COMP30026_Assignment_02_904904

by Xulin Yang

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COMP30026 Models of Computation
Assignment 2

XuLin Yang, 904904

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Challenge 1

Give context-free grammars for these languages:

a. The set A of odd-length strings in $\{\mathtt{a}, \mathtt{b}\}^*$ whose first, middle and last symbols are all the same. For example, b and ababa are in A, but ϵ, aaaa, and abbbb are not.

context-free grammar F is a 4-tuple (V, Σ, R, S) where
V is $\{T,\ X,\ Y\}$
Σ is $\{\mathtt{a},\ \mathtt{b}\}$
R is

$$\begin{aligned} T &\rightarrow \mathtt{a}\ X\ \mathtt{a} \mid \mathtt{b}\ Y\ \mathtt{b} \mid \mathtt{a} \mid \mathtt{b} \\ X &\rightarrow \mathtt{a}\ X\ \mathtt{b} \mid \mathtt{b}\ X\ \mathtt{a} \mid \mathtt{b}\ X\ \mathtt{b} \mid \mathtt{a}\ X\ \mathtt{a} \mid \mathtt{a} \\ Y &\rightarrow \mathtt{a}\ Y\ \mathtt{b} \mid \mathtt{b}\ Y\ \mathtt{a} \mid \mathtt{b}\ Y\ \mathtt{b} \mid \mathtt{a}\ Y\ \mathtt{a} \mid \mathtt{b} \end{aligned}$$

Excellent!

S is T

b. The set $B = \{\mathtt{a}^i\mathtt{b}\mathtt{a}^j \mid i \neq j\}$. For example, ab and abaaa are in B, but ϵ, a, b, and aabaa are not.

context-free grammar G is a 4-tuple (V, Σ, R, S) where
V is $\{T,\ X,\ Y,\ Z\}$
Σ is $\{\mathtt{a},\ \mathtt{b}\}$
R is

$$\begin{aligned} T &\rightarrow \mathtt{a}\ T\ \mathtt{a} \mid \mathtt{b}\ X \mid X\ \mathtt{b} \\ X &\rightarrow \mathtt{a}\ X \mid \mathtt{a} \end{aligned}$$

Excellent!

S is T

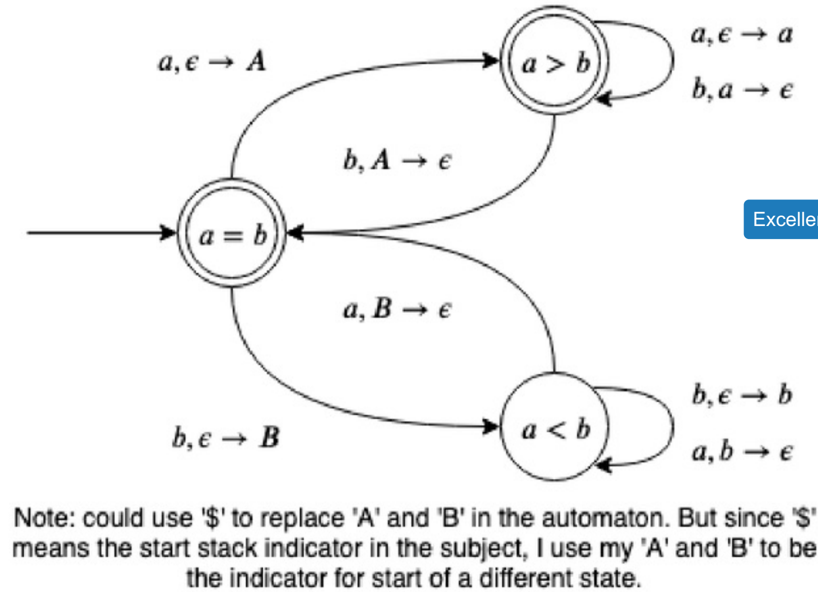
Challenge 2

Consider the language

$$C = \left\{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w \text{ contains at least as many as as bs}\right\}$$

For example, ϵ, aaa, aba, and bbaababaa are all in C, but bbb and bbaaabb are not.

a. Construct a 3-state push-down automaton to recognise C. Provide the solution as a transition diagram. Partial marks are given for a C recogniser with more than 3 states.



Excellent!

b. Prove formally that the following context-free grammar G generates C:

$$
\begin{aligned}
S \;\rightarrow\;& \epsilon \\
\mid\;& \mathtt{a} \\
\mid\;& \mathtt{a}\, S\, \mathtt{b} \\
\mid\;& \mathtt{b}\, S\, \mathtt{a} \\
\mid\;& S\, S
\end{aligned}
$$

Hint: Proceed in two steps; prove that every string in $L(G)$ is in C (by structural induction) and prove that every string in C is in $L(G)$ (by induction on the length of the string).

