Deep Learning for Search And Matching Models
(a.k.a. "DeepSAM")

Jonathan Payne   Adam Rebei   Yucheng Yang

Princeton   Stanford   Zurich & SFI

October 22, 2024

Frankfurt Workshop on Numerical Methods in Macroeconomics

Introduction

- Heterogeneity and aggregate shocks are important in markets with search frictions (e.g. labor and financial markets).
- Most search and matching (SAM) models with heterogeneous agents study:
  1. Deterministic steady state (e.g. Shimer-Smith '00),
  2. Aggregate fluctuations, but make assumptions to eliminate distribution from state space (e.g. "block recursivity" in Menzio-Shi '11, Lise-Robin '17; Lagos-Rocheteau '09).
- We present SAM models as high-dim. PDEs with distribution & agg. shocks as states

  ...and develop a new deep learning method, DeepSAM, to solve them globally.
- We also extend DeepSAM for calibration within efficient computational time.

This Paper

- Develop DeepSAM and apply to canonical search models with aggregate shocks:
  1. Shimer-Smith/Mortensen-Pissarides model with two-sided heterogeneity (today's focus).
  2. Lise-Robin on-the-job search model with worker bargaining power (at end).
  3. Duffie-Garleanu-Pederson OTC model with asset and investor heterogeneity (at end).
- High accuracy in "global" state space (including distribution); efficient compute time for both solution and calibration.
- We can study non-block recursive unemployment dynamics and wage dynamics:
  1. Lise-Robin style block recursive equilibria over-predict unemployment & vacancy IRF.
  2. Low-type worker wages more procyclical, especially those in high-type firms.
  3. Large impact of distribution on aggregates when aggregate shocks affect agents unevenly.
  4. Countercyclical sorting over business cycles; magnitude depends on bargaining power.

Literature

- Deep learning in macro; for incomplete market heterogeneous agent models (HAM) (e.g. Maliar et al '21, Azinovic et al '22, Kahou et al '21, Han-Yang-E '21 "DeepHAM"; Fernández-Villaverde et al '20, Huang '22, Gu-Laurière-Merkel-Payne '23, among others)
  - This paper: search and matching (SAM) models.

	Distribution	Distribution impact on decisions
HAM	Asset wealth and income	Via aggregate prices
SAM	Type (productivity) of agents in two sides of matching	Via matching probability with other types

- Search model with business cycle (e.g. Shimer '05, Menzio-Shi '11, Lise-Robin '17.)
  - This paper: keep distribution in the state vector.
- Integrate deep learning based solution methods with calibration and estimation (e.g., Chen et al '23, Kase et al '23, Friedl et al '23, Duarte & Fonseca '24)
  - This paper: standard calibration practice for quantitative macro.

Table of Contents

Shimer-Smith/Mortensen-Pissarides with Two-sided Heterogeneity

- Continuous time, infinite horizon environment.
- Workers $x \in [0,1]$ with exog density $g_t^w(x)$; Firms $y \in [0,1]$ with $g_t^f(y)$ by free entry:
  - Unmatched: unemployed workers get benefit b; vacant firms produce nothing.
  - Matched: type x worker and type y firm produce output $z_t f(x,y)$.
  - z_t: follows two-state continuous time Markov Chain (can be generalized).
  - Firms can pay entry cost c and draw a firm type y from uniform distribution $[0,1]$. More
- Meet randomly at rate $m(\mathcal{U}_t, \mathcal{V}_t)$, $\mathcal{U}_t$ is total unemployment, $\mathcal{V}_t$ is total vacancies.
- Upon meeting, agents choose whether to accept the match:
  - Match surplus $S_t(x,y)$ divided by generalized Nash bargaining: worker get fraction β.
  - Match acceptance decision $\alpha_t(x,y) = \mathbb{1}\{S_t(x,y) > 0\}$. Match dissolve rate $\delta(x,y,z)$.
- Equilibrium object: $g_t(x,y)$ mass of match $(x,y) \Rightarrow$ unemployed $g_t^u(x)$, vacant $g_t^v(y)$.

Recursive Equilibrium Part I: Unemployed Workers & KFE

- Idiosyncratic state $= x$, Aggregate states $= (z, g(x,y))$.
- Hamilton-Jacobi-Bellman equation for an unemployed worker's value $V^u(x,z,g)$:

$$\rho V^u(x,z,g) = b + \frac{m(z,g)}{\mathcal{U}(z,g)} \int \underbrace{\overbrace{\alpha(x,\tilde{y},z,g)}^{\text{acceptance decision}} \Big(\overbrace{V^e(x,\tilde{y},z,g)}^{\text{employed value}} - V^u(x,z,g)\Big)}_{\text{change of value conditional on match}} \frac{g^v(\tilde{y})}{\mathcal{V}(z,g)} d\tilde{y}$$

$$+ \lambda_{z\tilde{z}}(V^u(x,\tilde{z},g) - V^u(x,z,g)) + \underbrace{D_g V^u(x,z,g)}_{\substack{\text{Frechet derivative:} \\ \text{how change of } g \text{ affects } V}} \cdot \mu^g$$

- Dynamics of $g(x,y)$ is given by Kolmogorov forward equation (KFE):

$$\mu^g(x,y,z,g) := \frac{dg_t(x,y)}{dt} = -\delta(x,y,z)g(x,y) + \frac{m(z,g)}{\mathcal{U}(z,g)\mathcal{V}(z,g)}\alpha(x,y,z,g)g^v(y)g^u(x)$$

HJB for employed worker, vacant firm, producing firm

Recursive Characterization For Equilibrium Surplus

- Surplus from match $S(x, y, z, g) := V^p(x, y, z, g) - V^v(y, z, g) + V^e(x, y) - V^u(x, z, g)$.
- Characterize equilibrium with master equation for surplus: Free entry condition

$$\begin{aligned}
\rho S(x, y, z, g) &= zf(x, y) - \delta(x, y, z)S(x, y, z, g) \\
&\quad - (1-\beta)\frac{m(z, g)}{\mathcal{V}(z, g; S)}\int \alpha(\tilde{x}, y, z, g)S(\tilde{x}, y, z, g)\frac{g^u(\tilde{x})}{\mathcal{U}(z, g)}d\tilde{x} \\
&\quad - b - \beta\frac{m(z, g)}{\mathcal{U}(z, g)}\int \alpha(x, \tilde{y}, z, g)S(x, \tilde{y}, z, g)\frac{g^v(\tilde{y})}{\mathcal{V}(z, g; S)}d\tilde{y} \\
&\quad + \lambda(z)(S(x, y, \tilde{z}, g) - S(x, y, z, g)) + D_g S(x, y, z, g) \cdot \mu^g(z, g)
\end{aligned}$$

- Kolmogorov forward equation (KFE):

$$\frac{dg_t(x, y)}{dt} := \mu^g(x, y, z, g) = -\delta(x, y, z)g(x, y) + \frac{m(z, g)}{\mathcal{U}(z, g)\mathcal{V}(z, g)}\alpha(x, y, z, g)g^v(y)g^u(x)$$

- High-dim PDEs with distribution in state: hard to solve with conventional methods.

