实验三 神经网络-二分类

一、实验目的

1. 掌握本地Jupyter notebook编译环境；
2. 熟悉Python编程集成环境Pychram、vs code；
3. 掌握基于numpy与Tensorflow库的神经网络模型搭建。

本实验代码于吴恩达老师Coursera中的机器学习专项课程。

二、实验内容

1. 安装并配置本地Jupyter notebook编译环境；
2. 基于numpy与Tensorflow库的神经网络二分类模型搭建。

三、实验结果

1. 完成本实验中的Exercise 1-3, 直到显示All tests passed!;

四、实验心得(500字以内)

实验心得：通过本次实验，我学会了如何利用 Keras Sequential 模型和具有 sigmoid 激活的 Dense Layer 来构建一个简单的神经网络。随着实验的进行，我学会了Dense函数的运行原理，并根据其原理编写了两个my_dense函数替代 Dens Layer 给的模板函数。从Dense函数参数来看，它指定了一层网络中Units的个数和本层网络的激活函数；在函数内部来说，既可以利用for循环来编写矩阵运算的过程，也可以利用Numpy提供的函数np.matmul()来直接进行矩阵间的运算。构建好每一层的神经网络后，还需要选择损失函数和梯度下降策略。在载入训练集进行20次epoch训练过后，我得到了自己的0和1分类的模型。 本节课尚存在一些疑点：为什么建立一个25×15×1的神经网络，应当如何确定自己神经网络每一层的神经元个数，希望在以后的学习中可以找到答案。

Practice Lab: Neural Networks for Handwritten Digit Recognition, Binary

In this exercise, you will use a neural network to recognize the hand-written digits zero and one.

Outline

1 - Packages

First, let's run the cell below to import all the packages that you will need during this assignment.

- numpy (https://numpy.org/) is the fundamental package for scientific computing with Python.
- matplotlib (http://matplotlib.org) is a popular library to plot graphs in Python.
- tensorflow (https://www.tensorflow.org/) a popular platform for machine learning.

In [3]:
```
import numpy as np
import tensorflow as tf
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense
import matplotlib.pyplot as plt
from autils import *
%matplotlib inline

import logging
logging.getLogger("tensorflow").setLevel(logging.ERROR)
tf.autograph.set_verbosity(0)
```

Tensorflow and Keras
Tensorflow is a machine learning package developed by Google. In 2019, Google integrated Keras into Tensorflow and released Tensorflow 2.0. Keras is a framework developed independently by François Chollet that creates a simple, layer-centric interface to Tensorflow. This course will be using the Keras interface.

2 - Neural Networks

In Course 1, you implemented logistic regression. This was extended to handle non-linear boundaries using polynomial regression. For even more complex scenarios such as image recognition, neural networks are preferred.

2.1 Problem Statement

In this exercise, you will use a neural network to recognize two handwritten digits, zero and one. This is a binary classification task. Automated handwritten digit recognition is widely used today - from recognizing zip codes (postal codes) on mail envelopes to recognizing amounts written on bank checks. You will extend this network to recognize all 10 digits (0-9) in a future assignment.

This exercise will show you how the methods you have learned can be used for this classification task.

2.2 Dataset

You will start by loading the dataset for this task.

- The `load_data()` function shown below loads the data into variables `X` and `y`
- The data set contains 1000 training examples of handwritten digits [1], here limited to zero and one.
  - Each training example is a 20-pixel x 20-pixel grayscale image of the digit.
    - Each pixel is represented by a floating-point number indicating the grayscale intensity at that location.
    - The 20 by 20 grid of pixels is “unrolled” into a 400-dimensional vector.
    - Each training example becomes a single row in our data matrix `X`.
    - This gives us a 1000 x 400 matrix `X` where every row is a training example of a handwritten digit image.

$$X = \left(\begin{array}{cc} - - -(x^{(1)}) - -- \\ - - -(x^{(2)}) - -- \\ \vdots \\ - - -(x^{(m)}) - -- \end{array}\right)$$

- The second part of the training set is a 1000 x 1 dimensional vector `y` that contains labels for the training set
  - `y = 0` if the image is of the digit `0`, `y = 1` if the image is of the digit `1`.

[1] This is a subset of the MNIST handwritten digit dataset (http://yann.lecun.com/exdb/mnist/ (http://yann.lecun.com/exdb/mnist/))

In [4]:
```
# load dataset
X, y = load_data()
```

2.2.1 View the variables

Let's get more familiar with your dataset.

- A good place to start is to print out each variable and see what it contains.

The code below prints elements of the variables `X` and `y`.

In [5]:
```
print ('The first element of X is: ', X[0])
```

```
The first element of X is:  [ 0.00000000e+00  0.00000000e+00  0.00000000e+00
0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  0.00000000e+00
  0.00000000e+00  0.00000000e+00  0.00000000e+00  8.56059680e-06
  1.94035948e-06 -7.37438725e-04 -8.13403799e-03 -1.86104473e-02
```

In [6]:
```
print ('The first element of y is: ', y[0,0])
print ('The last element of y is: ', y[-1,0])
```

```
The first element of y is:  0
The last element of y is:  1
```

2.2.2 Check the dimensions of your variables

Another way to get familiar with your data is to view its dimensions. Please print the shape of `X` and `y` and see how many training examples you have in your dataset.

