

# Redistributive Inflation and Optimal Monetary Policy

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## Abstract

Inflation has heterogeneous impacts on households, which then affects optimal monetary policy design. I study optimal monetary policy rules in a quantitative heterogeneous agent New Keynesian (HANK) model where inflation has redistributive effects on households through their different (1) consumption baskets, (2) nominal wealth positions, and (3) earnings elasticities to business cycles. I parameterize the model based on the empirical analysis of these channels using the most recent data. Unlike in representative agent models, a utilitarian central bank should adopt an asymmetric monetary policy rule that is accommodative towards inflation and aggressive towards deflation. Specifically, by accommodating stronger demand and higher inflation, the central bank benefits low-income and low-wealth households through nominal debt devaluation and higher earnings growth.

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# 1 Introduction

Since its global surge in 2021, inflation has again become a central topic of discussion among economists, policymakers, and the public. The conventional view is that the central bank should attack inflation aggressively to bring it back to the target. However, recent empirical literature has established that inflation is not a neutral shock across households. At the same time, central banks around the world, including the US Federal Reserve and European Central Bank, have announced that they are pursuing an “inclusive goal” that places emphasis on the “low- and medium-income households” in their monetary policy design (Powell, 2020, 2021; Schnabel, 2021; Ioannidis et al., 2021). A key question is, therefore, how would the redistributive consequences of inflation affect the design of optimal monetary policy?

In this paper, I address this question by studying optimal nonlinear monetary policy *rules* in a quantitative equilibrium model featuring three redistributive channels of inflation. First, empirical evidence suggests that inflation could hurt lower-income households more because the prices of their consumption baskets increase more than those of higher-income households. Second, low-income households might benefit from inflation through the devaluation of nominal debt. Third, these households usually experience higher earnings growth in a strong economy that typically arises during inflationary episodes. To understand the net social welfare effect of different monetary policy rules, I develop a quantitative heterogeneous agent New Keynesian (HANK) model in which households have different consumption baskets, different portfolio positions in nominal assets, and different exposures of earnings growth to the business cycles. I show that a utilitarian central bank should adopt an asymmetric monetary policy rule that is accommodative towards inflation and aggressive towards deflation. As inflationary shocks benefit low-income households through nominal debt devaluation and higher earnings growth, the utilitarian central bank should react accommodatively. By contrast, deflationary shocks hurt low-income households, and the central bank should react more aggressively.

The paper proceeds in three steps. First, I provide a systematic revisiting of the empirical evidence on the redistributive channels of inflation. Second, I develop a two-sector quantitative HANK model that contains all these channels. Third, I use the model to evaluate social welfare and solve optimal monetary policy rules for the central bank.

Empirically, I use the most recent available data to systematically revisit the evidence on three redistributive channels of inflation. First, inflation is heterogeneous across products, and households differ in their consumption bundles, and thus in the inflation rates they experience through the *expenditure channel* (Cravino, Lan, and Levchenko, 2020). Using the Consumer Expenditure Survey and item-level price data until 2021, I construct monthly

consumer price indices for each income percentile in the US. I find that low- and middle-income households experience higher inflation after expansionary shocks, as they have higher expenditure shares in products with more flexible prices. Second, through the *revaluation channel*, which is also known as the *Fisher channel* (Fisher, 1933; Doepke and Schneider, 2006), unexpected inflation erodes the real value of nominal assets and liabilities and redistributes from the high-income net nominal creditors to net nominal borrowers. Using data from the 2019 Survey of Consumer Finances, I find that low- and middle-income households are the net nominal borrowers who might benefit from such inflationary episodes. Third, through the *earnings channel*, inflation shocks transmit to aggregate income growth via the Phillips curve, and households have different earnings growth elasticities to aggregate income (Guvenen, Schulhofer-Wohl, Song, and Yogo, 2017). I provide up-to-date estimates of households' earnings elasticities along the income distribution with data from Blanchet et al. (2022). In particular, I find that households at the bottom or very top of the income distribution have earnings growth that is more sensitive to aggregate income and inflation than households in the middle-income range.<sup>1</sup>

Next, I develop a two-sector heterogeneous agent New Keynesian (HANK) model for quantitative analysis. The model contains all three redistributive channels of inflation. First, households have non-homothetic preferences over goods produced in sectors with different levels of nominal rigidity. The expenditure share over products with more rigid prices increases with income, causing low-income households to experience a greater price increase following an inflationary shock. Second, households save and borrow in a nominal bond and cannot fully insure themselves against idiosyncratic shocks to their labor productivity. This generates a dispersion of nominal asset positions across households, and inflation redistributes wealth through the revaluation channel. Third, the earnings growth of households in different income groups has heterogeneous elasticity to the business cycle. I calibrate the parameters in the HANK model to match the key empirical moments of the US economy, including the above empirical facts on redistributive inflation.

I use the model to evaluate social welfare under different monetary policy *rules* adopted by the central bank. From the seminal work by Kydland and Prescott (1977) and Taylor (1993), the study of simple policy *rules* has generated interest in policymakers and economists. My study of optimal policy rules in HANK also echoes the tradition of optimal policy rule literature in representative agent New Keynesian (RANK) models (Taylor, 1993; Woodford, 2001; Schmitt-Grohé and Uribe, 2007). In particular, I focus on a family of nonlinear Taylor rules that set the nominal interest rate as a function of inflation and output deviations from

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<sup>1</sup>In a model with monetary policy rules, the revaluation channel only works with unexpected inflation, while the other two channels work with either expected or unexpected inflation.

the steady state. This nonlinearity means that the response coefficients against positive and negative inflation could differ. I solve for the optimal monetary policy rule by searching for the optimal nonlinear Taylor rule coefficients that maximize the expected social welfare. Under each policy rule and aggregate shock path, I solve for the perfect foresight nonlinear transition dynamics for all households and evaluate the social welfare defined as the present value of Pareto-weighted utility flow along the transition dynamics. Then I take the expectation over the aggregate shock processes to obtain the expected social welfare under each policy rule. The aggregate shocks in this economy are demand shocks akin to the fluctuations in the households' discount factor. I focus on demand shocks ([Smets and Wouters, 2007](#); [Bilbiie et al., 2022](#)) as they generate realistic business cycle dynamics, such as the positive comovements of real output and inflation.

My main finding is that the optimal monetary policy rule, from the perspective of a utilitarian central bank, is asymmetric. With redistributive channels of inflation, the central bank should be accommodative towards inflation, but aggressive towards deflation. In contrast, the utilitarian central bank in a RANK economy that is devoid of any redistributive channels should adopt a symmetrically aggressive Taylor rule towards both inflation and deflation. I investigate the contribution of each of the three channels by studying optimal monetary rules in counterfactuals where I shut down one channel at a time. I find that the revaluation and earnings channels cause the optimal policy to be more accommodative, while the expenditure channel makes the optimal policy more aggressive. Quantitatively, the revaluation and earnings channels dominate.

In the model featuring all three redistributive channels of inflation, I study the effect of the central bank moving from the optimal policy implied by a RANK model to the true optimal policy for this model. I find this policy change reduces the welfare cost due to inflationary shocks by 23%. Further, I analyze the distribution and sources of this welfare gain. I evaluate the welfare change at the individual level along the income and wealth distributions. The main winners under the policy change are households with low income and low wealth. Following an inflationary shock, the central bank only moderately raises the nominal rate to maintain a moderate level of inflation, allowing low-income and low-wealth households to pay less for their debt, and to enjoy higher earnings growth. Low- and middle-income households experience a greater price increase in their consumption baskets, though this downside does not offset the benefit from the revaluation of household debt and earnings growth. Furthermore, following [Benabou \(2002\)](#); [Floden \(2001\)](#); [Dyrda and Pedroni \(2021\)](#), I decompose the welfare gain along the transition path into three components. Of the aggregate welfare gain, around 70% is from the ex-ante redistribution, around 20% is from insurance against ex-post risk, and the remainder is due to an increase in average

consumption and leisure. The primary sources of welfare gain come through additional insurance and redistribution when the central bank accommodates inflation.

**Related Literature** This paper contributes to the long-standing literature on optimal monetary policy *rules*. From the seminal work of [Kydland and Prescott \(1977\)](#) and [Taylor \(1993\)](#), the study of optimal policy rules has generated enormous interest from both policymakers and researchers ([Clarida, Gali, and Gertler, 1999](#); [Woodford, 2003](#); [Giannoni and Woodford, 2003a,b](#); [Schmitt-Grohé and Uribe, 2007](#); [Galí, 2015](#)). [Schmitt-Grohé and Uribe \(2007\)](#) study optimal Taylor rules in a RANK model, which is the RANK counterpart to my exercise.

This project also relates to the rapidly growing literature on optimal monetary policy in HANK models by studying optimal policy rules with a comprehensive accounting of redistributive inflation. The HANK literature has made substantial progress on the *positive* side of monetary and fiscal policy ([Kaplan, Moll, and Violante, 2018](#); [Auclert, 2019](#)), while the *normative* HANK literature is still in its infancy. Generally, there are two types of optimal policy problems that can be studied in this normative HANK literature: first, the optimal unconstrained policy ([Bhandari, Evans, Golosov, and Sargent, 2021b](#); [Le Grand, Martin-Baillon, and Ragot, 2021](#); [Nuno and Thomas, 2021](#); [Dávila and Schaab, 2022a](#); [Acharya, Challe, and Dogra, 2020](#); [Bilbiie and Ragot, 2021](#)), and second, the optimal policy within a family of rules ([McKay and Wolf, 2022](#)). This paper falls into the second category. The recent contribution by [McKay and Wolf \(2022\)](#) also studies optimal policy rules in HANK, which my paper differs from in the following ways. First, my model incorporates three redistributive channels of inflation, which are not their focus. Second, they characterize the optimal policy rule as a “forecast targeting criterion”, which maps the future paths of aggregate variables that may not include monetary policy instruments to some forecasting targets. By contrast, I study the optimal generalized Taylor rule that sets monetary policy instruments as a function of observable aggregates, which is more practical and useful to guide monetary policy decisions. Third, I evaluate social welfare along the nonlinear transition dynamics with aggregate shocks, while they approximate it with first-order linear perturbation to equilibrium conditions. Their approximation allows them to derive an analytical decomposition of social welfare but also requires them to only approximate around the efficient steady state. As a result, they are restricted to studying the optimal policy problem with a particular set of Pareto weights that puts a higher weight on wealthier households in order to match the data.

With the multi-sector model setup, my work also relates to the studies of optimal monetary policy in multi-sector RANK economies such as [Aoki \(2001\)](#); [Guerrieri et al. \(2021\)](#),

which focus on how relative price changes and structural allocation affect optimal policy design.

This paper also contributes to the empirical literature on the redistributive channels of inflation by systematically reanalyzing these channels using recent available data. [Cravino and Levchenko \(2017\)](#); [Kaplan and Schulhofer-Wohl \(2017\)](#); [Cravino et al. \(2020\)](#); [Cavallo \(2020\)](#); [Argente and Lee \(2021\)](#); [Jaravel \(2021\)](#) document empirical facts on the expenditure channel, which is also referred to as “inflation inequality” across households. The revaluation channel of inflation through heterogeneous net nominal positions is studied by [Doepke and Schneider \(2006\)](#); [Pallotti \(2022\)](#) using the US data and by [Adam and Zhu \(2016\)](#) using data from Europe. For the earnings channel, [Guvenen et al. \(2017\)](#) uses the earnings data from the US Social Security Administration to document that the elasticity of workers’ annual earnings growth to real GDP exhibits a U-shape pattern along the earnings distribution. I extend their exercises to aggregate variables such as aggregate earnings and inflation until 2022, using publicly available data from [Blanchet et al. \(2022\)](#) and [Guvenen et al. \(2014\)](#). The three redistributive channels in this paper correspond to three key components of the household budget constraint. This approach I take is thus called the “budget set approach”, as in recent work by [Cardoso et al. \(2022\)](#) and [del Canto et al. \(2022\)](#). They derive an analytical decomposition of the impacts of aggregate shocks on household wealth and the first-order welfare impact, while my paper focuses on the nonlinear welfare impact and its implications for optimal monetary policy. I also account for general equilibrium channels, which they abstract from.

Lastly, this paper also relates to recent research that incorporates redistributive channels of inflation into quantitative HANK models, such as [Doepke, Schneider, and Selezneva \(2015\)](#) for the revaluation channel, [Cravino, Lan, and Levchenko \(2020\)](#) for the expenditure channel, [Alves and Violante \(2023\)](#) for the earnings channel, and [Zhou \(2022\)](#) for both the expenditure and revaluation channels. [Clayton, Jaravel, and Schaab \(2018\)](#) study both the expenditure and earnings channels along the dimension of education groups rather than income groups as in this paper. The HANK model in my paper features all three channels of redistributive inflation discussed above and derives lessons for the optimal design of monetary policy.