Finite Type Approximation

- Approximate $g(x, y)$ on finite types: $x \in \mathcal{X} = \{x_1, \dots, x_{n_x}\}$, $y \in \mathcal{Y} = \{y_1, \dots, y_{n_y}\}$.
- Finite state approximation $\Rightarrow$ analytical (approximate) KFE: $g \approx \{g_{ij}\}_{i \leq n_x, j \leq n_y}$
- Approximated master equation for surplus:

$$
\begin{aligned}
0 = \mathcal{L}^S S(x,y,z,g) &= -(\rho+\delta)S(x,y,z,g) + zf(x,y) - b \\
&- (1-\beta)\frac{m(z,g)}{\mathcal{V}(z,g)} \frac{1}{n_x} \sum_{i=1}^{n_x} \alpha(\tilde{x}_i, y, z, g) S(\tilde{x}_i, y, z, g) \frac{g^u(\tilde{x}_i)}{\mathcal{U}(z,g)} \\
&- \beta \frac{m(z,g)}{\mathcal{U}(z,g)} \frac{1}{n_y} \sum_{j=1}^{n_y} \alpha(x, \tilde{y}_j, z, g) S(x, \tilde{y}_j, z, g) \frac{g^v(\tilde{y}_j)}{\mathcal{V}(z,g)} \\
&+ \lambda(z)(S(x,y,\tilde{z},g) - S(x,y,z,g)) + \sum_{i=1}^{n_x}\sum_{j=1}^{n_y} \partial_{g_{ij}} S(x,y,z,\{g_{ij}\}_{i,j}) \mu^g(\tilde{x}_i, \tilde{y}_j, z, g)
\end{aligned}
$$

DeepSAM Algorithm for Solving the Model

- Approximate surplus by neural network $S(x,y,z,g) \approx \widehat{S}(x,y,z,g;\boldsymbol{\Theta})$. Function form
- Start with initial parameter guess $\boldsymbol{\Theta}^0$. At iteration n with $\boldsymbol{\Theta}^n$:
  1. Generate K sample points, $Q^n = \left\{\left(x_k, y_k, z_k, \{g_{ij,k}\}_{i\leq n_x, j\leq n_y}\right)\right\}_{k\leq K}$.
  2. Calculate the average mean squared error of surplus master equation on sample points:

$$L\left(\boldsymbol{\Theta}^n, Q^n\right) := \frac{1}{K}\sum_{k\leq K}\left|\mathcal{L}^S\widehat{S}\left(x_k, y_k, z_k, \{g_{ij,k}\}_{i\leq n_x, j\leq n_y}\right)\right|^2$$

  3. Update NN parameters with stochastic gradient descent (SGD) method:

$$\boldsymbol{\Theta}^{n+1} = \boldsymbol{\Theta}^n - \zeta^n \nabla_{\boldsymbol{\Theta}} L\left(\boldsymbol{\Theta}^n, Q^n\right)$$

  4. Repeat until $L\left(\boldsymbol{\Theta}^n, Q^n\right) \leq \epsilon$ with precision threshold ϵ.
- Once S is solved, we have α and can solve for worker and firm value functions.

DeepSAM for Calibration and Estimation

- DeepSAM for solving the model (e.g. 59 dimension PDE):

$$\mathcal{L}^{S} S(x, y, z, g) = 0 \tag{1}$$

- Include structural parameters directly in state space: DeepSAM for calibrating the model, solve (e.g. $59 + dim(\Omega)$ dimension PDE):

$$\mathcal{L}^{\widetilde{S}} \widetilde{S}(x, y, z, g, \Omega) = 0 \tag{2}$$

  Ω: structural parameters for internal calibration.

- Dimension of (2) is only marginally higher than (1). Solving (2), we obtain the model solution over a range of parameter space, enabling calibration through simulation.
  - We use simulation data to build a surrogate model mapping parameters to moments.
- Calibration only takes a marginally longer time than solving the model.

Table of Contents

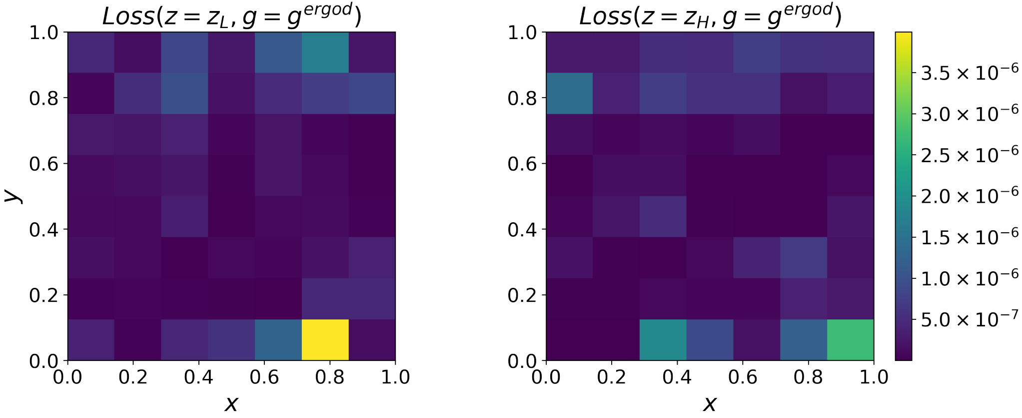
Calibration

Frequency: annual.