In [7]:
```
print ('The shape of X is: ' + str(X.shape))
print ('The shape of y is: ' + str(y.shape))
```

```
The shape of X is: (1000, 400)
The shape of y is: (1000, 1)
```

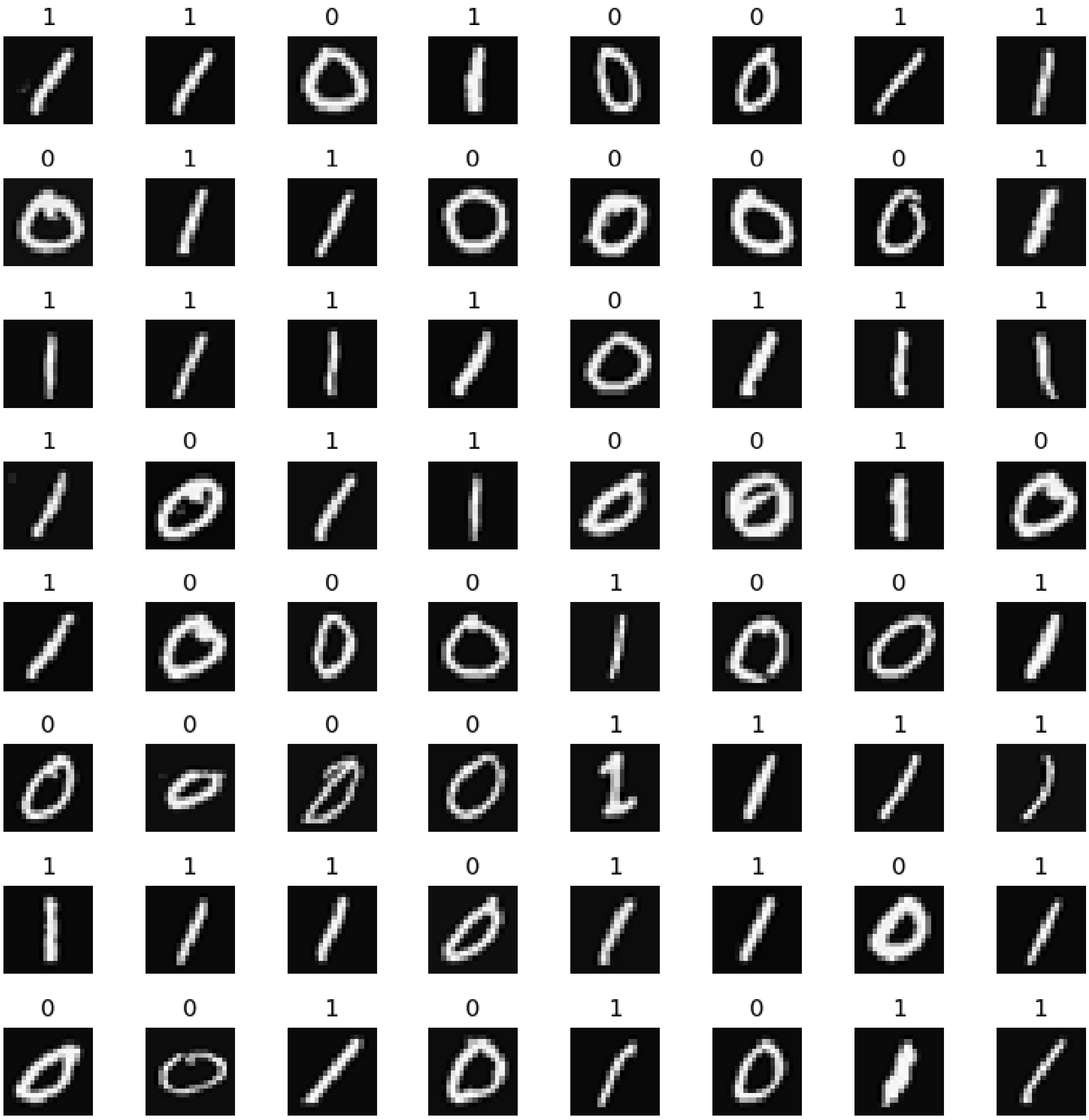
2.2.3 Visualizing the Data

You will begin by visualizing a subset of the training set.

- In the cell below, the code randomly selects 64 rows from `X`, maps each row back to a 20 pixel by 20 pixel grayscale image and displays the images together.
- The label for each image is displayed above the image

In [8]:

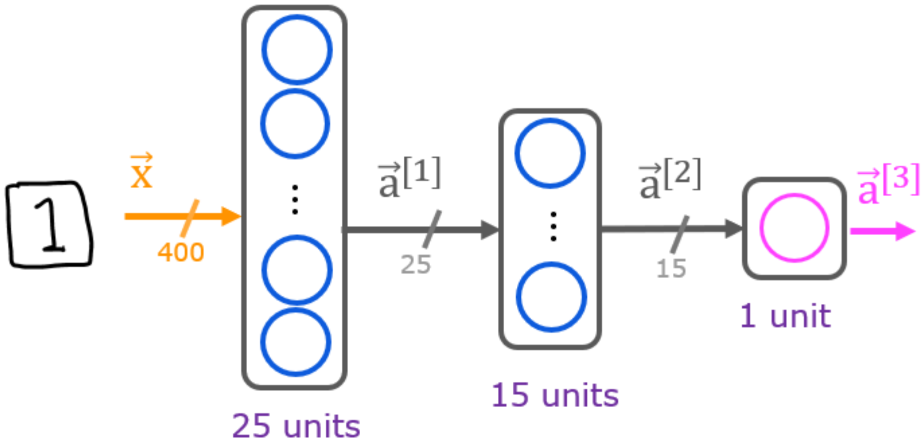
```python
import warnings
warnings.simplefilter(action='ignore', category=FutureWarning)
# You do not need to modify anything in this cell

m, n = X.shape

fig, axes = plt.subplots(8,8, figsize=(8,8))
fig.tight_layout(pad=0.1)

for i,ax in enumerate(axes.flat):
    # Select random indices
    random_index = np.random.randint(m)

    # Select rows corresponding to the random indices and
    # reshape the image
    X_random_reshaped = X[random_index].reshape((20,20)).T

    # Display the image
    ax.imshow(X_random_reshaped, cmap='gray')

    # Display the label above the image
    ax.set_title(y[random_index,0])
    ax.set_axis_off()
```



2.3 Model representation

The neural network you will use in this assignment is shown in the figure below.