The remainder of the paper is structured as follows. In [Section 2](#), I use the most recent data to systematically revisit the empirical facts on three redistributive channels of inflation. Then I develop a quantitative model and solution framework to study the optimal nonlinear monetary policy rules with redistributive consequences of inflation in [Section 3](#). I present the optimal policy results in [Section 4](#), and conclude in [Section 5](#).

## 2 Empirical Facts on the Redistributive Channels of Inflation

In this section, I use the most recent available data to provide a systematic revisiting of the empirical evidence on the three redistributive channels of inflation.

### 2.1 Expenditure Channel

For the expenditure channel, I construct the monthly consumer price indices for each income group in the US. Then I estimate the heterogeneous responses of the price indices for different income groups to monetary policy shocks. My measurement of price indices by income builds on previous work such as [Cravino et al. \(2020\)](#) with an extension to the most recent data and several adjustments in order to better match the model.

#### 2.1.1 Data

I link two datasets to construct income group level price indices. The first is the consumer expenditure survey (CEX) data from 1997 to 2021, which covers a representative sample of US households with rich information about household expenditure and income, among other characteristics. There are two parts of the CEX data, the interview survey and the diary survey, which cover two different samples of households. The interview part is a quarterly survey that typically covers relatively infrequent purchases, while the diary part is a weekly survey that covers the majority of daily purchases.

Using the pre-tax income information in both interview and diary surveys, I group all households into 100 percentiles for each year. Then I construct the expenditure shares for households in each income percentile, using the expenditure and sample weight information from both surveys. There are many overlaps between the expenditure classes in the interview and diary surveys, and I follow the CEX official guide to determine which survey to use for each expenditure class in a given year.<sup>2</sup> The details on constructing income group level expenditure shares can be found in Data Appendix [A.1](#).<sup>3</sup>

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<sup>2</sup>Detailed information can be found in the hierarchical grouping files at <https://www.bls.gov/cex/pumd/stubs.zip>

<sup>3</sup>My construction of expenditure shares mostly follows [Cravino et al. \(2020\)](#), except for the following differences. [Cravino et al. \(2020\)](#) follows the BLS handbook to make the adjustment on homeowners' equivalent rent of primary residence by imputing it as an expenditure class of homeowners. This generates a large expenditure share of high-income households, which is counterfactual. In this project, I do the imputation in the baseline index construction, but also check the robustness of my empirical results by not doing the imputation. The results are robust and do not change qualitatively.

For other adjustments on insurance reimbursement and transportation expenditure, I follow [Cravino et al. \(2020\)](#).



The second dataset I use is the monthly item-level consumer price data from 1969 to 2021, which is also published by the BLS.<sup>4</sup> Then I map the expenditure share I have constructed using the CEX data to the item-level price data to construct the consumer price indices for each income group by year. The details of constructing such a mapping are in Appendix A.2.

To estimate inflation responses for different income groups to monetary policy shocks, I use the Romer and Romer (2004) monetary policy shocks in the baseline analysis. I use the monthly shock series from 1969M3 to 2008M12 extended by Coibion et al. (2017).

### 2.1.2 Income group price index construction

Using the CEX data, I calculate the expenditure shares  $\omega_{j,\tilde{t}}^q$  for the households at income percentile  $q$  at time  $\tilde{t}$  for any expenditure class  $j \in J$ . Based on the update frequency of the CEX data, I set one period of  $\tilde{t}$  as one year. With proper expenditure class mapping detailed in Appendix A.2, I also obtain the price level  $P_{j,t}$  for expenditure class  $j \in J$  at time  $t$  from the BLS item level price data. Here one period of  $t$  is one month.

Then I construct the consumer price index  $\text{PIX}_t^q$  for income percentile  $q$  at time  $t$  with:

$$\log \frac{\text{PIX}_t^q}{\text{PIX}_{v(t)}^q} = \sum_{j \in J} \left( \omega_{j,\tilde{t}(t)}^q \times \log \frac{P_{j,t}}{P_{j,v(t)}} \right). \quad (1)$$

To calculate the price index in period  $t$ , we choose the pivot period  $v(t)$  and reference period  $\tilde{t}(t)$  of the expenditure shares. I choose  $v(t)$  as the last month in the calendar year before  $t$ . For the reference period, the BLS handbook and Cravino et al. (2020) use the expenditure share with a lag of two years, i.e.,  $\tilde{t}(t) \approx t^y(t) - 2$  years, where  $t^y(t)$  denotes the calendar year that month  $t$  belongs to.<sup>5</sup> To capture the impact effect of the expenditure share shift on household price indices, I follow the Törnqvist approach and use the concurrent expenditure weights in the current and pivot years. Namely, I define

$$\omega_{j,\tilde{t}(t)}^q \equiv \frac{1}{2} (\omega_{j,t^y(t)}^q + \omega_{j,t^y(t)-1}^q).$$

Using CEX and item-level price data from 1997 to 2021, I plot the time series of income

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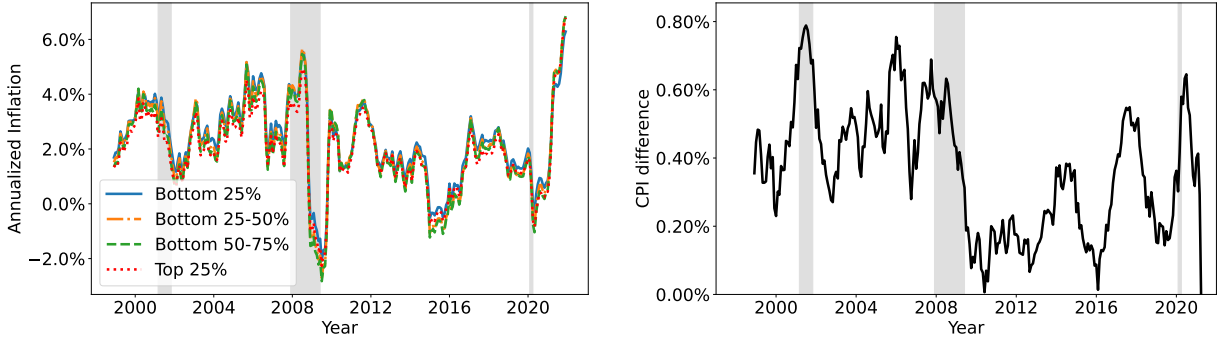
<sup>4</sup>Data from 1997 can be downloaded at <https://download.bls.gov/pub/time.series/cu/cu.data.0>. Current Data before 1997 can be downloaded at <https://download.bls.gov/pub/time.series/mu/> under each major consumption class.

<sup>5</sup>To be precise, the BLS and Cravino et al. (2020) uses a variant of price index Equation (1) without taking logs of prices, that is

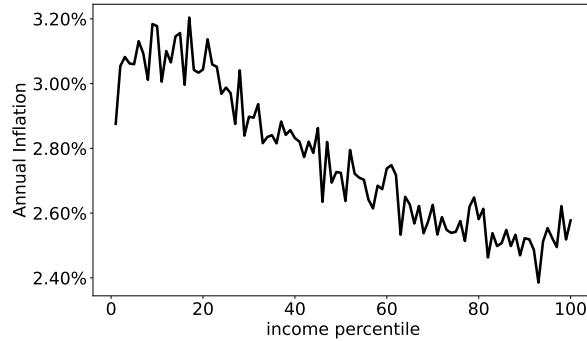
$$\frac{\text{PIX}_t^q}{\text{PIX}_{v(t)}^q} = \sum_{j \in J} \left( \omega_{j,\tilde{t}(t)}^q \times \frac{P_{j,t}}{P_{j,v(t)}} \right),$$



group level annualized inflation in Figure 1a, the difference of inflation between the bottom 25% and top 25% income groups in Figure 1b, and the 25-year average inflation along the income distribution in Figure 1c. During this period, average inflation is monotone decreasing with income percentile. On average, households in the top income quintile experience 0.5% percentage point lower inflation than those in the bottom income quintile. If we look at the time series plots in Figures 1a and 1b, the annualized inflation of the bottom 25% income group is persistently higher than that of the top 25% income group, until the recent change in 2021.



(a) Annualized CPI time series by income group      (b) CPI gap: bottom vs top 25% income



(c) Average annual inflation by income percentile

Figure 1: Inflation by income group: 1997-2021

For the analysis in the next subsection, I follow [Cravino et al. \(2020\)](#) to extend income group level price indices before 1997 to 1969 by choosing the reference period  $\tilde{t}(t) = 1997$  in Equation 1 for  $t < 1997$ . This is to extend the sample period for the short-run analysis.

### 2.1.3 Inflation responses for different income groups to monetary shocks

Using the consumer price indices for different income groups, I now look at the heterogeneous effect of inflationary shocks on households in the short run. As I am mostly interested

in optimal monetary policy rules with demand shocks in this paper, here I look at the heterogeneous response after monetary policy shocks. This exercise can be extended to other types of shocks, such as fiscal policy shocks, oil shocks, etc. Denote  $p_t^q = \log \text{PIX}_t^q$  as the log price level for income group  $q$  at time  $t$ . I run the following local projection (Jordà, 2005) with Romer-Romer monetary shock using monthly data from 1969 to 2008:<sup>6</sup>

$$p_{t+s}^q - p_t^q = \alpha_s + \theta_s \text{shock}_t^{RR} + \sum_{j=1}^J \beta_{s,j} (p_{t-j}^q - p_{t-j-1}^q) + \sum_{i=1}^I \gamma_{s,i} \text{shock}_{t-i}^{RR} + \epsilon_{t+s},$$

where the lag period  $J = 6$ , and I control shocks in the past  $I = 24$  periods. The local projection estimates with 1% monetary policy shocks are in Figure 2.

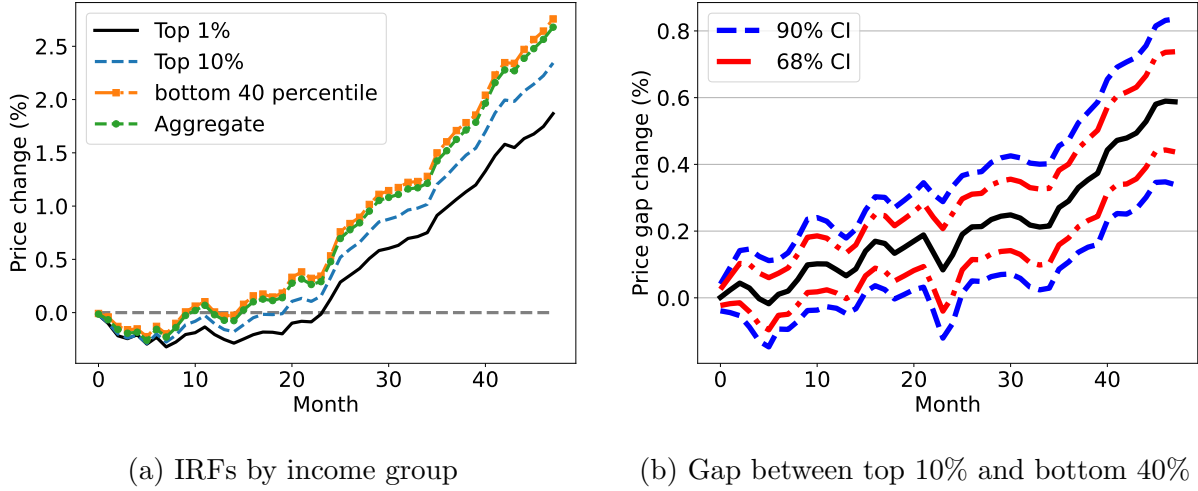


Figure 2: IRF of Income group price index to 1% expansionary monetary shock

Following an unexpected 1% expansionary monetary shock, which is a demand shock, the consumer price index of households in the bottom 40% income group increases by 2.5% after four years. In contrast, the consumer price index of households in the top 10% income group only increases by 2.0%. The 90% and 86% confidence intervals of their price change difference are plotted in Figure 2b, which are significantly different from 0.

For robustness check, I also run the local projection with different model specifications and different series of monetary shocks. I report these results in Appendix C.1.

<sup>6</sup>I use the Romer-Romer monetary shocks extended by Coibion et al. (2017), which estimates the monthly monetary policy shock from 1969M3 to 2008M12. As a result, my local projection exercise only covers this time period.

## 2.2 Revaluation Channel

For the revaluation channel, I measure the average net nominal position, including its decomposition in various instruments, for households in each income group in the US.