Parameter	Interpretation	Value	Target/Source
ρ	Discount rate	0.05	Kaplan, Moll, Violante '18
δ	Job destruction rate	0.2	BLS job tenure 5 years
ξ	Extreme value distribution for α choice	2.0	
$f(x,y)$	Production function for match (x,y)	$0.6 + 0.4\left(\sqrt{x}+\sqrt{y}\right)^2$	Hagedorn et al '17
β	Surplus division factor	0.72	Shimer '05
c	Entry cost	4.86	Steady state $\mathcal{V}/\mathcal{U} = 1$
$z, \tilde{z}$	TFP shocks	1 ± 0.015	Lise Robin '17
$\lambda_z, \lambda_{\tilde{z}}$	Poisson transition probability	0.08	Shimer '05
$\delta, \tilde{\delta}$	Separation shocks	0.2 ± 0.02	Shimer '05
$\lambda_\delta, \lambda_{\tilde{\delta}}$	Poisson transition probability	0.08	Shimer '05
$m(\mathcal{U},\mathcal{V})$	Matching function	$\kappa \mathcal{U}^\nu \mathcal{V}^{1-\nu}$	Hagedorn et al '17
ν	Elasticity parameter for meeting function	0.5	Hagedorn et al '17
κ	Scale parameter for meeting function	5.4	Unemployment rate 5.9%
b	Worker unemployment benefit	0.5	Shimer '05
n_x	Discretization of worker types	7	
n_y	Discretization of firm types	8	

Numerical Performance: Accuracy I Calibration

- Mean squared loss as a function of type in the master equations of S (at ergodic g).



Numerical Performance: Accuracy II Calibration

- Compare steady state solution without aggregate shocks to solution using conventional methods.

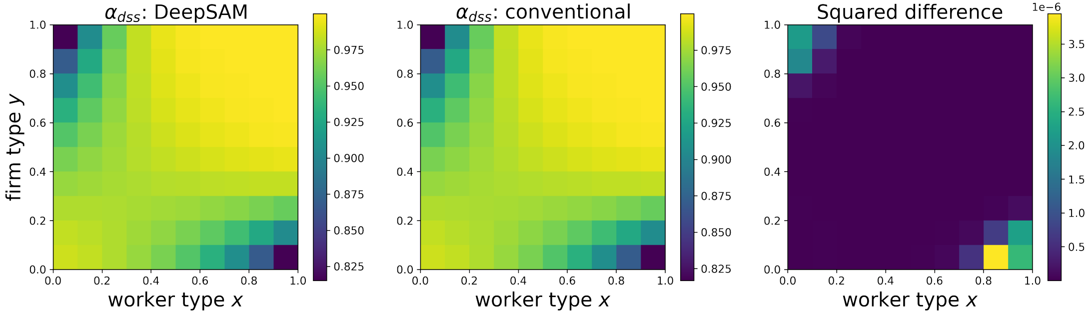


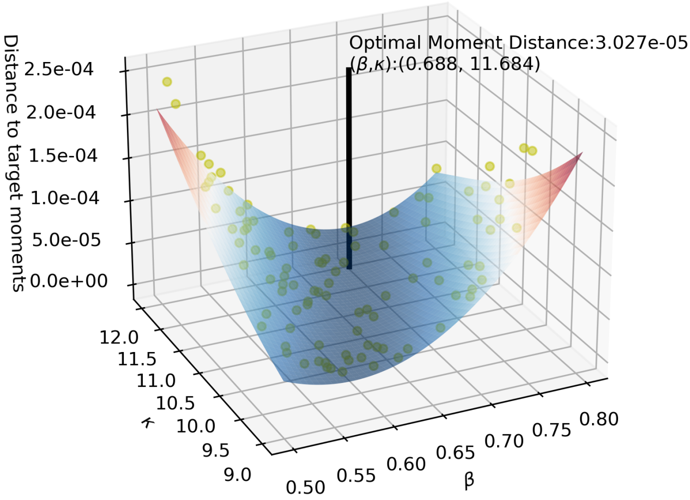
Figure: Comparison with steady-state solution

Comparison for discrete α

Numerical Performance: Speed

- Solving the 59-dimensional surplus function takes 57 minutes on an A100 GPU, which is easily accessible to everyone on Google Colab.
- To our knowledge, it's infeasible to use any conventional methods to solve the problem globally with 59 dimensions.
- Calibrating 3 parameters based on the solution over 40,000 parameter combinations takes 90% more computational time than solving the model over 1 given parameter combination.

Calibration: Visualization in 2D



Target moment: $\mathbb{E}[U], \mathbb{E}[V]$. Parameter: matching efficiency κ, worker bargaining power β.

Table of Contents

Q1. How do block recursive models restrict aggregate dynamics?

(IRF to negative TFP shock for block recursive vs other calibrations)

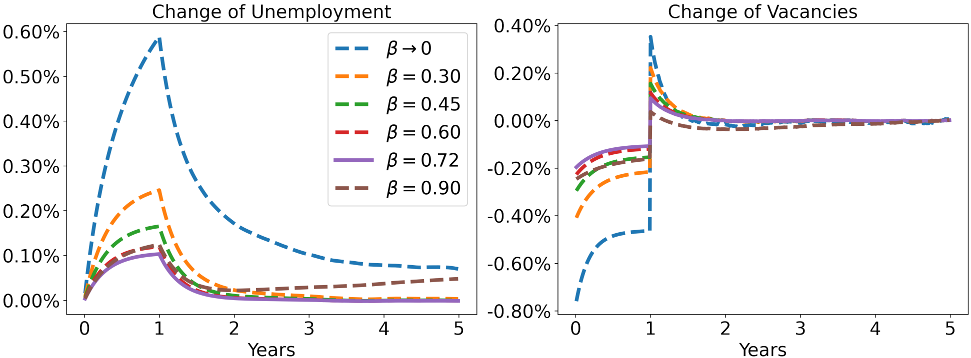
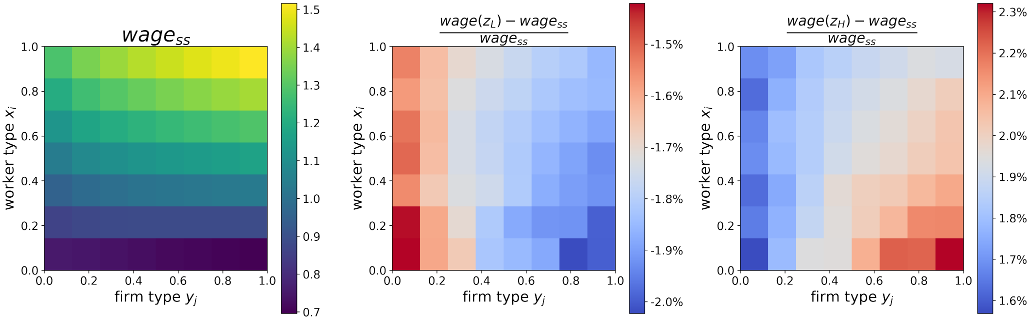


Figure: IRF with different β's vs. block-recursive model with $\beta = 0$

- By assuming firms get all surplus, block recursive models predict high U_t response (because firms' vacancy posting is very elastic to aggregate shocks).