Given:

context-free language C:

$$C = \{w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w \text{ contains at least as many } \mathtt{a}\text{s as } \mathtt{b}\text{s}\}$$

and

context-free grammar G:

$$
\begin{aligned}
S \;\rightarrow\;& \epsilon \\
\mid\;& \mathtt{a} \\
\mid\;& \mathtt{a}\, S\, \mathtt{b} \\
\mid\;& \mathtt{b}\, S\, \mathtt{a} \\
\mid\;& S\, S
\end{aligned}
$$

To be proved: G generates C

Proof:

(a) **Step 1**: prove that every string in $L(G)$ is in C (by structural induction)
Let string $x \in L(G)$, string $S' \in L(G)$, string $S'' \in L(G)$.
$A(string)$ = number of $\texttt{a}s$ in the $string$
$B(string)$ = number of $\texttt{b}s$ in the $string$

i. **base case:**
   A. 1^{st} **base case:** $x = \epsilon$
      As $\epsilon \in C$ by definition, so $x \in C$.
      $\therefore 1^{st}$ base case is in C.
   B. 2^{nd} **base case:** $x = \texttt{a}$
      As $\texttt{a} \in C$ by definition, so $x \in C$.
      $\therefore 2^{nd}$ base case is in C.

ii. **inductive case:**
   A. 1^{st} **inductive case:** $x = \texttt{a}S'\texttt{b}$ where $S' \in L(G)$
      As $S' \in C$, S' has at least as many $\texttt{a}$s as $\texttt{b}$s.
      So add one $\texttt{a}$ at the start of S' and one $\texttt{b}$ at the end of S' still makes x has at least as many $\texttt{a}$s as $\texttt{b}$s.
      In other words, as $A(x) = 1 + A(S')$, $B(x) = 1 + B(S')$ and $A(S') \geq B(S')$, so $A(x) \geq B(x)$
      $\therefore 1^{st}$ inductive case is in C.
   B. 2^{nd} **inductive case:** $x = \texttt{b}S'\texttt{a}$ where $S' \in L(G)$
      Symmetrically, as $S' \in C$, S' has at least as many $\texttt{a}$s as $\texttt{b}$s.
      So add one $\texttt{b}$ at the start of S' and one $\texttt{a}$ at the end of S' still makes x has at least as many $\texttt{a}$s as $\texttt{b}$s.
      In other words, as $A(x) = 1 + A(S')$, $B(x) = 1 + B(S')$ and $A(S') \geq B(S')$, so $A(x) \geq B(x)$
      $\therefore 2^{nd}$ inductive case is in C.
   C. 3^{rd} **inductive case:** $x = S'S''$ where $S' \in L(G)$, $S'' \in L(G)$
      As $A(x) = A(S') + A(S'')$, $B(x) = B(S') + B(S'')$, $A(S') \geq B(S')$, $A(S'') \geq B(S'')$, so $A(x) \geq B(x)$
      $\therefore 3^{rd}$ inductive case is in C.

Hence, all of context free grammer rules in G generate string in C.
Therefore, every string in $L(G)$ is in C.

(b) **Step 2**: prove that every string in C is in $L(G)$ (by induction on the length of the string)
Let string $w \in C$, $|w| = \text{length}(w)$.
Assumption (Inductive hypothesis): If $|w| = n$ where $n \geq 0$, $w \in L(G)$. We can construct string of length $= |w| + 1$ or length $= |w| + 2$ that also holds by using the rules defined in $L(G)$.

i. **base case:**
   A. 1^{st} **base case:** $n = 0$, $w = \epsilon$.
      As $\epsilon \in L(G)$ by G's context free rule, so $w \in L(G)$.
      $\therefore 1^{st}$ base case is in $L(G)$.
   B. 2^{nd} **base case:** $n = 1$, $w = a$.
      As $a \in L(G)$ by G's context free rule, so $w \in L(G)$.
      $\therefore 2^{nd}$ base case is in $L(G)$.

i. **inductive case:**

A. 1^{st} **inductive case:** $|w'| = n+1$, $w' = aw$.
As $a \in L(G)$ by G's context free rule and $w \in L(G)$ by assumption, so $w' \in L(G)$ by G's context free rule: $S = S'S'', S \in L(G)$ where $S' \in L(G)$ and $S'' \in L(G)$.
$\therefore$ 1^{st} inductive case is in $L(G)$.

B. 2^{nd} **inductive case:** $|w'| = n+1$, $w' = wa$.
Symmetrically, as $a \in L(G)$ by definition and $w \in L(G)$ by assumption, so $w' \in L(G)$ by G's context free rule: $S = S'S'', S \in L(G)$ where $S' \in L(G)$ and $S'' \in L(G)$.
$\therefore$ 2^{nd} inductive case is in $L(G)$.