### 2.2.1 Data and Method

My measurement of average net nominal positions for each income group is based on the 2019 Survey of Consumer Finances (SCF) data and the US Financial Accounts (FA) data, which are both published by the US Federal Reserve Board. The SCF data cover detailed information on income and wealth for a representative sample of US households. The FA data cover a detailed breakdown of assets and liabilities for major US sectors, including household, business, foreign, and government sectors, as well as various financial intermediaries.

My estimates combine my own work following the methodology in [Doepke and Schneider \(2006\)](#) and the estimates from [Pallotti \(2022\)](#).<sup>7</sup> Following [Doepke and Schneider \(2006\)](#), the net nominal position (NNP) of a household  $h$  is her direct nominal position (DNP) net of the (negative) nominal position she held through equity:

$$\text{NNP}_{h,t} = \text{DNP}_{h,t} - \tilde{\lambda}_t \times \text{equity\_held}_{h,t},$$

where  $\tilde{\lambda}_t$  is the net nominal leverage ratio for the business sector at time  $t$ , which is measured using the FA data by year. The direct nominal positions of households  $\text{DNP}_{h,t}$  is sum of directly held nominal assets plus nominal assets held through investment intermediaries less nominal liabilities, measured using the SCF data. Following [Doepke and Schneider \(2006\)](#), the nominal assets can be classified into the following categories: short-term nominal assets, bonds, mortgages, and equity. More details about the measurement of net nominal positions can be found in [Appendix A.3](#).

### 2.2.2 Results on net nominal positions by income groups

Table [1](#) presents the average net nominal positions (NNP) of US households in each income group in 2019, as well as the decomposition to various instruments.

In 2019, households in the top 10% of the US income distribution hold \$524,700 in net nominal assets on average. In contrast, the bottom 20% US households hold \$1,300 in net nominal debt, while the remaining middle-income households hold \$12,400 in net nominal debt. The dispersion in net nominal positions generates a large revaluation effect when we

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<sup>7</sup>I thank Filippo Pallotti for sharing the household level estimates behind the results reported in [Pallotti \(2022\)](#).

| Income groups | Bottom 20% | Middle income | Top 10% |
|---------------|------------|---------------|---------|
| Short-term    | 7.9        | 32.8          | 346.7   |
| Bonds         | 9.4        | 45.1          | 589.9   |
| Mortgages     | -17        | -91.4         | -341.8  |
| Equity        | -1.7       | 1.1           | -70.1   |
| Total NNP     | -1.3       | -12.4         | 524.7   |

Table 1: Net nominal position of US households by income group in 2019. Unit: \$1000.

have unexpected inflation or deflation.

## 2.3 Earnings Channel

In this section, I estimate the earnings growth elasticity of households in different income groups with regard to aggregate economic variables, including GDP, aggregate earnings, and inflation.

### 2.3.1 Data

I need real earnings growth data at the income group level to estimate the elasticity. There are two such datasets available. The first is the “real-time inequality” data provided by [Blanchet, Saez, and Zucman \(2022\)](#),<sup>8</sup> who estimate the monthly real labor income growth data for several income groups (four quartiles, top 1%, top 10%, top 1-10%, top 10-25%) in the US from January 1976 to September 2022. Using the methodology developed in [Piketty et al. \(2018\)](#), they construct the estimates with survey data, including the Quarterly Census of Employment and Wages (QCEW) and the Current Population Survey, adjusted with aggregate variables in the National Income and Product Accounts (NIPA).

The second is made available by [Guvenen, Ozkan, and Song \(2014\)](#) and covers the annual real earnings growth for each income percentile (and for smaller income groups at the top 0.1%, 0.3%, 0.6%, 0.9% percentiles) in the US from 1979 to 2010. The data is constructed using microdata from the US Social Security Administration’s Master Earnings file.<sup>9</sup>

These two datasets have their own advantages and limitation. The “real-time inequality” data from [Blanchet et al. \(2022\)](#) provide estimates in the monthly frequency, and include more recent data up to 2022, covering the recent surge of inflation. The data from [Guvenen et al. \(2014\)](#) is directly measured with the microdata at an annual frequency. They provide

<sup>8</sup>The data is available for download at <https://realtimeinequality.org/>. Last accessed: October 5, 2022.

<sup>9</sup>The data is available for download at <https://www.fatihguvenen.com/s/gos-jpe2014-data.xlsx>. I use the data in Table Add. 1 for 1-100 percentile and Table 16 for the top 0.1%, 0.3%, 0.6%, 0.9% percentiles. Last accessed: September 27, 2022.

estimates for finer-level income groups, especially at the very top income distribution. They divide the top 1% income percentile into 10 quantiles and provide estimates for the top 0.1%, 0.3%, 0.6%, and 0.9% percentiles. I use the [Blanchet et al. \(2022\)](#) data for the baseline estimates in the paper and use [Guvenen et al. \(2014\)](#) data as a robustness check in Appendix [C.1.1](#).

To estimate the elasticity to business cycle variables, I obtain aggregate output (GDP for annual data, industrial output for monthly data) and inflation from the Federal Reserve Economic Data (FRED), and use the aggregate earnings data from the two sources above.

### 2.3.2 Results on earnings growth elasticity by income groups

The main specification for earnings growth elasticity estimation at the income group level is

$$\Delta y_{q,t} = \alpha_q + \beta_q \Delta X_t + \epsilon_{q,t} \quad (2)$$

$\Delta y_{q,t}$  is log difference of average earnings for households in income percentile  $q$ .  $t$  is at the annual or monthly frequency, depending on which dataset to use.  $\Delta X_t$  is the log difference of aggregate variables, which could be (1) aggregate output, (2) inflation, or (3) aggregate earnings. We are interested in the earnings growth elasticity for each group  $\beta_q$ , which is called “worker betas” in [Guvenen et al. \(2017\)](#).

Using the 1976-2022 monthly “real time inequality” data from [Blanchet et al. \(2022\)](#), I estimate Equation (2) for six labor income groups: bottom 25%, 25-50%, 50-75%, 75-90%, top 1-10%, top 1%, and for three aggregate variables. For the estimation with regard to aggregate inflation, I only use the data since 1985 to exclude the unusual period of the Great Inflation. The results are in Figure [3](#).

As seen in Figure [3](#), the earnings growth elasticity exhibits a U-shape pattern along the income distribution for all three aggregate variables. The earnings growth is very elastic at the bottom income quartile and at the very top (1%) income percentiles, while it is less sensitive in the middle-income range. For calibration purposes, we pay special attention to the earnings growth elasticity to aggregate output in Figure [3a](#). The earnings growth elasticity to aggregate output is around 2 for households at the bottom 25% of the income distribution and is smaller than 0.5 for households in the top 1-50% of the distribution. For households in the top 1% income distribution, their earnings growth elasticity is around 0.9.

In Appendix [C.1.1](#), I use the earnings growth data from [Guvenen et al. \(2014\)](#) to estimate the elasticity, and the results with regard to output and aggregate earnings are very similar. The elasticity with aggregate inflation differs somewhat since we can only run an annual regression for a sample ending in 2010 with the [Guvenen et al. \(2014\)](#) data. Thus, it does

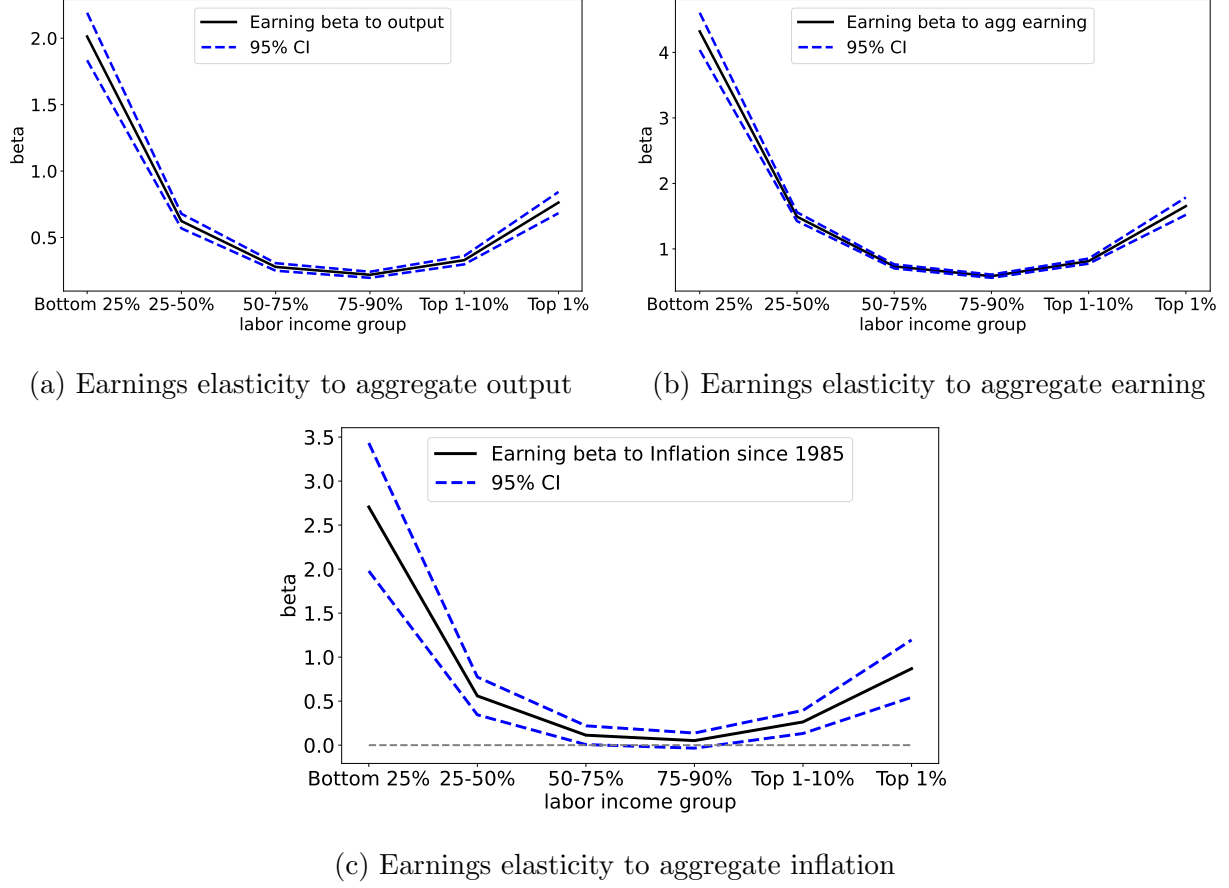


Figure 3: Elasticity of earnings growth to aggregate variables by income group.

not cover the dynamics of inflation since the recovery from the Great Recession, or the recent surge of inflation since 2021. Such a difference does not affect the quantification of the model, since we will model the income process with earnings growth elasticity directly to aggregate output rather than inflation.

### 3 Quantitative Model and Numerical Solution

In this section, I present an infinite horizon two-sector HANK model to study optimal monetary policy rule with redistributive consequences of inflation. The model features (1) uninsurable idiosyncratic shocks on labor productivity, (2) the heterogeneous elasticity of earnings growth to aggregate output, (3) aggregate demand shocks akin to the fluctuations in the households' discount factor, (4) nominal debt, (5) two production sectors, (6) non-homothetic preferences and heterogeneous expenditure shares over goods produced in different sectors, and (7) the heterogeneous levels of price rigidity in different sectors.

### 3.1 Model Setup

#### 3.1.1 Households

**Households' preferences.** The economy is populated by a continuum of ex ante identical households indexed by  $h \in [0, 1]$ . Household  $h$  chooses consumption  $c_{h,t}$ , labor supply  $\ell_{h,t}$ , real value of nominal bond position  $b_{h,t+1}$  to maximize the present value of utility:

$$\max_{\{c_{h,t}, \ell_{h,t}, B_{h,t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_{h,t}, \ell_{h,t}), \quad (3)$$

where  $u(c_{h,t}, \ell_{h,t}) = \frac{c_{h,t}^{1-\gamma}}{1-\gamma} - \varphi \frac{\ell_{h,t}^{1+\psi}}{1+\psi}$ . There are two types of consumption goods in this economy, produced by two sectors  $s \in \{A, B\}$  to be specified later. Households' real consumption  $c_{h,t}$  is the generalized Stone-Geary bundle of consumption over two types of goods  $A$  and  $B$ :

$$c_{h,t} \equiv \left[ (1-\alpha)^{\frac{1}{\eta}} (c_{h,t}^A - \underline{c})^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} (c_{h,t}^B)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}. \quad (4)$$

$A$  is the necessity good with consumption subsistence level  $\underline{c} > 0$ .  $\eta$  governs the elasticity of substitution across products.