Q2. Are wage dynamics heterogeneous across distribution?

- In Lise-Robin: "wages cannot be solved for exactly... need to solve worker values where the distribution of workers across jobs is a state variable."
- DeepSAM can solve wage dynamics with rich heterogeneity.
- Low-type worker wages more procyclical, especially those in high-type firms.



Q3. Is the feedback from g to α important? Evidence from COVID

- Workers and firms have heterogeneous exposure to aggregate shocks.
- We calibrate separation rate $\delta(x, y, z)$ to match the heterogeneous employment effect of COVID on different workers/firms (Cajner et al., 2020).
- Study aggregate dynamics with and without distribution feedback to agent decision:

$$\text{Full dynamics: } \frac{dg_t(x,y)}{dt} = -\delta(x,y,z_t)g_t(x,y) + \frac{m_t(z,g_t)}{\mathcal{U}_t(g_t)\mathcal{V}_t(g_t)}\alpha(x,y,z_t,g_t)g_t^u(x)g_t^v(y)$$

$$\text{No distribution feedback: } \frac{dg_t(x,y)}{dt} = -\delta(x,y,z_t)g_t(x,y) + \frac{m_t(z,g_t)}{\mathcal{U}_t(g_t)\mathcal{V}_t(g_t)}\alpha(x,y,z_t,g^{\text{ergodic}})g_t^u(x)g_t^v(y)$$

A3. Feedback from g to α matter for asymmetric shocks.

$$\text{Full dynamics: } \frac{dg_t(x,y)}{dt} = -\delta(x,y,z_t)g_t(x,y) + \frac{m_t(z,g_t)}{\mathcal{U}_t(g_t)\mathcal{V}_t(g_t)}\alpha(x,y,z_t,g_t)g_t^u(x)g_t^v(y)$$

$$\text{No distribution feedback: } \frac{dg_t(x,y)}{dt} = -\delta(x,y,z_t)g_t(x,y) + \frac{m_t(z,g_t)}{\mathcal{U}_t(g_t)\mathcal{V}_t(g_t)}\alpha(x,y,z_t,g^{\text{ergodic}})g_t^u(x)g_t^v(y)$$

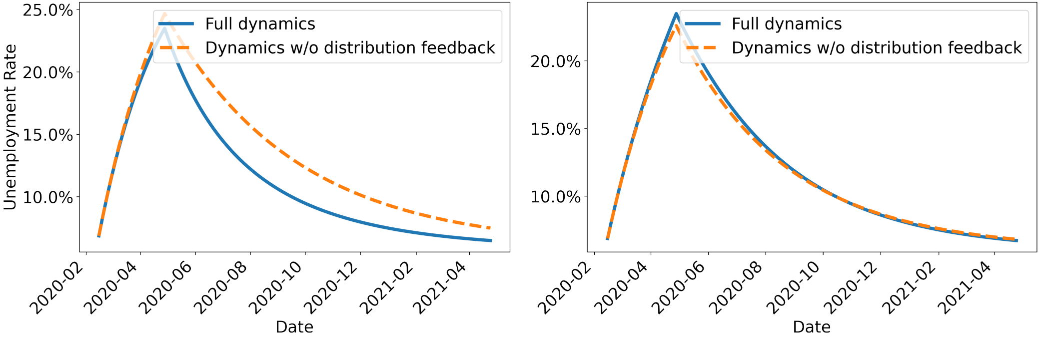


Figure: Unemployment U_t after (left) true COVID shock, (right) counterfactual "symmetric" shock.

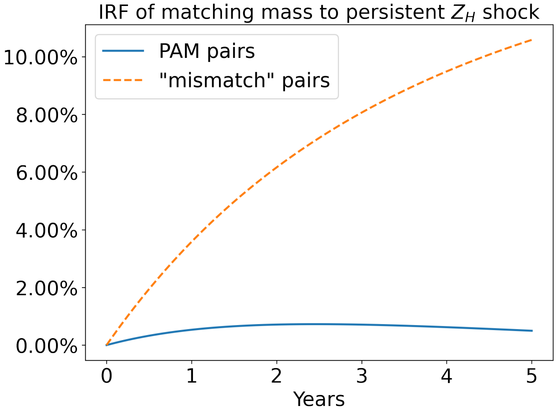
Table of Contents

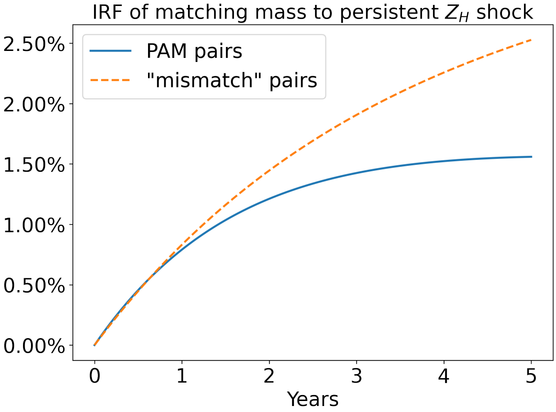
More Applications in the Paper

1. SAM model with on-the-job search and endogenous separation. details
   - Similar to Lise-Robin '17, but allow for $\beta \in (0, 1)$.
   - We also do not assume that vacancies are destroyed if not filled. Vacancy is a stock variable.
2. OTC financial market with heterogeneous investors, different bond maturities, and aggregate default risk. details

Q4. How do agents sort over the business cycle (On-the-Job Search)?

- Countercyclicality of sorting depends on bargaining power.





Left: $\beta = 0$ (Lise-Robin '17). Right: $\beta = 0.72$ (benchmark).

Conclusion and Future Work

- We develop an integrated global solution and calibration method, DeepSAM, to search and matching models with heterogeneity and aggregate shocks.
- We apply DeepSAM to canonical labor search models, and find important interaction between heterogeneity and aggregate shocks that we cannot study before.
- A powerful tool to be combined with rich data of heterogeneous workers & firms over business cycles!
- More applications:
  - Spatial models with aggregate uncertainty.
  - Network models with aggregate uncertainty.