- This has three dense layers with sigmoid activations.
  - Recall that our inputs are pixel values of digit images.
  - Since the images are of size 20×20, this gives us 400 inputs



- The parameters have dimensions that are sized for a neural network with 25 units in layer 1, 15 units in layer 2 and 1 output unit in layer 3.
  - Recall that the dimensions of these parameters are determined as follows:
    - If network has s_{in} units in a layer and s_{out} units in the next layer, then
      - W will be of dimension $s_{in} \times s_{out}$.
      - b will a vector with s_{out} elements
  - Therefore, the shapes of `W`, and `b`, are
    - layer1: The shape of `W1` is (400, 25) and the shape of `b1` is (25,)
    - layer2: The shape of `W2` is (25, 15) and the shape of `b2` is: (15,)
    - layer3: The shape of `W3` is (15, 1) and the shape of `b3` is: (1,)

> **Note:** The bias vector `b` could be represented as a 1-D (n,) or 2-D (n,1) array. Tensorflow utilizes a 1-D representation and this lab will maintain that convention.

2.4 Tensorflow Model Implementation

Tensorflow models are built layer by layer. A layer's input dimensions (s_{in} above) are calculated for you. You specify a layer's *output dimensions* and this determines the next layer's input dimension. The input dimension of the first layer is derived from the size of the input data specified in the `model.fit` statment below.

Exercise 1

Below, using Keras [Sequential model (https://keras.io/guides/sequential_model/)](https://keras.io/guides/sequential_model/) and [Dense Layer (https://keras.io/api/layers/core_layers/dense/)](https://keras.io/api/layers/core_layers/dense/) with a sigmoid activation to construct the network described above.

In [19]:
```
# UNQ_C1
# GRADED CELL: Sequential model

model = Sequential(
    [
        tf.keras.Input(shape=(400,)),    #specify input size
        ### START CODE HERE ###
        Dense(25,activation='sigmoid',name='layer1'),
        Dense(15,activation='sigmoid',name='layer2'),
        Dense(1,activation='sigmoid',name='layer3')
        ### END CODE HERE ###
    ], name = "my_model"
)
```

In [20]:
```
model.summary()
```

```
Model: "my_model"
_________________________________________________________________
 Layer (type)                Output Shape              Param #
=================================================================
 layer1 (Dense)              (None, 25)                10025

 layer2 (Dense)              (None, 15)                390

 layer3 (Dense)              (None, 1)                 16

=================================================================
Total params: 10431 (40.75 KB)
Trainable params: 10431 (40.75 KB)
Non-trainable params: 0 (0.00 Byte)
_________________________________________________________________
```

Expected Output (Click to Expand)

In [21]:
```
# UNIT TESTS
from public_tests import *

test_c1(model)
```

```
All tests passed!
```

The parameter counts shown in the summary correspond to the number of elements in the weight and bias arrays as shown below.

In [22]:
```
L1_num_params = 400 * 25 + 25  # W1 parameters  + b1 parameters
L2_num_params = 25 * 15 + 15   # W2 parameters  + b2 parameters
L3_num_params = 15 * 1 + 1     # W3 parameters  + b3 parameters
print("L1 params = ", L1_num_params, ", L2 params = ", L2_num_params, ",  L3 params
```

```
L1 params =  10025 , L2 params =  390 ,  L3 params =  16
```

Let's further examine the weights to verify that tensorflow produced the same dimensions as we calculated above.

In [23]:
```
[layer1, layer2, layer3] = model.layers
```

In [24]:
```
#### Examine Weights shapes
W1,b1 = layer1.get_weights()
W2,b2 = layer2.get_weights()
W3,b3 = layer3.get_weights()
print(f"W1 shape = {W1.shape}, b1 shape = {b1.shape}")
print(f"W2 shape = {W2.shape}, b2 shape = {b2.shape}")
print(f"W3 shape = {W3.shape}, b3 shape = {b3.shape}")
```

```
W1 shape = (400, 25), b1 shape = (25,)
W2 shape = (25, 15), b2 shape = (15,)
W3 shape = (15, 1), b3 shape = (1,)
```

Expected Output

```
W1 shape = (400, 25), b1 shape = (25,)
W2 shape = (25, 15), b2 shape = (15,)
W3 shape = (15, 1), b3 shape = (1,)
```

`xx.get_weights` returns a NumPy array. One can also access the weights directly in their tensor form. Note the shape of the tensors in the final layer.

In [25]:
```
print(model.layers[2].weights)
```

```
[<tf.Variable 'layer3/kernel:0' shape=(15, 1) dtype=float32, numpy=
array([[ 0.41469365],
       [ 0.17289668],
       [ 0.6021336 ],
       [ 0.07787865],
       [-0.10991949],
       [-0.37612113],
       [ 0.08926427],
       [ 0.06583095],
       [ 0.32765186],
       [-0.45328483],
       [-0.512994  ],
       [ 0.09385496],
       [ 0.31334978],
       [-0.43881935],
       [ 0.15837032]], dtype=float32)>, <tf.Variable 'layer3/bias:0' shape=(1,)
dtype=float32, numpy=array([0.], dtype=float32)>]
```

The following code will define a loss function and run gradient descent to fit the weights of the model to the training data. This will be explained in more detail in the following week.

