Spectral Independence Beyond Total Influence on Trees and Related Graphs

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Xiongxin Yang (Northeast Normal University)
Yitong Yin (Nanjing University)

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Sampling from High Dimensional Distributions

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Distribution of graphical models

$\mathrm{supp}(\mu) \subseteq \{0,1\}^U$

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Hardcore model (weighted independent sets)

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$\mathcal{I}$: collection of independent sets $I \subseteq V$

Monomer-dimer model (weighted matchings)

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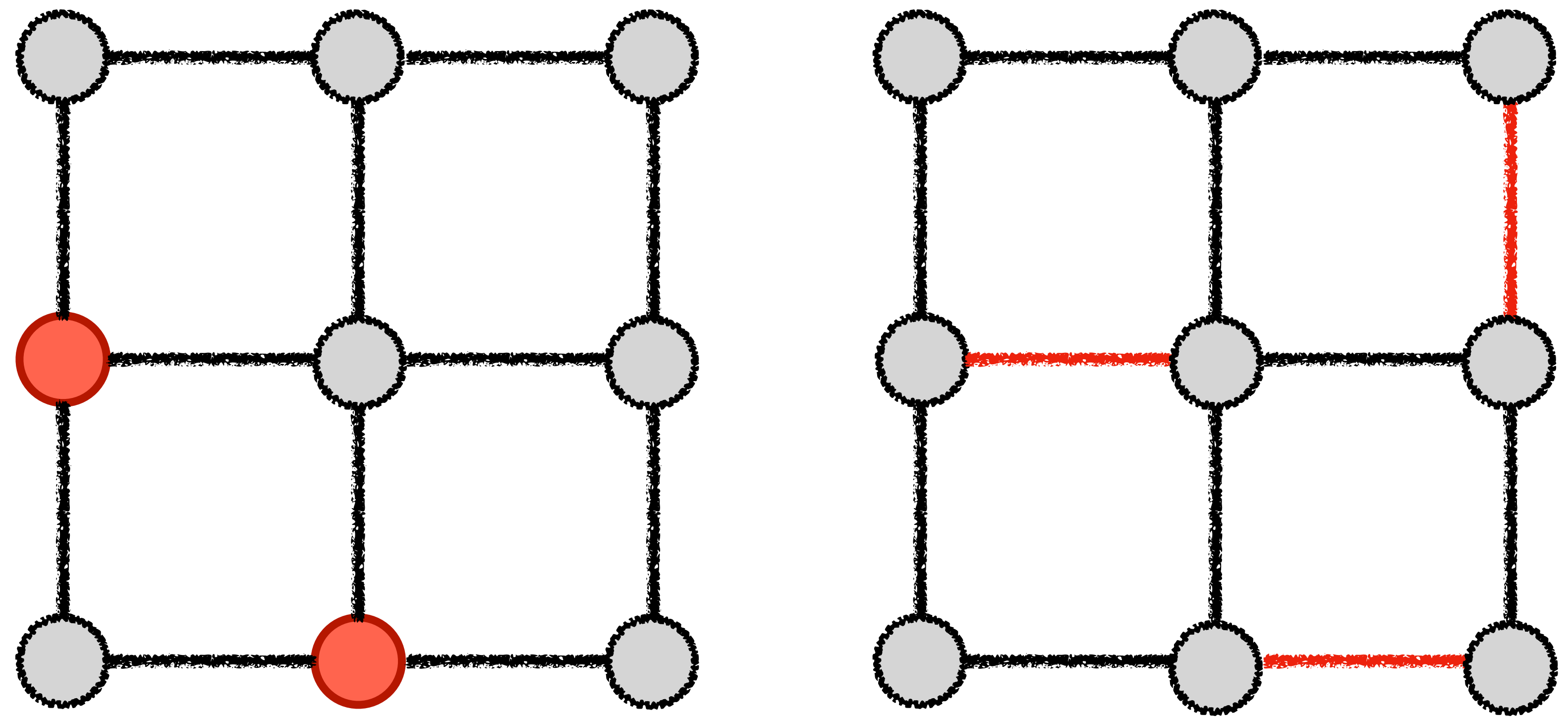

Examples: Independent set and matching

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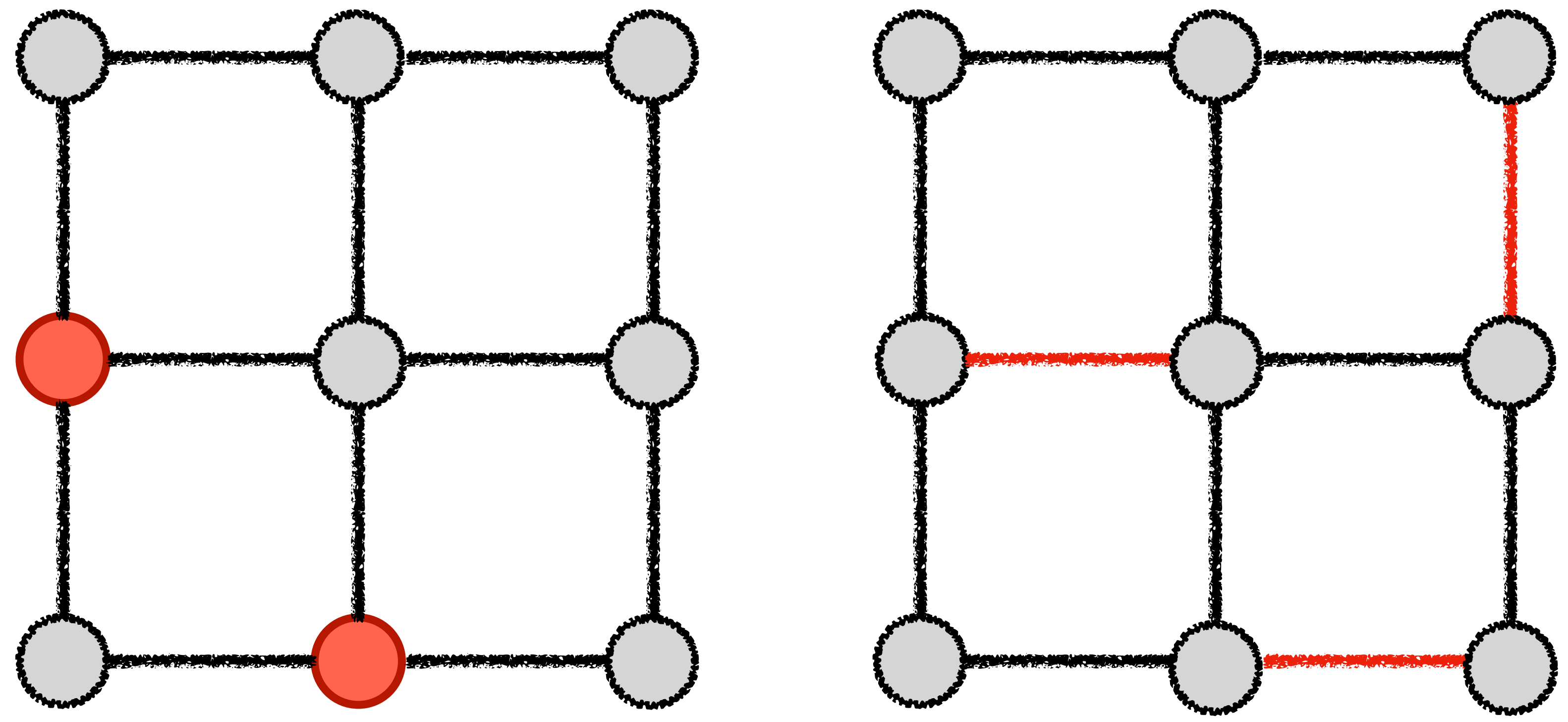


Examples: Independent set and matching

Challenge

Efficiently draw an (approximate) sample from the target distribution

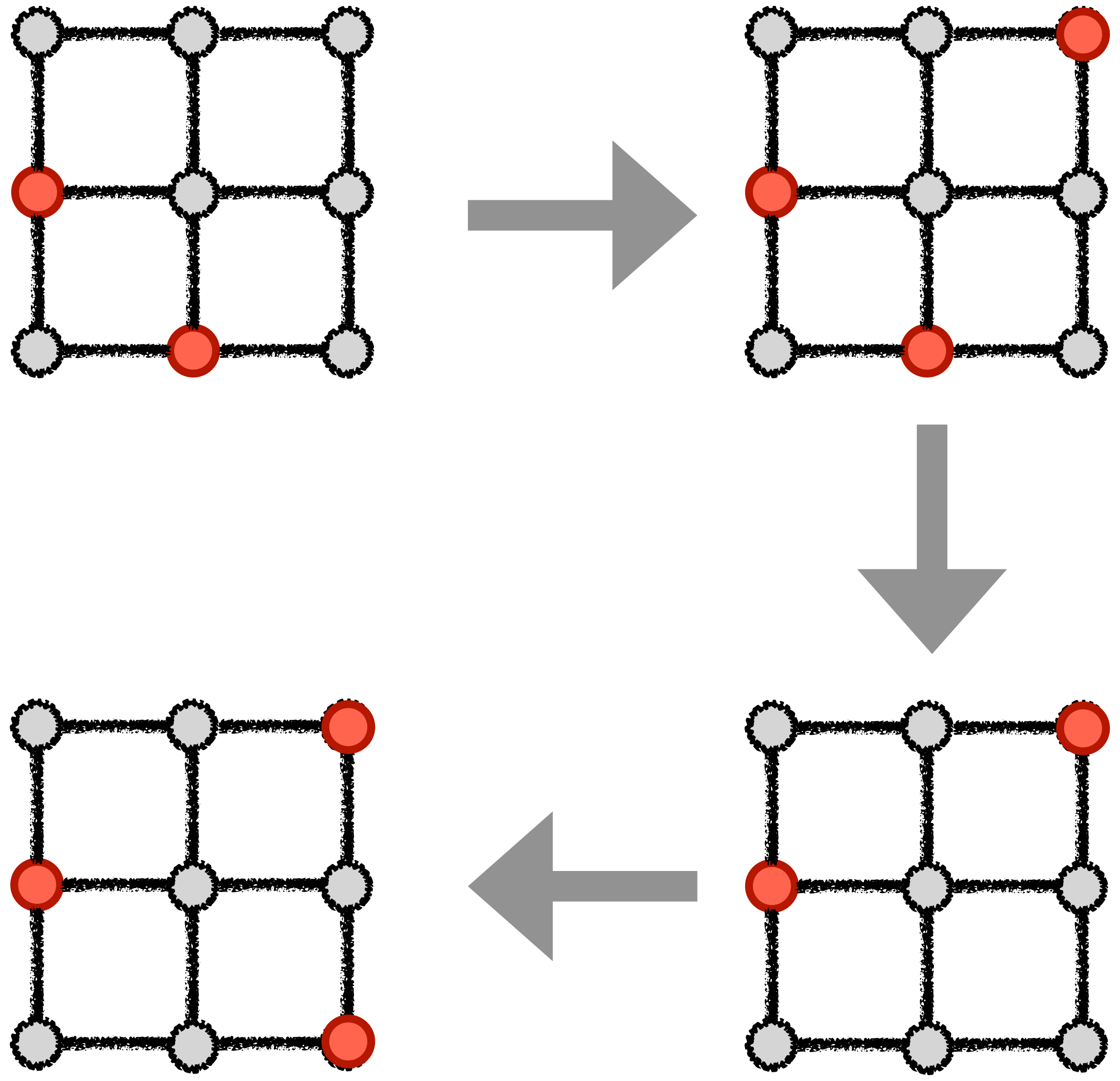
Glauber Dynamics

Glauber dynamics on hardcore model

(single-site dynamics)

In each transition step, update independent set I as follows:

- Pick a vertex $v \in V$ u.a.r.;
- If $I \cup \{v\}$ is an independent set, then
  - update $I \leftarrow I \cup \{v\}$ with probability $\frac{\lambda}{1+\lambda}$;
  - update $I \leftarrow I \setminus \{v\}$ with remaining probability;
- Otherwise, we do not update the independent set.



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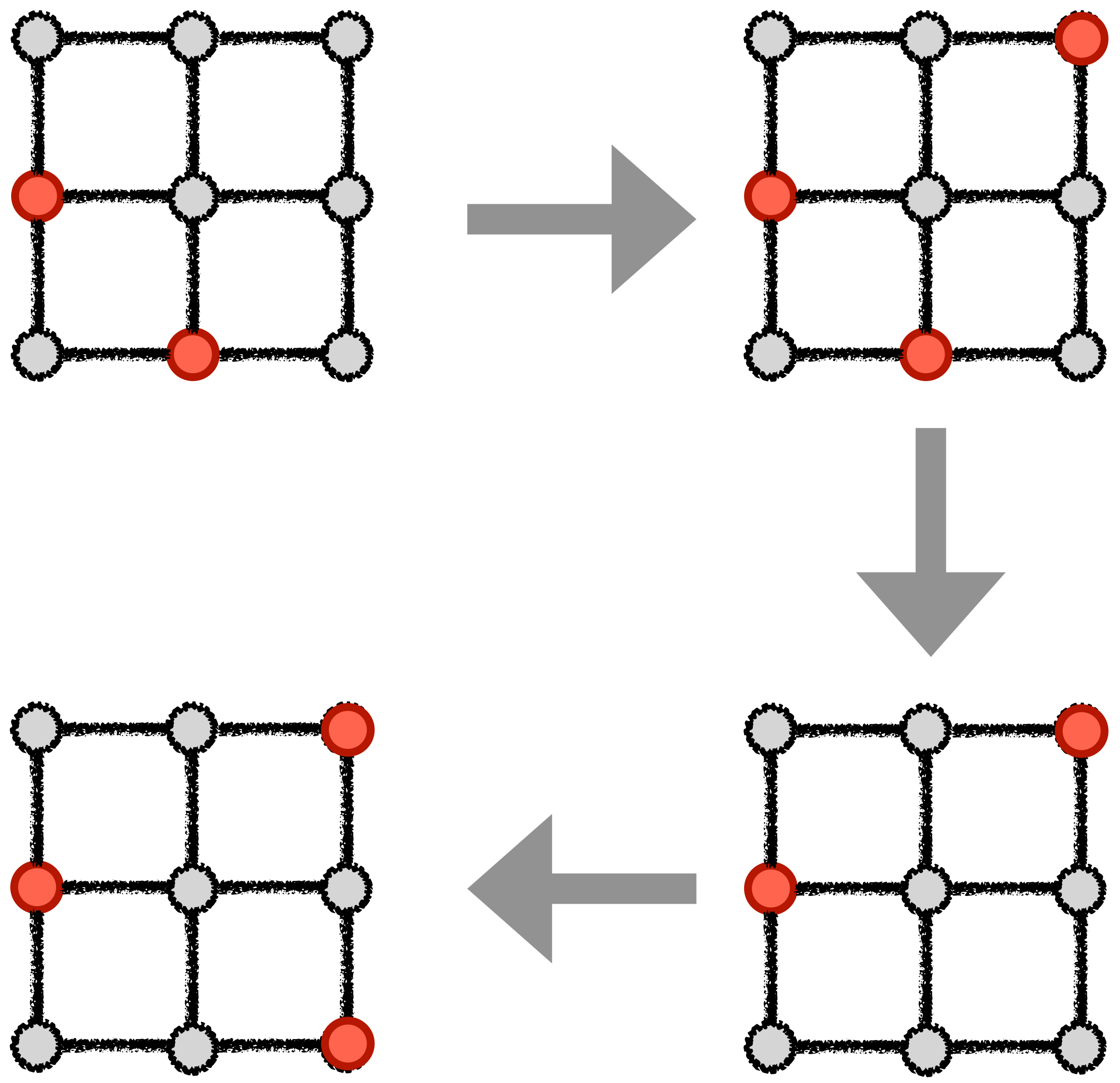

Computational phase transition

$\lambda \le (1-\delta)\lambda_c(\Delta)$: Optimal $O(n \log n)$ mixing time [Chen-Eldan, 22; Chen-Feng-Yin-Z., 22]

$\lambda = \lambda_c(\Delta)$: Polynomial mixing time [Chen-Chen-Yin-Z., 24]

$\lambda \ge (1+\delta)\lambda_c(\Delta)$: No poly-time sampler unless **NP=RP** [Sly, 10]