Denote  $P_t^s$  as the price index in sector  $s$ . Define the aggregate price index corresponding to the nonhomothetic preference (4):

$$P_t \equiv \left[ (1-\alpha) (P_t^A)^{1-\eta} + \alpha (P_t^B)^{1-\eta} \right]^{\frac{1}{1-\eta}}. \quad (5)$$

Define nominal price index of  $c_{h,t}$  as  $P_{h,t}$ , i.e.  $P_{h,t} c_{h,t} \equiv P_t^A c_{h,t}^A + P_t^B c_{h,t}^B$ . Following [Herrendorf, Rogerson, and Valentinyi \(2014\)](#) we can show that

$$P_{h,t} c_{h,t} \equiv P_t^A c_{h,t}^A + P_t^B c_{h,t}^B = P_t c_{h,t} + P_t^A \underline{c}. \quad (6)$$

The generalized Stone-Geary preference also implies the following demand system:

$$\frac{P_t^A (c_{h,t}^A - \underline{c})}{P_t^B c_{h,t}^B} = \frac{1-\alpha}{\alpha} \left( \frac{P_t^A}{P_t^B} \right)^{1-\eta}$$

At the steady state with constant prices, households' expenditure share in sector  $A$  is decreasing in households' income.

**Household budget constraint and earnings process.** Households save and borrow in nominal asset  $b_{h,t+1}$  with the fixed aggregate supply by the government, subject to the



budget constraint in nominal terms:

$$P_{h,t}c_{h,t} + P_tb_{h,t+1} = (1 + i_t)P_{t-1}b_{h,t} + (1 - \tau)P_tw_te_{h,t}\ell_{h,t} + P_tT_t + P_td_t(\xi_{h,t}). \quad (7)$$

The nominal asset  $b_{h,t+1}$  is determined in period  $t$ , and pays the nominal rate of return  $1 + i_t$  in period  $t + 1$ . Households are exposed to idiosyncratic shock  $\xi_{h,t}$ , and their effective labor productivity  $e_{h,t}$  depends on both  $\xi_{h,t}$  and aggregate output  $Y_t$  as:

$$\log e_{h,t} = \underbrace{\log \xi_{h,t}}_{\text{idiosyncratic state}} + \underbrace{\zeta(\xi_{h,t}) \log(Y_t)}_{\text{heterogeneous response to aggregate output}} - \underbrace{\log \mathbb{E}_\xi[\xi_{h,t} Y_t^{\zeta(\xi_{h,t})}]}_{\text{normalization}}. \quad (8)$$

Here the elasticity  $\zeta(\xi_{h,t})$  of labor productivity to aggregate output depends on the idiosyncratic income state of households, and I calibrate the elasticity to match the empirical facts of the earnings channels documented in Section 2.3. The normalization term in Equation (8) guarantees that  $\int e_{h,t} dh$  is a constant for any  $t$ .  $w_t$  is the real wage. There is a proportional labor income tax with a constant rate  $\tau$ , and the government conduct uniform lump-sum transfer  $T_t$ , which is common across all households.  $d_t(\xi_{h,t})$  is the allocation of dividends from the producers, which is a function of the idiosyncratic state.

The household also faces the borrowing constraint  $b_{h,t} \geq \underline{b}$ . Using Equation (6), we can write Equation (7) as

$$P_tc_{h,t} + P_tb_{h,t+1} = (1 + i_t)P_{t-1}b_{h,t} + (1 - \tau)P_tw_te_{h,t}\ell_{h,t} + P_tT_t + P_td_t(\xi_{h,t}) - \frac{P_t^A}{P_t}\underline{c}.$$

where aggregate inflation  $\pi_t = \frac{P_t}{P_{t-1}} - 1$ . We can further write the household budget constraint in real terms:

$$c_{h,t} + b_{h,t+1} = \frac{1 + i_t}{1 + \pi_t}b_{h,t} + (1 - \tau)w_te_{h,t}\ell_{h,t} + T_t + d_t(\xi_{h,t}) - \frac{P_t^A}{P_t}\underline{c}. \quad (9)$$

### 3.1.2 Production side

The production side follows a two-sector New Keynesian setup, with heterogeneous levels of price rigidity in each sector. There are two sectors  $s \in \{A, B\}$  in this economy. Products in different sectors differ in (1) income elasticity and (2) price rigidity. In the calibration, sector  $A$  produces necessity goods that have more flexible prices, while sector  $B$  produces luxury goods that have stickier prices.

**Final goods producers.** In each sector  $s \in \{A, B\}$ , there is a retailer who purchases differentiated inputs  $y_{j,t}^s$  from intermediate goods firms  $j \in [0, 1]$  and bundles them into a sector-specific, final consumption good. This aggregation technology is given by

$$Y_t^s = \left( \int_0^1 (y_{j,t}^s)^{\frac{1}{\mu_t}} dj \right)^{\mu_t}.$$

where  $Y_t^s$  denotes real final goods production in sector  $s$ .  $\mu_t \equiv \mu$  is the markup parameter. Each retailer demands intermediate input  $j$  according to the standard demand function with the elasticity of substitution  $\epsilon_t = \frac{\mu_t}{\mu_t - 1}$ :

$$Y_{j,t}^s = \left( \frac{P_{j,t}^s}{P_t^s} \right)^{-\epsilon_t} Y_t^s, \quad \text{where} \quad P_t^s = \left( \int_0^1 (P_{j,t}^s)^{1-\epsilon_t} dj \right)^{\frac{1}{1-\epsilon_t}}. \quad (10)$$

**Intermediate goods producers.** Intermediate good producers in sector  $s$  produce good  $j$  in a monopolistically competitive market with production function:

$$y_{j,t}^s = Z_t n_{j,t}^s \quad (11)$$

with productivity  $Z_t \equiv Z$  that is common across sectors. They hire labor with a common nominal wage  $W_t$ . The fiscal authority provides a linear labor subsidy  $\tau_p$  and collects lump-sum tax  $T_p$  from the intermediate producers. The cost minimization problem is:

$$\min_{\{n_{j,t}^s\}} (1 - \tau_p) W_t n_{j,t}^s,$$

subject to production function (11) and demand schedule (10). It implies that the relative marginal cost  $m_t$  is common across all intermediate goods producers in sector  $s$ :

$$m_t^s = w_t^s = \frac{(1 - \tau_p) W_t}{P_t^s}.$$

The intermediate goods producers set prices with the Rotemberg price adjustment cost:

$$\max_{\{P_{j,t}^s\}} \mathbb{E}_0 \sum_{t=0}^{\infty} \frac{1}{\Pi_{s=1}^t (1 + r_s)} \left\{ \left( \frac{P_{j,t}^s}{P_t^s} - \frac{(1 - \tau_p) W_t}{P_t^s} \right) \left( \frac{P_{j,t}^s}{P_t^s} \right)^{-\epsilon_t} Y_t^s - \phi_s \left( \frac{P_{j,t}^s}{P_{j,t-1}^s} \right) Y_t^s - T_p \right\} \quad (12)$$

Here I choose the stochastic discount factor following [Auclert et al. \(2021\)](#). The price adjustment cost for sector  $s$  is quadratic as in [Rotemberg \(1982\)](#):

$$\phi_s(\Pi_{j,t}^s) \equiv \frac{\mu_t}{\mu_t - 1} \frac{1}{2\kappa^s} (\log \Pi_{j,t}^s)^2$$

Gross inflation rate  $\Pi_{j,t}^s = \frac{P_{j,t}^s}{P_{j,t-1}^s}$ .  $\kappa^A > \kappa^B$  implies that the price in sector  $A$  is more flexible and less costly to adjust.

Solving for the symmetric equilibrium where  $P_{j,t}^s = P_t^s$ , the first-order condition of (12) delivers the New Keynesian Phillips curve (NKPC) for each sector as:

$$\log(1 + \pi_t^s) = \kappa^s \left( \frac{(1 - \tau_p)w_t^s}{Z_t} - \frac{1}{\mu_t} \right) + \frac{1}{1 + r_{t+1}} \frac{Y_{t+1}^s}{Y_t^s} \log(1 + \pi_{t+1}^s).$$

with real wage in each sector  $w_t^s = \frac{W_t}{P_t^s}$ .

Aggregate nominal monopoly profit in sector  $s$  in period  $t$  is

$$D_t^s = P_t^s \left[ \left( 1 - \frac{(1 - \tau_p)W_t}{P_t^s} \right) Y_t^s - \phi_s(\Pi_t^s) Y_t^s - T_p \right],$$

which sums up to aggregate nominal profit  $D_t = \sum_s D_t^s$ . Real profits from the two sectors are distributed to households:  $D_t = P_t \int_h d_t(\xi_{h,t}) dh$ .

**Fiscal authority.** The fiscal authority collects linear labor income tax  $\tau$  and make a lump-sum transfer  $T_t$  to households. On the firm side, the fiscal authority provides a linear labor subsidy  $\tau_p$  and collects lump-sum tax  $T_t^p$  on intermediate producers. The government supplies a fixed amount of nominal debt  $B_g$ , has constant expenditure  $G$ , and the fiscal authority uses lump-sum transfer  $T_t$  to balance the government budget in real terms:

$$\tau_t w_t N_t + B_{g,t+1} + T_t^p = \frac{1 + i_t}{1 + \pi_t} B_{g,t} + G_t + T_t + \tau_p w_t N_t. \quad (13)$$

On the production side, I assume that the fiscal authority sets the labor subsidy to intermediate producers  $\tau_p = 1 - \frac{1}{\mu}$  using the lump-sum tax  $T_t^p = \tau_p w_t N_t$  on the same producers, in order to fix the monopoly power distortion in the steady state. This is an important assumption following the normative RANK literature ([Galí, 2015](#)). Without this assumption, the optimal policy would always try to mitigate this distortion, regardless of the aggregate shocks that hit the economy. So the effective government budget constraint in real terms is

$$\tau_t w_t N_t + B_{g,t+1} = \frac{1 + i_t}{1 + \pi_t} B_{g,t} + G_t + T_t.$$

In the main exercise of the paper, I will take the fiscal policy rules calibrated to the US economy as given and only solve for optimal monetary policy.

**Monetary authority.** Monetary authority sets nominal interest rate  $i_t$  within a family of nonlinear Taylor rules  $\Phi = \{\phi_\pi^+, \phi_\pi^-, \phi_y\}$  that leads to equilibrium determinacy:

$$i_{t+1} = r^* + \phi_\pi^+ \pi_t^+ + \phi_\pi^- \pi_t^- + \phi_y \hat{y}_t, \quad (14)$$

$r^*$  is a constant intercept,  $\pi_t^+ = \max\{\pi_t, 0\}$  and  $\pi_t^- = \min\{\pi_t, 0\}$  are the positive and negative parts of inflation  $\pi_t = \frac{P_t}{P_{t-1}} - 1$  with aggregate price index defined in Equation (5).  $\phi_\pi^+$  and  $\phi_\pi^-$  capture the aggressiveness of the monetary authority towards positive and negative inflation. Aggregate output deviation from the steady state is defined as  $\hat{y}_t = \log \frac{Y_t}{Y_{ss}}$ . Note  $i_{t+1}$  is the nominal return between  $t$  and  $t+1$  and is set at  $t$  against inflation  $\pi_t$  and output deviation  $\hat{y}_t$ .<sup>10</sup>

**Aggregate shocks.** In the baseline analysis, I focus on the optimal monetary policy rules against demand shocks  $\{\epsilon_t^r\}$ , which is modeled as the fluctuations of the wedge to the Euler equation.

**Definition of the equilibrium** Now I present the formal definition of the equilibrium. The list of the equilibrium conditions is in Appendix B.1.

**Definition 1.** *An equilibrium in this economy consists of exogenous aggregate shocks  $\{\epsilon_t^r\}$ , prices  $\{\pi_t^A, \pi_t^B, w_t^A, w_t^B, r_t, i_t\}$ , quantities  $\{T_t, d_t^A, d_t^B, N_t^A, N_t^B, T_t^p, Y_t^A, Y_t^B, C_t^A, C_t^B\}$ , individual household policy rules  $\{c_t, c_t^A, c_t^B, b_{t+1}, \ell_t\}$ , and a path of household distributions over the asset position and the idiosyncratic state  $\{\lambda_t(b, \xi)\}$  such that*

- *Given aggregate shocks, prices, and quantities, the individual household policy rules solve the household's problem (3) subject to (9) (8);*
- *Given aggregate shocks, the prices and quantities satisfy the equilibrium conditions of the producer's problem;*
- *The monetary authority follows monetary policy rule (14);*
- *Government budget constraint (13) holds;*

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<sup>10</sup>Note that under non-homothetic preference, the price index defined in (5) is not the exact price index of the aggregate consumption basket. Nevertheless, it can serve as a proxy of the price index (Redding and Weinstein, 2020) for the Taylor rule.