In [26]:
```
model.compile(
    loss=tf.keras.losses.BinaryCrossentropy(),
    optimizer=tf.keras.optimizers.Adam(0.001),
)

model.fit(
    X,y,
    epochs=20
)
```

```
Epoch 1/20
32/32 [==============================] - 1s 1ms/step - loss: 0.6244
Epoch 2/20
32/32 [==============================] - 0s 935us/step - loss: 0.4763
Epoch 3/20
32/32 [==============================] - 0s 903us/step - loss: 0.3279
Epoch 4/20
32/32 [==============================] - 0s 871us/step - loss: 0.2236
Epoch 5/20
32/32 [==============================] - 0s 903us/step - loss: 0.1602
Epoch 6/20
32/32 [==============================] - 0s 903us/step - loss: 0.1224
Epoch 7/20
32/32 [==============================] - 0s 871us/step - loss: 0.0978
Epoch 8/20
32/32 [==============================] - 0s 871us/step - loss: 0.0811
Epoch 9/20
32/32 [==============================] - 0s 871us/step - loss: 0.0690
Epoch 10/20
```

To run the model on an example to make a prediction, use [Keras `predict` (https://www.tensorflow.org/api_docs/python/tf/keras/Model)](https://www.tensorflow.org/api_docs/python/tf/keras/Model). The input to `predict` is an array so the single example is reshaped to be two dimensional.

In [27]:
```
prediction = model.predict(X[0].reshape(1,400))  # a zero
print(f" predicting a zero: {prediction}")
prediction = model.predict(X[500].reshape(1,400))  # a one
print(f" predicting a one:  {prediction}")
```

```
1/1 [==============================] - 0s 71ms/step
 predicting a zero: [[0.02494038]]
1/1 [==============================] - 0s 15ms/step
 predicting a one:  [[0.98850954]]
```

The output of the model is interpreted as a probability. In the first example above, the input is a zero. The model predicts the probability that the input is a one is nearly zero. In the second example, the input is a one. The model predicts the probability that the input is a one is nearly one. As in the case of logistic regression, the probability is compared to a threshold to make a final prediction.

In [28]:
```
if prediction >= 0.5:
    yhat = 1
else:
    yhat = 0
print(f"prediction after threshold: {yhat}")
```

```
prediction after threshold: 1
```

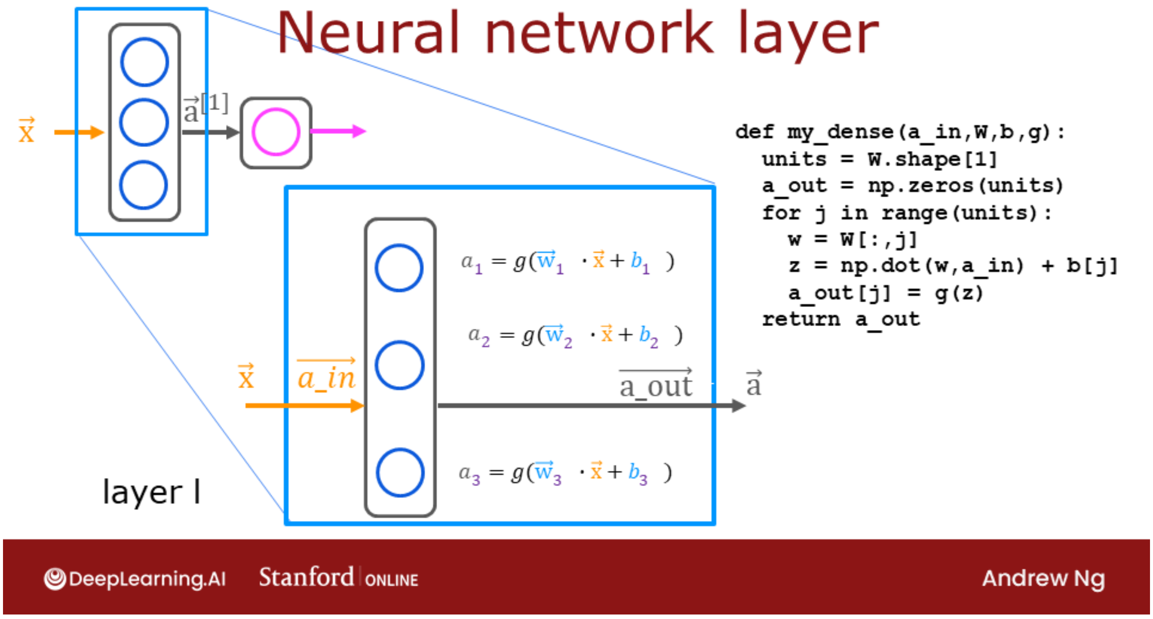
Let's compare the predictions vs the labels for a random sample of 64 digits. This takes a moment to run.

In [29]:
```python
import warnings
warnings.simplefilter(action='ignore', category=FutureWarning)
# You do not need to modify anything in this cell

m, n = X.shape

fig, axes = plt.subplots(8,8, figsize=(8,8))
fig.tight_layout(pad=0.1,rect=[0, 0.03, 1, 0.92]) #[left, bottom, right, top]

for i,ax in enumerate(axes.flat):
    # Select random indices
    random_index = np.random.randint(m)

    # Select rows corresponding to the random indices and
    # reshape the image
    X_random_reshaped = X[random_index].reshape((20,20)).T

    # Display the image
    ax.imshow(X_random_reshaped, cmap='gray')

    # Predict using the Neural Network
    prediction = model.predict(X[random_index].reshape(1,400))
    if prediction >= 0.5:
        yhat = 1
    else:
        yhat = 0

    # Display the label above the image
    ax.set_title(f"{y[random_index,0]},{yhat}")
    ax.set_axis_off()
fig.suptitle("Label, yhat", fontsize=16)
plt.show()
```

```
1/1 [==============================] - 0s 18ms/step
1/1 [==============================] - 0s 17ms/step
1/1 [==============================] - 0s 15ms/step
1/1 [==============================] - 0s 16ms/step
1/1 [==============================] - 0s 15ms/step
1/1 [==============================] - 0s 16ms/step
1/1 [==============================] - 0s 16ms/step
1/1 [==============================] - 0s 13ms/step
1/1 [==============================] - 0s 16ms/step
1/1 [==============================] - 0s 17ms/step
1/1 [==============================] - 0s 16ms/step
1/1 [==============================] - 0s 17ms/step
1/1 [==============================] - 0s 17ms/step
1/1 [==============================] - 0s 17ms/step
1/1 [==============================] - 0s 15ms/step
1/1 [==============================] - 0s 19ms/step
1/1 [==============================] - 0s 17ms/step
1/1 [==============================] - 0s 27ms/step
1/1 [==============================] - 0s 16ms/step
```

2.5 NumPy Model Implementation (Forward Prop in NumPy)

As described in lecture, it is possible to build your own dense layer using NumPy. This can then be utilized to build a multi-layer neural network.