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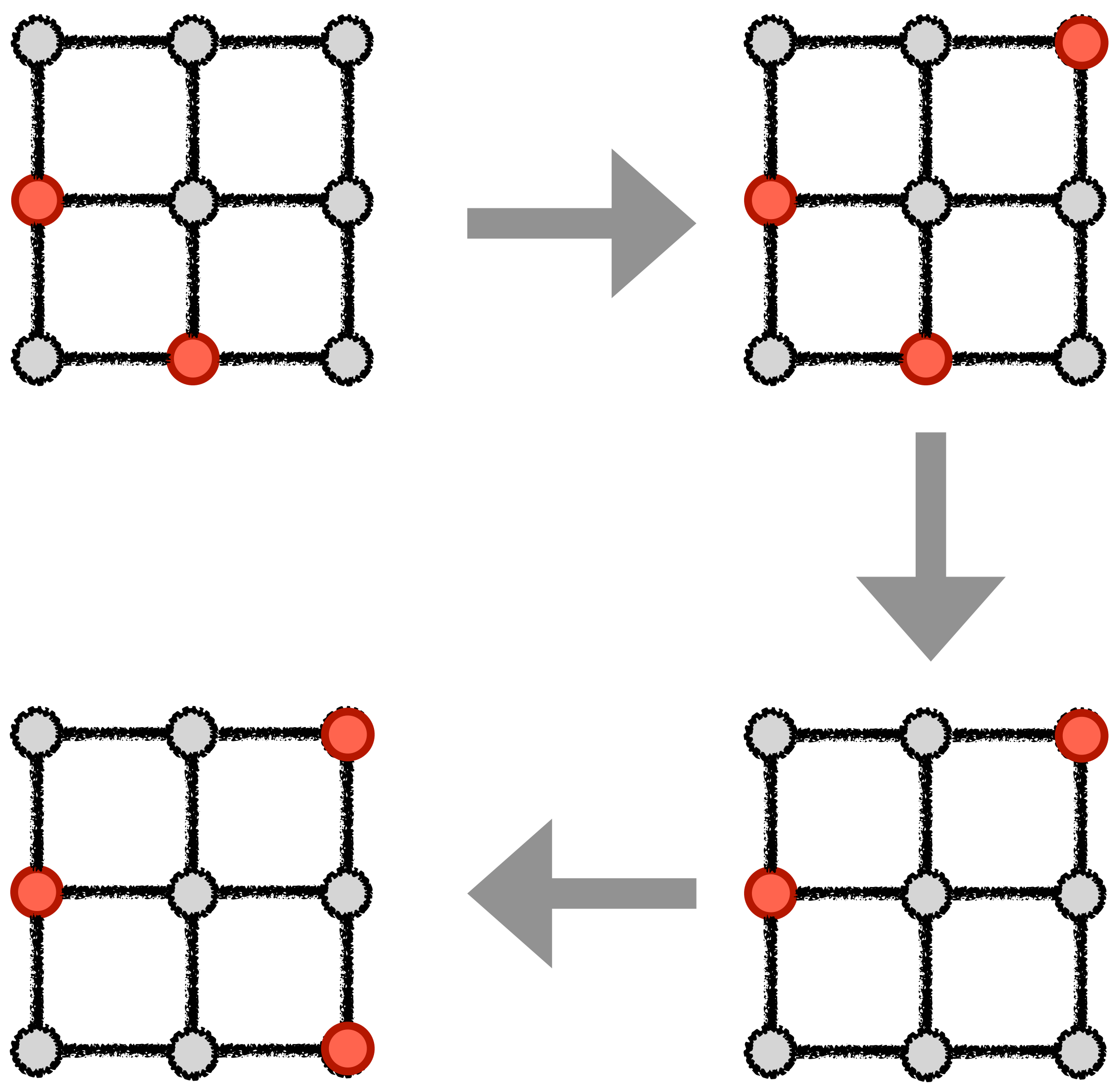

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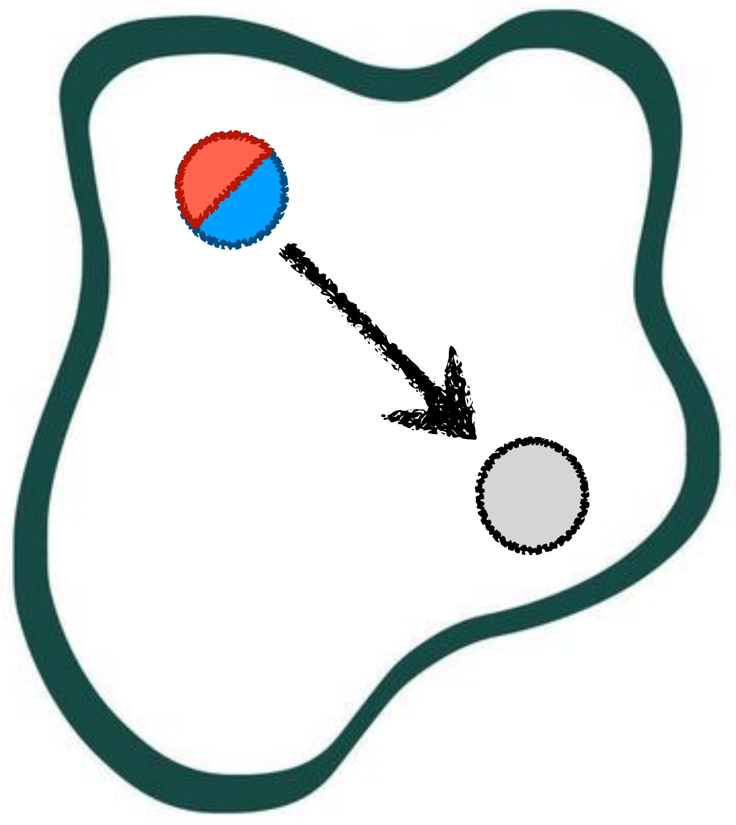
Based on the framework of **spectral independence**!

Spectral Independence and Rapid Mixing

(η-) **Spectral independence (SI)**
[Anari-Liu-Oveis Gharan, 20]

$\lambda_{\max}(\Psi_\mu) \leq \eta$, where $\Psi_\mu(i,j) = \mu(j \mid i) - \mu(j \mid \bar{i})$.



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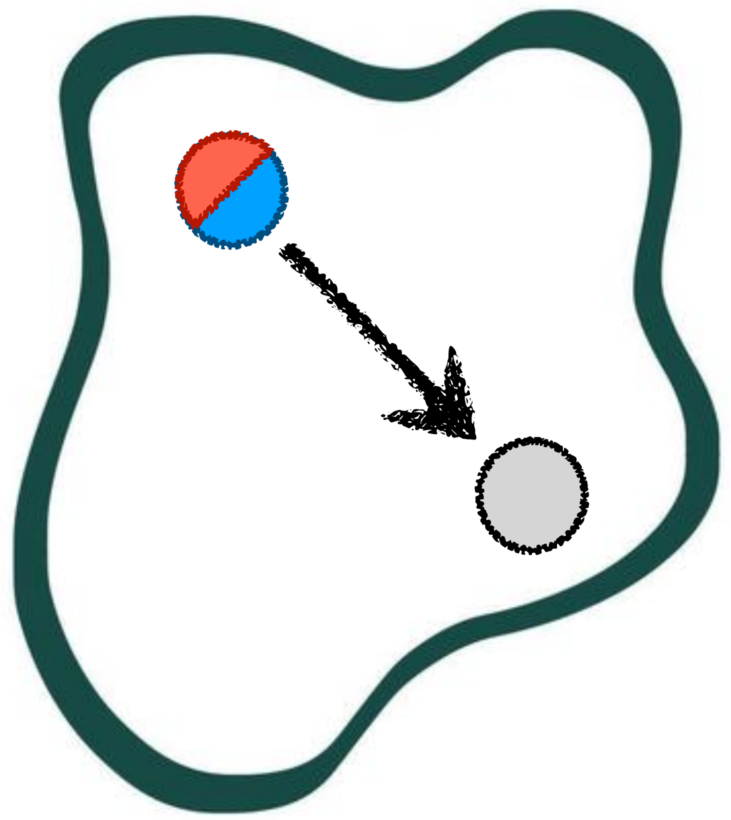

Rapid mixing via spectral independence (informal)

[Chen-Liu-Vigoda, 21]
Assuming constant maximum degree and bounded marginal bounds,
Constant SI implies an optimal $O(n \log n)$ mixing time bound for Glauber dynamics;

[Chen-Feng-Yin-Z., 21]
Assuming rapid mixing of Glauber dynamics in a sub-critical regime,
Constant SI implies an optimal $O(n)$ relaxation time bound for Glauber dynamics.

Previous Techniques on Establishing Spectral Independence

Challenge

Establishing constant spectral independence for the target distributions

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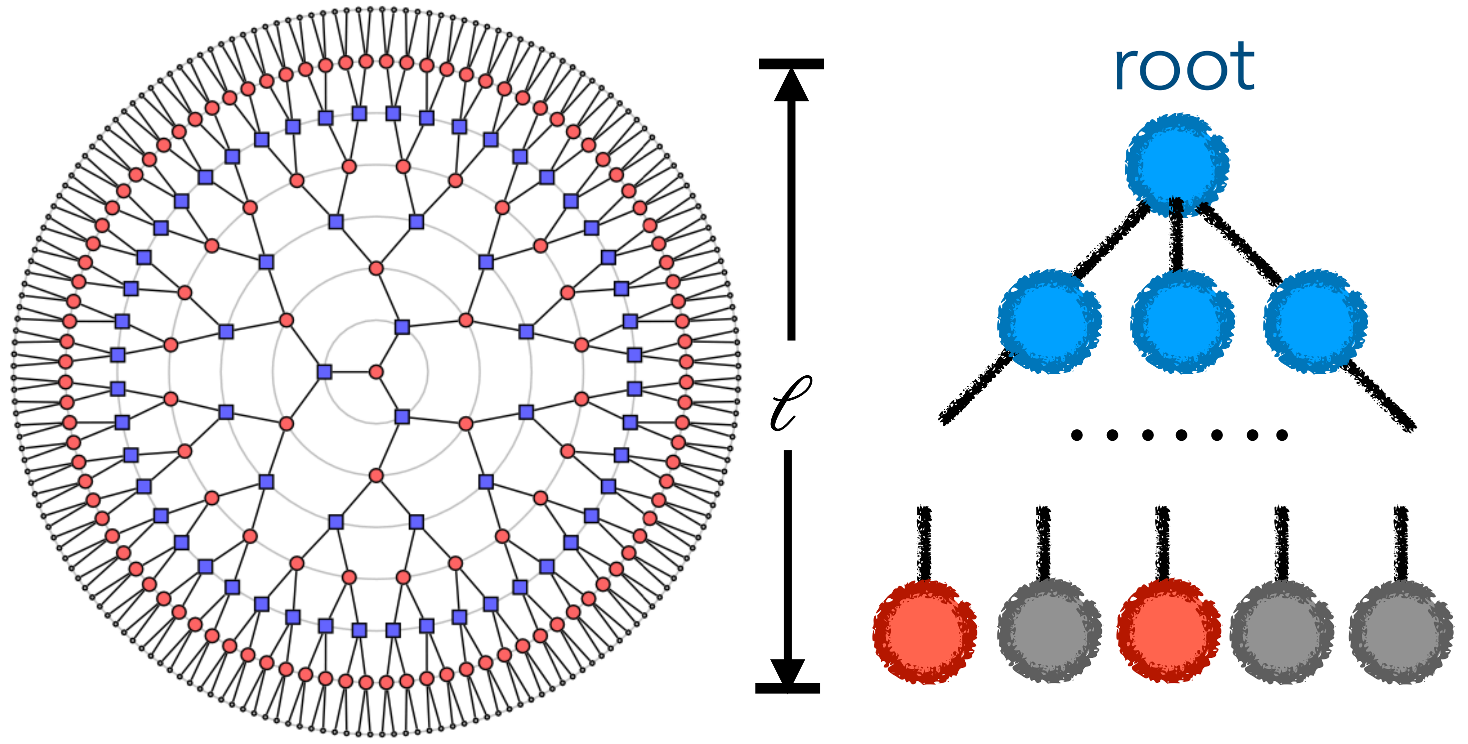
Decay of correlation

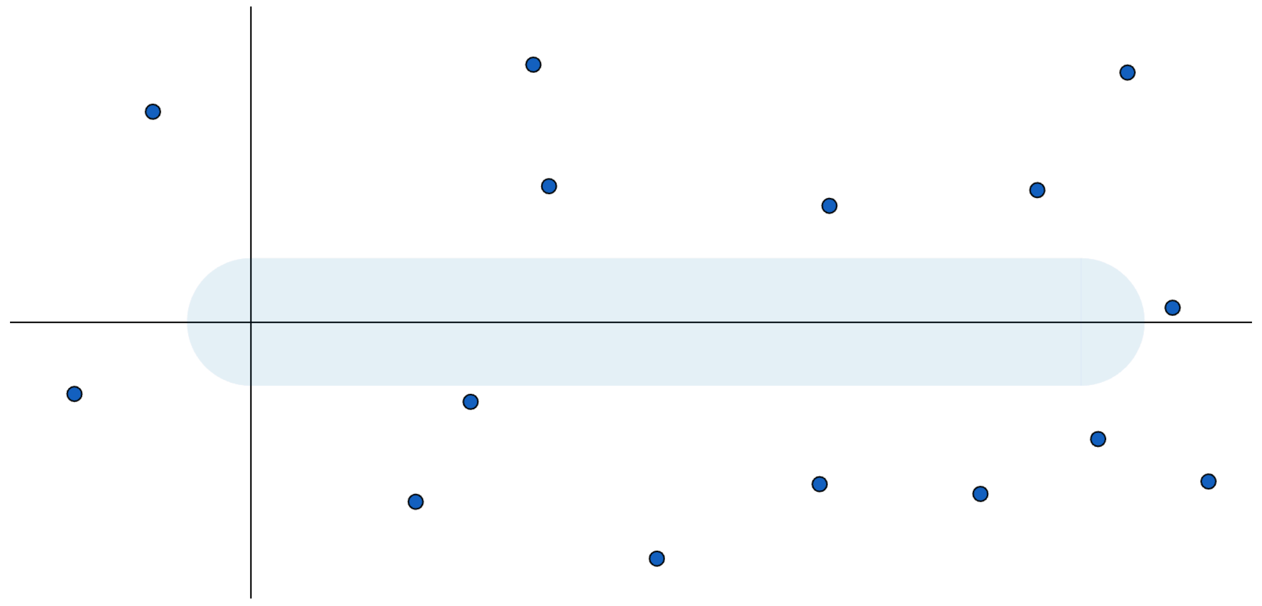


Bethe lattice
(infinite regular tree)

Correlation decay

Applications: two-spin systems, colorings…
[Anari-Liu-Oveis Gharan, 20],
[Chen-Liu-Vigoda,21], …

Stability of Polynomials



Applications: Holant problem, k-DPP, …
[Alimohammadi-Anari-Shiragur-Vuong, 21],
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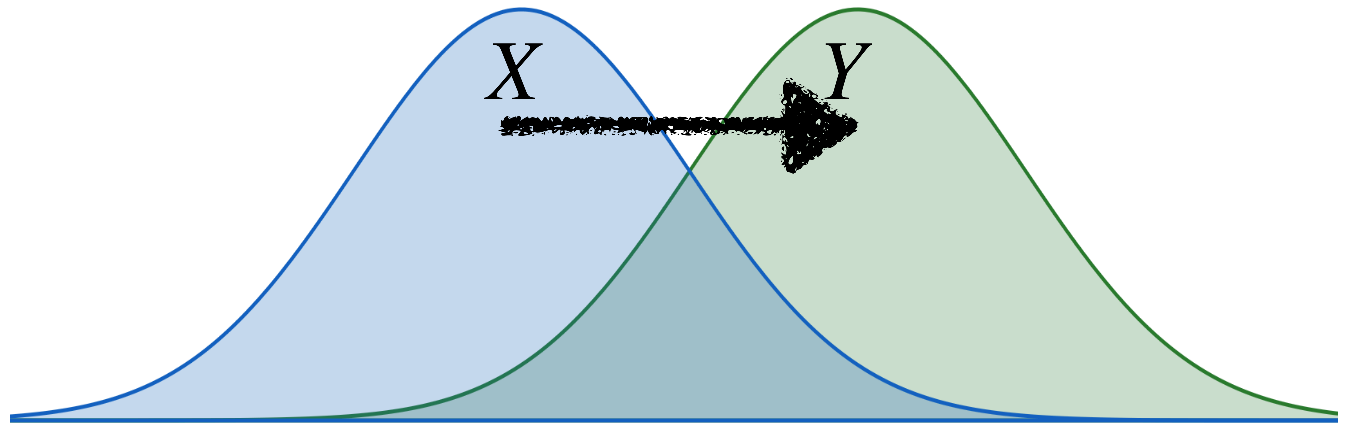
Coupling Independence