- *The sequence of distributions satisfies aggregate consistency conditions;*
- *Markets clear for labor, goods in both sectors, and nominal bond.*

**Market Clearing.** Goods markets in each sector:

$$Y_t^s = C_t^s + G^s + \frac{\epsilon_t}{2\kappa^s} Y_t^s (\log \Pi_t^s)^2, \quad \forall s \in \{A, B\},$$

where  $C_t^s = \int_h c_{h,t}^s dh$ ,  $Y_t^s = ZN_t^s$ .

The labor market clears:

$$N_t^A + N_t^B = \int_h e_{h,t} \ell_{h,t} dh$$

The bond market clears:

$$B_g + \int_h b_{h,t} dh = 0.$$

### 3.2 Welfare and optimal monetary policy rule

The main exercise in this project is to evaluate social welfare under each policy rule along the paths of aggregate shocks. For a given Taylor rule  $\Phi = \{\phi_\pi^+, \phi_\pi^-, \phi_y\}$ , with aggregate shock path  $\{\mathcal{E}_t\}$  hitting the steady state, I solve for the perfect foresight nonlinear transition dynamics  $\{c_{h,t}, \ell_{h,t}, \pi_t\}$  for all households with the sequence-space Jacobian method developed by [Auclert et al. \(2021, Section 6\)](#). Then I plug the solution into households' utility functions, and obtain their utility flows as  $u(c_{h,t}, \ell_{h,t})$ .<sup>11</sup>

For arbitrary Pareto weight  $\omega_h$ , the central bank's objective includes two components:

$$\Omega(\Phi, \{\mathcal{E}_t\}) \equiv \mathbb{E} \left[ \int_h \omega_h \sum_{t=0}^{\infty} \beta^t u(c_{h,t}, \ell_{h,t}) dh \middle| \{\mathcal{E}_t\}; \Phi \right] - \chi \sum_{t=0}^{\infty} \pi_t^2 \quad (15)$$

The first component is social welfare as the present value of Pareto-weighted household utility. The second component is the social cost of inflation.  $\chi \geq 0$  allows for possible special emphasis on price stability like the ECB ([Ioannidis et al., 2021](#)).<sup>12</sup> In the baseline calibration, I choose the value of  $\chi$  such that the second term in Equation (15) can capture the welfare cost of inflation that is ignored by New Keynesian models. In the Appendix, I also study optimal monetary policy when  $\chi = 0$  and (15) reduces to the standard social

<sup>11</sup>[Auclert et al. \(2021\)](#) develops the nonlinear solution of aggregate variables, and I adapt their code to obtain the nonlinear transition dynamics of individual households.

<sup>12</sup>Also see "Protocol on the statute of the European system of Central Banks and of the European Central Bank": "In accordance with Article 105(1) of this Treaty, the primary objective of the ESCB shall be to maintain price stability." [https://www.ecb.europa.eu/ecb/pdf/orga/escbstatutes\\_en.pdf](https://www.ecb.europa.eu/ecb/pdf/orga/escbstatutes_en.pdf)

welfare objective.<sup>13</sup>

Optimal monetary policy rule  $\Phi^*$  solves:

$$\max_{\Phi} \mathbb{E}_{\{\mathcal{E}_t\}} \Omega(\Phi, \{\mathcal{E}_t\}),$$

with expectation taken over different paths of aggregate shocks  $\{\mathcal{E}_t\}$ . The aggregate shock paths are exponential decay shocks  $\{\mathcal{E}_t\} = \{\xi_0, \rho\xi_0, \rho^2\xi_0, \dots, 0\}$  with initial magnitude  $\xi_0$  simulated from the calibrated shock process, and  $\rho$  is the persistence parameter calibrated in Section 3.3.

Following [Schmitt-Grohé and Uribe \(2007\)](#), the search range for the Taylor rule coefficients are:  $\phi_{\pi}^+, \phi_{\pi}^-, \phi_y \in [\text{determinacy bound}, 3]$ . 3 is a small positive constant that is larger than the determinacy boundary but still realistically small for a Taylor rule.<sup>14</sup>

**Remarks on the solution method** I make two remarks on the solution methods in this project as follows.

First, I solve for the perfect foresight nonlinear transition dynamics after the unanticipated aggregate shocks hit the steady state, rather than the first-order perturbation solution with regard to aggregate shocks. Though by certainty equivalence, the two solution results are close to each other when the aggregate shocks are small, they have different interpretations and allow for different choices of Pareto weights. The nonlinear transition dynamics provide a nonlinear global solution to the problem with unanticipated aggregate shocks, while the first-order perturbation solution is a linear approximation to the problem with aggregate shocks. Using transition dynamics from a nonlinear global solution, I plug the solved dynamics  $\{c_{h,t}, \ell_{h,t}\}$  into the utility function to obtain households' utility flow and thus the social welfare for arbitrary Pareto weights. In this sense, I obtain accurate social welfare value that is not subject to the Taylor approximation error.

In contrast, if we use the first-order perturbation solution to approximate social welfare up to the second-order accuracy, we can only approximate around the steady state that is efficient from the perspective of the planner. This is analog to the classic results in linear-quadratic problems in the RANK literature ([Woodford, 2003](#); [Benigno and Woodford, 2005](#)), but obtaining the efficient steady state in HANK is more complicated due to the

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<sup>13</sup>As is known in the literature, the cost of inflation is too small in the New Keynesian models. In the HANK models, inflation brings additional benefits through various redistributive channels, so one would need to model the additional costs of inflation such that the optimal monetary policy can be attained at an interior solution. Some recent papers on optimal monetary policy ([Nuno and Thomas, 2021](#); [Dávila and Schaab, 2022a](#)) also introduce a reduced-form inflation cost in the households' preferences.

<sup>14</sup>The determinacy region is solved with the winding number criterion ([Onatski, 2006](#)) using the algorithm of [Auclert et al. \(2021, 2019 working paper version\)](#).

existence of the incomplete market. [McKay and Wolf \(2022\)](#) provides a good discussion on this issue and addresses the problem by introducing a certain set of Pareto weights that make the deterministic steady state efficient. The first-order condition of the planner’s problem implies that the Pareto weight times the marginal utility of consumption would equalize across all households, which means the Pareto weights should be higher for high-income people with higher levels of consumption. In [Appendix C.2.3](#), I show that the main results of [McKay and Wolf \(2022\)](#) crucially depend on their choice of Pareto weights, which is quite different from the perspective of an “inclusive” central bank.

Second, I calculate the social welfare after the exponential decaying aggregate shocks hit the steady state, rather than solve for social welfare along the ergodic distribution. Thus, the solution does not capture the effect of the aggregate risks on households’ portfolio choices and precautionary motives. Future work can use a global solution method, such as DeepHAM ([Han, Yang, and E, 2021](#)), to incorporate those effects and investigate their implications on optimal policy.

### 3.3 Calibration

I calibrate the parameters for the three redistributive channels of inflation to match the empirical facts documented in [Section 2](#). For the remaining parameters in the HANK model, I calibrate them following the literature, such as [McKay et al. \(2016\)](#); [Kaplan et al. \(2018\)](#); [Auclert et al. \(2021\)](#). The calibration results are summarized in [Table 2](#). One period in the model is one quarter.

I calibrate the discount factor  $\beta$  such that the annualized real interest rate is 2% at the steady state. The coefficient of risk aversion  $\gamma = 1$ , and I choose the Frisch elasticity of labor supply  $1/\psi = 1/2$ , which is in line with the literature. The supply of government bonds  $B = 5.6$  to match the ratio of aggregate liquid assets to GDP, which is close to 1.4 annually ([McKay et al., 2016](#)). The borrowing constraint  $\underline{b}$  is chosen as -1 times quarterly average labor income following [Kaplan et al. \(2018\)](#). The desired markup of intermediate firms at the steady state  $\mu = 1.2$ .

**Expenditure channel.** In the two-sector model, I take sector  $B$  as the service sector less shelter and energy service, and sector  $A$  as the composition of other sectors. I calibrate  $\alpha = 0.3$  to match the consumption share in these two sectors in the US economy. Following [Herrendorf, Rogerson, and Valentinyi \(2013\)](#), I calibrate  $\underline{c} = 0.2$  at the steady state. I calibrate  $\kappa_A = 0.15, \kappa_B = 0.05$  to match two moments: (1) the average slope of the Phillips curve, and (2) the ratio of average price adjustment frequencies of goods and service sectors, as reported in [Nakamura and Steinsson \(2008, Table II\)](#).



Table 2: Model calibration

| Parameter                    | Interpretation                         | Value      | Target                                   |
|------------------------------|--|------------|--|
| Households                   |  |            |  |
| $\beta$                      | Discount factor                        | 0.982      | $r^* = 0.5\%$                            |
| $\varphi$                    | Disutility of labor                    | 0.786      | $N = 1$                                  |
| $\gamma$                     | Inverse IES                            | 2          |  |
| $\psi$                       | Inverse Frisch                         | 2          |  |
| $\underline{b}$              | Borrowing constraint                   | -1         | Quarterly labor income                   |
| $\underline{c}$              | Subsistence level                      | 0.2        | Expenditure shares                       |
| $\alpha$                     | Expenditure share parameter            | 0.3        | Expenditure shares                       |
| Markov chain for $\xi_{h,t}$ | Idiosyncratic shock process            | see text   |  |
| $p$                          | Probability to enter superstar state   | 1/99       | top 1%                                   |
| $q$                          | Probability to stay in superstar state | 0.99       | top 1%                                   |
| $e_{\text{star}}$            | labor productivity in superstar state  | 20         | <a href="#">Boar and Midrigan (2022)</a> |
| $\zeta(\xi_{h,t})$           | Earnings growth elasticity             | see text   | Empirics in Section <a href="#">2.3</a>  |
| Firms                        |  |            |  |
| $\mu$                        | Steady-state markup                    | 1.2        |  |
| $Z$                          | Steady-state TFP                       | 1          |  |
| $\kappa^A, \kappa^B$         | Adjustment cost                        | 0.15, 0.05 | Adjustment frequency                     |
| Gov't policy                 |  |            |  |
| $B$                          | Bond supply                            | 5.6        | Liquid assets/GDP                        |
| $\tau$                       | Labor tax rate                         | 0.3        | <a href="#">Kaplan et al. (2018)</a>     |
| $T$                          | Lump-sum transfer to HH at ss          | 0.27       | Balance gov't budget                     |
| $G$                          | Government spending                    | 0          |  |
| $\tau_p$                     | Labor subsidy to intermediate firms    | 1/6        | $\tau_p = 1 - \frac{1}{\mu}$             |
| $T_p$                        | Lump-sum tax on intermediate firms     | 1/6        | $T_p = \tau_p w N$                       |
| $\phi_\pi^+, \phi_\pi^-$     | Baseline Taylor rule coefficients      | 1.5        |  |
| $\phi_y$                     | Baseline Taylor rule coefficient       | 0          |  |
| Aggregate shocks             |  |            |  |
| $\rho_r$                     | Persistence for $\log \epsilon_t^r$    | 0.55       | US inflation persistence                 |
| $\sigma_r$                   | Volatility for $\log \epsilon_t^r$     | 1.175%     | US inflation volatility                  |
| $\chi$                       | Social cost of inflation               | 120        | <a href="#">Lagos and Wright (2005)</a>  |

Using the average price adjustment frequency reported in Table II of [Nakamura and Steinsson \(2008\)](#), I compute the average price adjustment frequency for service less shelter

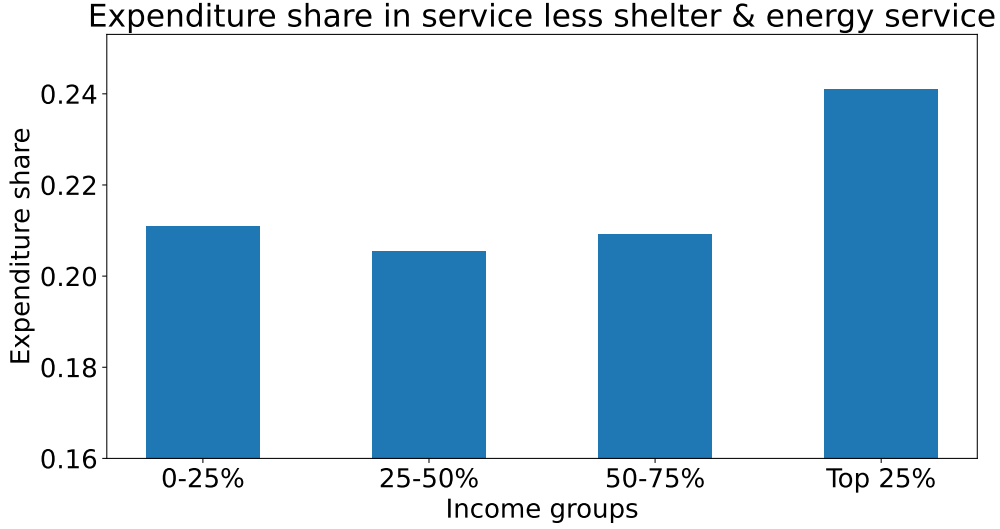


Figure 4: Elasticity of individual earning to aggregate earning & inflation, by income group.

and energy service as 15.6% every month, while the average frequency for other categories (including durable goods, non-durable goods, energy services, shelter) is 29.7%.