Exercise 2

Below, build a dense layer subroutine. The example in lecture utilized a for loop to visit each unit (j) in the layer and perform the dot product of the weights for that unit (`W[:, j]`) and sum the bias for the unit (`b[j]`) to form z . An activation function $g(z)$ is then applied to that result. This section will not utilize some of the matrix operations described in the optional lectures. These will be explored in a later section.

In [44]:
```
# UNQ_C2
# GRADED FUNCTION: my_dense

def my_dense(a_in, W, b, g):
    """
    Computes dense layer
    Args:
      a_in (ndarray (n, )) : Data, 1 example
      W    (ndarray (n, j)) : Weight matrix, n features per unit, j units
      b    (ndarray (j, )) : bias vector, j units
      g    activation function (e.g. sigmoid, relu..)
    Returns
      a_out (ndarray (j,))  : j units
    """
    units = W.shape[1]
    a_out = np.zeros(units)
### START CODE HERE ###
    for j in range (units):
        w=W[:, j]
        z=np.dot(w, a_in)+b[j]
        a_out[j]=g(z)

### END CODE HERE ###
    return(a_out)
```

In [45]:
```
# Quick Check
x_tst = 0.1*np.arange(1,3,1).reshape(2,)  # (1 examples, 2 features)
W_tst = 0.1*np.arange(1,7,1).reshape(2,3) # (2 input features, 3 output features)
b_tst = 0.1*np.arange(1,4,1).reshape(3,)  # (3 features)
A_tst = my_dense(x_tst, W_tst, b_tst, sigmoid)
print(A_tst)
```

```
[0.54735762 0.57932425 0.61063923]
```

Expected Output

```
[0.54735762 0.57932425 0.61063923]
```

In [46]:
```
# UNIT TESTS
test_c2(my_dense)
```

```
All tests passed!
```

The following cell builds a three-layer neural network utilizing the `my_dense` subroutine above.

In [47]:
```
def my_sequential(x, W1, b1, W2, b2, W3, b3):
    a1 = my_dense(x,  W1, b1, sigmoid)
    a2 = my_dense(a1, W2, b2, sigmoid)
    a3 = my_dense(a2, W3, b3, sigmoid)
    return(a3)
```

We can copy trained weights and biases from Tensorflow.

In [48]:
```
W1_tmp,b1_tmp = layer1.get_weights()
W2_tmp,b2_tmp = layer2.get_weights()
W3_tmp,b3_tmp = layer3.get_weights()
```

In [49]:
```
# make predictions
prediction = my_sequential(X[0], W1_tmp, b1_tmp, W2_tmp, b2_tmp, W3_tmp, b3_tmp )
if prediction >= 0.5:
    yhat = 1
else:
    yhat = 0
print( "yhat = ", yhat, " label= ", y[0,0])
prediction = my_sequential(X[500], W1_tmp, b1_tmp, W2_tmp, b2_tmp, W3_tmp, b3_tmp )
if prediction >= 0.5:
    yhat = 1
else:
    yhat = 0
print( "yhat = ", yhat, " label= ", y[500,0])
```

```
yhat =  0  label=  0
yhat =  1  label=  1
```

Run the following cell to see predictions from both the Numpy model and the Tensorflow model. This takes a moment to run.

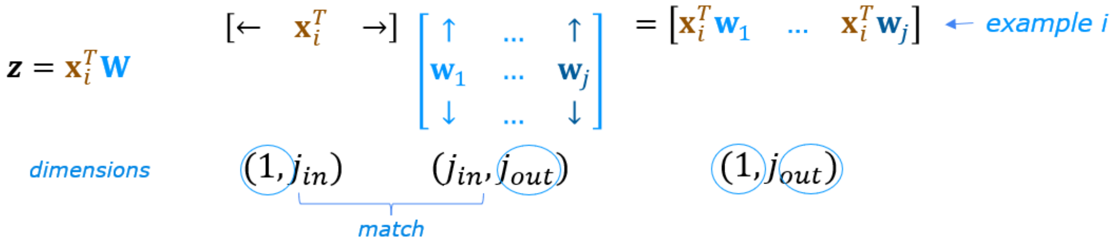
In [50]:

```
import warnings
warnings.simplefilter(action='ignore', category=FutureWarning)
# You do not need to modify anything in this cell

m, n = X.shape

fig, axes = plt.subplots(8,8, figsize=(8,8))
fig.tight_layout(pad=0.1,rect=[0, 0.03, 1, 0.92]) #[left, bottom, right, top]

for i,ax in enumerate(axes.flat):
    # Select random indices
    random_index = np.random.randint(m)

    # Select rows corresponding to the random indices and
    # reshape the image
    X_random_reshaped = X[random_index].reshape((20,20)).T

    # Display the image
    ax.imshow(X_random_reshaped, cmap='gray')

    # Predict using the Neural Network implemented in Numpy
    my_prediction = my_sequential(X[random_index], W1_tmp, b1_tmp, W2_tmp, b2_tmp,
    my_yhat = int(my_prediction >= 0.5)

    # Predict using the Neural Network implemented in Tensorflow
    tf_prediction = model.predict(X[random_index].reshape(1,400))
    tf_yhat = int(tf_prediction >= 0.5)

    # Display the label above the image
    ax.set_title(f"{y[random_index,0]},{tf_yhat},{my_yhat}")
    ax.set_axis_off()
fig.suptitle("Label, yhat Tensorflow, yhat Numpy", fontsize=16)
plt.show()
```

```
1/1 [==============================] - 0s 16ms/step
1/1 [==============================] - 0s 15ms/step
1/1 [==============================] - 0s 13ms/step
1/1 [==============================] - 0s 15ms/step
1/1 [==============================] - 0s 22ms/step
1/1 [==============================] - 0s 17ms/step
1/1 [==============================] - 0s 16ms/step
1/1 [==============================] - 0s 15ms/step
1/1 [==============================] - 0s 19ms/step
1/1 [==============================] - 0s 17ms/step
1/1 [==============================] - 0s 18ms/step
1/1 [==============================] - 0s 15ms/step
1/1 [==============================] - 0s 15ms/step
1/1 [==============================] - 0s 15ms/step
1/1 [==============================] - 0s 16ms/step
1/1 [==============================] - 0s 18ms/step
1/1 [==============================] - 0s 24ms/step
1/1 [==============================] - 0s 15ms/step
1/1 [==============================] - 0s 16ms/step
```

