall z\ (P(x,y) \land P(y,z) \Rightarrow P(x,z)) \\
G' &= \forall x\ \forall y\ \forall z\ (P(x,y) \land P(y,z) \Rightarrow \neg P(x,z)) \\
H &= \forall x\ \forall y\ (P(x,y) \Rightarrow \neg P(y,x))
\end{aligned}
$$

a. Show that $F \lor G \lor G' \lor H$ is not valid.

Explain how you're showing non-

note: **f** = false, **t** = true in the following answers.

domain	$P(a,b)$	formula			
		F	G	G'	H
{-1, 0, 1}	$P(a,b) = 2a + b \geq 0$	$x = 1$ **f**	$x = 0, y = 1, z = -1$ **f**	$x = 0, y = 0, z = 0$ **f**	$x = 0, y = 0$ **f**

So we have one intepretation gives $F \lor G \lor G' \lor H = \mathbf{f}$.
It shows $F \lor G \lor G' \lor H$ is not valid.

b. Show that $F \land G' \land H$ is satisfiable.

Explain how you're showing

domain	$P(a,b)$	formula			
		F	G'	H	$F \land G' \land H$
{0}	$P(a,b) = \mathbf{f}$	**t**	**t**	**t**	**t**

Universal quantifier?

The listed intepretation shows $F \land G' \land H$ is satisfiable.

c. Show that $(F \land G) \Rightarrow H$ is valid.
Let $K = (F \land G) \Rightarrow H$,

Explain how you're showing

Proof. K is valid

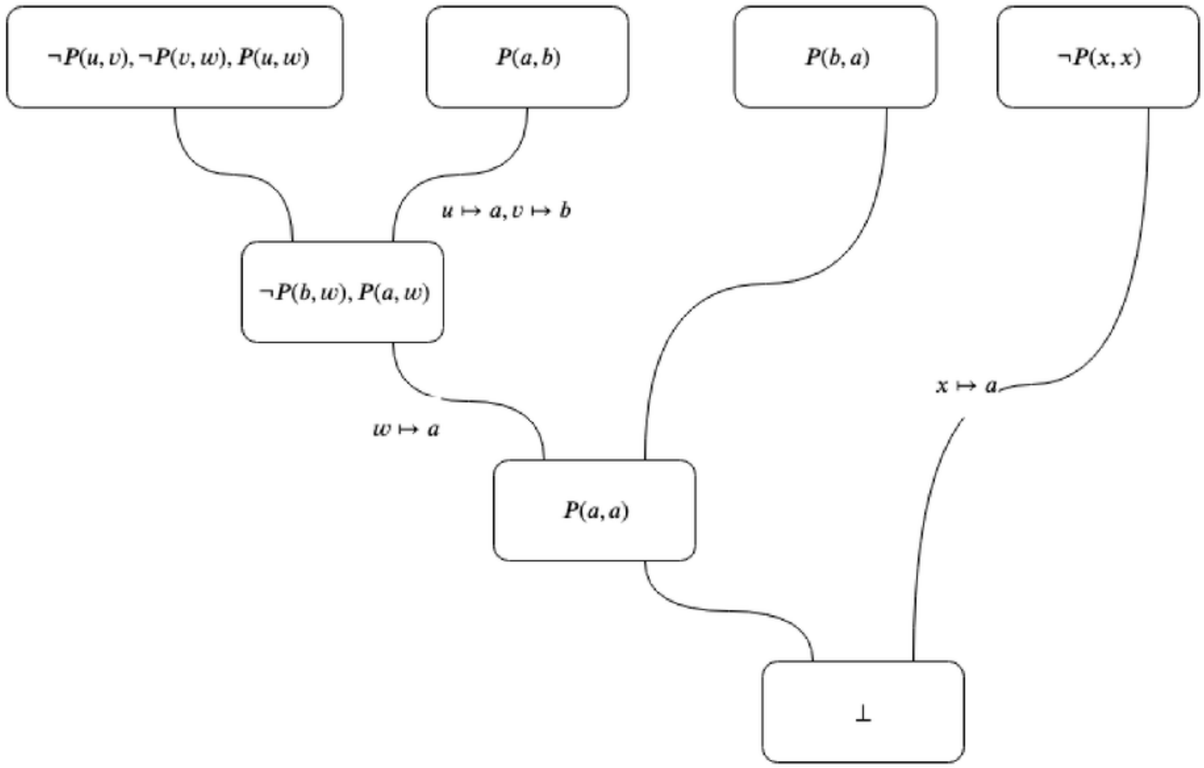
is equivalent to prove $\neg K$ is unsatisfiable.