**Income process and earnings channel.** The idiosyncratic shocks on labor productivity follow a four-state Markov chain, corresponding to four income groups: bottom 25%, 25-75%, top 1-25%, and top 1%. The transition matrix across income states outside the top 1% comes from the discretization of a log-AR(1) process, such as the one in [Auclert et al. \(2021\)](#). I introduce the superstar state to match the top 1% of the income distribution and to capture the high earnings growth elasticity in the right tail of the income distribution, as in Figure 3.

For the earnings channel, I calibrate the earnings growth elasticity to aggregate output in Equation (8) to match the estimates I obtain in Section 2.3. To match these moments, I obtain  $\zeta(\xi_{h,t}) = 6$  for the poor state, -2 for the middle-income states, and -1 for the superstar state.<sup>15</sup>

**Aggregate shock.** The aggregate shocks in this economy are demand shocks akin to the fluctuations in the households' discount factor. The demand shock  $\epsilon_t^r$  follows a log-AR(1) process with quarterly persistence and volatility  $\rho_r = 0.55, \sigma_r = 1.175\%$ .  $\rho_r, \sigma_r$  are calibrated to match the persistence (0.572) and volatility (0.855%) of US quarterly inflation from 1947 to 2019.

<sup>15</sup> $\zeta(\xi_{h,t}) < 0$  for middle-income households since real wage elasticity is quite large in this model with price rigidity but flexible wages. As a result, a negative  $\zeta(\xi_{h,t})$  can still generate positive real earning elasticity.

**Social cost of inflation** In the baseline calibration, I choose the value of  $\chi$  such that the second term in Equation (15) can capture the welfare cost of inflation that is ignored by New Keynesian models. I refer to the money search literature, such as Lagos and Wright (2005), to calibrate  $\chi = 120$ , which implies that a 10% inflation would lead to a social welfare loss equivalent to all the households reducing consumption by 4.6%.

## 4 Results on Optimal Monetary Policy Rules

In this section, I present optimal monetary policy rule solutions from the perspective of a utilitarian central bank that sets the Pareto weights  $\omega_h \equiv 1$ . As discussed above, my framework also allows for other choices of Pareto weights. I will first introduce a set of counterfactual models for comparison. Then I will present the optimal policy rules in the full models, compared with optimal rules in the counterfactual models. To better understand the results, I will compare the impulse responses under the optimal rule and other rules, and study the distribution and decomposition of welfare gain when the central bank moves towards the optimal rule.

### 4.1 Models for comparison

In this section, I present the solution results to the optimal monetary policy rule problem. To gauge the contribution of each channel of redistributive inflation, besides the *full model*, I also solve the optimal monetary policy rules in three *counterfactuals*, with one channel shut down each time. The counterfactual models and the redistributive channels included are:

1. Counterfactual I. Set  $\zeta(\xi_{h,t}) \equiv 0$ . This model contains the revaluation and expenditure channels;
2. Counterfactual II. Set  $\zeta(\xi_{h,t}) \equiv 0$  and  $\underline{c} = 0$ . This model only contains the revaluation channel;
3. Counterfactual III. I shut down the idiosyncratic shocks by letting  $\xi_{h,t} \equiv 1$ , and set  $\zeta(\xi_{h,t}) \equiv 0, \underline{c} = 0$ . This would become a two-sector RANK model which does not contain any redistributive channels of inflation.

### 4.2 Optimal policy rule in the full model

For the benchmark full model, I calculate the expected social welfare  $\mathbb{E}_{\{\mathcal{E}_t\}} \Omega(\Phi, \{\mathcal{E}_t\})$  for each choice of the Taylor rule parameters  $\Phi = \{\phi_\pi^+, \phi_\pi^-, \phi_y\}$ . Then I solve for the optimal parameters  $\phi_\pi^{+,*}, \phi_\pi^{-,*}, \phi_y^*$  that maximize the expected social welfare.

First, as in Figure 6, I find the coefficient on output deviation has a negligible effect on the expected social welfare, which is consistent with the literature (Schmitt-Grohé and Uribe, 2007). As a result, I set  $\phi_y = 0$  for the remainder of the paper. For the coefficients on inflation, I plot the expected social welfare change compared to the steady state, as a function of  $\phi_\pi^+$  and  $\phi_\pi^-$  when setting  $\phi_y = 0$  in Figure 5. Since I study exponential decay shocks  $\{\mathcal{E}_t\} = \{\xi_0, \rho\xi_0, \rho^2\xi_0, \dots, 0\}$ , we can classify shocks into two categories, inflationary and deflationary, based on the sign of initial shock  $\xi_0$ . Then the expected social welfare conditional on inflationary shocks mostly depends on  $\phi_\pi^+$ , while that conditional on deflationary shocks mostly depends on  $\phi_\pi^-$ .<sup>16</sup>

In Figures 5a and 5b, I plot the expected social welfare change compared to the steady state, conditional on inflationary and deflationary shocks, as functions of  $\phi_\pi^+$  and  $\phi_\pi^-$ . The optimal values are  $\phi_\pi^{+,*} = 1.36$ ,  $\phi_\pi^{-,*} = 3$ , which means that a utilitarian central bank should adopt an asymmetric monetary policy rule that is accommodative towards inflation and aggressive towards deflation.

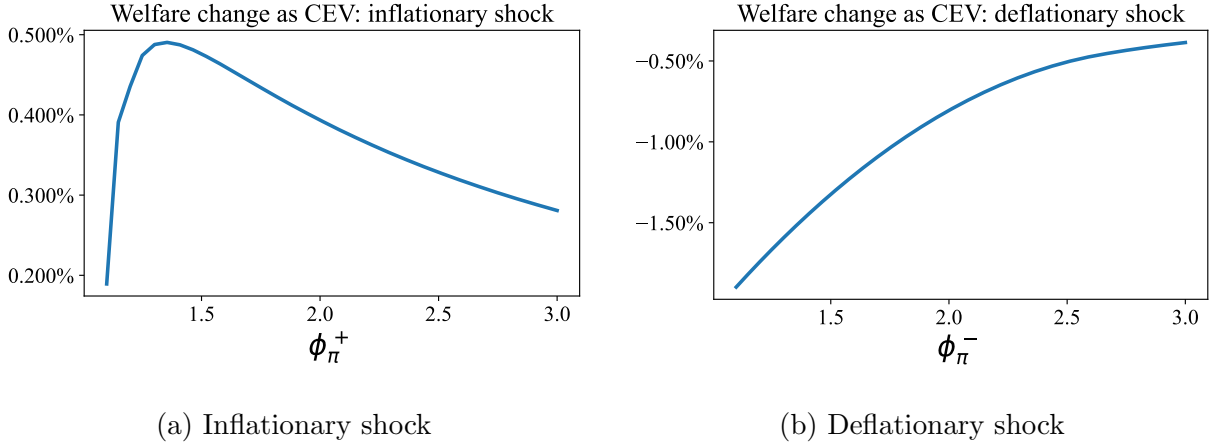


Figure 5: Welfare loss with  $\phi_\pi^+$ ,  $\phi_\pi^-$ , measured as consumption loss relative to steady state

Conditional on inflationary shocks, under the optimal policy regime, the expected welfare loss compared to the steady state in the absence of aggregate shocks is equivalent to all agents' consumption increase by 0.48%. In contrast, if we choose  $\phi_\pi^+ = 3$ , which is the optimal policy parameter implied by a RANK model (see Section 4.3), the welfare change is equivalent to a 0.3% consumption increase. In other words, moving from the policy with  $\phi_\pi^+ = 3$  to the optimal policy rule, the welfare improvement due to inflationary shocks drops by 40%.

The asymmetry in optimal monetary policy is robust when we shut down the quadratic

<sup>16</sup>With inflationary shocks, there could also be deflation after several periods, depending on the persistence of the shock. However, the magnitude of deflation after inflationary shocks is extremely small, so the value of  $\phi_\pi^-$  does not have a significant impact on the expected social welfare conditional on inflationary shocks.

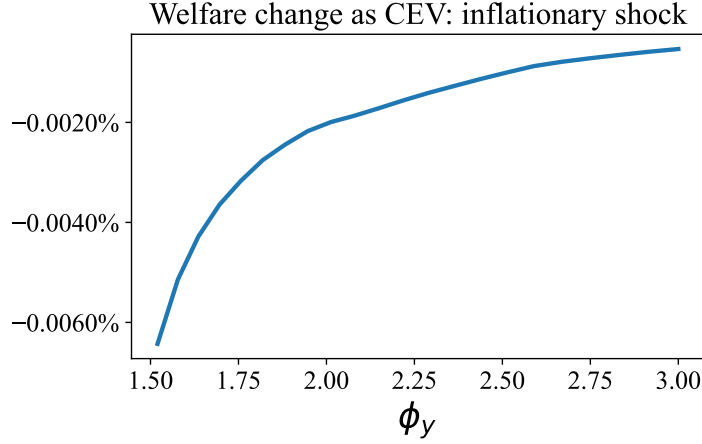


Figure 6: Welfare change with  $\phi_y$  as consumption equivalent variation (CEV) to steady state

inflation cost term in the objective function, see Figure 7, which plots the social welfare objective (15) with  $\chi = 0$ , namely with no additional term for the cost of inflation in the objective function.

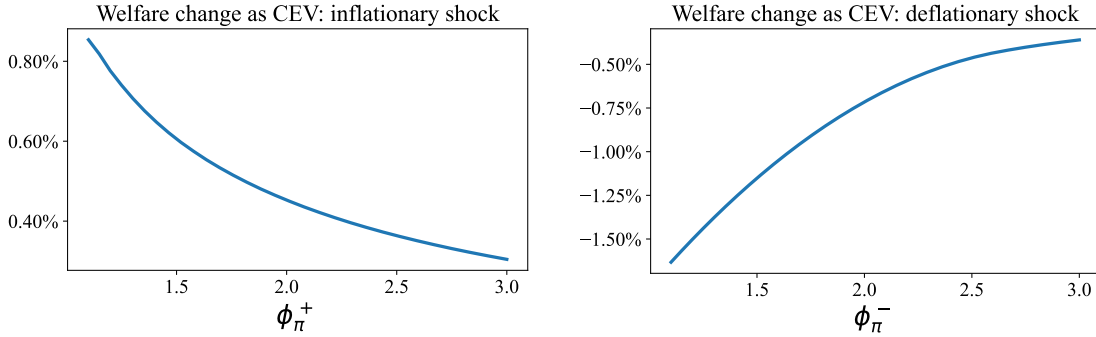


Figure 7: With  $\chi = 0$ , welfare change with  $\phi_\pi^+, \phi_\pi^-$  as consumption equivalent variation (CEV) to steady state

### 4.3 Optimal policy rule: model comparison

To better understand the optimal Taylor coefficients in the full model, I solve the optimal policy rules in the counterfactual models and report the optimal coefficients on inflation in Table 3.<sup>17</sup>

<sup>17</sup>More results of the expected social welfare and optimal policy rules in the counterfactual models are in Appendix C.2.

Table 3: Optimal monetary policy rules in different models

|                        | Redistributive channels            | Optimal $\phi_{\pi}^{+,*}$ | Optimal $\phi_{\pi}^{-,*}$ |
|------------------------|------------------------------------|----------------------------|----------------------------|
| Full model             | Revaluation, expenditure, earnings | 1.36                       | 3                          |
| Counterfactual 1       | Revaluation, expenditure           | 1.52                       | 3                          |
| Counterfactual 2       | Revaluation                        | 1.41                       | 3                          |
| Counterfactual 3: RANK | None                               | 3 (boundary)               | 3 (boundary)               |

The main findings are as follows.