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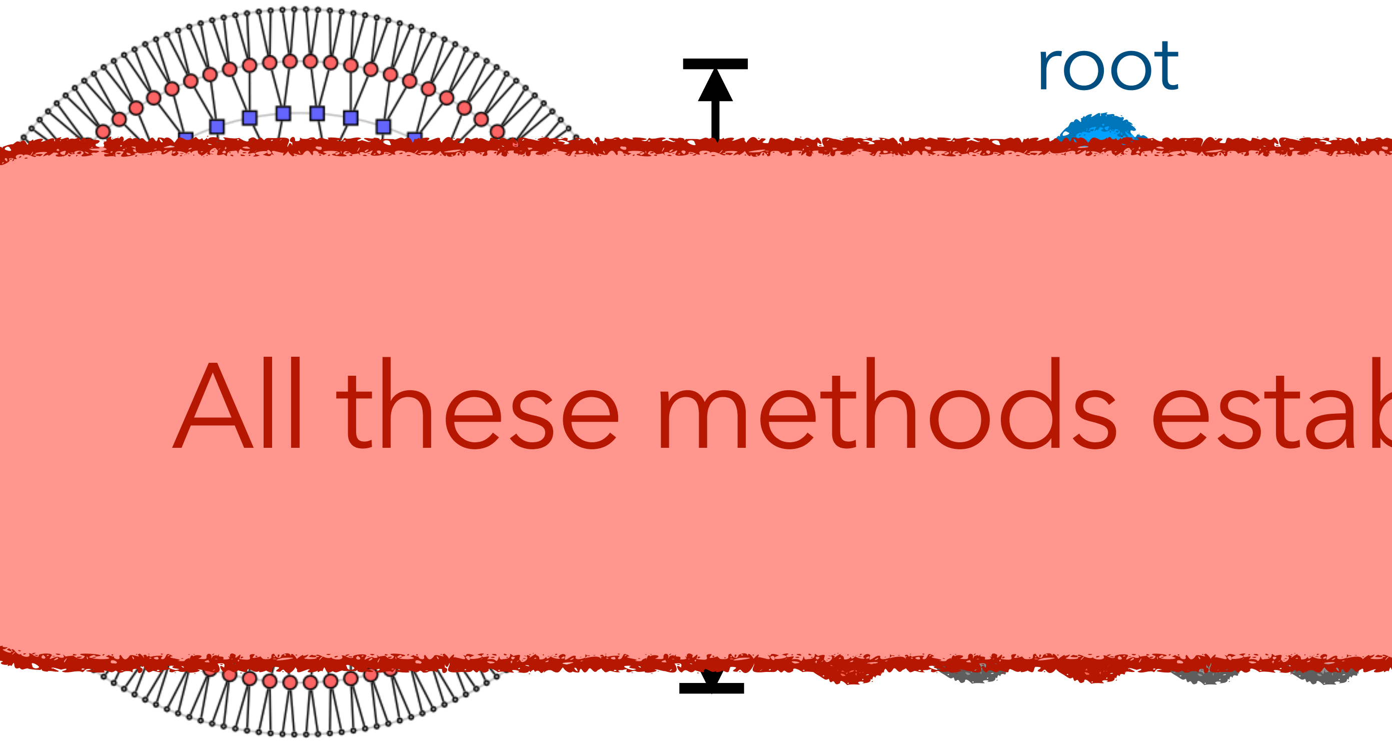

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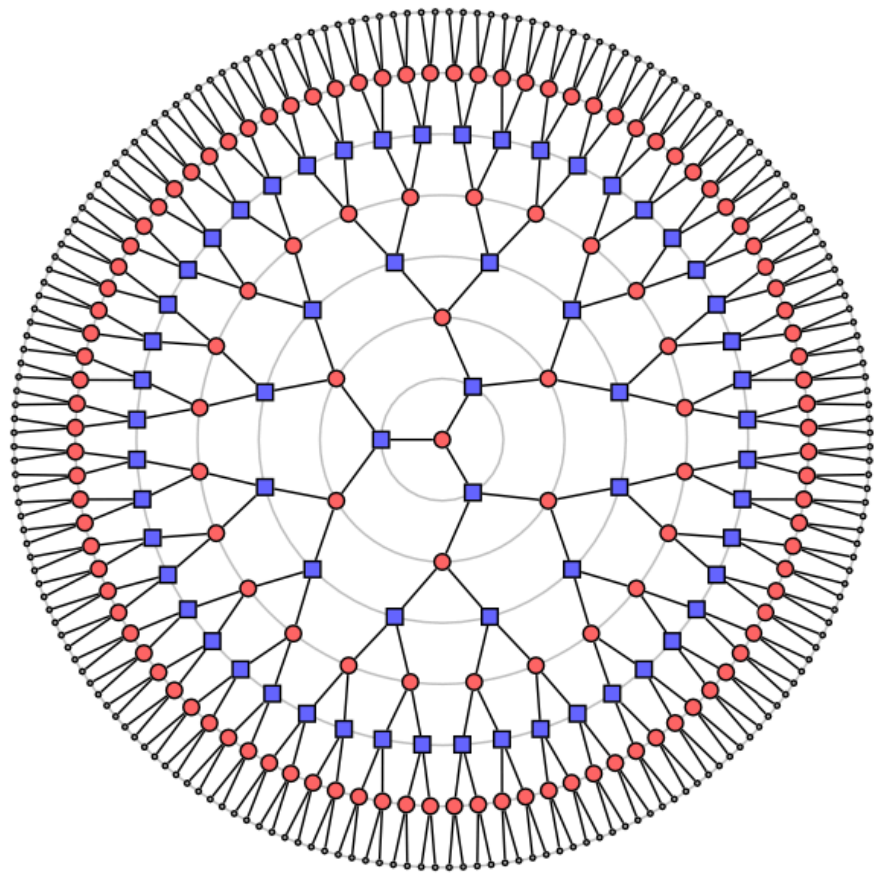
All these methods establish upper bounds for $\|\Psi_\mu\|_1$ or $\|\Psi_\mu\|_\infty$ **(total influence)**

Spectral Independence beyond Total Influence



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Hardcore: Total influence on
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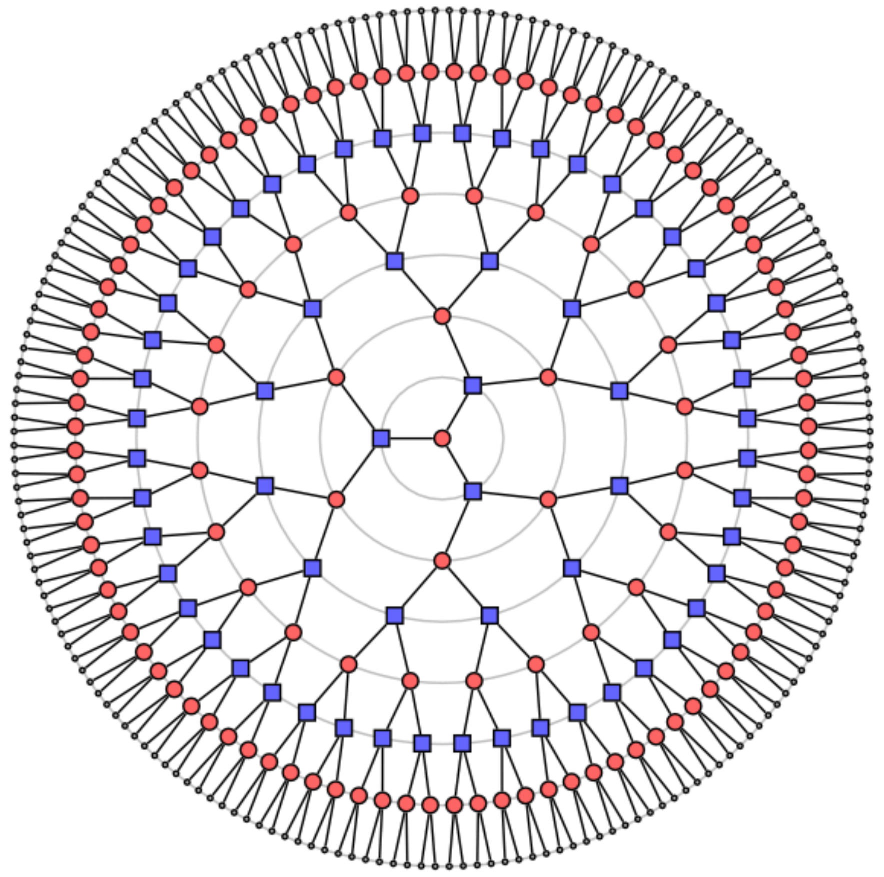
$\|\Psi_\mu\|_1$ and $\|\Psi_\mu\|_\infty$ are unbounded, when $\lambda > \lambda_c(\Delta)$.



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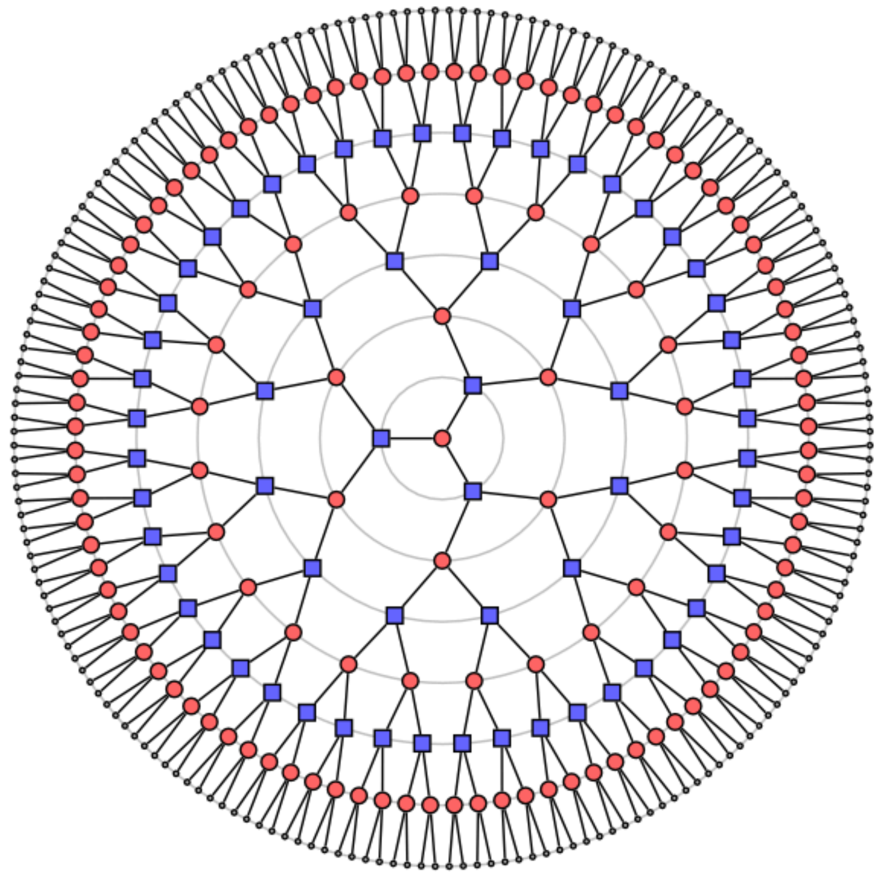
Fact: Glauber dynamics **optimally mixes** on trees even when $\lambda = 1.01\lambda_c(\Delta)$



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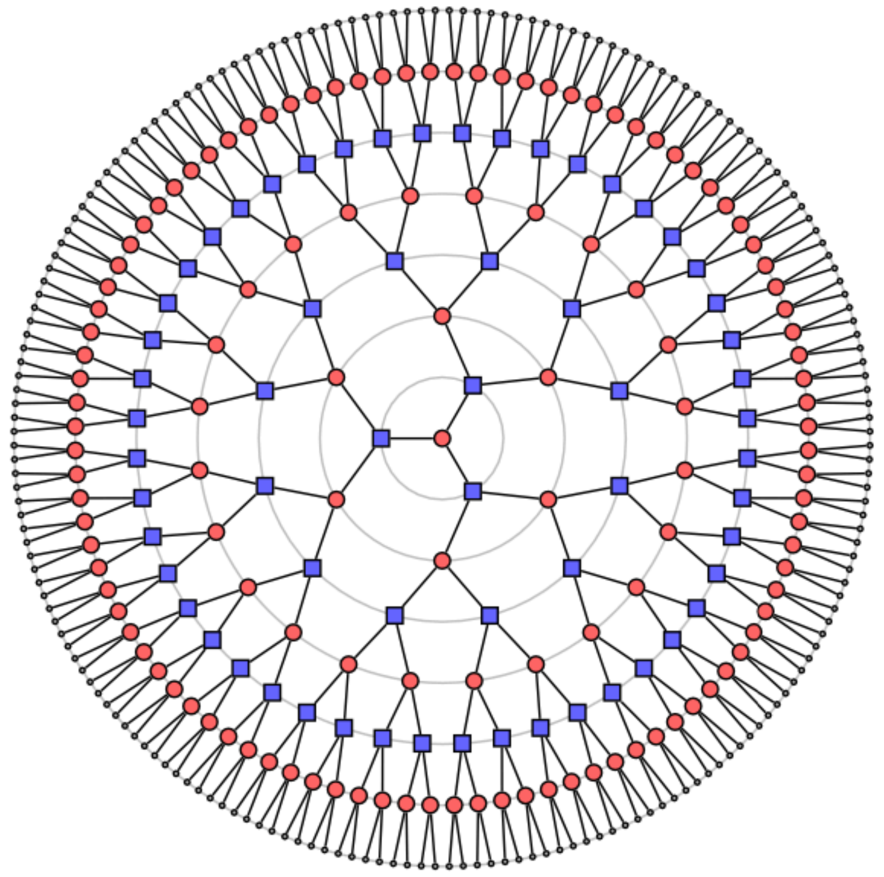

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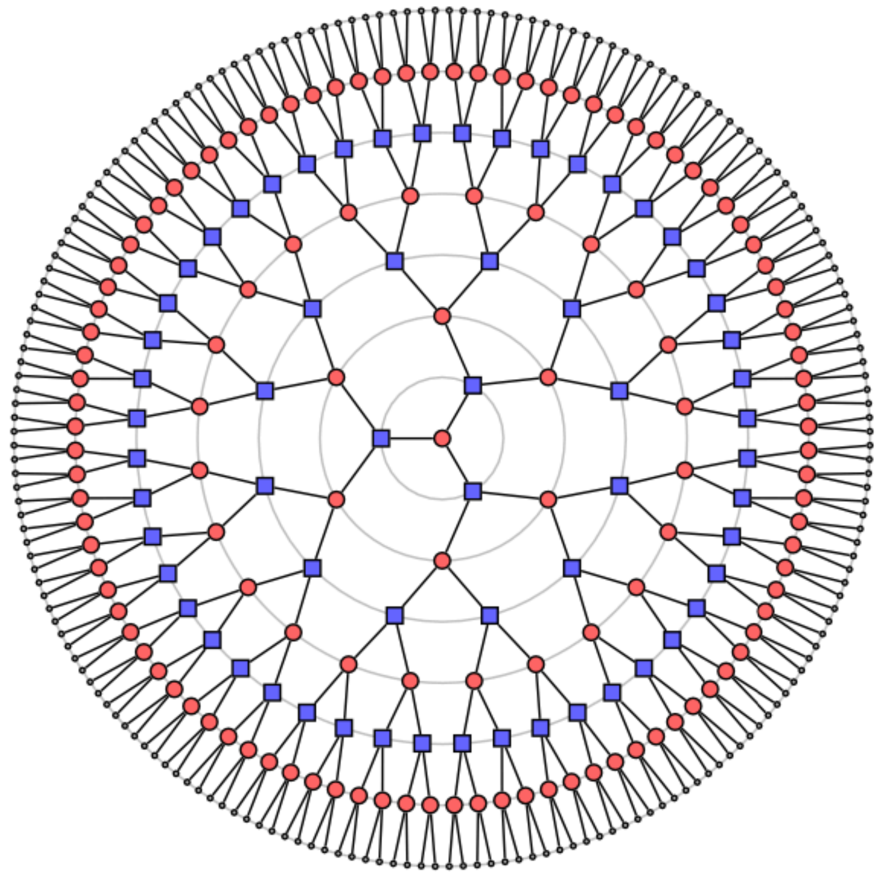

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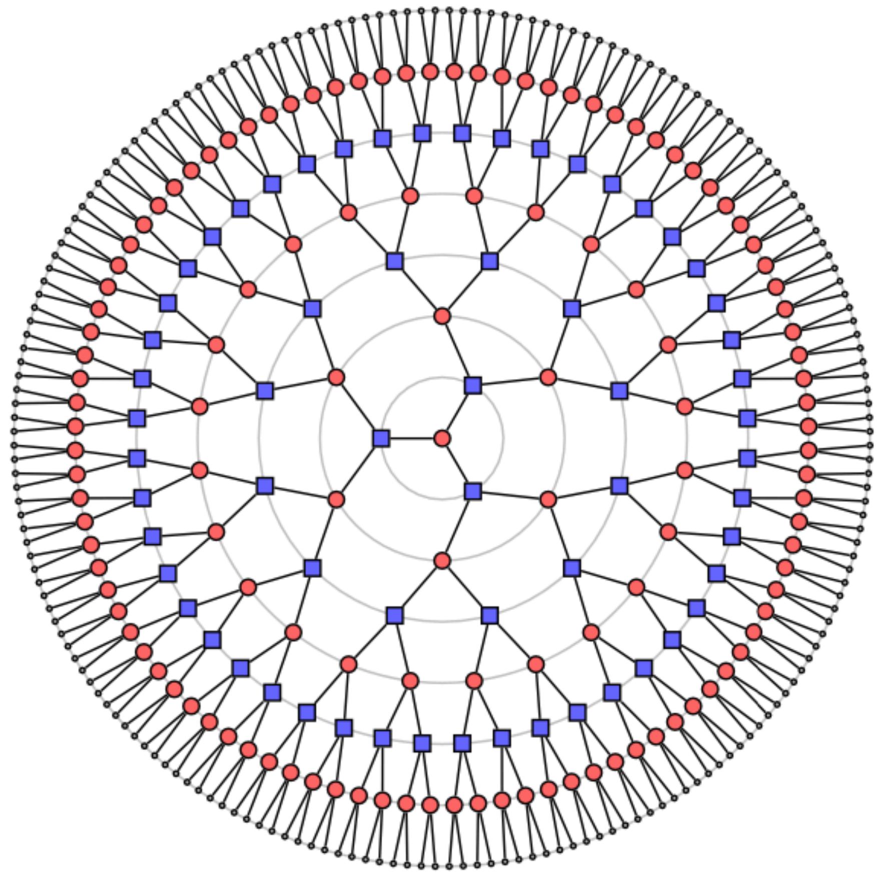