$$
\begin{aligned}
\neg K &\equiv \neg[(F \wedge G) \Rightarrow H] \\
&\equiv \neg[\neg(F \wedge G) \vee H] && \text{(by implication)} \\
&\equiv F \wedge G \wedge \neg H && \text{(by deriving negation in)} \\
&\equiv [\forall x\ (\neg P(x,x))] \wedge \{\forall x\ \forall y\ \forall z\ [(\neg(P(x,y) \wedge P(y,z)) \vee P(x,z)]\} \wedge [\neg \forall x\ \forall y\ (\neg P(x,y) \vee \neg P(y,x))] \\
& && \text{(by replacing occurrence of } \Rightarrow) \\
&\equiv [\forall x\ (\neg P(x,x))] \wedge [\forall x\ \forall y\ \forall z\ (\neg P(x,y) \vee \neg P(y,z) \vee P(x,z))] \wedge [\exists x\ \exists y\ (P(x,y) \wedge P(y,x))] \\
& && \text{(by deriving negation in)} \\
&\equiv [\forall x\ (\neg P(x,x))] \wedge [\forall u\ \forall v\ \forall w\ (\neg P(u,v) \vee \neg P(v,w) \vee P(u,w))] \wedge [\exists z\ \exists y\ (P(z,y) \wedge P(y,z))] \\
& && \text{(by standarizing apart)} \\
&\rightsquigarrow [\forall x\ (\neg P(x,x))] \wedge [\forall u\ \forall v\ \forall w\ (\neg P(u,v) \vee \neg P(v,w) \vee P(u,w))] \wedge P(a,b) \wedge P(b,a) \\
& && \text{(by skolemization)} \\
&\equiv \neg P(x,x) \wedge (\neg P(u,v) \vee \neg P(v,w) \vee P(u,w)) \wedge P(a,b) \wedge P(b,a) \\
& && \text{(by dropping universal quantifiers)}
\end{aligned}
$$

and written as a set of sets of literals:

$$
\left\{
\begin{array}{l}
\{\neg P(x,x)\}, \\
\{\neg P(u,v), \neg P(v,w), P(u,w)\}, \\
\{P(a,b)\}, \\
\{P(b,a)\}
\end{array}
\right\}
$$

and do the resolution:



As resolution shows: $\neg K$ is unsatisfiable.

$\therefore K$ is valid. $\square$

Challenge 2

- $F(x)$, which stands for "the force is with x"
- $J(x)$, which stands for "x is a Jedi master"
- $E(x, y)$, which stands for "x exceeds y"
- $P(x, y)$, which stands for "x is a pupil of y"
- $V(x)$, which stands for "x is venerated"

and use the constant 'a' to denote Yoda.

a. For each of the statements S_1, ..., S_4, express it *as a formula in first-order predicate logic* (not clausal form):

   $S_1 = \forall x(J(x) \Rightarrow F(x))$

   $S_2 = \exists y(J(y) \wedge E(a, y))$

   $S_3 = \forall z\{[J(z) \wedge F(z) \wedge (\forall u(P(u, z) \Rightarrow E(u, z)))] \Rightarrow V(z)\}$

   $S_4 = \forall v\{[J(v) \wedge (\neg\exists w(P(w, v)))] \Rightarrow V(v)\}$

b. Translate S_1-S_3 to clausal form.

$$
\begin{aligned}
S_1 &\equiv \forall x(J(x) \Rightarrow F(x)) \\
&\equiv \forall x(\neg J(x) \vee F(x)) && \text{(by replacing occurrence of } \Rightarrow) \\
&\equiv \neg J(x) \vee F(x) && \text{(by dropping universal quantifiers)}
\end{aligned}
$$

and written as a set of sets of literals:

$$\{\ \{\neg J(x), F(x)\}\ \ \}$$

$$
\begin{aligned}
S_2 &\equiv \exists y(J(y) \wedge E(a,y)) \\
&\rightsquigarrow J(b) \wedge E(a,b) && \text{(by skolemization)}
\end{aligned}
$$

and written as a set of sets of literals:

$$\left\{\begin{array}{l}\{J(b)\}, \\ \{E(a,b)\}\end{array}\right\}$$

$$
\begin{aligned}
S_3 &\equiv \forall z\{[J(z) \wedge F(z) \wedge (\forall u(P(u,z) \Rightarrow E(u,z)))] \Rightarrow V(z)\} \\
&\equiv \forall z\{\neg[J(z) \wedge F(z) \wedge (\forall u(\neg P(u,z) \vee E(u,z)))] \vee V(z)\} && \text{(by replacing occurrence of } \Rightarrow) \\
&\equiv \forall z[\neg J(z) \vee \neg F(z) \vee \neg(\forall u(\neg P(u,z) \vee E(u,z))) \vee V(z)] && \text{(by deriving negation in)} \\
&\equiv \forall z[\neg J(z) \vee \neg F(z) \vee (\exists u(P(u,z) \wedge \neg E(u,z))) \vee V(z)] && \text{(by deriving negation in)} \\
&\rightsquigarrow \forall z[\neg J(z) \vee \neg F(z) \vee (P(f(z),z) \wedge \neg E(f(z),z)) \vee V(z)] && \text{(by skolemization)} \\
&\equiv \neg J(z) \vee \neg F(z) \vee (P(f(z),z) \wedge \neg E(f(z),z)) \vee V(z) && \text{(by dropping universal quantifiers)} \\
&\equiv (\neg J(z) \vee \neg F(z) \vee V(z)) \vee (P(f(z),z) \wedge \neg E(f(z),z)) && \text{(by commutavity)} \\
&\equiv (\neg J(z) \vee \neg F(z) \vee V(z) \vee P(f(z),z)) \wedge (\neg J(z) \vee \neg F(z) \vee V(z) \vee \neg E(f(z),z)) \\
& && \text{(by distributivity)}
\end{aligned}
$$

I see you are being careful renaming variables to make sure you don't have any double ups. If you want to be consistent with that, be sure you rename across these two clauses too! Even though they came from the same universally quantified statement originally.

and written as a set of sets of literals:

$$\left\{\begin{array}{l}\{\neg J(z), \neg F(z), V(z), P(f(z),z)\}, \\ \{\neg J(z), \neg F(z), V(z), \neg E(f(z),z)\}\end{array}\right\}$$

1

c. Translate *the negation of* S_4 to clausal form.

$$
\begin{aligned}
\neg S_4 &\equiv \neg\forall v\{[J(v) \wedge (\neg\exists w(P(w,v)))] \Rightarrow V(v)\} \\
&\equiv \neg\forall v\{\neg[J(v) \wedge (\neg\exists w(P(w,v)))] \vee V(v)\} && \text{(by replacing occurrence of } \Rightarrow) \\
&\equiv \exists v\{J(v) \wedge [\forall w(\neg P(w,v))] \wedge \neg V(v)\} && \text{(by deriving negation in)} \\
&\rightsquigarrow J(c) \wedge [\forall w(\neg P(w,c))] \wedge \neg V(c) && \text{(by skolemization)} \\
&\equiv J(c) \wedge \neg P(w,c) \wedge \neg V(c) && \text{(by dropping universal quantifiers)}
\end{aligned}
$$

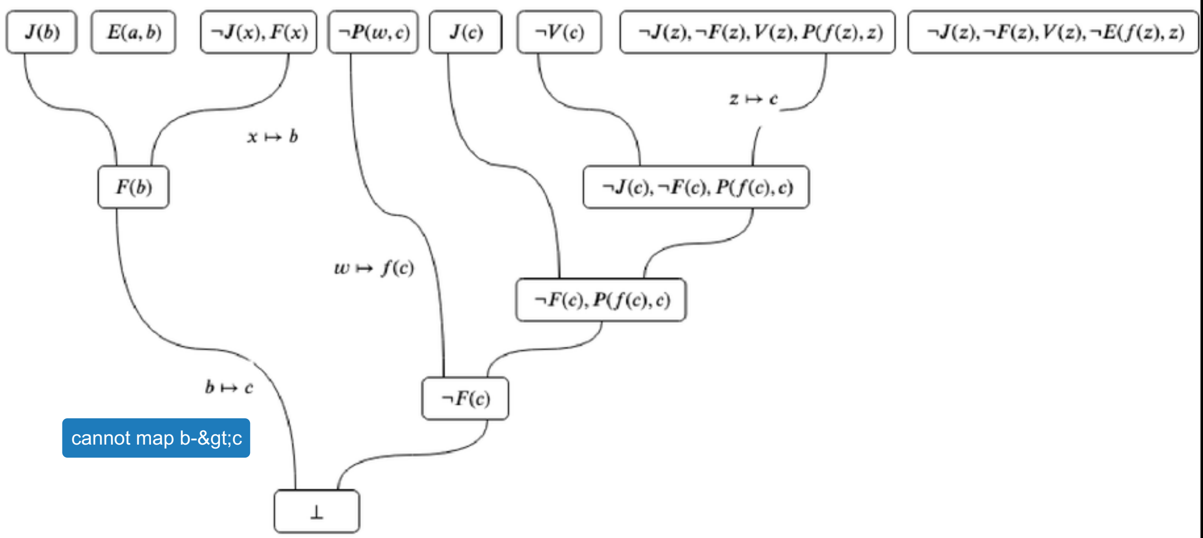
and written as a set of sets of literals:

$$\left\{\begin{array}{l}\{J(c)\},\\ \{\neg P(w,c)\},\\ \{\neg V(c)\}\end{array}\right\}$$

d. Give a proof by resolution to show that S_4 follows from the other statements.

Proof. $S_1 \wedge S_2 \wedge S_3 \models S_4$

is equivalent to prove: $S_1 \wedge S_2 \wedge S_3 \wedge \neg S_4$ is unsatisfiable.



As resolution shows: $S_1 \wedge S_2 \wedge S_3 \wedge \neg S_4$ is unsatisfiable

$\therefore S_1 \wedge S_2 \wedge S_3 \models S_4$ $\square$

Challenge 3

a. Consider the Boolean vector function $\mathbf{f}_a(b_1, b_2) = (\neg b_1, b_1 \oplus b_2)$. Show that this function is reversible.

Note: 0 = false, 1 = true in the following answers.

b_1	b_2	$\neg b_1$	$b_1 \oplus b_2$
0	0	1	0
0	1	1	1
1	0	0	1
1	1	0	0

As we can see every input has an unique output, $\mathbf{f}_a(b_1, b_2)$ is reversible.

b. Consider the Boolean vector function $\mathbf{f}_b(b_1, b_2, b_3) = (b_1 \oplus b_2, b_2 \oplus b_3, b_3 \oplus b_1)$. Show that this function is *not* reversible.

b_1	b_2	b_3	$b_1 \oplus b_2$	$b_2 \oplus b_3$	$b_3 \oplus b_1$
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	1	1	0
0	1	1	1	0	1
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	0	1	1
1	1	1	0	0	0

As we can see $\mathbf{f}_b(0,0,0) = \mathbf{f}_b(1,1,1)$, $\mathbf{f}_b(b_1,b_2,b_3)$ is not reversible.

c. Is the Boolean vector function $\mathbf{f}_c(b_1,b_2,b_3) = (b_1 \oplus b_3, \neg b_3, b_1 \oplus b_2)$ reversible? Justify your answer.

b_1	b_2	b_3	$b_1 \oplus b_3$	$\neg b_3$	$b_1 \oplus b_2$
0	0	0	0	1	0
0	0	1	1	0	0
0	1	0	0	1	1
0	1	1	1	0	1
1	0	0	1	1	1
1	0	1	0	0	1
1	1	0	1	1	0
1	1	1	0	0	0

As we can see every input has an unique output, $\mathbf{f}_c(b_1,b_2,b_3)$ is reversible.

d. Give a formula, in terms of n, for the fraction of distinct n-dimensional Boolean vector functions that are reversible.

note: # means the number of

#permutation without replacement of k input $= k!$

#permutation with replacement of k input $= k^k$

A n-dimensional boolean vector function has 2^n inputs.

To make a k input function invertible is just to choose k samples from input and each input picked once. This is equivalent to a permute without replacement of k input.

$\therefore$ #invertible n-dimensional boolean vector function
=#invertible function with 2^n inputs
=#permutation without replacement of 2^n inputs
$=(2^n)!$

To enumerate all possible ways to mapping k input function with k output is equivalent to find distinct choices of sampling from inputs. This is achieved by permuting with replacement.

Excellent!

$$
\begin{aligned}
\therefore \quad & \text{\#n-dimensional boolean vector function} \\
&= \text{\#function with } 2^n \text{ inputs mapped to } 2^n \text{ with replacement} \\
&= \text{\#permutation with replacement of } 2^n \text{ inputs} \\
&= (2^n)^{2^n}
\end{aligned}
$$

With all discussed above,

$$
\begin{aligned}
& \text{fraction of distinct n-dimensional invertible Boolean vector functions} \\
&= \frac{\text{\#n-dimensional invertible Boolean vector functions}}{\text{\#n-dimensional Boolean vector functions}} \\
&= \frac{(2^n)!}{(2^n)^{2^n}}
\end{aligned}
$$

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GRADEMARK REPORT

FINAL GRADE

6/6

GENERAL COMMENTS

Instructor

PAGE 1

QM **Explain how you're showing non-validity**

To disprove validity, you are providing an interpretation that makes the formula false (a "counter-model"). You should explicitly state up-front that this is the approach you're taking.

