COMP30026 Models of Computation
Assignment 2

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Challenge 1

Give context-free grammars for these languages:

a. The set A of odd-length strings in $\{\mathtt{a}, \mathtt{b}\}^*$ whose first, middle and last symbols are all the same. For example, b and ababa are in A, but ϵ, aaaa, and abbbb are not.

b. The set $B = \{\mathtt{a}^i\mathtt{b}\mathtt{a}^j \mid i \neq j\}$. For example, ab and abaaa are in B, but ϵ, a, b, and aabaa are not.

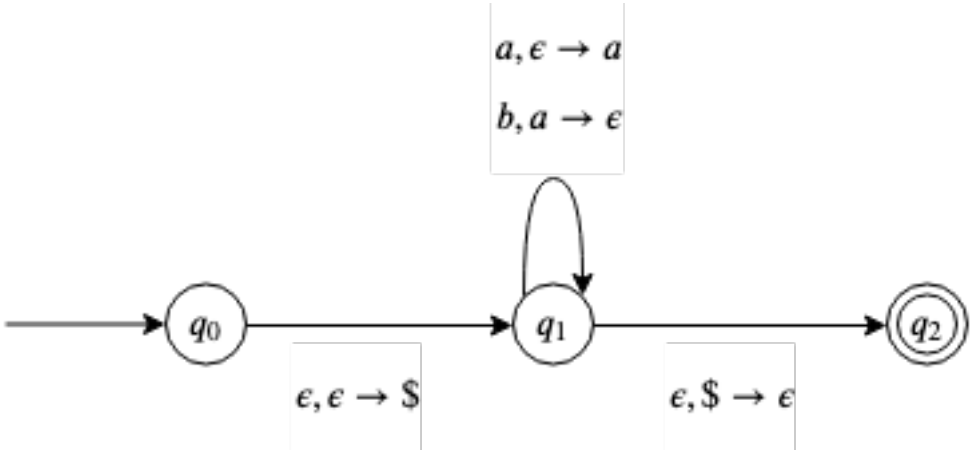
Challenge 2

Consider the language

$$C = \{\ w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w \text{ contains at least as many a}\mathtt{s} \text{ as b}\mathtt{s}\ \}$$

For example, ϵ, aaa, aba, and bbaababaa are all in C, but bbb and bbaaabb are not.

a. Construct a 3-state push-down automaton to recognise C. Provide the solution as a transition diagram. Partial marks are given for a C recogniser with more than 3 states.



b. Prove formally that the following context-free grammar G generates C:

$$\begin{array}{rcl} S & \to & \epsilon \\ & | & \mathtt{a} \\ & | & \mathtt{a}\ S\ \mathtt{b} \\ & | & \mathtt{b}\ S\ \mathtt{a} \\ & | & S\ S \end{array}$$

Hint: Proceed in two steps; prove that every string in $L(G)$ is in C (by structural induction) and prove that every string in C is in $L(G)$ (by induction on the length of the string).

Challenge 3

Consider the two language-transformer functions *triple* and *snip* defined as follows:

$$\begin{array}{rcl} \mathit{triple}(L) & = & \{www \mid w \in L\} \\ \mathit{snip}(L) & = & \{xz \mid xyz \in L \text{ and } |x| = |y| = |z|\} \end{array}$$

Note that $\mathit{snip}(L)$ discards a string w from L unless w has length $3k$ for some $k \in \mathbb{N}$ (possibly 0), and then the strings whose lengths are multiples of 3 have their middle thirds removed. For example, if $L = \{\mathsf{ab}, \mathsf{bab}, \mathsf{bbb}, \mathsf{babba}, \mathsf{aabbaa}\}$ then $\mathit{snip}(L) = \{\mathsf{bb}, \mathsf{aaaa}\}$.

a. Let R be a regular language. Is $R^3 = R \circ R \circ R$ necessarily regular? Justify your answer.

b. Let R be a regular language. Is $\mathit{triple}(R)$ necessarily regular? Justify your answer.

c. Let R be a regular language. Show that $\mathit{snip}(R)$ is not necessarily regular.