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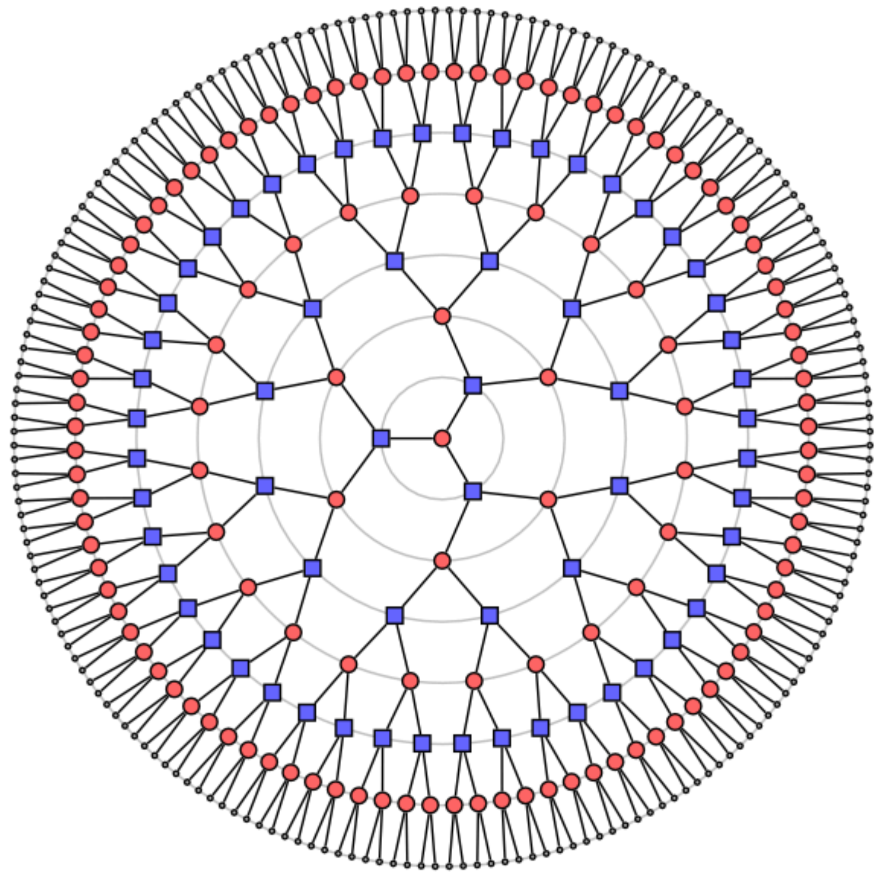

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🤔 An improved bound (independent of Δ) for matchings on general graphs
(open problem in [Chen-Liu-Vigoda, 21])

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Establish constant spectral independence beyond total influence (row/column sums)

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Trickle-down Theorem

Applications: matroid, random cluster model,…
[Anari-Liu-Ovies Gharan-Vinzant, 20],
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Matrix trickle-down

Applications: edge colorings
[Abdolazimi-Liu-Oveis Gharan, 21],
[Wang-Zhang-Zhang, 24]

Trickle-down in localization schemes

Applications: SK model, …
[Anari-Koehler-Vuong, 24]

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Lack of systematic ways to apply this technique

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New systematic technique on establishing SI beyond total influence

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Hardcore model on trees

Constant spectral independence when $\lambda < e^2$, implying an optimal mixing time bound for bounded degree trees.

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Monomer-dimer model on graphs with large girth

Constant spectral independence (independent of Δ), implying an optimal relaxation time bound for trees.

- Partly answers the question in [Chen-Liu-Vigoda, 21];
- Provide a counterexample for multi-graphs.

In this talk, we only consider trees.

Spectral Independence via Matrix Inverse

Inverse of influence matrix

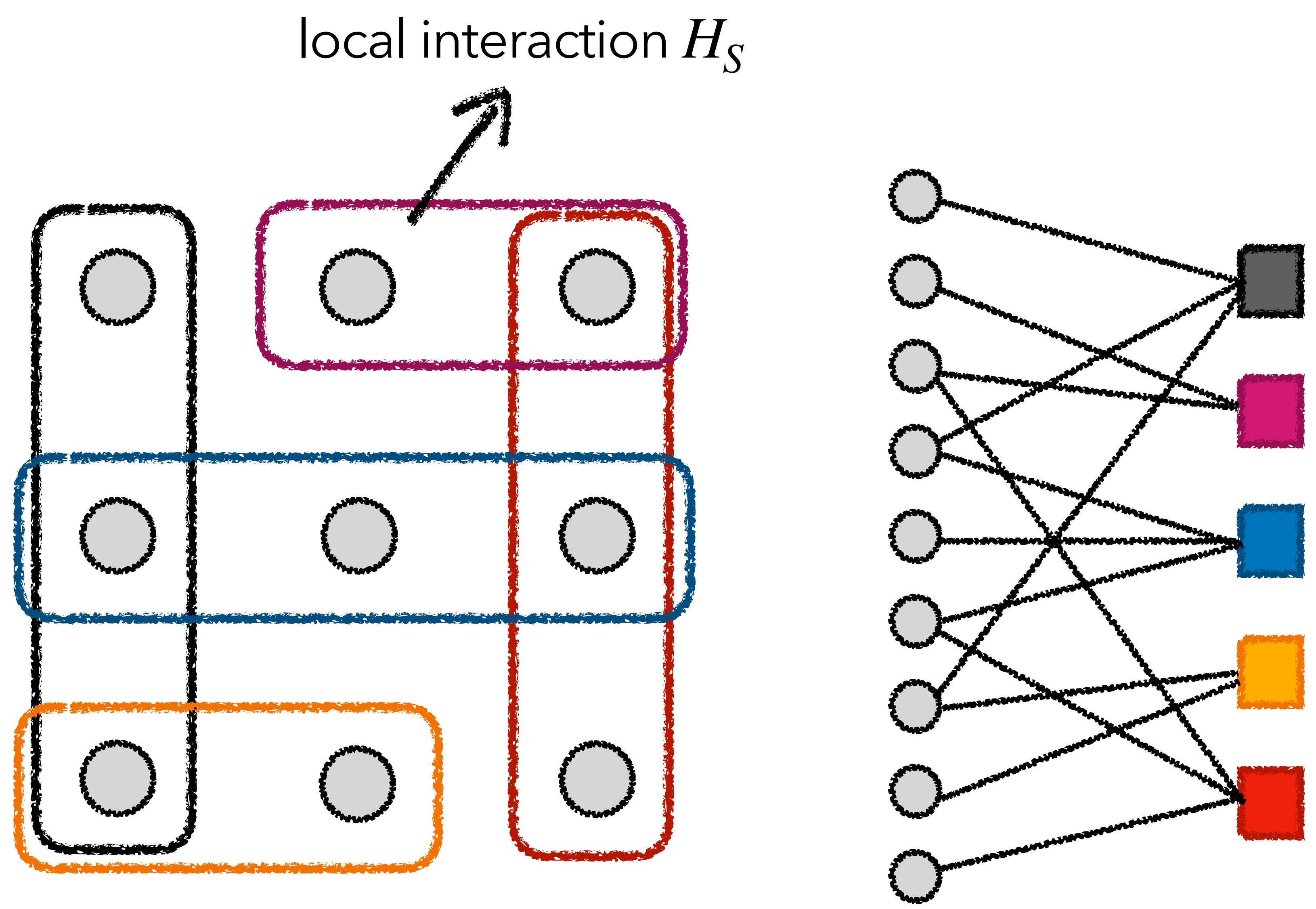
Let μ be a distribution over $\{0,1\}^V$ satisfying

$$\forall x \in \{0,1\}^n, \quad \mu(x) \propto \exp\left(\sum_{S \in \mathcal{S}} H_S(x_S)\right)$$

If the factor graph is a tree, the inverse of Ψ_μ satisfies

$$\Psi_\mu^{-1} = \sum_{S \in \mathcal{S}} \widetilde{\Psi_{\mu_S}^{-1}} - \mathrm{diag}(d_i - 1)_{i \in V}$$

$H_S : \{0,1\}^S \to \mathbb{R} \cup \{+\infty\}$
factor graph: bipartite graph $(V, \mathcal{S}, E)$
with edge $(i, S) \in E$ iff $i \in S$



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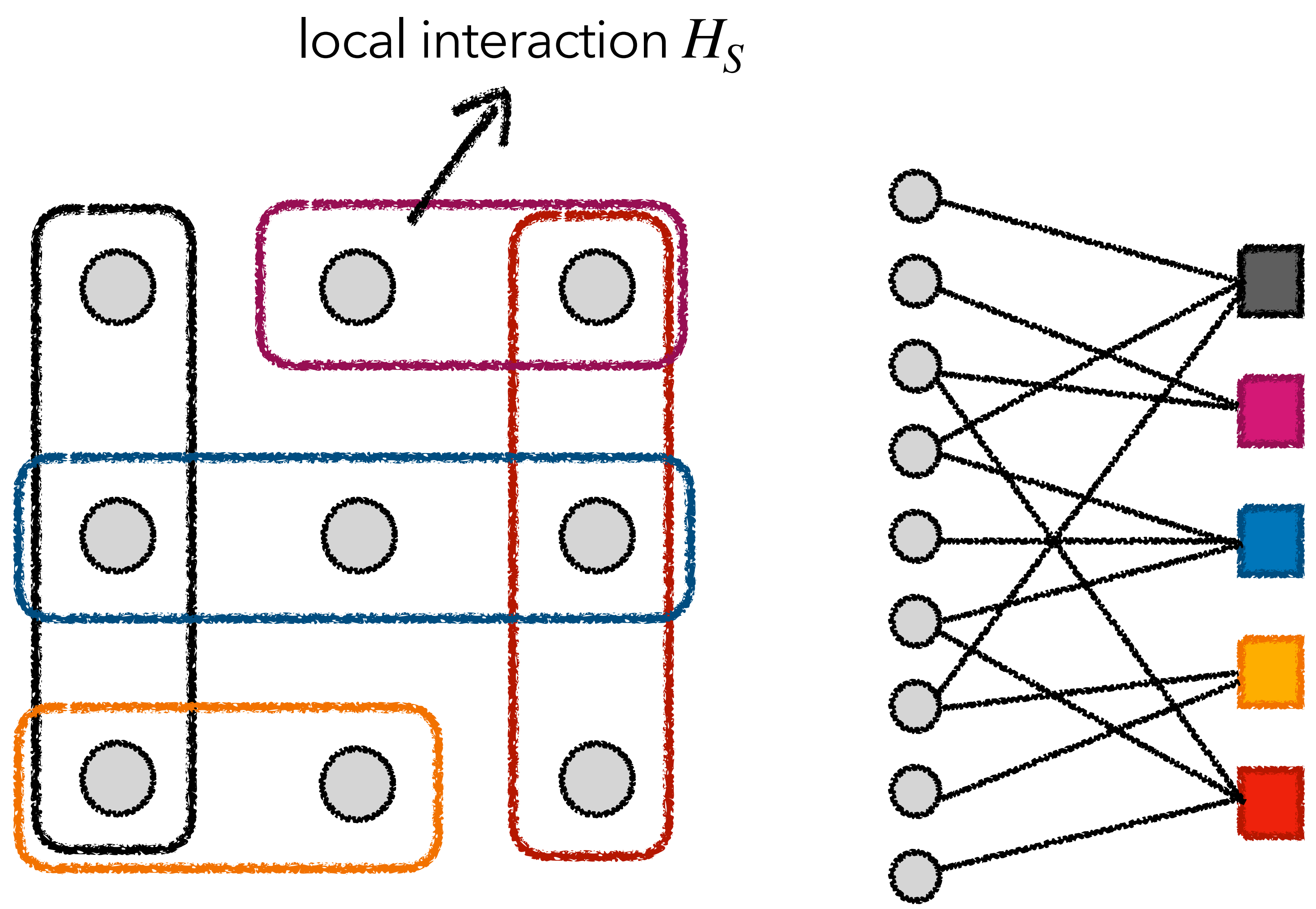

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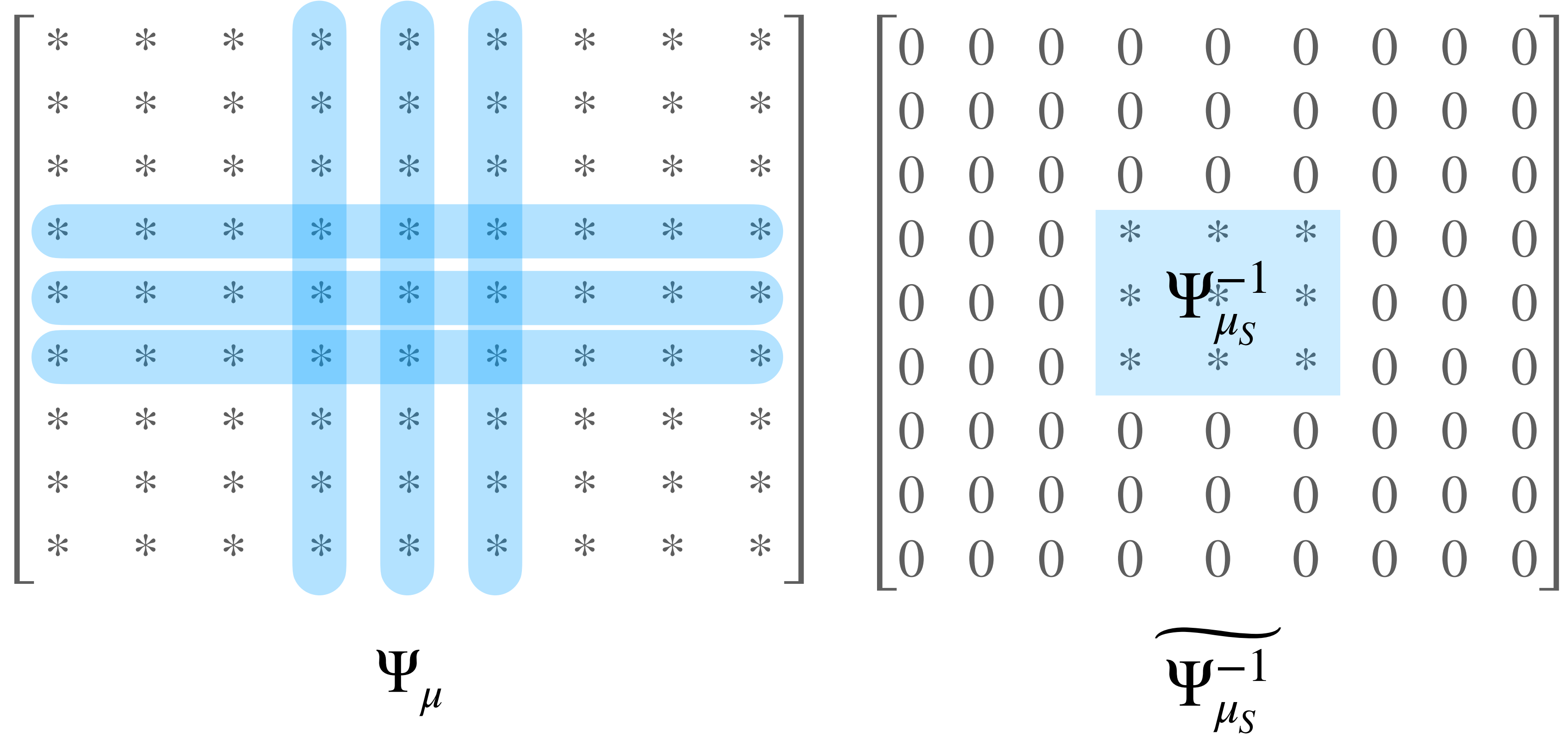