Thank You!

Deep Learning for Economic Models

- Deep learning has been successful in high-dimensional scientific computing problems.
- We can use deep learning to solve high-dim value & policy functions in economics:
  1. Use deep neural networks to approximate value function $V : \mathbb{R}^N \to \mathbb{R}$

$$V(\mathbf{x}) \approx \mathcal{L}^P \circ \cdots \circ \mathcal{L}^p \circ \cdots \circ \mathcal{L}^1(\mathbf{x}), \quad \mathbf{x}\text{: high-dim state vector,}$$
$$\mathbf{h}_p = \mathcal{L}^p(\mathbf{h}_{p-1}) = \sigma\big(\mathbf{W}_p \mathbf{h}_{p-1} + \mathbf{b}_p\big), \quad \mathbf{h}_0 = \mathbf{x},$$

σ : element-wise nonlinear fn, e.g. Tanh$(\cdot)$. Want to solve unknown parameters $\boldsymbol{\Theta} = \{\mathbf{W}_p, \mathbf{b}_p\}_p$.

  2. Cast high-dim function into a loss function, e.g. Bellman equation residual.
  3. Optimize unknown parameters, $\boldsymbol{\Theta}$, to minimize average loss on a "global" state space, using stochastic gradient descent (SGD) method.

- Similar procedure to polynomial "projection", but more efficient in practice. back

Methodology Q & A

- ***Q.*** *What about dimension reduction?*
  - Krusell-Smith '98 suggest approximating distribution by mean.
  - For random search, not clear what moment enables approximation of:

$$\int \alpha(\tilde{x}, y, z, g) S(\tilde{x}, y, z, g) \frac{g^u(\tilde{x})}{\mathcal{U}(z, g)} d\tilde{x}, \quad \text{and} \quad \int \alpha(x, \tilde{y}, z, g) S(x, \tilde{y}, z, g) \frac{g^v(\tilde{y})}{\mathcal{V}(z, g)} d\tilde{y}$$

- ***Q.*** *How do we choose where to sample?*
  - We start by drawing distributions "between" steady states for different fixed z.
  - Can move to ergodic sampling once error is small.
  - Can increase sampling in regions of the state space where errors are high.
- ***Q.*** *Why are SAM models hard to solve?*
  - Compared to PINNs, we have feedback between agent optimization and distribution.
  - Difficult when feedback is strong & $\widehat{S}(x, y, z, g; \boldsymbol{\Theta})$ has sharp curvature. Use "homotopy".

Table of Contents

Comparison to Other Heterogeneous Agent Search Models

- Lise-Robin '17: sets $\beta = 0$ (and other conditions, including Postal-Vinay Robin style Bertrand competition for workers searching on-the-job)

$$S(x, y, z, g) = S(x, y, z), \quad \alpha(x, y, z, g) = \alpha(x, y, z)$$

- Menzio-Shi '11: competitive search (directed across a collection of sub-markets):

$$S(x, y, z, g) = S(x, y, z)$$

- We look for a solution for S and α in terms of the distribution g.

Modification 1: Finite Type Approximation

- Approximate $g(x, y)$ on finite types: $x \in \mathcal{X} = \{x_1, \ldots, x_{n_x}\}$, $y \in \mathcal{Y} = \{y_1, \ldots, y_{n_y}\}$.
- Finite state approximation $\Rightarrow$ analytical (approximate) KFE: $g \approx \{g_{ij}\}_{i \le n_x, j \le n_y}$
- Approximated master equation for surplus:

$$
\begin{aligned}
0 = \mathcal{L}^S S(x, y, z, g) &= -(\rho + \delta) S(x, y, z, g) + z f(x, y) - b \\
&- (1 - \beta) \frac{m(z, g)}{\mathcal{V}(z, g)} \frac{1}{n_x} \sum_{i=1}^{n_x} \alpha(\tilde{x}_i, y, z, g) S(\tilde{x}_i, y, z, g) \frac{g^u(\tilde{x}_i)}{\mathcal{U}(z, g)} \\
&- \beta \frac{m(z, g)}{\mathcal{U}(z, g)} \frac{1}{n_y} \sum_{j=1}^{n_y} \alpha(x, \tilde{y}_j, z, g) S(x, \tilde{y}_j, z, g) \frac{g^v(\tilde{y}_j)}{\mathcal{V}(z, g)} \\
&+ \lambda(z) (S(x, y, \tilde{z}, g) - S(x, y, z, g)) + \sum_{i=1}^{n_x} \sum_{j=1}^{n_y} \partial_{g_{ij}} S(x, y, z, \{g_{ij}\}_{i,j}) \mu^g(\tilde{x}_i, \tilde{y}_j, z, g)
\end{aligned}
$$

Modification 2: Approximate Discrete Choice

- In the original model,

$$\alpha(x, y, z, g) = \mathbb{1}\{S(x, y, z, g) > 0\}$$

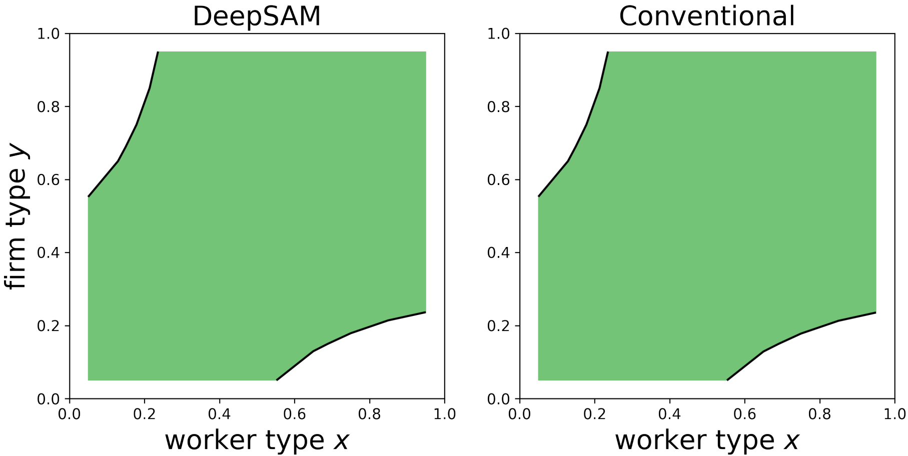
- Discrete choice $\alpha \Rightarrow$ discontinuity of $S(x, y, z, g)$ at some g.