2.6 Vectorized NumPy Model Implementation (Optional)

The optional lectures described vector and matrix operations that can be used to speed the calculations. Below describes a layer operation that computes the output for all units in a layer on a given input example:

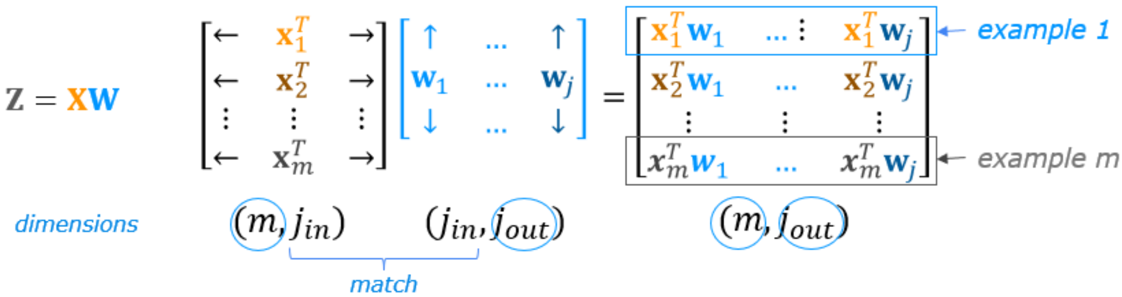


We can demonstrate this using the examples `X` and the `W1` , `b1` parameters above. We use `np.matmul` to perform the matrix multiply. Note, the dimensions of x and W must be compatible as shown in the diagram above.

```
In [51]: x = X[0].reshape(-1,1)         # column vector (400,1)
         z1 = np.matmul(x.T,W1) + b1    # (1,400)(400,25) = (1,25)
         a1 = sigmoid(z1)
         print(a1.shape)
```

```
(1, 25)
```

You can take this a step further and compute all the units for all examples in one Matrix-Matrix operation.



The full operation is $\mathbf{Z}=\mathbf{XW}+\mathbf{b}$. This will utilize NumPy broadcasting to expand $\mathbf{b}$ to m rows. If this is unfamiliar, a short tutorial is provided at the end of the notebook.

Exercise 3

Below, compose a new `my_dense_v` subroutine that performs the layer calculations for a matrix of examples. This will utilize `np.matmul()` .

In [63]:
```
# UNQ_C3
# GRADED FUNCTION: my_dense_v

def my_dense_v(A_in, W, b, g):
    """
    Computes dense layer
    Args:
      A_in (ndarray (m,n)) : Data, m examples, n features each
      W    (ndarray (n,j)) : Weight matrix, n features per unit, j units
      b    (ndarray (1,j)) : bias vector, j units
      g    activation function (e.g. sigmoid, relu..)
    Returns
      A_out (ndarray (m,j)) : m examples, j units
    """
### START CODE HERE ###
    Z=np.matmul(A_in,W)+b
    A_out=g(Z)
### END CODE HERE ###
    return(A_out)
```

In [64]:
```
X_tst = 0.1*np.arange(1,9,1).reshape(4,2) # (4 examples, 2 features)
W_tst = 0.1*np.arange(1,7,1).reshape(2,3) # (2 input features, 3 output features)
b_tst = 0.1*np.arange(1,4,1).reshape(1,3) # (1, 3 features)
A_tst = my_dense_v(X_tst, W_tst, b_tst, sigmoid)
print(A_tst)
```

```
tf.Tensor(
[[0.54735762 0.57932425 0.61063923]
 [0.57199613 0.61301418 0.65248946]
 [0.5962827  0.64565631 0.6921095 ]
 [0.62010643 0.67699586 0.72908792]], shape=(4, 3), dtype=float64)
```

Expected Output

```
[[0.54735762 0.57932425 0.61063923]
 [0.57199613 0.61301418 0.65248946]
 [0.5962827  0.64565631 0.6921095 ]
 [0.62010643 0.67699586 0.72908792]]
```

In [65]:
```
# UNIT TESTS
test_c3(my_dense_v)
```

```
All tests passed!
```

The following cell builds a three-layer neural network utilizing the `my_dense_v` subroutine above.

In [66]:
```
def my_sequential_v(X, W1, b1, W2, b2, W3, b3):
    A1 = my_dense_v(X,  W1, b1, sigmoid)
    A2 = my_dense_v(A1, W2, b2, sigmoid)
    A3 = my_dense_v(A2, W3, b3, sigmoid)
    return(A3)
```

We can again copy trained weights and biases from Tensorflow.

In [67]:
```
W1_tmp,b1_tmp = layer1.get_weights()
W2_tmp,b2_tmp = layer2.get_weights()
W3_tmp,b3_tmp = layer3.get_weights()
```

Let's make a prediction with the new model. This will make a prediction on *all of the examples at once*. Note the shape of the output.