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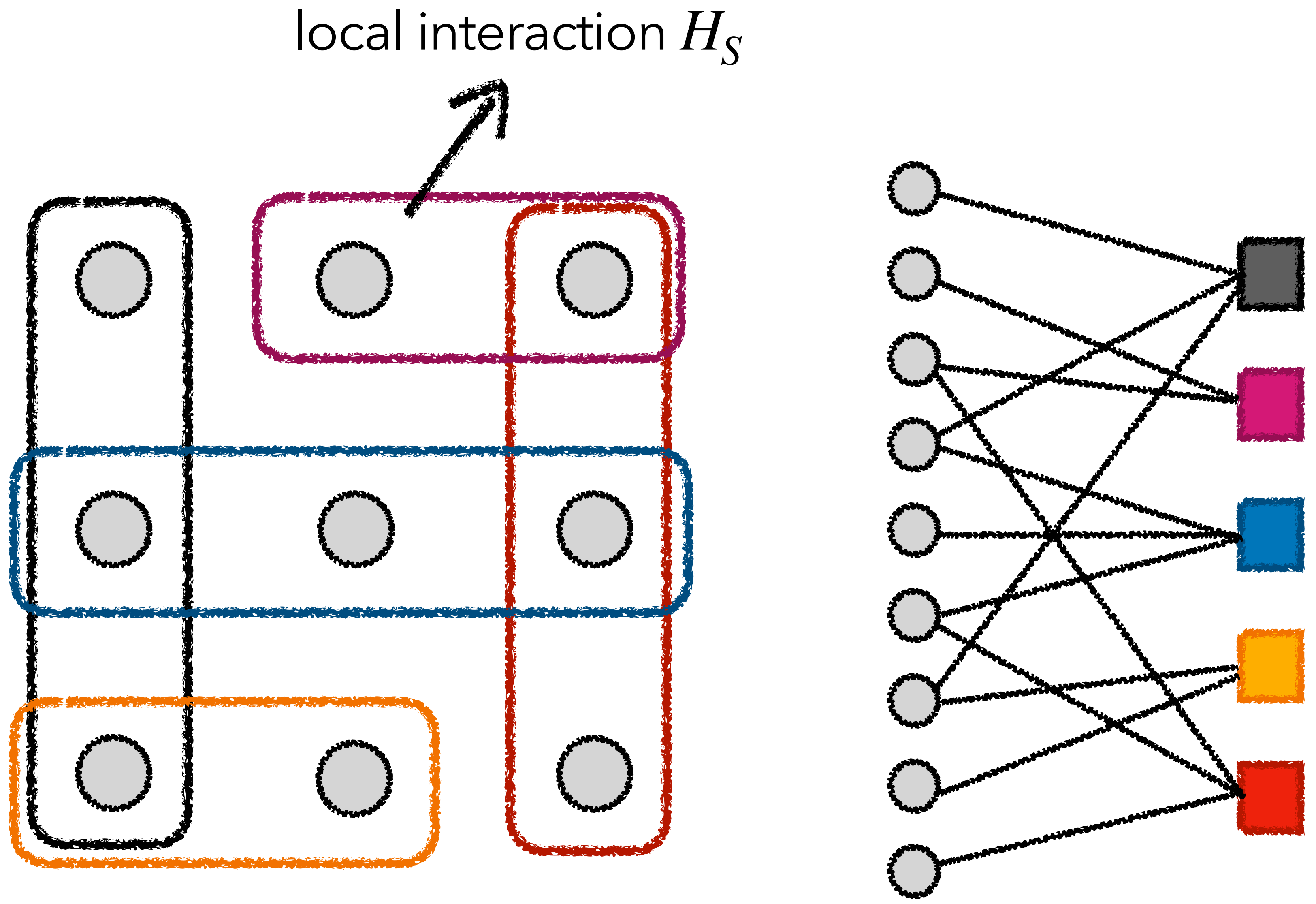

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$$\begin{bmatrix} * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \\ * & * & * & * & * & * & * & * & * \end{bmatrix} \qquad \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & * & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & * & 0 & 0 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & * & 0 & 0 & * & 0 & 0 & * \end{bmatrix}$$

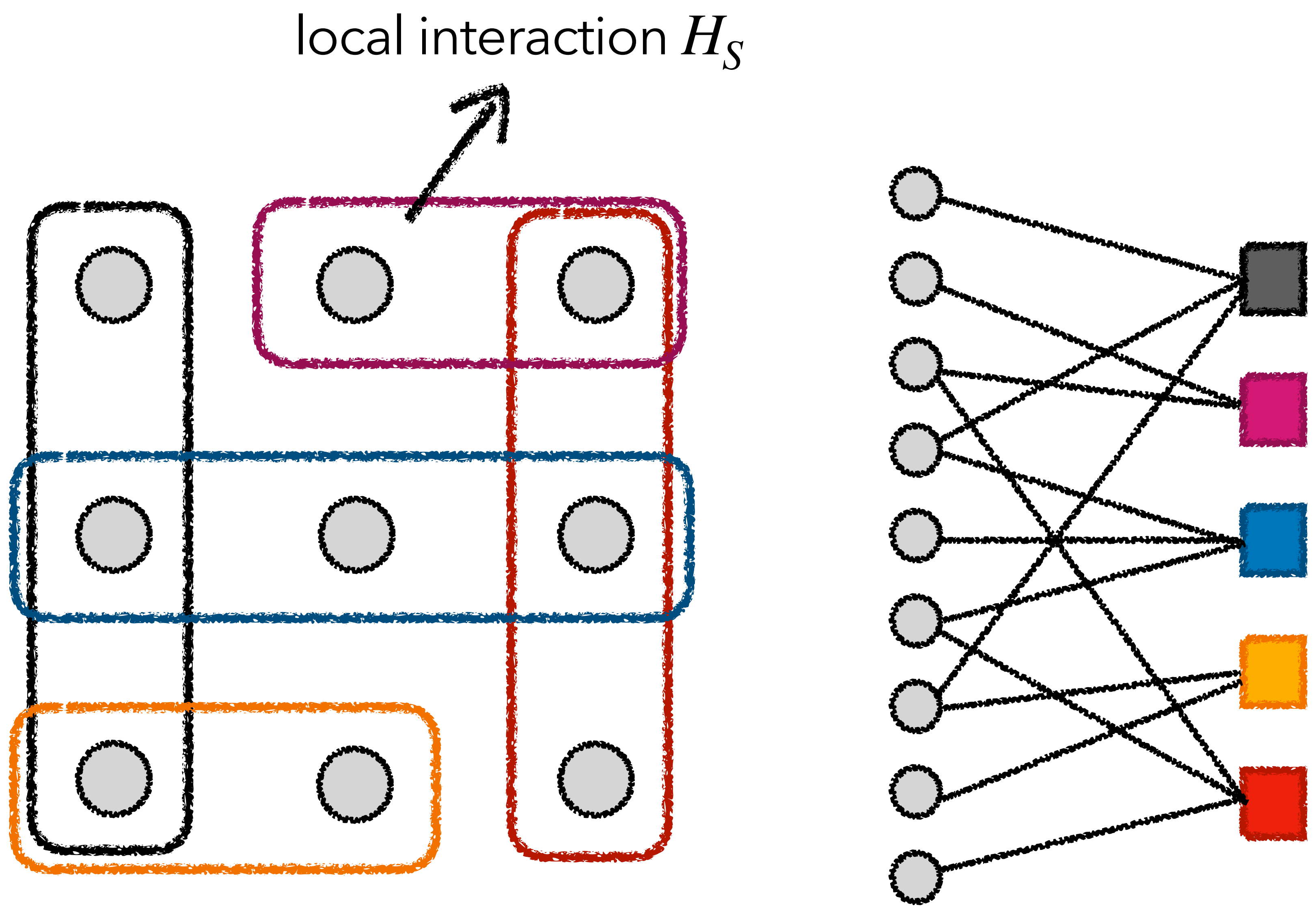
Ψ_μ $\qquad$ $\widetilde{\Psi_{\mu_S}^{-1}}$

Influence Matrix as Product Distance Matrix

Product distance matrix

Let $G = (V, E)$ be an undirected graph and $P \in \mathbb{R}^{V \times V}$.
Matrix P is a *product distance matrix* if

- For any $u, v, w \in V$ satisfying that any path from u to v must pass w, we have $P(u, v) = P(u, w) \cdot P(w, v)$.



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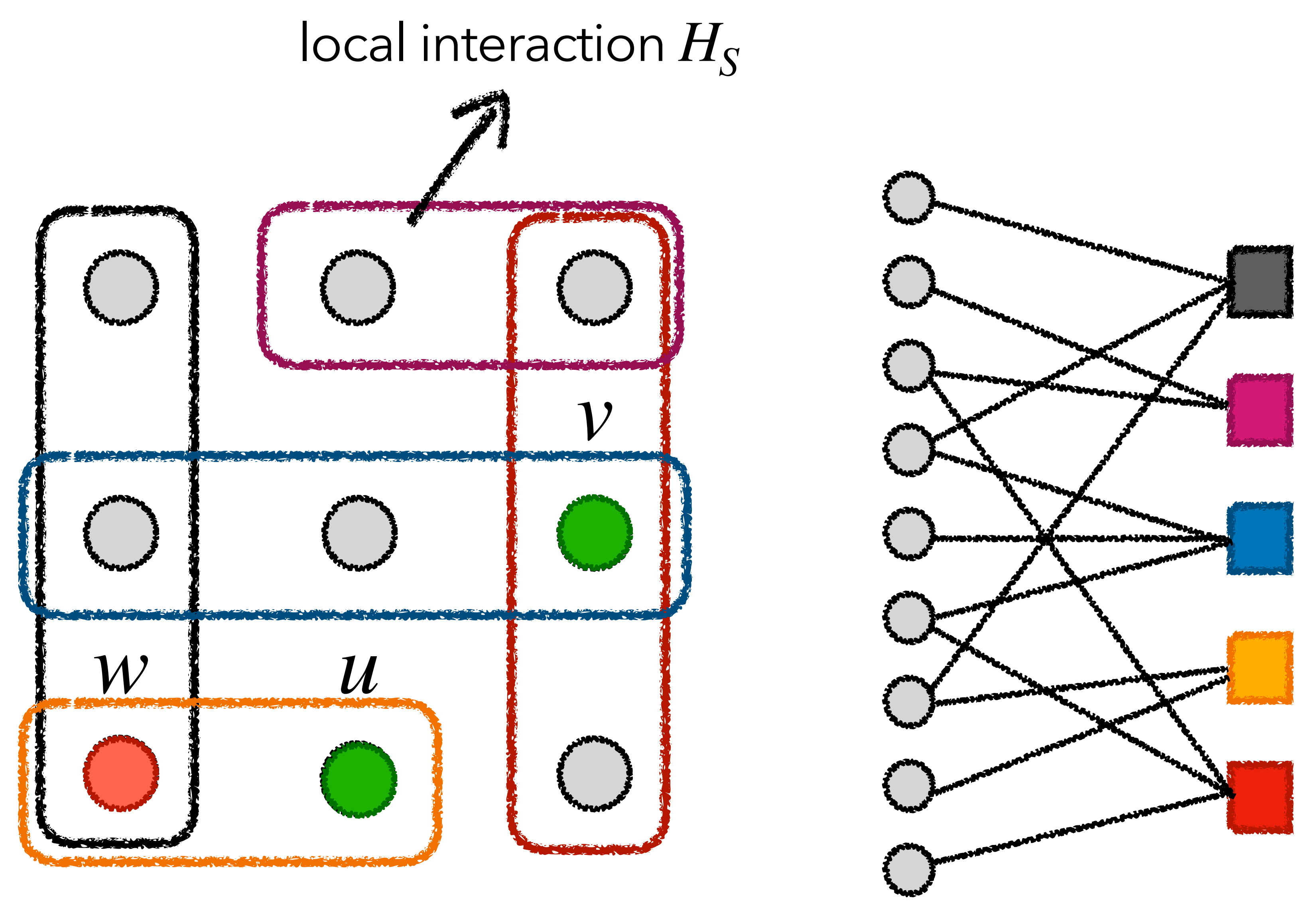

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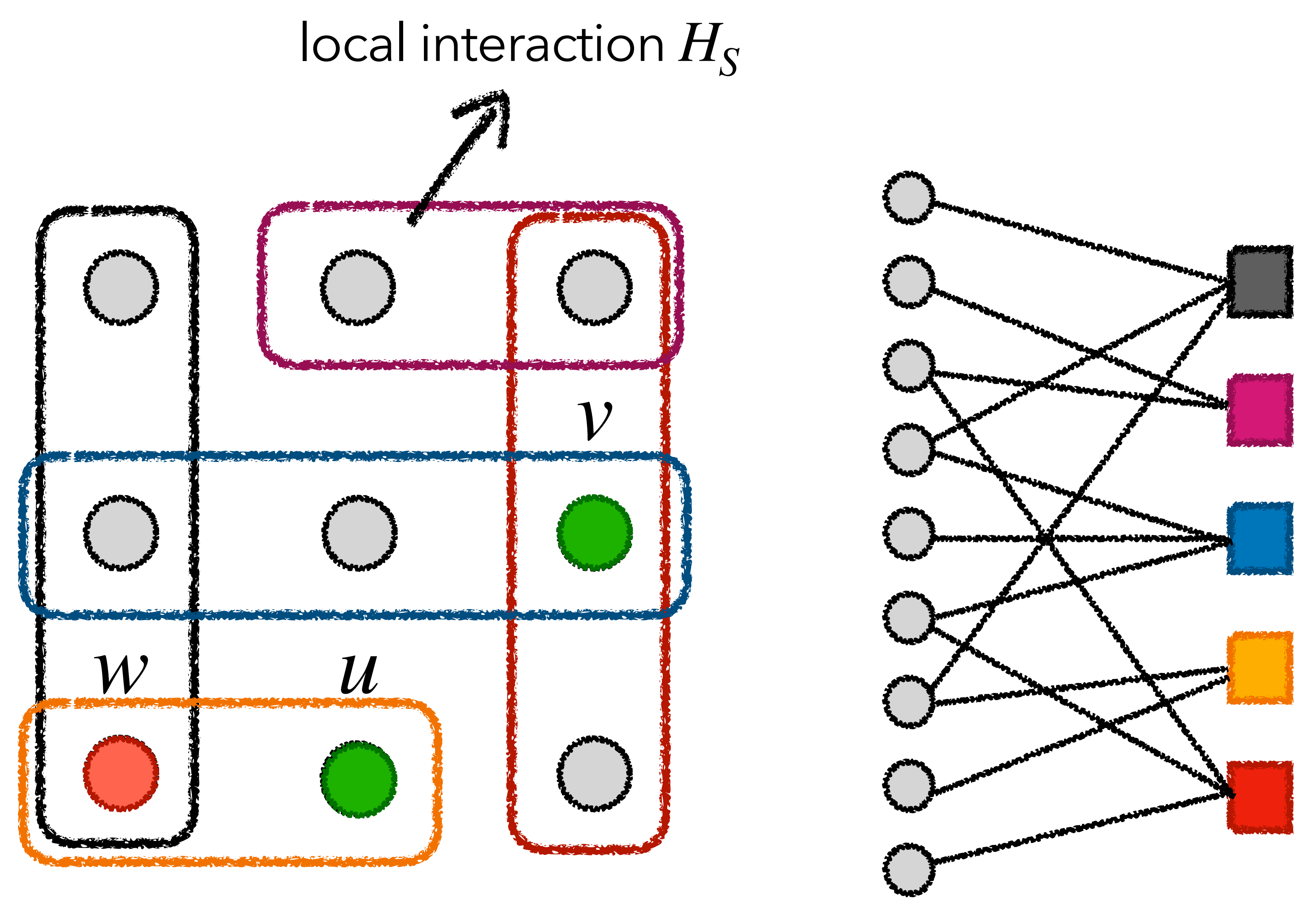


distribution and factor graph

If the factor graph is a tree, the influence matrix Ψ_μ is a product distance matrix!
(We treat each interaction set $S \in \mathcal{S}$ as a clique)

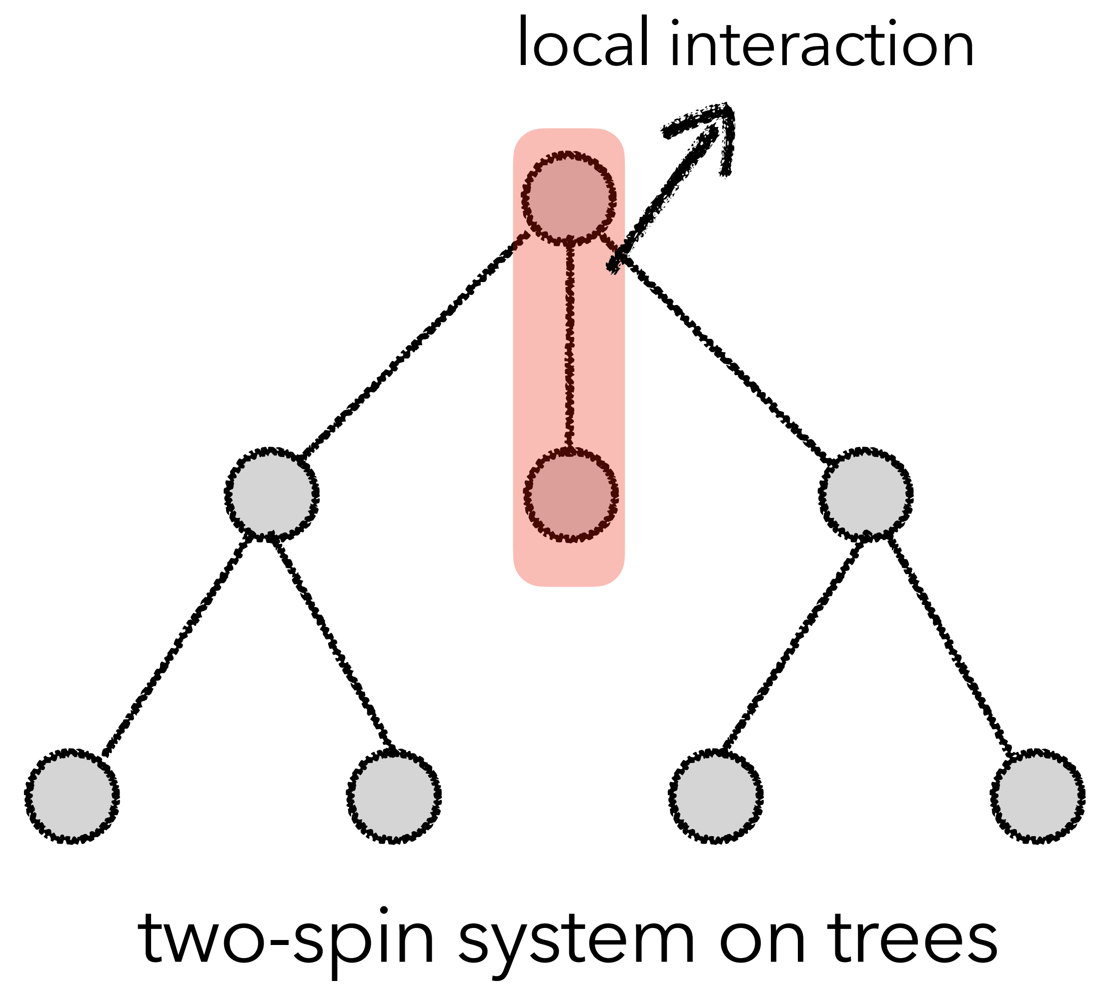
Applications: Two-Spin Systems on Trees

SI for two-spin system on trees (informal)

If for any non-root vertex $v \in V$, we have

$$(\star) \qquad \sum_{w \in \mathscr{C}(v)} \Psi_\mu(v, w)\Psi_\mu(w, v) < 1 - \delta,$$

then $\lambda_{\max}(\Psi_\mu) = O(1/\delta^2)$.