- To ensure master equation well defined & NN algorithm works, we approximate with

$$\alpha(x, y, z, g) = \frac{1}{1 + e^{-\xi S(x,y,z,g)}}$$

- Interpretation: logit choice model with utility shocks $\sim$ extreme value distribution. ($\xi \to \infty \Rightarrow$ discrete choice α.)

DeepSAM vs Conventional method at DSS: discrete case



back

Free Entry Condition

- Firms can pay entry cost c and draw a firm type y from uniform distribution $[0, 1]$:

$$c = \mathbb{E}[V_t^v] = \int V^v(\tilde{y}, z, g)d\tilde{y}. \tag{3}$$

- As the matching function is homothetic $\frac{m(z_t, g_t)}{\mathcal{V}_t} = \widehat{m}\left(\frac{\mathcal{V}_t}{\mathcal{U}_t}\right)$, combining free entry condition with HJB equation for V^v gives:

$$\widehat{m}\left(\frac{\mathcal{V}_t}{\mathcal{U}_t}\right) = \frac{\rho c}{\int\int \alpha(\tilde{x}, \tilde{y})\frac{g_t^u(\tilde{x})}{\mathcal{U}_t}(1-\beta)S_t(\tilde{x}, \tilde{y})d\tilde{x}d\tilde{y}} \Rightarrow \quad \mathcal{V}_t = \mathcal{U}_t\widehat{m}^{-1}(\cdots) \tag{4}$$

  where $g_t^u = g_t^w - \int g_t^m(x, y)dy$ and so the RHS can be computed from g_t^m and S_t.
- $g_t^f = \mathcal{V}_t + \mathcal{P}_t$, where $\mathcal{V}_t$ and $\mathcal{P}_t$ can be expressed in terms of g and S.
- With free entry condition, the master equation expression for surplus takes the same form as without free entry, but with different expressions of $g^f(y)$.

back

Recursive Equilibrium Part II: Other Equations

- Hamilton-Jacobi-Bellman equation (HJBE) for employed worker's value $V^e(x, y, z, g)$:

$$\rho V^e(x,y,z,g) = w(x,y,z,g) + \delta(x,y,z)\left(V^u(x,z,g) - V^e(x,y,z,g)\right) + \lambda_{z\tilde{z}}(V^e(x,y,\tilde{z},g) - V^e(x,y,z,g)) + D_g V^e(x,y,z,g) \cdot \mu^g$$

- HJBE for a vacant firm's value $V^v(y, z, g)$:

$$\rho V^v(y,z,g) = \frac{m(z,g)}{\mathcal{V}(z,g)} \int \alpha(\tilde{x},y,z,g)(V^p(\tilde{x},y,z,g) - V^v(y,z,g))\frac{g^u(\tilde{x})}{\mathcal{U}(z,g)} d\tilde{x} + \lambda_{z\tilde{z}}(V^v(x,\tilde{z},g) - V^v(x,z,g)) + D_g V^v(y,z,g) \cdot \mu^g$$

- HJBE for a producing firm's value $V^p(x, y, g)$:

$$\rho V^p(x,y,z,g) = zf(x,y) - w(x,y,z,g) + \delta(x,y,z)(V^v(y,z,g) - V^p(x,y,z,g)) + \lambda_{z\tilde{z}}(V^p(x,y,\tilde{z},g) - V^p(x,y,z,g)) + D_g V^p(x,y,z,g) \cdot \mu^g$$

back

Variation in α as the Distribution Varies

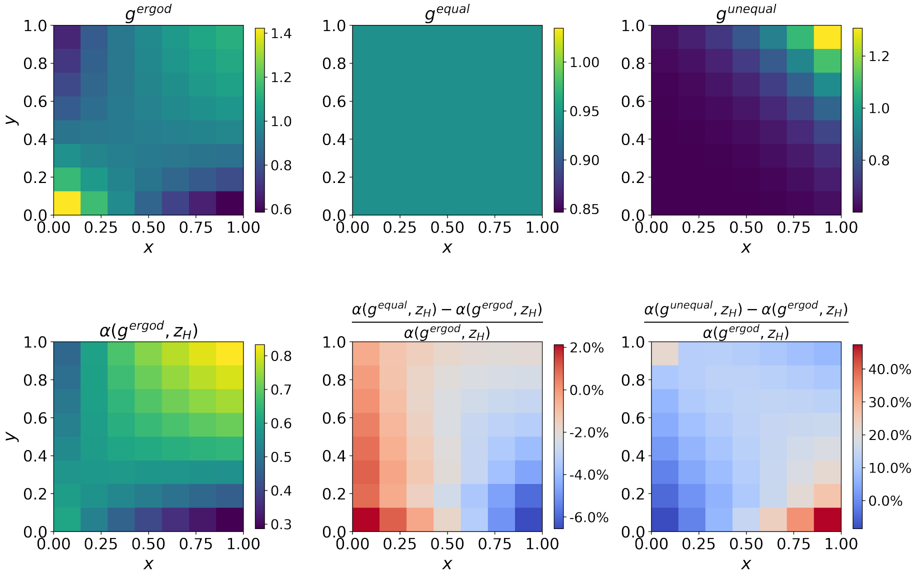


Table of Contents

On-The-Job Search: Environment Features

- Same worker types, firm types, and production function.
- Now all workers search; meeting rate is $m(\mathcal{W}_t, \mathcal{V}_t)$; total search effort is $\mathcal{W}_t := \mathcal{U}_t + \phi \mathcal{E}_t$
- Terms of trade when a vacant $\tilde{y}$-firm meets:
  - Unemployed x-worker: Nash bargaining where workers get surplus fraction β,
  - Worker in (x, y) match: Nash bargaining over incremental surplus.

    If $S_t(x, \tilde{y}) > S_t(x, y)$, worker moves to firm $\tilde{y}$ and gets additional $\beta(S_t(x, \tilde{y}) - S_t(x, y))$.
- Endogenous separation $\alpha_t^b(x, y) = 1$ when $S_t(x, y) < 0$.