C. 3^{rd} **inductive case:** $|w'| = n+2$, $w' = awb$.
As $w \in L(G)$ by assumption, by G's context free rule: $S = aS'b$ where $S' \in L(G)$, therefore $S \in L(G)$.
So $w' \in L(G)$.
$\therefore$ 3^{rd} inductive case is in $L(G)$.

D. 4^{th} **inductive case:** $|w'| = n+2$, $w' = bwa$.
Symmetrically, as $w \in L(G)$ by assumption, by G's context free rule: $S = bS'a$ where $S' \in L(G)$, therefore $S \in L(G)$.
1 So $w' \in L(G)$.
$\therefore$ 4^{th} inductive case is in $L(G)$.

Hence, in $|w'| = n+1$, $|w'| = n+2$ inductive cases and base cases, the assumption if $|w| = n$ where $n \geq 0$, $w \in L(G)$ is true.
Therefore, every string in $L(G)$ is in C.

Conclusion: Since every string in $L(G)$ is in C and every string in C is in $L(G)$, we can say that context-free grammar G generates C.

Challenge 3

Consider the two language-transformer functions *triple* and *snip* defined as follows:

$$\begin{aligned} \mathit{triple}(L) &= \{www \mid w \in L\} \\ \mathit{snip}(L) &= \{xz \mid xyz \in L \text{ and } |x| = |y| = |z|\} \end{aligned}$$

Note that $\mathit{snip}(L)$ discards a string w from L unless w has length $3k$ for some $k \in \mathbb{N}$ (possibly 0), and then the strings whose lengths are multiples of 3 have their middle thirds removed. For example, if $L = \{\mathsf{ab}, \mathsf{bab}, \mathsf{bbb}, \mathsf{babba}, \mathsf{aabbaa}\}$ then $\mathit{snip}(L) = \{\mathsf{bb}, \mathsf{aaaa}\}$.

a. Let R be a regular language. Is $R^3 = R \circ R \circ R$ necessarily regular? Justify your answer.

According to the theorem: The class of regular languages is closed under $\circ$.
As R is regular, so does $R \circ R$, and so does $R \circ R \circ R$.
$\therefore$ $R^3 = R \circ R \circ R$ is regular when R is regular. Excellent!

b. Let R be a regular language. Is $\mathit{triple}(R)$ necessarily regular? Justify your answer.

To be proved: $\mathit{triple}(R)$ is not necessarily regular, if R is regular.
Let $R = \{a^*b\}$, R is regular.
We have $\mathit{triple}(R) = \{a^iba^iba^ib \mid i \geq 0\}$.
Pumping Lemma:
L is regular $\Rightarrow \exists p \in \mathbb{N}_0\ \forall s \in L(\,|s| \geq p \Rightarrow \exists x, y, z(xyz = s \wedge |xy| \leq p \wedge |y| > 0 \wedge\ \forall i \in \mathbb{N}_0 |xy^iz \in L))$

Assume: $triple(R)$ is regular.
Let p be the pumping length, where $p \in \mathbb{N}_0$.
Pick $s = a^p ba^p ba^p b,\ s \in triple(R)$
$|s| = 3p + 3$ and $|s| \geq p$
Let x, y, z be strings
Assume $xyz = s,\ |xy| \leq p,\ |y| > 0$
Pick $i = 2$
$y = a^k,\ k > 0$ becuase $|xy| \leq p$ means y is made up of a

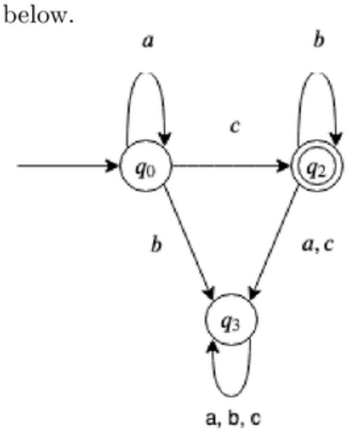
$$\begin{aligned} xy^i z &= xyyz & (1) \\ &= a^{p+k} ba^p ba^p b & (2) \end{aligned}$$

$\therefore |xy^i z| \notin S.$
$\therefore$ $triple(R)$ is not regular. Excellent!

In conclusion: $triple(R)$ is not necessarily regular if R is regular.

c. Let R be a regular language. Show that $snip(R)$ is not necessarily regular.

Let $R = \{a^* cb^* \cup \epsilon\}$, where $\Sigma = \{a, b, c, \epsilon\}$. And R is regular as there is a dfa that recognizes R given below.



Let S be a language, $S = \{a^n b^n | n \geq 0\}$, where $\Sigma = \{a, b, \epsilon\}$.
Let T be a regular language, $T = \{a^* b^*\}$, where $\Sigma = \{a, b, \epsilon\}$.