QM **Explain how you're showing satisfiability**

To show satisfiability, you're providing a model for the formula. You should explicitly state up-front that this is the approach you're taking.

QM **Universal quantifier?**

How does this result follow from the universal quantification of x etc? Explain.

QM **Explain how you're showing validity**

To show validity, you're showing that the *negation* of the formula is a contradiction through resolution refutation. You should explicitly state up-front that this is the approach you're taking.

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PAGE 3

PAGE 4

Comment 1

I see you are being careful renaming variables to make sure you don't have any double ups. If you want to be consistent with that, be sure you rename across these two clauses too! Even though they came from the same universally quantified statement originally.

PAGE 5

QM

cannot map b->c

Incorrect mapping. You cannot map a constant to a constant.

PAGE 6

Text Comment. Excellent!

PAGE 7

CHALLENGE 1A 0.66 / 0.66

POOR (0)	No solution or a token answer; limited evidence of understanding
OKAY (0.33)	Clear evidence of understanding and grasp of concepts; but also some definite error or excessively messy or unclear presentation
EXCELLENT (0.66)	Correct and well presented solution

CHALLENGE 1B 0.66 / 0.66

POOR (0)	No solution or a token answer; limited evidence of understanding
OKAY (0.33)	Clear evidence of understanding and grasp of concepts; but also some definite error or excessively messy or unclear presentation
EXCELLENT (0.66)	Correct and well presented solution

CHALLENGE 1C 0.66 / 0.66

POOR (0)	No solution or a token answer; limited evidence of understanding
OKAY (0.33)	Clear evidence of understanding and grasp of concepts; but also some definite error or excessively messy or unclear presentation
EXCELLENT (0.66)	Correct and well presented solution

CHALLENGE 2A 0.50 / 0.5

POOR (0)	No solution or a token answer; limited evidence of understanding
OKAY (0.25)	Clear evidence of understanding and grasp of concepts; but also some definite error or excessively messy or unclear presentation
EXCELLENT (0.50)	Correct and well presented solution

CHALLENGE 2B 0.50 / 0.5

POOR	No solution or a token answer; limited evidence of understanding

(0)

OKAY (0.25)	Clear evidence of understanding and grasp of concepts; but also some definite error or excessively messy or unclear presentation
EXCELLENT (0.50)	Correct and well presented solution

CHALLENGE 2C — 0.50 / 0.5

POOR (0)	No solution or a token answer; limited evidence of understanding
OKAY (0.25)	Clear evidence of understanding and grasp of concepts; but also some definite error or excessively messy or unclear presentation
EXCELLENT (0.50)	Correct and well presented solution

CHALLENGE 2D — 0.25 / 0.5

POOR (0)	No solution or a token answer; limited evidence of understanding
OKAY (0.25)	Clear evidence of understanding and grasp of concepts; but also some definite error or excessively messy or unclear presentation
EXCELLENT (0.50)	Correct and well presented solution

CHALLENGE 3A — 0.50 / 0.5

POOR (0)	No solution or a token answer; limited evidence of understanding
OKAY (0.25)	Clear evidence of understanding and grasp of concepts; but also some definite error or excessively messy or unclear presentation
EXCELLENT (0.50)	Correct and well presented solution

CHALLENGE 3B — 0.50 / 0.5

POOR (0)	No solution or a token answer; limited evidence of understanding
OKAY (0.25)	Clear evidence of understanding and grasp of concepts; but also some definite error or excessively messy or unclear presentation
EXCELLENT	Correct and well presented solution

(0.50)

CHALLENGE 3C 0.50 / 0.5

POOR (0)	No solution or a token answer; limited evidence of understanding
OKAY (0.25)	Clear evidence of understanding and grasp of concepts; but also some definite error or excessively messy or unclear presentation
EXCELLENT (0.50)	Correct and well presented solution

CHALLENGE 3D 0.50 / 0.5

POOR (0)	No solution or a token answer; limited evidence of understanding
OKAY (0.25)	Clear evidence of understanding and grasp of concepts; but also some definite error or excessively messy or unclear presentation
EXCELLENT (0.50)	Correct and well presented solution