Recursive Characterization For Equilibrium Surplus

- Can characterize equilibrium with the master equation for the surplus:

$$
\begin{aligned}
\rho S(x,y,z,g) &= zf(x,y) - (\delta + \alpha^b(x,y,z,g))S(x,y,z,g) \\
&- \frac{m(z,g)}{\mathcal{W}(z,g)\mathcal{V}(z,g)}\Bigg[(1-\beta)\int \alpha(\tilde{x},y,z,g)S(\tilde{x},y,z,g)g^u(\tilde{x})d\tilde{x} \\
&-\phi(1-\beta)\int \alpha^p(\tilde{x},y,\tilde{y},z,g)(S(\tilde{x},y,z,g) - S(\tilde{x},\tilde{y},z,g))g(\tilde{x},\tilde{y})d\tilde{x}d\tilde{y} \\
&+\phi\beta\int \alpha^p(x,\tilde{y},y,z,g)S(x,y,z,g)g^v(\tilde{y})d\tilde{y}\Bigg] \\
&- b - \beta\frac{m(z,g)}{\mathcal{W}(z,g)\mathcal{V}(z,g)}\int \alpha(x,\tilde{y},z,g)S(x,\tilde{y},z,g)g^v(\tilde{y})d\tilde{y} \\
&+ \lambda(z)(S(x,y,\tilde{z},g) - S(x,y,z,g)) + D_g S(x,y,z,g)\cdot\mu^g(z,g)
\end{aligned}
$$

where:

$$
\alpha^p(\tilde{x},y,\tilde{y},z,g) := \mathbb{1}\{S(\tilde{x},y,z,g) \geq S_t(\tilde{x},\tilde{y},z,g) \geq 0\}
$$

KFE

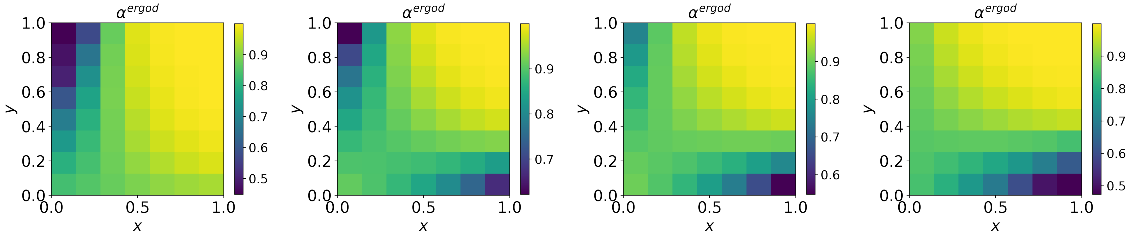
On-the-job-search: KFE

- The KFE becomes:

$$
\begin{aligned}
dg_t^m(x,y) = &-\delta g_t^m(x,y)dt \\
&-\phi\frac{m(\mathcal{W}_t,\mathcal{V}_t)}{\mathcal{W}_t\mathcal{V}_t}g_t^m(x,y)\int \alpha_t^p(x,y,\tilde{y})g_t^v(\tilde{y})d\tilde{y}dt \\
&+\frac{m(\mathcal{W}_t,\mathcal{V}_t)}{\mathcal{W}_t\mathcal{V}_t}\alpha_t(x,y)g_t^u(x)g_t^v(y)dt \\
&+\phi\frac{m(\mathcal{W}_t,\mathcal{V}_t)}{\mathcal{W}_t\mathcal{V}_t}\int \alpha_t^p(\tilde{x},\tilde{y},y)g_t^v(y)\frac{g_t^m(\tilde{x},\tilde{y})}{\mathcal{E}_t}d\tilde{x}d\tilde{y}dt
\end{aligned}
$$

back

Worker Bargaining Power Influences Assortative Matching

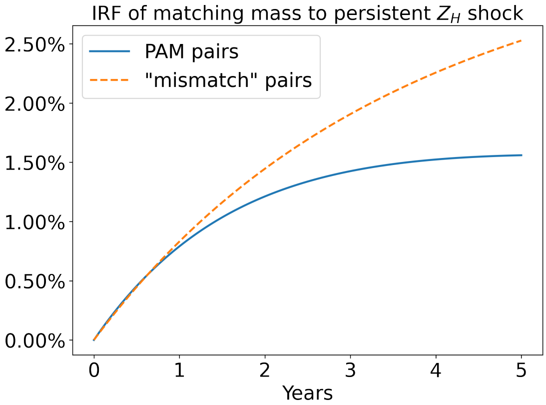


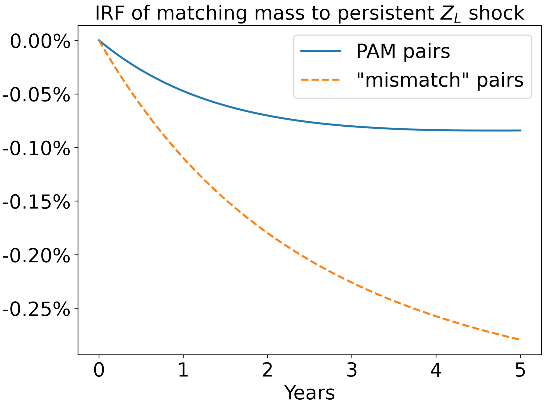
Sorting at the ergodic distribution for different worker bargaining power β. Left to right $\beta = 0$ (Lise-Robin '17), $0.5, 0.72$ (benchmark), 1.

Additional parameter calibration: $\phi = 0.2$.

Sorting Over Business Cycles

- Study how "mismatch" changes over the business cycle. back





"PAM" pairs: pairs where x & y are close. "Mismatch": pairs where x & y are not close.

Table of Contents

Environment: Setting, Bonds, and Households

- Continuous time, infinite horizon environment.
- There are many bonds, $k \in \{1, \ldots, K\}$, in positive net supply s_k:
  - Every bond pays the same dividend $\delta > 0$.
  - Bond k matures at rate $1/\tau_k$ (so it has average maturity τ_k).
- Populated by a unit-mass continuum of infinitely-lived and risk-neutral investors:
  - An investor can hold either zero or one share of at most one type of asset.
  - Investor type $j \in \{1, \ldots, J\}$ gets flow utility $\delta - \psi(j, k)$ from holding bond k.
  - Agents switch from type i to j at rate $\lambda_{i,j}$.
- Aggregate (default) state $z \in \{z_1, \ldots, z_n\}$, switches at rate $\zeta_{z,z'}$.
  At state z, asset k pays a fraction $\phi(k, z)$ of the coupon and the principal.