First, from the perspective of a utilitarian central bank, the optimal policy rules in all three HANK models (full model, counterfactuals 1 and 2) are asymmetric. The central bank should be accommodative towards inflation and aggressive towards deflation.

Second, in the RANK model devoid of any redistributive channels, the utilitarian central bank should adopt a symmetric Taylor rule that is aggressive towards both inflation and deflation. Interestingly, the optimal monetary policy in the RANK model is very close to the optimal policy in a full HANK model with price stabilization as the dominant policy objective (i.e. a very large  $\chi$  in Equation (15)), which is plotted in Appendix C.2.2.

Third, as mentioned in Section 4.2, in the model featuring all three redistributive channels of inflation, if the central bank moves from the optimal policy implied by a RANK model to the true optimal policy for this model, the welfare cost due to inflationary shocks decrease by 23%.

Lastly, by solving for the optimal policy rules in the counterfactual models where I shut down one channel at a time, I find that the revaluation and earnings channels cause the optimal policy to be more accommodative. In contrast, the expenditure channel makes the optimal policy more aggressive. Quantitatively, the revaluation and earnings channels dominate.

#### 4.4 Comparing policy rules with impulse responses

To understand the welfare difference between the optimal policy rule and other policy rules, in the model featuring all three redistributive channels of inflation, I plot the impulse responses of various variables under (1) optimal policy rule, and (2) optimal RANK policy rule. Since the optimal policy parameter against deflationary shock is the same for these two policies, I compare impulse responses to positive inflationary shocks.

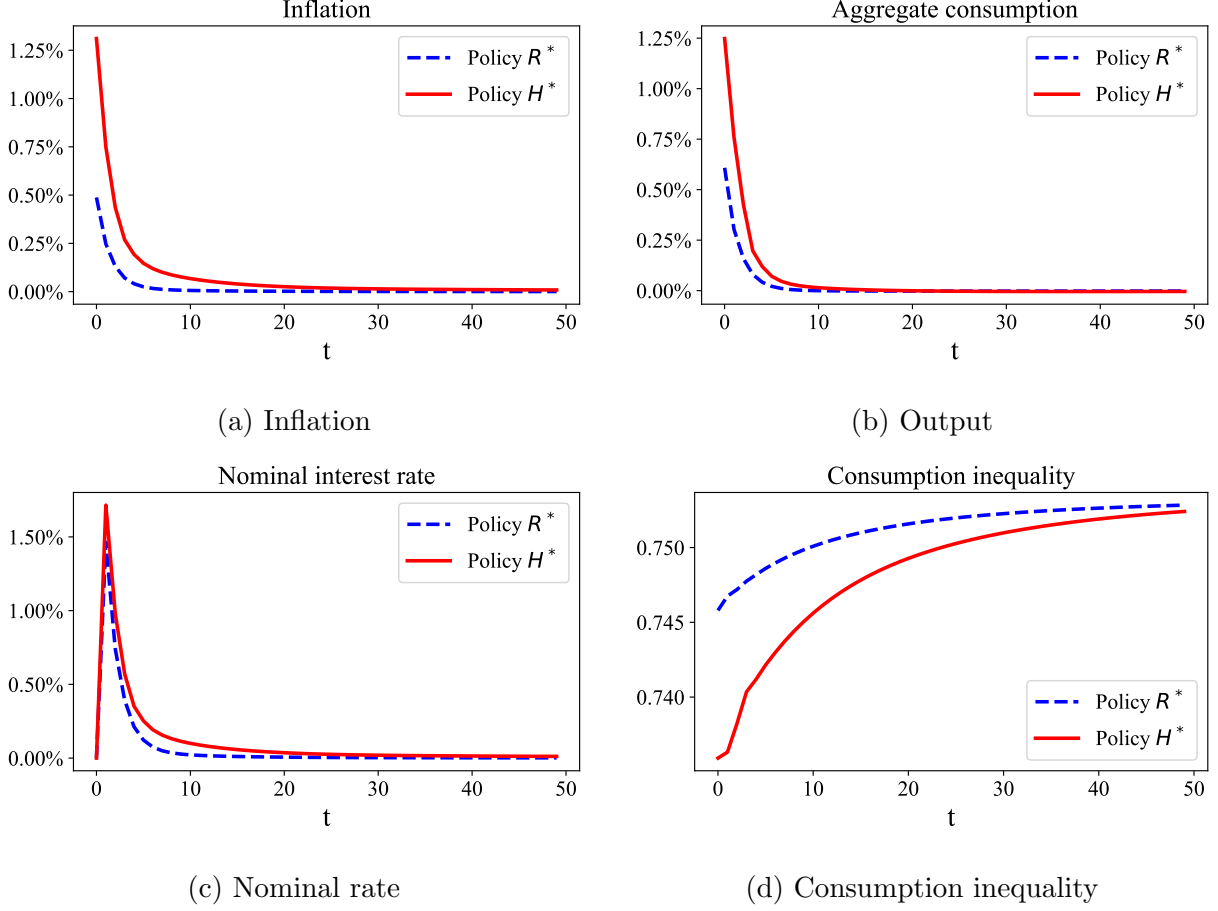


Figure 8: IRF to positive demand shock with different Taylor rule coefficients  $\phi_{\pi}^{+}$

Figures 8a to 8c plot the impulse response functions of aggregate output, inflation, and nominal interest rate after a 1.427% positive demand shock with different Taylor rule coefficients. After the same magnitude of aggregate shocks, the nominal rate would increase more when  $\phi_{\pi}^{+} = 3$ . Inflation and output respond less with a larger Taylor coefficient. As a result, there will be a larger welfare cost of inflation in households' preference (3) when we choose the optimal policy parameter.

Besides the aggregate variables, in Figure 8d, I plot the impulse responses of consumption inequality under each policy rule. The consumption inequality at time  $t$  is defined as

$$\int_h (c_{h,t} - C_t)^2 dh$$

where  $C_t$  is the aggregate real consumption plotted in Figure 8b. Under the optimal policy parameter  $\phi_{\pi}^{+,*} = 1.36$ , the consumption increase more, and the consumption inequality increases less. Both of these effects would offset the welfare loss due to inflation.



## 4.5 Distribution of welfare gain

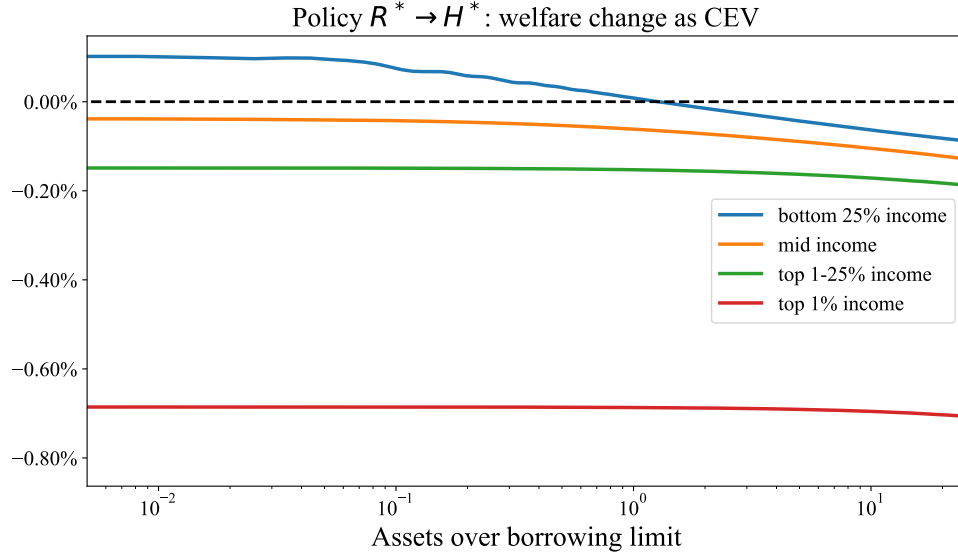
In the model featuring all three redistributive channels of inflation, I now look into the distribution of the welfare change when the central bank moves from the optimal policy implied by a RANK model (called policy “R”) to the true optimal policy for this full model (called policy “H”).

For households with individual state  $(b, \xi)$ , I measure the difference of their individual welfare under policies R and H, using consumption equivalent variation  $\lambda(b, \xi)$ , defined as

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^H(b, \xi), \ell_t^H(b, \xi)) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u((1 + \lambda(b, \xi))c_t^R(b, \xi), \ell_t^R(b, \xi)).$$

The consumption equivalent variation for welfare change  $\lambda(b, \xi)$  is plotted in Figure 9.

Figure 9: Welfare change as CEV from optimal RANK to optimal HANK policy



The main winners under the optimal policy regime are households with low income and low wealth. Households at the bottom 25% income distribution enjoy a welfare gain equivalent to increasing consumption by 0.16%. In contrast, households in the top 75% income group experience a welfare loss.

Following an inflationary shock, the central bank only moderately raises the nominal rate to keep some inflation, allowing low-income and low-wealth households to pay less for their debt, and to enjoy higher earnings growth. Low- and middle-income households would experience a greater price increase in their consumption baskets, though this downside does

not offset the benefit from the revaluation of household debt and earnings growth.

## 4.6 Decomposition of welfare gain

In the full model, I now investigate the source of the welfare change when the central bank moves from the optimal policy implied by a RANK model (policy “R”) to the true optimal policy for this full HANK model (policy “H”). I follow [Benabou \(2002\)](#); [Floden \(2001\)](#); [Dyrda and Pedroni \(2021\)](#) to decompose the welfare gain along the transition path into three components: (1) the efficiency gain on average consumption and leisure, (2) the insurance against ex-post risk, and (3) the ex-ante redistribution. As we see in the impulse response functions in Section 4.4 and welfare metric (15), households bear a higher welfare cost under policy “HH” directly due to higher inflation. Here we only decompose the welfare gain  $\Delta$  due to components in the household’s utility, as in

$$\int \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u((1 + \Delta)c_t^R(b, \xi), \ell_t^R(b, \xi)) d\lambda_0 = \int \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t^H(b, \xi), \ell_t^H(b, \xi)) d\lambda_0,$$

where  $\lambda_0$  is the initial distribution over states  $(b, \xi)$ . The aggregate welfare gain  $\Delta$  can be decomposed into three components as below.<sup>18</sup>

**1. Efficiency gain.** Under policy  $j \in \{H, R\}$ , let the aggregate level of  $c_t$  and  $\ell_t$  at each  $t$  be

$$C_t^j \equiv \int c_t^j(b, \xi) d\lambda_t^j(b, \xi), \quad L_t^j \equiv \int \ell_t^j(b, \xi) d\lambda_t^j(b, \xi),$$

where  $\lambda_t^j(b, \xi)$  is the distribution over  $(b, \xi)$ . The efficiency gain,  $\Delta_E$ , is then given by

$$\sum_{t=0}^{\infty} \beta^t u((1 + \Delta_E) C_t^R, L_t^R) = \sum_{t=0}^{\infty} \beta^t u(C_t^H, L_t^H).$$

**2. Insurance effect.** Given that households inherently exhibit risk aversion, their welfare increases if the uncertainty associated with their future consumption and labor trajectories is minimized, conditioned upon their initial assets and productivity status. To define this component precisely, first let  $\{\bar{c}_t^j(b_0, \xi_0), \bar{\ell}_t^j(b_0, \xi_0)\}_{t=0}^{\infty}$  denote a certainty-equivalent sequence

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<sup>18</sup>There are a few alternative approaches of welfare gain decomposition, including [Bhandari et al. \(2021a\)](#); [Dávila and Schaab \(2022b\)](#).

of consumption and labor conditional on a household's initial state that satisfies<sup>19</sup>

$$\sum_{t=0}^{\infty} \beta^t u(\bar{c}_t^j(b_0, \xi_0), \bar{\ell}_t^j(b_0, \xi_0)) = \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t^j, \ell_t^j) \right].$$

Then define  $\bar{C}_t^j$  and  $\bar{L}_t^j$  as

$$\bar{C}_t^j = \int \bar{c}_t^j(b_0, \xi_0) d\lambda_0, \quad \text{and} \quad \bar{L}_t^j = \int \bar{\ell}_t^j(b_0, \xi_0) d\lambda_0, \quad \text{for } j \in \{H, R\}. \quad (16)$$

The insurance effect,  $\Delta_I$ , is defined by

$$1 + \Delta_I \equiv \frac{1 - p_{risk}^H}{1 - p_{risk}^R}, \quad \text{where} \quad \sum_{t=0}^{\infty} \beta^t u((1 - p_{risk}^j) C_t^j, L_t^j) = \sum_{t=0}^{\infty} \beta^t u(\bar{C}_t^j, \bar{L}_t^j).$$

Here  $p_{risk}^j$  is the welfare cost of risk under policy regime  $j \in \{H, R\}$ .

