

A NOTE ON "KELVIN" WAVES IN THE ATMOSPHERE¹

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ABSTRACT

A special solution of the linearized equations for an equatorial β -plane is described. The meridional velocity is set identically zero and it is shown that the solution is a vertically propagating gravity wave in the x - z plane which is in geostrophic balance in the meridional plane. These waves are shown to be similar to Kelvin waves in a bounded ocean. Some implications of the atmospheric "Kelvin" waves for equatorial dynamics are discussed.

1. INTRODUCTION

Lindzen [1] has recently shown that oscillations of the atmosphere in response to forcing of a specified zonal wave number and specified period (in excess of $\frac{1}{2}$ day) are characterized by two classes of modes. The modes of one class have their greatest amplitudes outside the Tropics, while the modes of the other class have their greatest amplitudes near the Equator. In general the only modes of the former class that will propagate vertically are waves whose zonal phase speeds are slightly easterly relative to the mean zonal flow³, while all modes of the second class propagate vertically as gravity waves regardless of their zonal phase speeds. In the present note we describe an important member of the latter class which was not included in the analysis of [1].

2. THE "KELVIN" WAVE SOLUTION

If in equations (10)–(14) of [1] $v' (= \rho_0^{1/2} v)$ where v is the northward velocity) is set *identically equal to zero*, then a nontrivial solution is possible only if

$$\frac{\partial u'}{\partial y} - \frac{k}{\omega} (f + \beta y) u' = 0 \quad (1)$$

where $u' = \rho_0^{1/2} u$, and u is the eastward velocity.⁴ For an equatorially centered β -plane where $f=0$, equation (1) has a solution

$$u' = F(z) \exp\left(\frac{k\beta}{2\omega} y^2\right) \quad (2)$$

where for an isothermal basic state and in the absence of thermal sources $F(z)$ is a solution of

$$\frac{d^2 F}{dz^2} + \left(\frac{\kappa g k^2}{H \omega^2} - \frac{1}{4H^2} \right) F = 0 \quad (3)$$

which is identical in form to equation (23) of [1].

The solution (2) is the gravest mode for which u is symmetric about the Equator (i.e., the mode of largest latitudinal scale). Comparison with equations (22)–(32) of [1] suggests that this mode should be assigned the index $n = -1$. The solution can then be written as

$$u'_{-1} = F(z) e^{-\xi^2/2} \quad (4)$$

where $v'_{-1} \equiv 0$ and

$$\xi^2 = \beta (gh_{-1})^{-1/2} y^2 \quad (5)$$

where

$$h_{-1} = \frac{1}{g} \frac{\omega^2}{k^2} \quad (6)$$

is the equivalent depth. Comparing equations (2) and (4) the appropriate square root in (5) is seen to be negative so that

$$\sqrt{gh_{-1}} = \frac{-\omega}{k} \quad (7)$$

The boundedness of u'_{-1} then requires that $\omega/k < 0$, i.e. that the phase speed of the waves be westerly relative to the mean zonal flow. Equation (7), it may be noted, gives the equivalent depth for a gravity wave of phase velocity $c = -\omega/k$ in the absence of rotation.

3. DISCUSSION

An analogous solution for a barotropic atmosphere has been derived by Matsuno [3] who pointed out that this

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³ These are the modes described by Charney and Drazin [2].

⁴ All other notation is as defined in [1].

$n = -1$ mode is similar to a Kelvin wave. Kelvin waves are generally defined as shallow water gravity waves which propagate parallel to a coastline and have no velocity component normal to the coastal boundary. The latter condition implies that the pressure gradient normal to the coastline be in geostrophic balance with the velocity field, which in turn requires that the amplitude of the wave decay exponentially away from the coast. In view of the similarity of the present solution to Kelvin waves, it seems reasonable to call the wave of the solution (2) an atmospheric "Kelvin" wave, noting that the Equator plays the same role as a coastal boundary.

4. IMPLICATIONS FOR EQUATORIAL DYNAMICS

The importance of the present solution is that as the gravest symmetric, westerly, equatorial mode it is likely to be strongly excited. This has been confirmed in recent data analyses by Wallace and Kousky [4]. Their data indicate vertically propagating oscillations in the zonal wind of the tropical stratosphere between about 80 mb. and 20 mb. with periods of the order 12–15 days and a vertical wavelength of ~ 10 km. Letting λ denote the vertical wave number, equation (3) indicates that

$$\frac{\omega^2}{k^2} \approx \frac{\kappa g}{\lambda^2 H}. \quad (8)$$

For $H = 6$ km., equation (8) gives a zonal phase speed of $\approx +34$ m./sec., so that a period of ~ 12 days corresponds to zonal wave number one. From equation (2) the half width of the waves may be expressed as

$$L_y \approx \sqrt{2\omega/\beta k} \approx 1700 \text{ km.}$$

Thus, the "Kelvin" waves are trapped within about 20° of the Equator. Since these waves propagate westerly momentum upwards, and since their latitudinal extent is similar to that of the quasi-biennial oscillation in the zonal winds of the equatorial stratosphere (Reed [5]), these waves may be the primary mechanism for generating westerly accelerations in the quasi-biennial oscillation. This possibility is currently being studied by the authors and by J. M. Wallace and V. Kousky of the University of Washington.

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