Distribution and Bargaining

- An investor's state is made up of her holding cost $j \in \{1, \ldots, J\}$ and her ownership status, for each asset type $k \in \{1, \ldots, K\}$ (owner o or non-owner n). Hence the set of investor idiosyncratic states is:

$$A = \{1n, 2n, \ldots, Jn, 1o1, \ldots, 1oK, 2o1, \ldots 2oK, Jo1, \ldots, JoK\} \tag{5}$$

- The rate of contact between investors with states a and b is:

$$\mathcal{M}_{a,b} = \kappa_{a,b} g_a g_b \tag{6}$$

- Agents a, b engage in Generalized Nash bargaining with bargaining power $\beta_{a,b}$.

Value Function: Non-Owners

- The value function for non-owner with type i, $V(in, g, z)$, is given by:

$$\begin{aligned}\rho_i V(in, g, z) = & \sum_a \kappa_{in,a} \alpha(in, a, g, z) \beta_{in,a} S(in, a, z, g) \\ & + \sum_k \xi_{i,k} (V(iok, g, z) - V(in, g, z)) \\ & + \sum_{j \neq i} \lambda_{i,j} (V(jn, g, z) - V(in, g, z)) \\ & + \sum_{z'} \zeta_{z,z'} (V(in, g, z') - V(in, g, z)) + \sum_{a \in A} \partial_{g_a} V(in, g, z) \mu^g(a, z)\end{aligned}$$

where $\alpha(in, jok, g, z)$ is an indicator for whether the surplus from the trade is positive $S(in, jok, g, z) > 0$ and the trade is accepted upon matching.

Value Function: Owners

- Value function for an investor of type i holding asset k, $V(iok, g, z)$, is given by:

$$\begin{aligned}
\rho_i V(iok, g, z) = \delta\phi(k, z) - \psi(i, k) + \frac{1}{\tau_k}(V(in, g, z) + \pi(k, z) - V(iok, g, z)) \\
+ \sum_a \kappa_{iok,a}\alpha(iok, a, g, z)g_a\beta_{iok,a}S(iok, a, g, z) \\
+ \sum_{j\neq i} \lambda_{i,j}(V(jok, g, z) - V(iok, g, z)) \\
+ \sum_{z'} \zeta_{z,z'}(V(iok, g, z') - V(iok, g, z)) + \sum_{a\in A} \partial_{g_a} V(iok, g, z)\mu^g(a, z).
\end{aligned}$$

Parameter Values: Holding Costs

Agent Type (i)	Maturity (τ): $\tau_1 = 0.25$	$\tau_2 = 1.0$	$\tau_3 = 5$	$\tau_4 = 10$
A	$\delta\phi(1, z)$	$\delta\phi(2, z)$	$\delta\phi(3, z)$	$\delta\phi(4, z)$
B	0.02	0.02	0.02	0.02
C	0.0	0.0	0.0	0.0
D	0.02	0.02	0.01	0.00

Table: Holding costs: $\psi(i, \tau)$.

Parameter Values: Switching Rates

Parameter Values: Participation in Primary Market

		Maturity (τ)			
		$\tau_1 = 0.25$	$\tau_2 = 1.0$	$\tau_3 = 5$	$\tau_4 = 10$
Agent Type (i)	A	ξ_1	ξ_2	ξ_3	ξ_4
	B	–	–	–	–
	C	–	–	–	–
	D	–	–	–	–

Table: Primary market participation: $\xi(i, \tau)$.

Parameter Values: Mathing Rates and Bargaining

$$\kappa_{a,b} = \begin{cases} 50, & \text{if } (a,b) = (in, jok) \text{ and } i, j \neq A, \\ 50, & \text{if } (a,b) = (iok, jok) \text{ and } i, j \neq A, \\ 75, & \text{if } (a,b) = (in, Aok) \text{ and } i \neq A, \\ 0, & \text{if } (a,b) = (iok, Aol) \text{ and } \forall i, \\ 0, & \text{if } (a,b) = (in, jn) \text{ and } \forall i, j, \end{cases} \tag{7}$$

$$\beta_{a,b} = \begin{cases} 0.5, & \text{if } (a,b) = (in, jok) \text{ and } i, j \neq A, \\ 0.5, & \text{if } (a,b) = (iok, jol) \text{ and } i, j \neq A, \\ 0.05, & \text{if } (a,b) = (in, Aok) \text{ and } i, j \neq A, \end{cases} \tag{8}$$

Parameter Values: Other Values

Parameter	Interpretation	Value	Target/Source
ρ	Discount rate	0.05	Chen at al. (2017)
δ	Bond Coupon Rate	0.01	
Aggregate State: $z \in \{z_L, z_M, z_H\}$			
$\phi(z)$	Coupon haircut	$(0.986, 0.991, 0.997)$	Chen at al. (2017)
$\pi(z)$	Principal haircut	$(0.986, 0.991, 0.997)$	Chen at al. (2017)
$\zeta_{M,L}$, $\zeta_{M,H}$	Rate from 2 to 1 and 2 to 3	0.1	Crisis every 10 years
$\zeta_{L,M}$, $\zeta_{H,M}$	Rate from 1 to 2 and 3 to 2	0.5	Average crisis duration 2 years

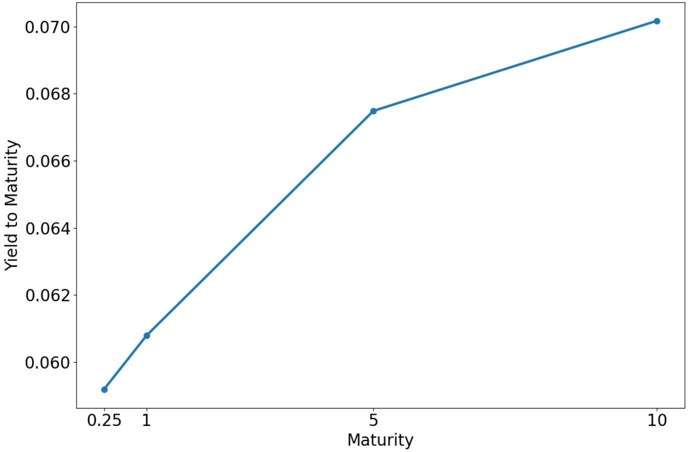
Table: Economic Parameters.

Neural Network Parameter Values

Parameter	Value
Number of layers	8
Neurons per layer	100
Activation function	$\mathrm{GELU}(\cdot)$
Initial learning rate	10^{-4}
Final learning rate	10^{-6}
Initial sample size per epoch	256
Sample size per epoch	1024
Convergence threshold for target calibration	10^{-6}

Table: Neural network parameters

Endogenous Yield Curve back



Yeild IRF back