Assume: $snip(R)$ is regular.
And $snip(R) \cap T = S$. Because if c is not in $xz \in snip(R)$ where $xyz \in R$ means that there is enough number of as and bs near c such that when the middle part y is removed, the x is made up only of as and z is made up only of bs otherwise $xz \notin T$.
As $|x| = |z|$ and x, z are made up of a and b respectively, so $snip(R) \cap T = S$.

By theorem: The class of regular languages is closed under $\cap$. As $snip(R)$ and T are regular, so S is regular.
However, S is not regular.
Pumping Lemma:
L is regular $\Rightarrow \exists p \in \mathbb{N}_0\ \forall s \in L(|s| \geq p \Rightarrow \exists x, y, z(xyz = s \wedge |xy| \leq p \wedge |y| > 0 \wedge\ \forall i \in \mathbb{N}_0 |xy^i z \in L))$
Assume: S is regular.
Let p be the pumping length, where $p \in \mathbb{N}_0$.

Pick $s = a^p b^p,\ s \in S$
$|s| = 2p$ and $|s| \geq p$
Let x, y, z be strings
Assume $xyz = s,\ |xy| \leq p,\ |y| > 0$
Pick $i = 2$
$y = a^k,\ k > 0$ becuase $|xy| \leq p$ means y is made up of a

$$\begin{aligned} xy^i z &= xyyz && (3) \\ &= a^{p+k} b^p && (4) \end{aligned}$$

$\therefore |xy^i z| \notin S$.
$\therefore S$ is not regular.

As S is not regular, T is regular, by the theorem mentioned above, $snip(R)$ is not regular when R is regular.

In conclusion: $snip(R)$ is not necessarily regular if R is regular.

Excellent!

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GRADEMARK REPORT

FINAL GRADE

/6

GENERAL COMMENTS

Instructor

PAGE 1

QM **Excellent!**

QM **Excellent!**

PAGE 2

QM **Excellent!**

PAGE 3

PAGE 4

Comment 1

How about **bwb**?

Text Comment. Excellent!

PAGE 5

Text Comment. Excellent!

PAGE 6

Text Comment. Excellent!

CHALLENGE 1A 1 / 1

UNSATISFACTORY (0)	Missing answer, or an answer that suggests no real knowledge of CFGs.
SOME MERIT (0.33)	A meaningful but clearly wrong CFG.
GOOD (0.67)	A CFG which is either close to correct, or correct but overly messy or complex.
EXCELLENT (1)	Fully correct and well presented solution.

CHALLENGE 1B 1 / 1

UNSATISFACTORY (0)	Missing answer, or an answer that suggests no real knowledge of CFGs.
SOME MERIT (0.33)	A meaningful but clearly wrong CFG.
GOOD (0.67)	A CFG which is either close to correct, or correct but overly messy or complex.
EXCELLENT (1)	Fully correct and well presented solution.

CHALLENGE 2 2 / 2

UNSATISFACTORY (0)	Missing answer, or an answer that suggests no real knowledge of PDAs/structural induction, whichever applies.
SOME MERIT (0.67)	A meaningful but clearly wrong PDA, or alternatively, a proof with considerable flaws.
GOOD (1.33)	A PDA with a minor flaw (such as missing indication of start/accept), or alternatively, a proof that either requires minor correction, or is too messy/complex.
EXCELLENT (2)	Fully correct and well presented solution.

CHALLENGE 3A 0.67 / 0.67

UNSATISFACTORY (0)	Missing answer, or an answer that suggests no real knowledge of regular languages.
SOME MERIT (0.22)	Answers 'no', but provides some plausible, if wrong, justification.

GOOD (0.44)	Answers 'yes' without adequate justification, or using overly complex reasoning.
EXCELLENT (0.67)	Fully correct and well presented solution.

CHALLENGE 3B — 0.67 / 0.67

UNSATISFACTORY (0)	Missing answer, or an answer that suggests no real knowledge of regular languages.
SOME MERIT (0.22)	Answers 'yes', but provides some plausible, if wrong, justification.
GOOD (0.44)	Answers 'no' without adequate justification (typically through a flawed proof by pumping), or using overly complex reasoning.
EXCELLENT (0.67)	Fully correct and well presented solution.

CHALLENGE 3C — 0.67 / 0.67

UNSATISFACTORY (0)	Missing answer, or an answer that suggests no real knowledge of regular languages.
SOME MERIT (0.22)	Argues without producing a counter-example, but in terms that show some understanding of regular languages.
GOOD (0.44)	A faulty proof, but based on a (claimed) counter-example, and showing ability to present a formal argument.
EXCELLENT (0.67)	Fully correct and well presented solution.