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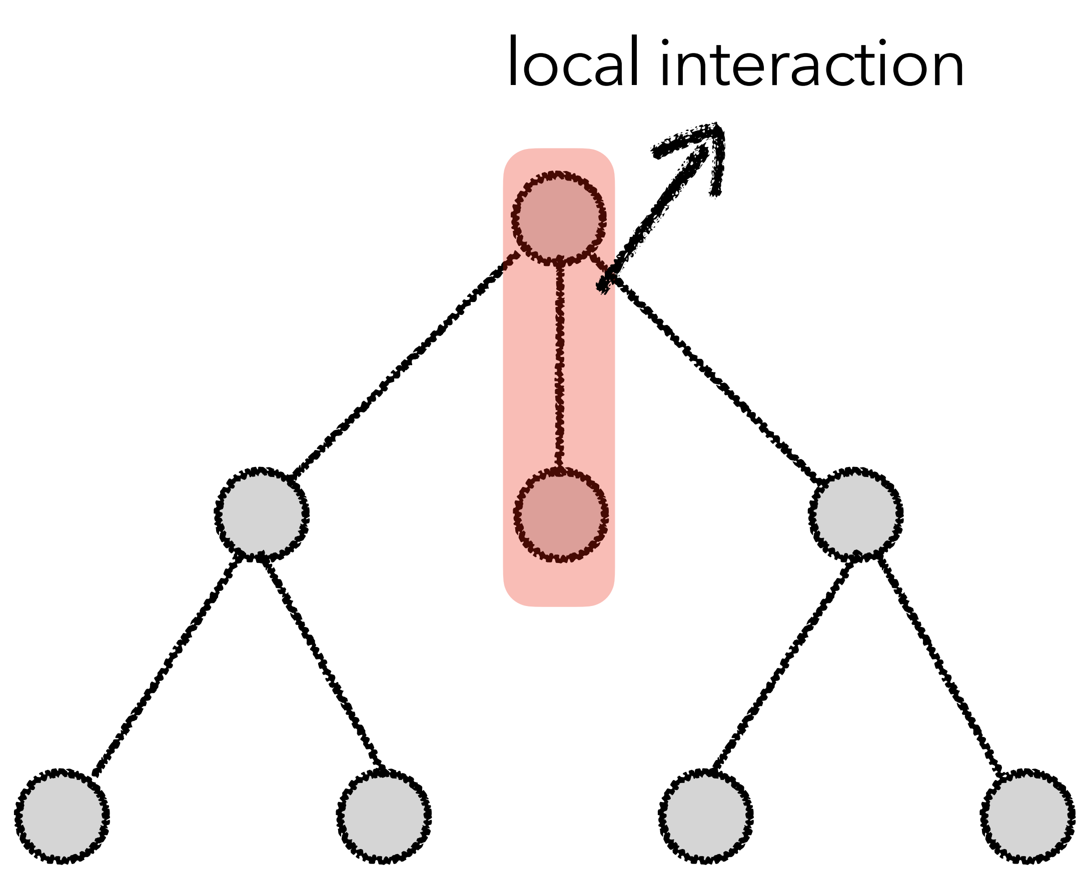

two-spin system on trees

Bounding total influence via contraction (simplified)

Total influence is bounded, if for any non-root vertex $v \in V$, we have

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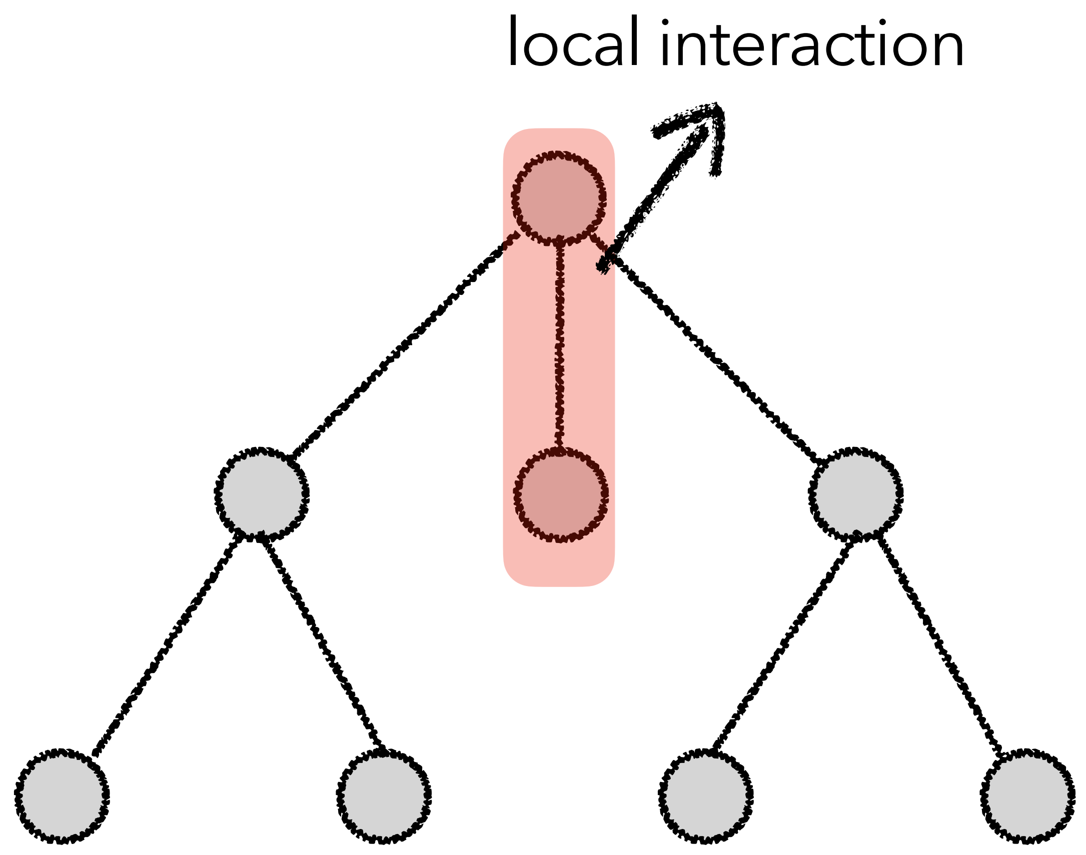

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Constant spectral independence via ℓ_2-norm instead of ℓ_1-norm on trees!

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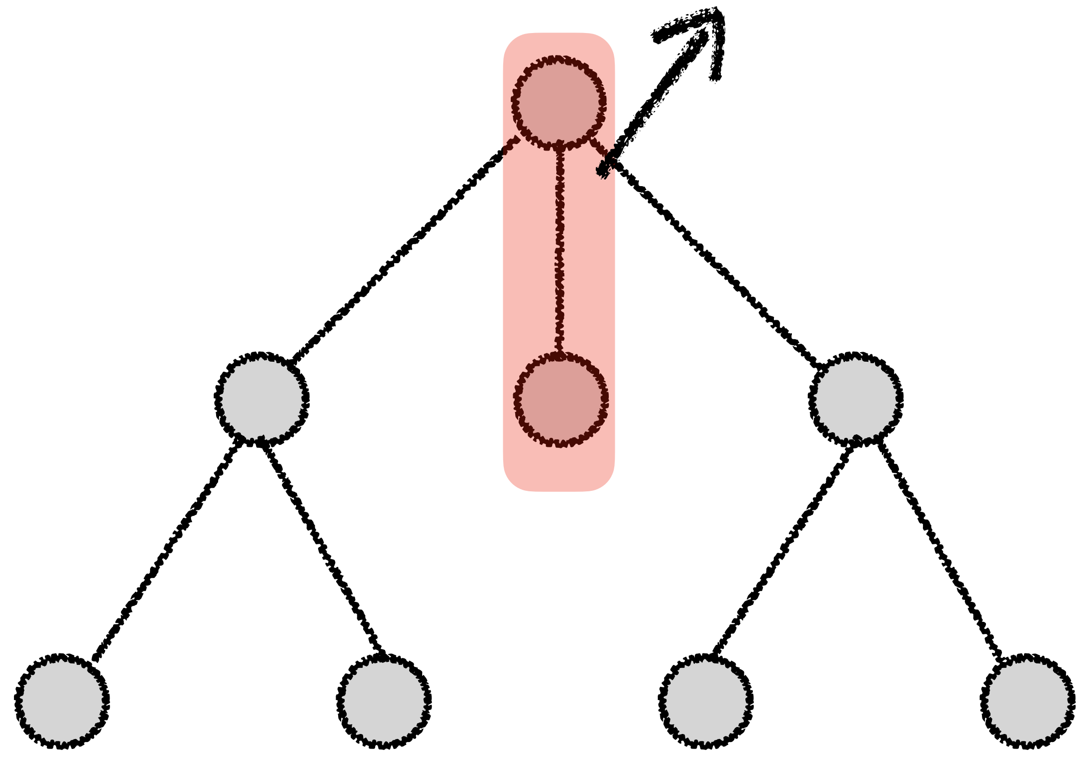
$$(\star) \qquad \sum_{w \in \mathscr{C}(v)} \Psi_\mu(v, w)\Psi_\mu(w, v) < 1 - \delta,$$

then $\lambda_{\max}(\Psi_\mu) = O(1/\delta^2)$.



Applications

(Ferro) Ising model: $\beta < \beta_1(\Delta)$ (spin-glass critical point) implies a constant SI;
Hardcore model: $\lambda < \mathrm{e}^2$ implies a constant SI.

Applications: Two-Spin Systems on Trees

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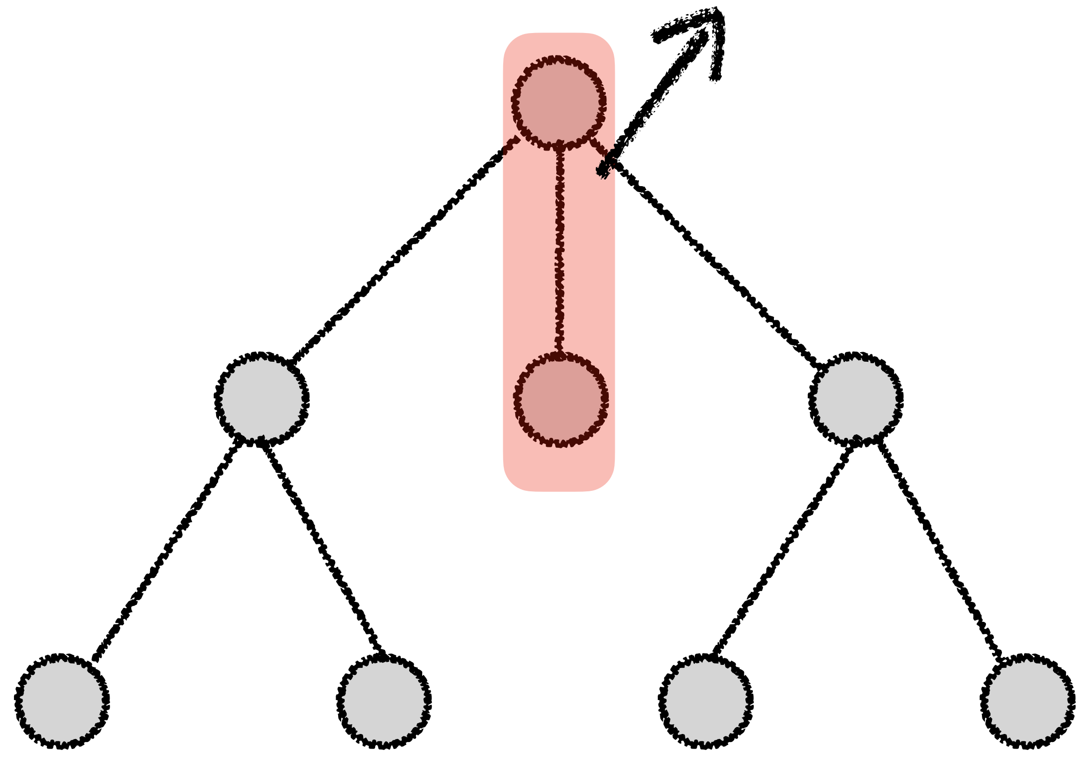

two-spin system on trees

Proof Sketch (informal)

- Symmetrize Ψ_μ^{-1} as $\widehat{\Psi_\mu^{-1}}$;
- It suffices to show that $x^\intercal(\widehat{\Psi_\mu^{-1}} - \delta^2 I)x \geq 0$ for all $x \in \mathbb{R}^V$;
- Write the quadratic form into a sum of squares from bottom to top.

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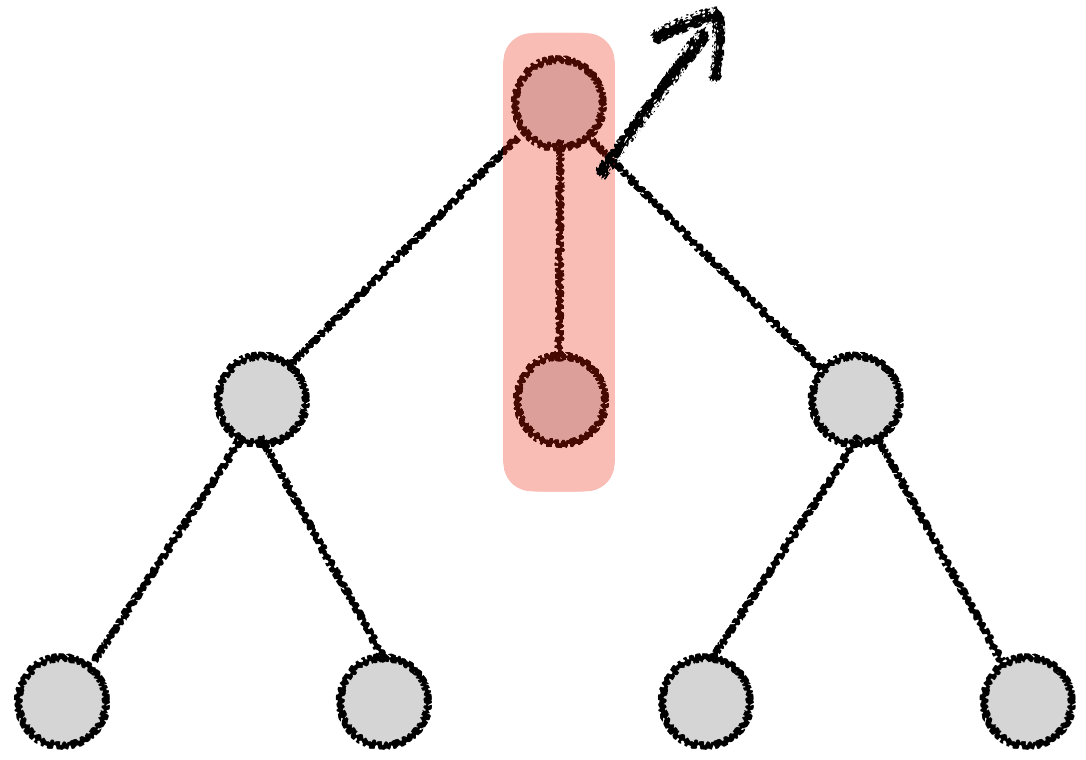