In [68]:
```
Prediction = my_sequential_v(X, W1_tmp, b1_tmp, W2_tmp, b2_tmp, W3_tmp, b3_tmp )
Prediction.shape
```

Out[68]:
```
TensorShape([1000, 1])
```

We'll apply a threshold of 0.5 as before, but to all predictions at once.

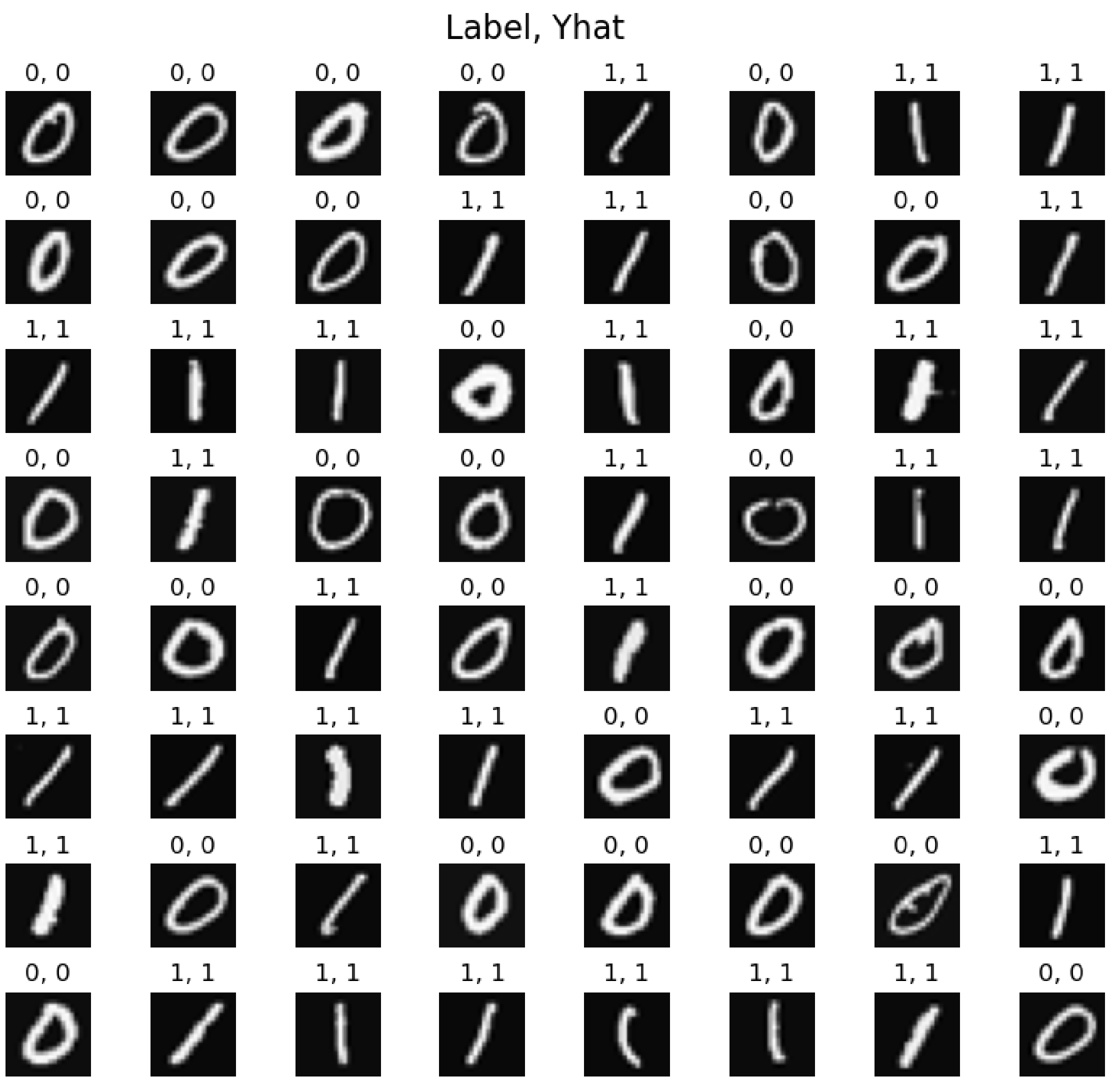
In [69]:
```
Yhat = (Prediction >= 0.5).numpy().astype(int)
print("predict a zero: ",Yhat[0], "predict a one: ", Yhat[500])
```

```
predict a zero:  [0] predict a one:  [1]
```

Run the following cell to see predictions. This will use the predictions we just calculated above. This takes a moment to run.

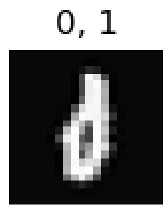
In [70]:

```python
import warnings
warnings.simplefilter(action='ignore', category=FutureWarning)
# You do not need to modify anything in this cell

m, n = X.shape

fig, axes = plt.subplots(8, 8, figsize=(8, 8))
fig.tight_layout(pad=0.1, rect=[0, 0.03, 1, 0.92]) #[left, bottom, right, top]

for i, ax in enumerate(axes.flat):
    # Select random indices
    random_index = np.random.randint(m)

    # Select rows corresponding to the random indices and
    # reshape the image
    X_random_reshaped = X[random_index].reshape((20, 20)).T

    # Display the image
    ax.imshow(X_random_reshaped, cmap='gray')

    # Display the label above the image
    ax.set_title(f"{y[random_index,0]}, {Yhat[random_index, 0]}")
    ax.set_axis_off()
fig.suptitle("Label, Yhat", fontsize=16)
plt.show()
```



You can see how one of the misclassified images looks.

In [71]:
```
fig = plt.figure(figsize=(1, 1))
errors = np.where(y != Yhat)
random_index = errors[0][0]
X_random_reshaped = X[random_index].reshape((20, 20)).T
plt.imshow(X_random_reshaped, cmap='gray')
plt.title(f"{y[random_index,0]}, {Yhat[random_index, 0]}")
plt.axis('off')
plt.show()
```



2.7 Congratulations!

You have successfully built and utilized a neural network.