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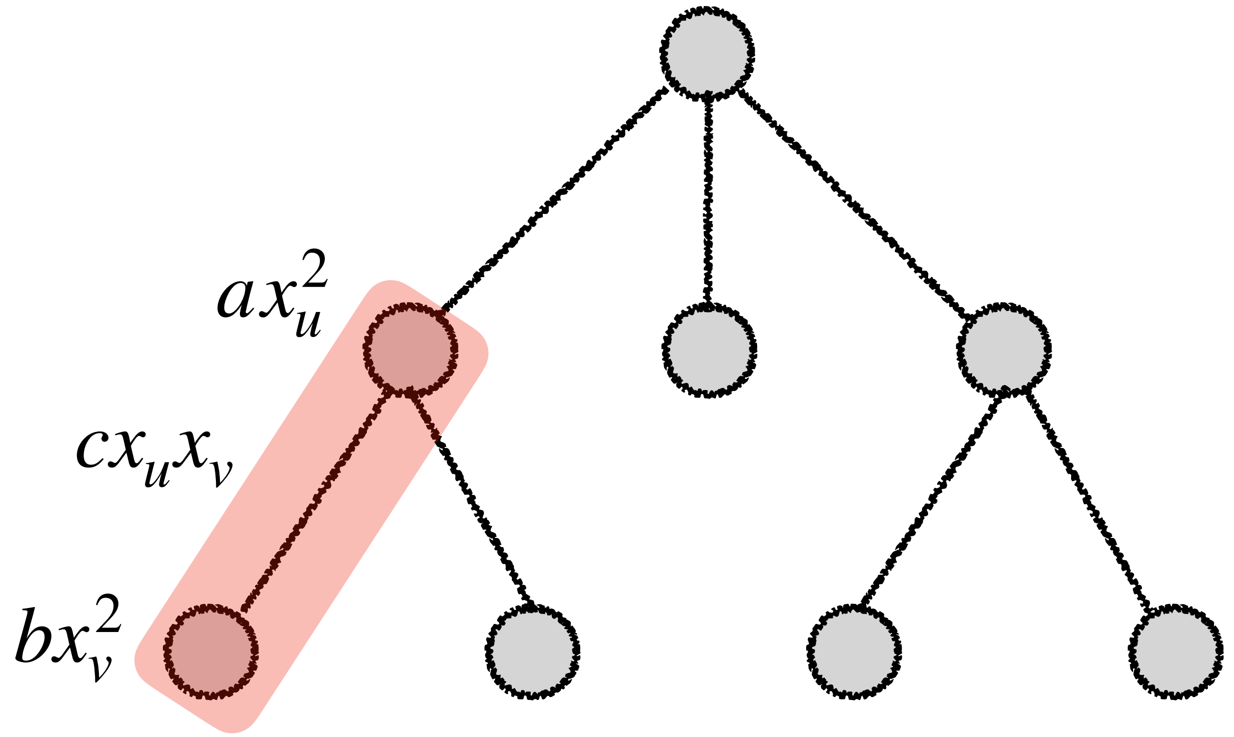

Construction of SOS

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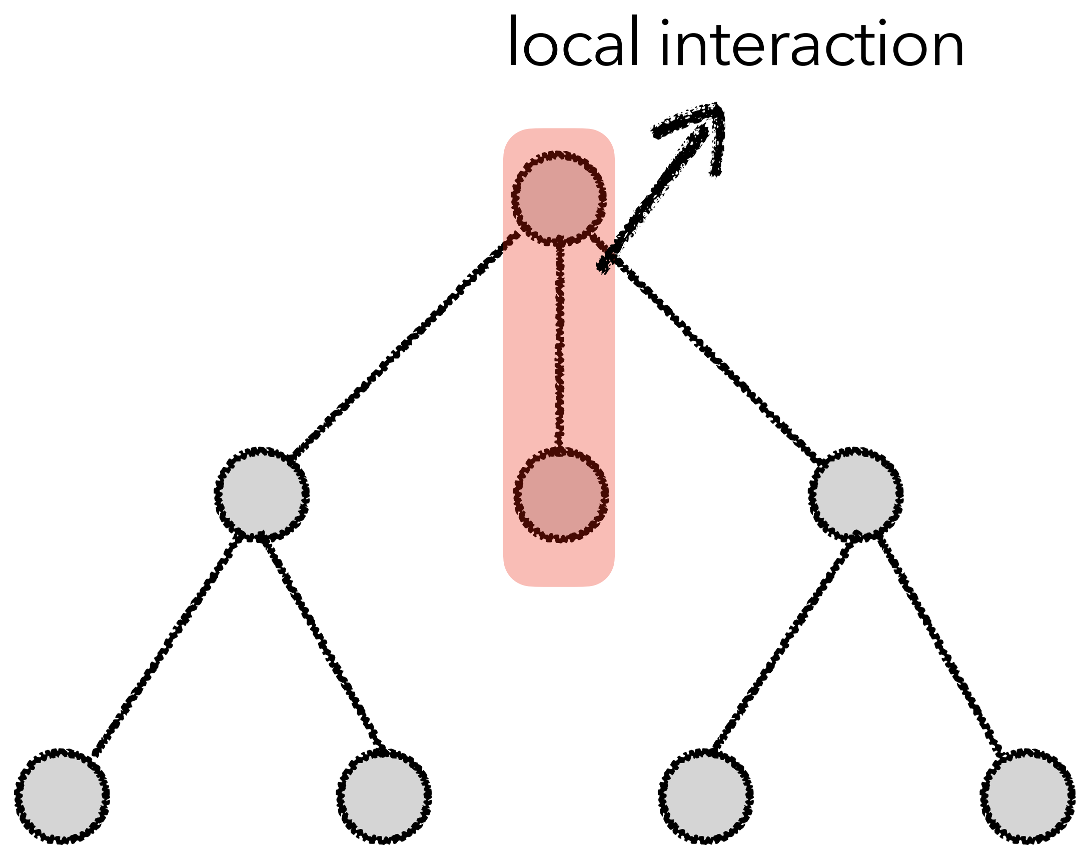

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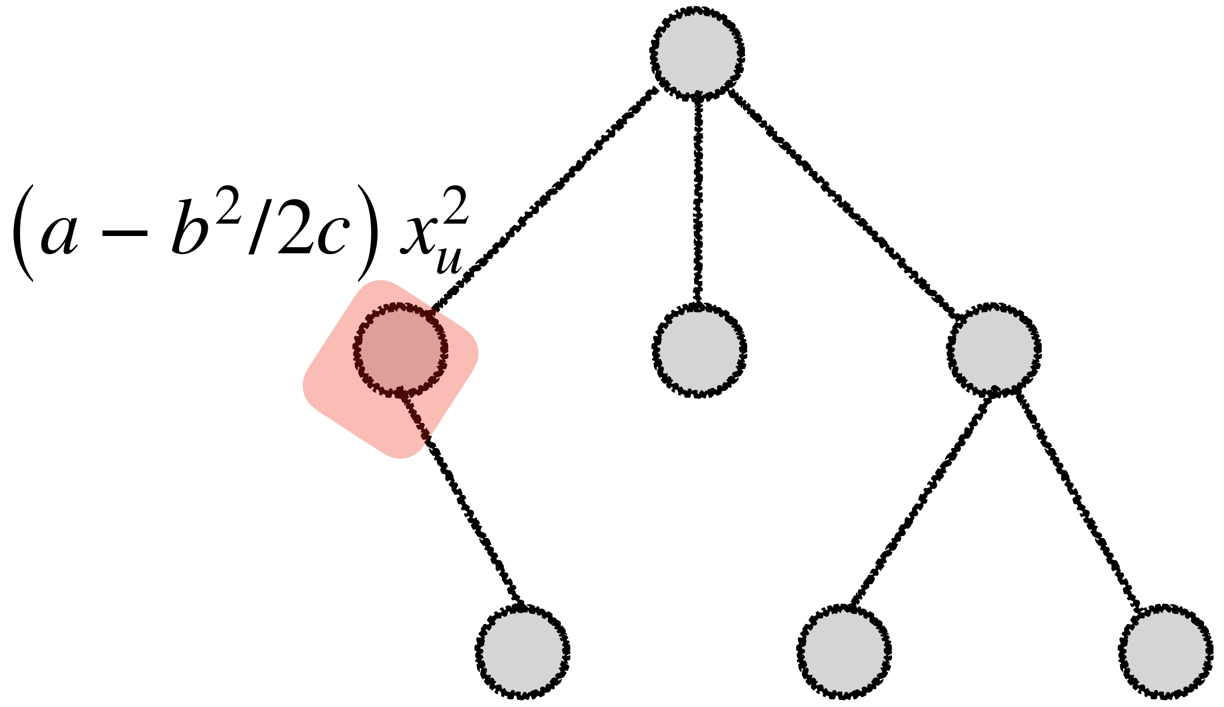


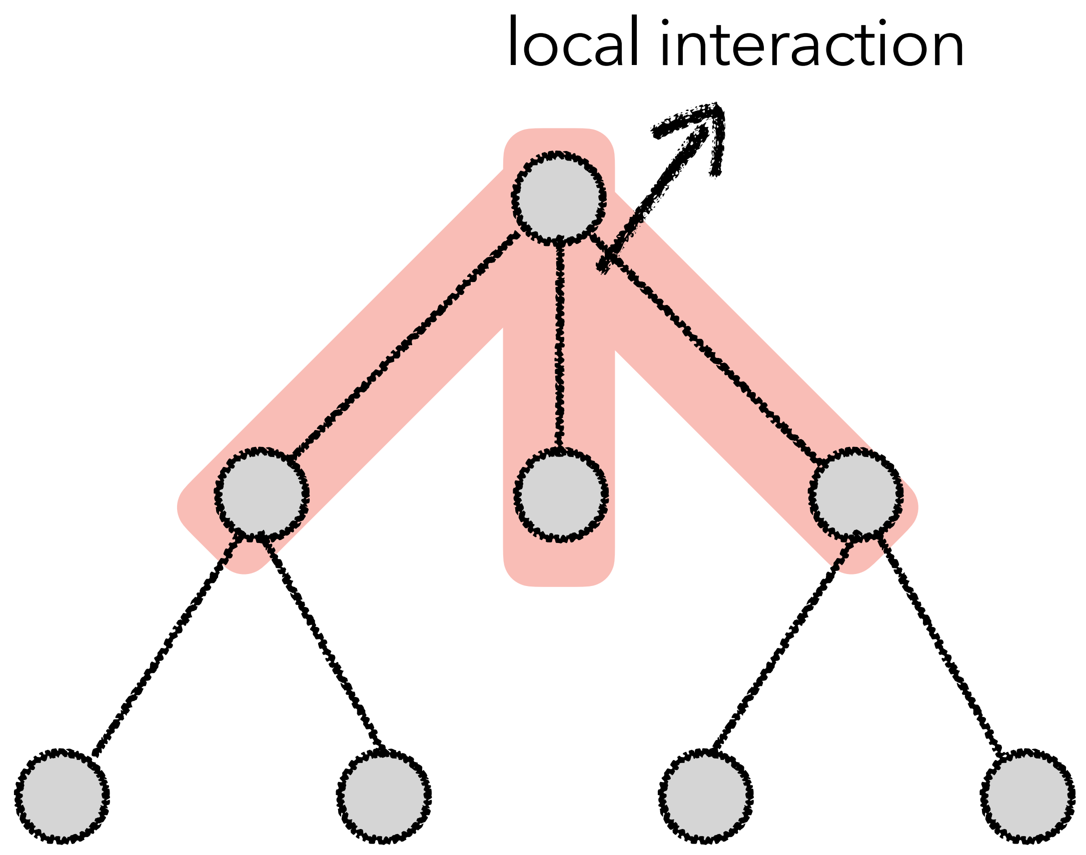
Construction of SOS

Applications: Matching on Trees

SI for monomer-dimer model on trees

$O(\lambda)$-spectral independence, i.e. $\lambda_{\max}(\Psi_\mu) \leq 2(\lambda + 1)$

Inverse of influence matrix: $\Psi_\mu^{-1} = \sum_{S \in \mathcal{S}} \widetilde{\Psi_{\mu_S}^{-1}} - \mathrm{diag}(d_i - 1)_{i \in E}$



matching on trees

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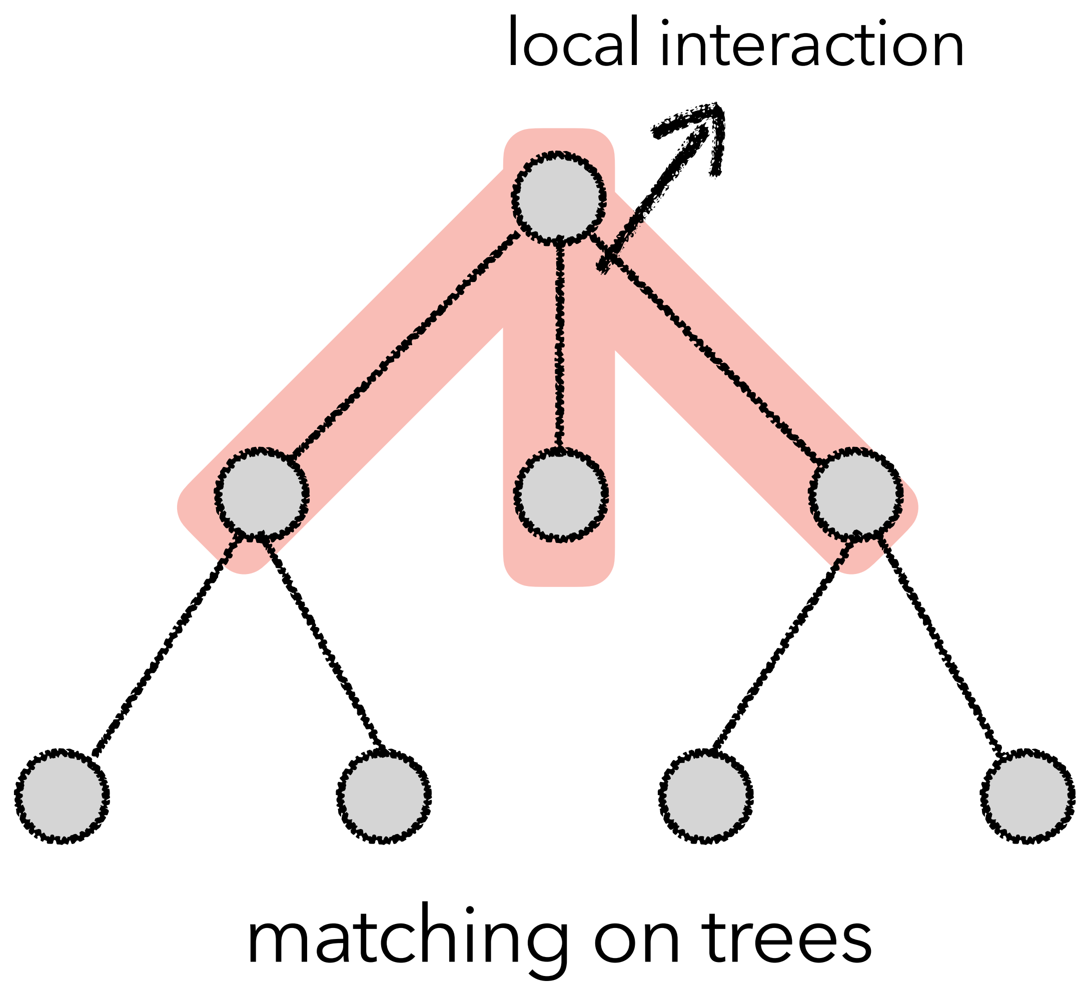

Inverse of influence matrix: $\Psi_\mu^{-1} = \sum_{S \in \mathcal{S}} \widetilde{\Psi_{\mu_S}^{-1}} - \mathrm{diag}(d_i - 1)_{i \in E}$

Each edge contains two vertices (constraints)

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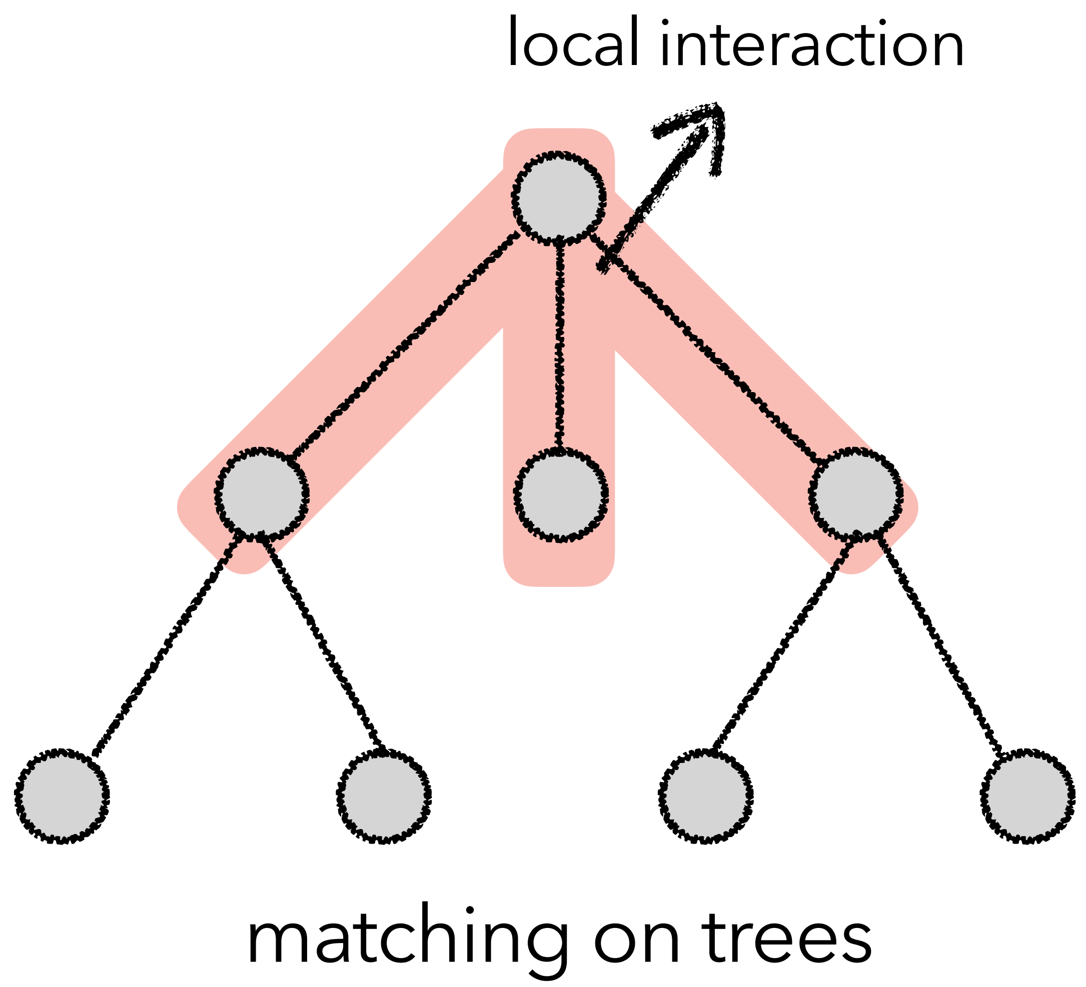

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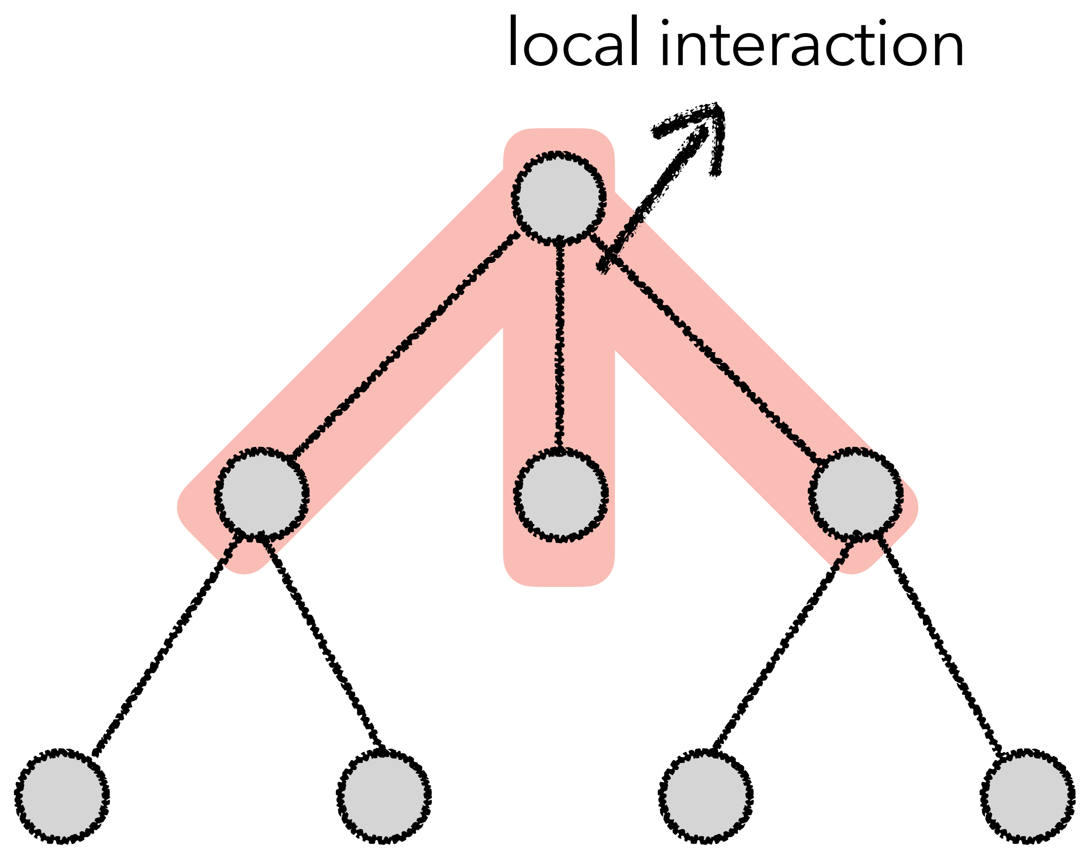
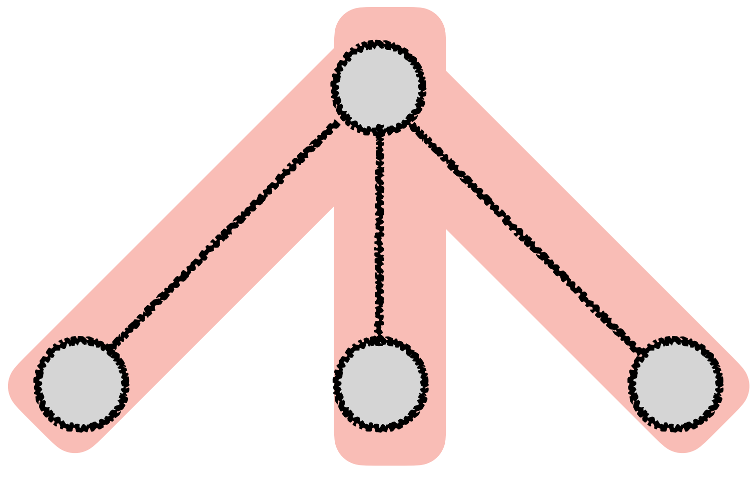


matching on trees

Proof Sketch (informal)

- It suffices to show that $\Psi_{\mu_S}^{-1} \succeq (1/2 + \epsilon) I$ for all local interactions S;
- Analyze $\Psi_{\mu_S}^{-1}$ via Schur's theorem.



$$\begin{bmatrix} 1 & \mu_2/(1-\mu_1) & \mu_3/(1-\mu_1) \\ \mu_1/(1-\mu_2) & 1 & \mu_3/(1-\mu_2) \\ \mu_1/(1-\mu_3) & \mu_2/(1-\mu_3) & 1 \end{bmatrix} = D - uv^\top$$

Conclusion and Open Problems

Spectral independence beyond total influence

Technique

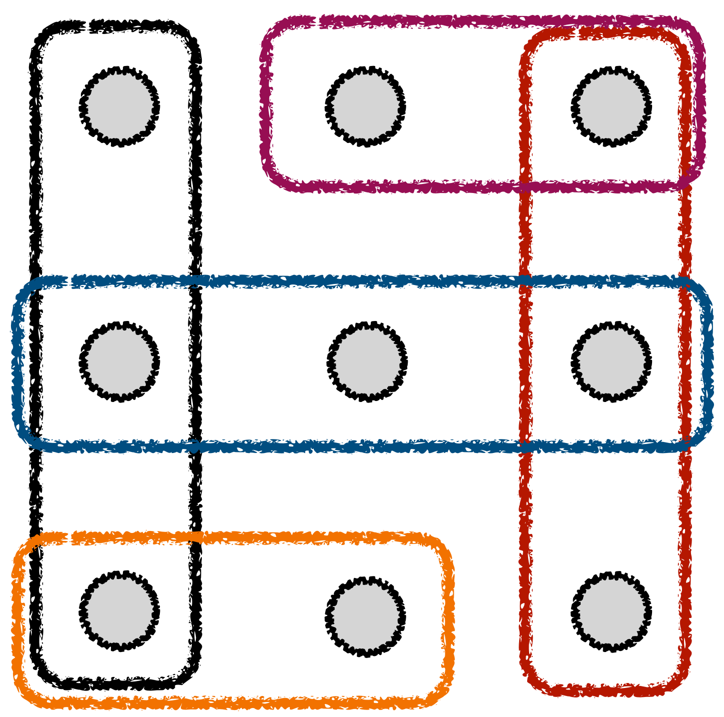
Approximate inverse of influence matrix

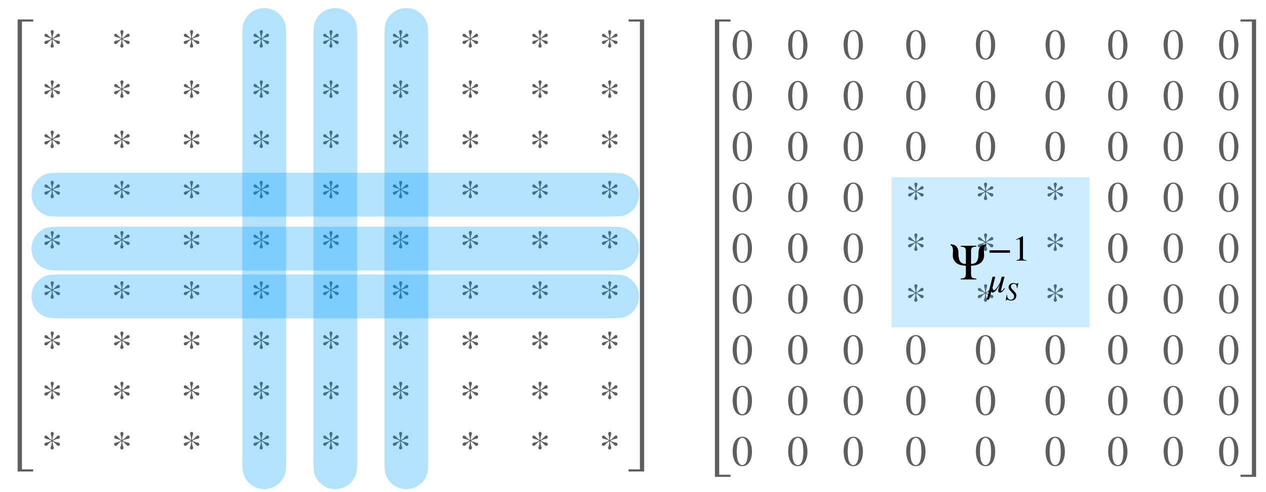
Hardcore model on trees

Optimal mixing time bound for bounded degree trees when $\lambda < \mathrm{e}^2$

Monomer-dimer model

$O(\lambda)$-spectral independence for graphs with large girth





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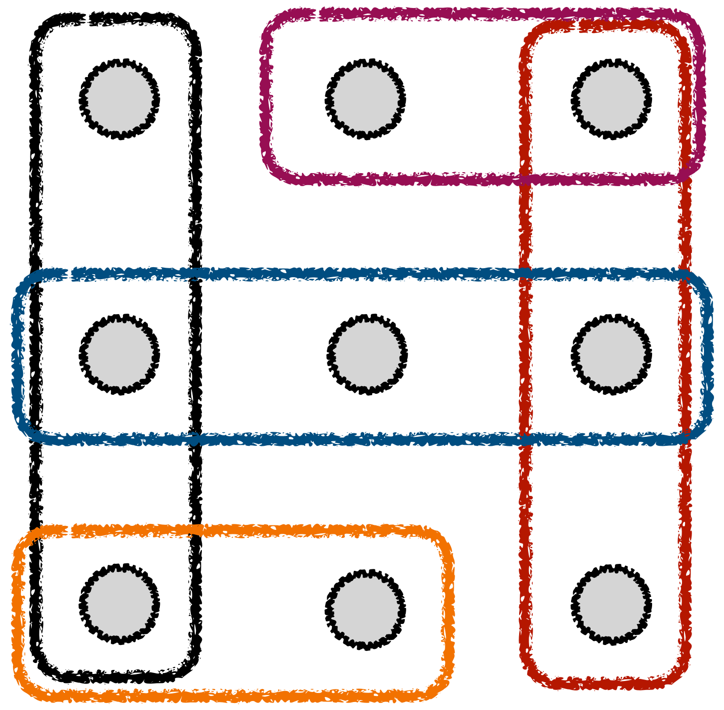

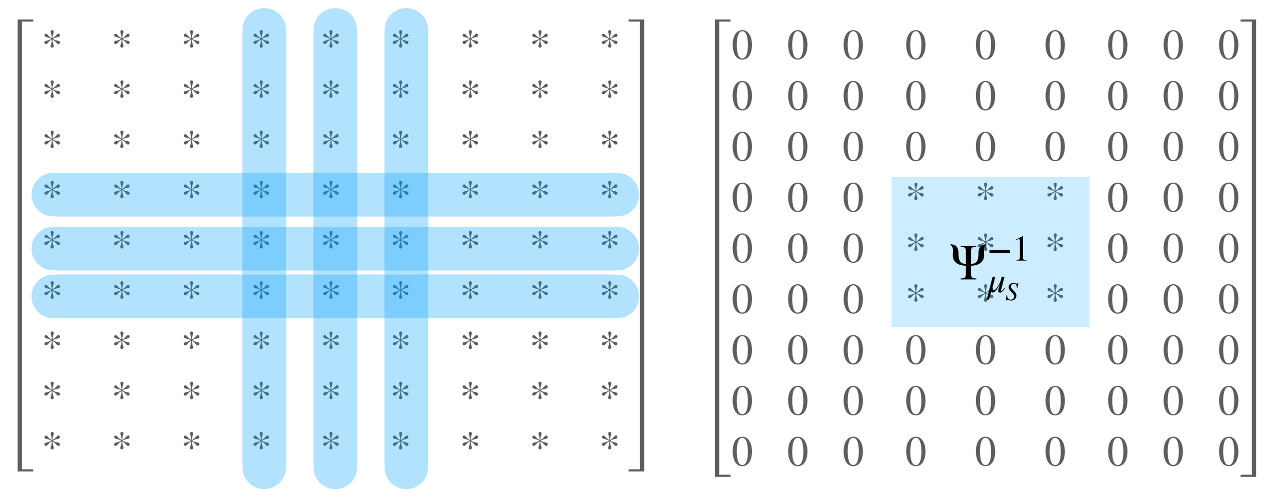

Open problems

- Spectral Independence and mixing time for monomer-dimer model **on general graphs**;
- Approximate inverse **beyond two-state systems** and **further applications**;
- …

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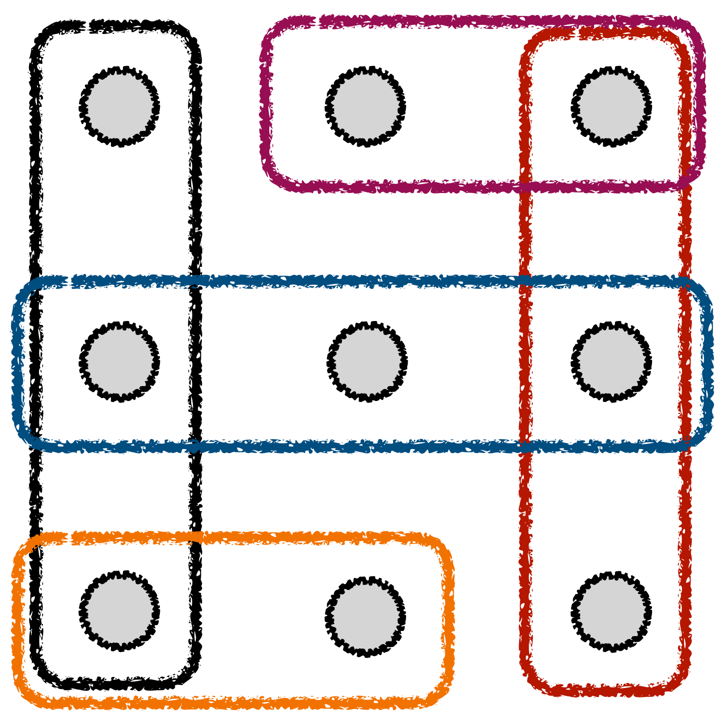

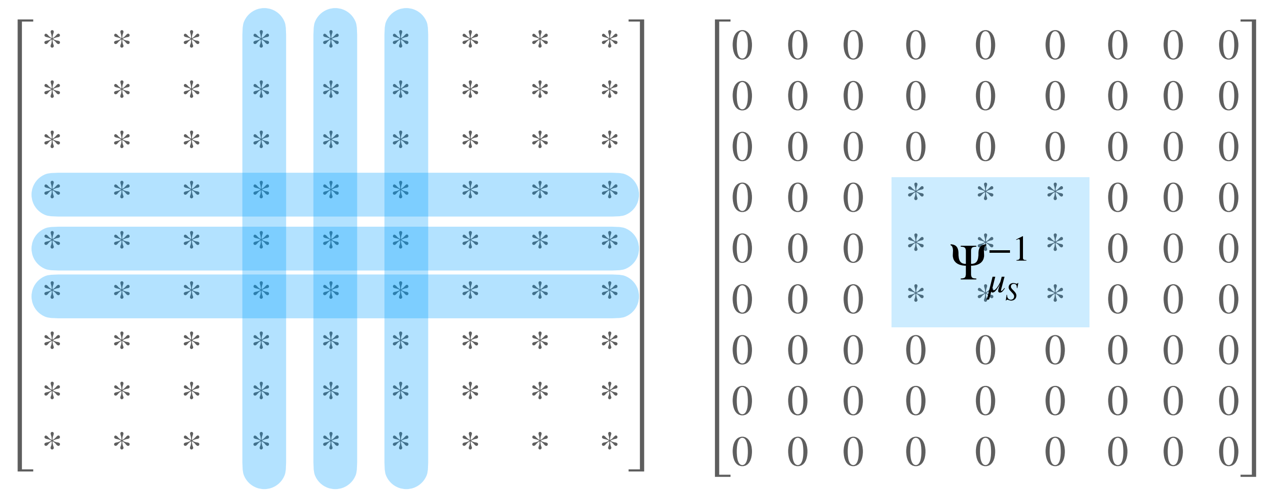

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Thanks!