2.8 NumPy Broadcasting Tutorial (Optional)

In the last example, $\mathbf{Z} = \mathbf{XW} + \mathbf{b}$ utilized NumPy broadcasting to expand the vector $\mathbf{b}$. If you are not familiar with NumPy Broadcasting, this short tutorial is provided.

$\mathbf{XW}$ is a matrix-matrix operation with dimensions $(m, j_1)(j_1, j_2)$ which results in a matrix with dimension (m, j_2). To that, we add a vector $\mathbf{b}$ with dimension $(1, j_2)$. $\mathbf{b}$ must be expanded to be a (m, j_2) matrix for this element-wise operation to make sense. This expansion is accomplished for you by NumPy broadcasting.

Broadcasting applies to element-wise operations.
Its basic operation is to 'stretch' a smaller dimension by replicating elements to match a larger dimension.

More [specifically (https://NumPy.org/doc/stable/user/basics.broadcasting.html)](https://NumPy.org/doc/stable/user/basics.broadcasting.html): When operating on two arrays, NumPy compares their shapes element-wise. It starts with the trailing (i.e. rightmost) dimensions and works its way left. Two dimensions are compatible when

- they are equal, or
- one of them is 1

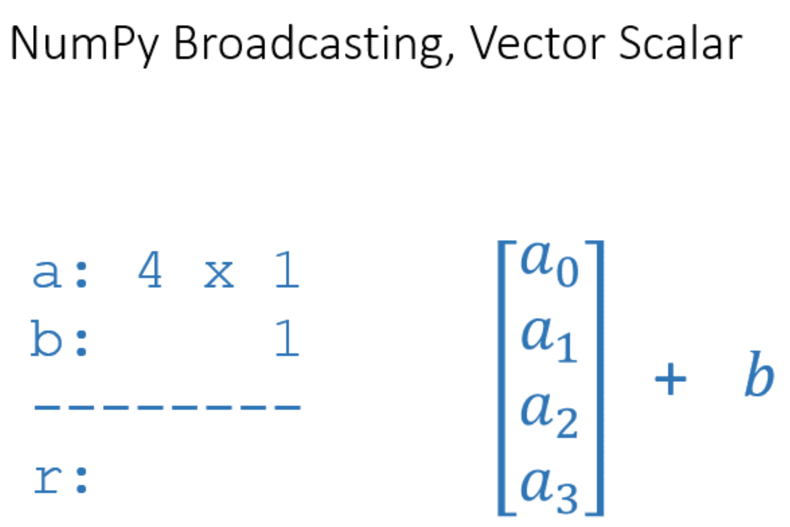
If these conditions are not met, a ValueError: operands could not be broadcast together exception is thrown, indicating that the arrays have incompatible shapes. The size of the resulting array is the size that is not 1 along each axis of the inputs.

Here are some examples:

```
a:        m x 1     1 x n     4 x 1
b:            1         1     1 x 3
-----     -----     -----     -----
result:   m x 1     1 x n     4 x 3
```

Calculating Broadcast Result shape

The graphic below describes expanding dimensions. Note the red text below:



Broadcast notionally expands arguments to match for element wise operations

The graphic above shows NumPy expanding the arguments to match before the final operation. Note that this is a notional description. The actual mechanics of NumPy operation choose the most efficient implementation.

For each of the following examples, try to guess the size of the result before running the example.

In [72]:
```
a = np.array([1,2,3]).reshape(-1,1)  #(3,1)
b = 5
print(f"(a + b).shape: {(a + b).shape}, \na + b = \n{a + b}")
```

```
(a + b).shape: (3, 1),
a + b =
[[6]
 [7]
 [8]]
```

Note that this applies to all element-wise operations:

```
In [73]: a = np.array([1,2,3]).reshape(-1,1)   #(3,1)
         b = 5
         print(f"(a * b).shape: {(a * b).shape}, \na * b = \n{a * b}")
```

```
(a * b).shape: (3, 1),
a * b =
[[ 5]
 [10]
 [15]]
```

$$\boldsymbol{a} = \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix} \qquad a + b = b + a = \begin{array}{cc} & \begin{matrix} [\quad b_0 & b_1 & b_2 \quad] \end{matrix} \\ \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{bmatrix} & \begin{bmatrix} a_0 + b_0 & a_0 + b_1 & a_0 + b_2 \\ a_1 + b_0 & a_1 + b_1 & a_1 + b_2 \\ a_2 + b_0 & a_2 + b_1 & a_2 + b_2 \\ a_3 + b_0 & a_3 + b_1 & a_3 + b_2 \end{bmatrix} \end{array}$$

Row-Column Element-Wise Operations

```
In [74]: a = np.array([1,2,3,4]).reshape(-1,1)
         b = np.array([1,2,3]).reshape(1,-1)
         print(a)
         print(b)
         print(f"(a + b).shape: {(a + b).shape}, \na + b = \n{a + b}")
```

```
[[1]
 [2]
 [3]
 [4]]
[[1 2 3]]
(a + b).shape: (4, 3),
a + b =
[[2 3 4]
 [3 4 5]
 [4 5 6]
 [5 6 7]]
```

This is the scenario in the dense layer you built above. Adding a 1-D vector *b* to a (m,j) matrix.

```
a:  4  x  3
b:        3
-----------
```

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix} \begin{bmatrix} b_0 & b_1 & b_2 \end{bmatrix}$$

B is a 1-D Vector

```
a:  4  x  3
b:  4  x  3
-----------
r:  4  x  3
```

$$\begin{bmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \\ a_{30} & a_{31} & a_{32} \end{bmatrix} \begin{bmatrix} b_0 & b_1 & b_2 \\ b_0 & b_1 & b_2 \\ b_0 & b_1 & b_2 \\ b_0 & b_1 & b_2 \end{bmatrix}$$

Matrix + 1-D Vector

In [ ]:

```

```