**3. Redistribution effect.** Utilitarian welfare also increases if the inequality across households with different initial states  $(b_0, \xi_0)$  is reduced. Formally the redistribution effect,  $\Delta_R$ , can be defined as

$$1 + \Delta_R \equiv \frac{1 - p_{ineq}^H}{1 - p_{ineq}^R}, \quad \text{where} \quad \sum_{t=0}^{\infty} \beta^t u((1 - p_{ineq}^j) \bar{C}_t^j, \bar{L}_t^j) = \int \sum_{t=0}^{\infty} \beta^t u(\bar{c}_t^j(b_0, \xi_0), \bar{\ell}_t^j(b_0, \xi_0)) d\lambda_0$$

Here  $p_{ineq}^j$  denotes the welfare cost of inequality under policy regime  $j$ .

With the three components defined as above, we have the following proposition.

**Proposition 1** (Floden (2001); Dyrda and Pedroni (2021)). *For balanced-growth-path preferences with  $\gamma = 1$ , the components defined above satisfy the following relationship:*

$$1 + \Delta = (1 + \Delta_E)(1 + \Delta_I)(1 + \Delta_R).$$

Using consumption and labor supply paths under R and H policy rules, I find 72.23% of the aggregate welfare gain  $\Delta$  comes from the ex-ante redistribution, 21.68% comes from insurance against ex-post risk, and the remaining 6.09% is due to efficiency improvement in aggregate consumption and leisure. The primary sources of welfare gain are as a result of additional insurance and redistribution when the central bank accommodates inflation.

<sup>19</sup>The certainty-equivalent sequence of consumption and labor is not unique. Here I follow Dyrda and Pedroni (2021) to construct  $\bar{c}_t^j(b_0, \xi_0) = \eta^j(b_0, \xi_0) C_t^j$ , and  $1 - \bar{\ell}_t^j(b_0, \xi_0) = \eta^j(b_0, \xi_0) (1 - H_t^j)$ , for some function  $\eta^j(b_0, \xi_0)$ . The paths  $\{C_t^j\}_{t=0}^{\infty}$ , and  $\{H_t^j\}_{t=0}^{\infty}$  are computed in Equation (16).

## 5 Conclusion

Central banks around the world have announced that monetary policy should be more inclusive. In this paper, I study optimal nonlinear monetary policy rules in a quantitative HANK model where inflation has heterogeneous effects on households through their consumption baskets, nominal wealth positions, and earnings elasticity to business cycles. After calibrating the model to the empirical facts on these redistributive channels, I find that from the perspective of a utilitarian central bank, the optimal policy rule should be asymmetric. The central bank should accommodate inflationary shocks that benefit low-income households through nominal debt revaluation and higher earnings growth. In contrast, the central bank should react aggressively against deflationary shocks that hurt low-income households.

Besides the three channels in this paper, future work can incorporate other channels of redistributive inflation to study optimal monetary policy. For example, recent empirical work by [Fang, Liu, and Roussanov \(2022\)](#) suggests that real asset returns comove with the core inflation rate. As a result, it would be interesting to study optimal monetary policy design when households hold assets with heterogeneous levels of risk premium ([Kekre and Lenel, 2022](#)). Furthermore, inflation has different redistributive impacts under different fiscal regimes ([Leeper, 1991](#); [Bayer et al., 2019](#)), which brings new opportunities for the study of optimal monetary-fiscal policy.

In terms of methodology, although I adopt a different approach from the previous work, this paper generally belongs to the literature that solves optimal monetary policy around the deterministic steady state. As a result, it shares the limitations of these local solution approaches. In the steady state, households do not expect future inflation to come from the asymmetric policy rules and would not adjust their portfolio choice or consumption decision to that. One way to address this limitation is to adopt a global solution method. In ongoing work, I use DeepHAM, which is a class of machine learning-based global solution methods for heterogeneous agent models with aggregate shocks that I have developed with my coauthors ([Han, Yang, and E, 2021](#)), to study the global dynamics in HANK, and to evaluate the welfare effects of various monetary policy rules in the stochastic steady state. This is among one of the first attempts to study optimal policy problems in the HANK models beyond the local solutions.

# Appendix

## A Data Appendix

### A.1 Procedure of construct expenditure shares by income group

1. Use the quarterly pre-tax income data at the household level from the interview survey to calculate the income percentile cutoffs by year. Group households in the interview survey into 100 percentiles every year.
2. Put the quarterly expenditure, income, and household weights data from the interview survey of each year. Using the compiled interview data to calculate annual household level interview expenditure and income interval it belongs to. Make adjustments for the homeowner's equivalent rents, medical expenditures, and transportation.
3. Apply the income percentile cutoffs to households in the diary survey. Combine the interview and diary expenditures to get the expenditure shares for all the UCCs. For overlapped UCCs in both interview and diary data, use the hierarchical groupings guide from BLS to drop duplicates.
4. Aggregate households into income percentiles and calculate expenditure shares at the UCC level for 100 income groups.

### A.2 Map expenditure share to item-level price data

To link the expenditure shares that I have constructed with the CEX data to the item-level price data, I need a common classification of product classes. The expenditure class in the CEX data is based on the Universal Classification Code (UCC) categories, and the item class in the BLS price data is identified by “series ID”. We follow the BLS guidance<sup>20</sup> to use the “item code” in series ID to map them to the BLS Entry Level Item (ELI) classification. Then I construct a mapping between ELI and UCC classes. As both UCC and ELI are time-varying, I need a UCC-ELI concordance by year. The BLS provides a snapshot of UCC-ELI Concordance in Appendix 5 of the CPI Handbook of Methods.<sup>21</sup> However, this is only a recent snapshot and does not work for the data that are not so recent. I construct a year-by-year UCC-ELI Concordance by mapping the names of classes, and the mapping is available upon request. With the UCC-ELI Concordance by year, we can calculate the

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<sup>20</sup><https://www.bls.gov/help/hlpforma.htm#CU>

<sup>21</sup>The concordance snapshot can be accessed here <https://www.bls.gov/cpi/additional-resources/ce-cpi-concordance.htm>

monthly price index and annual expenditure shares, at both the UCC level and the ELI level.

### A.3 Measuring net nominal positions with SCF and FA data

Following [Doepke and Schneider \(2006\)](#), the net nominal position (NNP) of a household  $h$  is her direct nominal position (DNP) net of the (negative) nominal position she held through equity:

$$\text{NNP}_{h,t} = \text{DNP}_{h,t} - \tilde{\lambda}_t \times \text{equity\_held}_{h,t},$$

where  $\tilde{\lambda}_t$  is the net nominal leverage ratio for the business sector at time  $t$ , defined as the nominal debt position of the business sector per dollar of equity held:

$$\tilde{\lambda}_t = -\frac{\text{DNP of business sector}_t}{\text{net equity}_t}.$$

The direct nominal positions of households  $\text{DNP}_{h,t}$  is the sum of directly held nominal assets plus nominal assets held through investment intermediaries less nominal liabilities, measured using the SCF data.  $\tilde{\lambda}_t$  is measured with the FA data.

## B Model Appendix

### B.1 Equilibrium definition and conditions

In this section, I list the equilibrium conditions of the model in [Section 3](#).

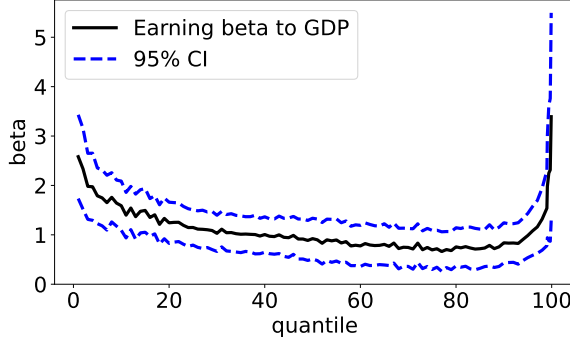
## C Additional Results

### C.1 Additional empirical results

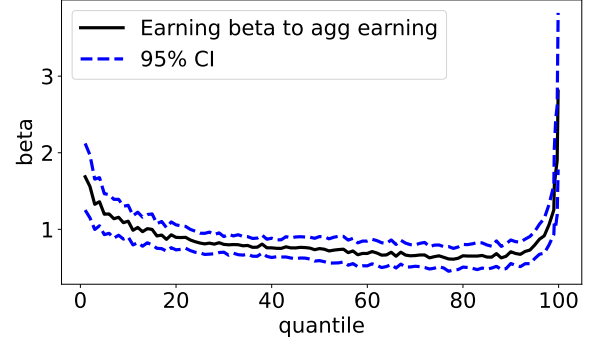
#### C.1.1 Earnings channel

Using the 1979-2011 annual data from [Guvenen et al. \(2014\)](#), the income groups, determined by households' income in the past year, are those at percentiles  $q \in \{1, \dots, 99, 99.1, 99.4, 99.9\}$ .

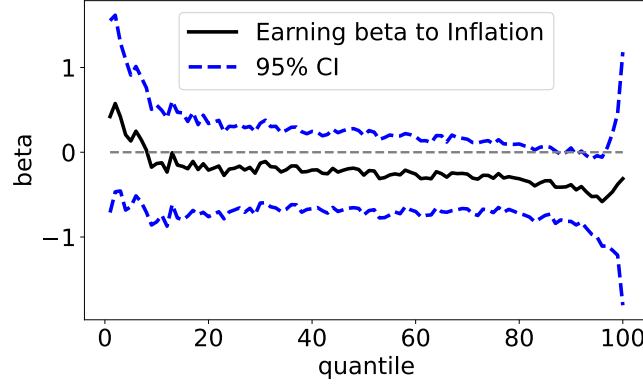
Figures [10a](#) and [10b](#) plot percentile-wise earnings growth elasticity to aggregate GDP growth and aggregate earnings growth. Earnings growth is very elastic to aggregate economic condition at the bottom and very top (0.5%) income percentiles, while insensitive in the middle income. These plots are similar to those in [Guvenen, Schulhofer-Wohl, Song, and Yogo \(2017\)](#) and [Alves, Kaplan, Moll, and Violante \(2020\)](#), though my confidence intervals



(a) earnings elasticity to aggregate output



(b) earnings elasticity to aggregate earning



(c) earnings elasticity to aggregate inflation

Figure 10: Elasticity of earnings growth to aggregate variables by income group.

are much wider as I run a yearly regression while they directly regress with individual level earnings data.

Figure 10c plots elasticity to aggregate inflation (end-of-year annualized inflation). The key finding is that the earnings growth of households at the bottom of the income distribution responds positively to inflation, while others are not very sensitive. The result is robust to adding percentile-specific time trend, or using middle-year annualized inflation.

## C.2 Additional results on optimal monetary policy

### C.2.1 Optimal monetary policy in counterfactual models

In this section, I report the results of optimal monetary policies in reduced HANK models with (1) revaluation and expenditure channels, and (2) only the revaluation channel. The summary table that compares all models is 3.



**Optimal monetary policy in HANK with revaluation channel**  $\phi_{\pi}^{+,*} = 1.41, \phi_{\pi}^{-,*} = 3$ : accommodative to inflation/expansion, but aggressive to deflation.

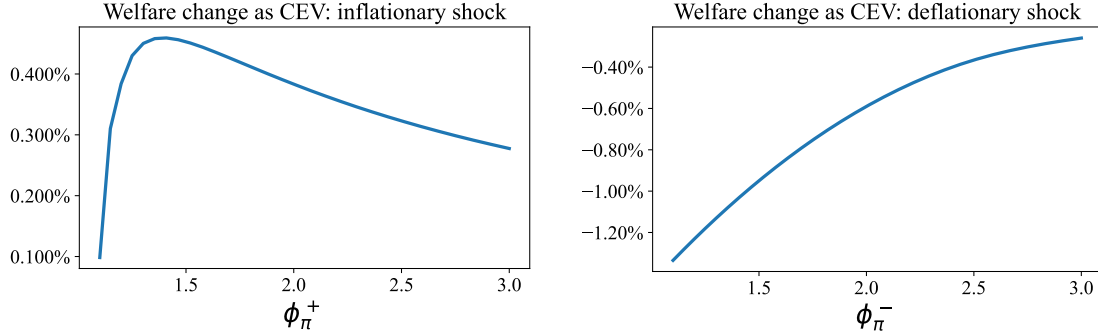


Figure 11: Welfare loss with  $\phi_{\pi}^{+}, \phi_{\pi}^{-}$ , measured as consumption loss relative to steady state

**Optimal monetary policy in HANK with revaluation and expenditure channels**  $\phi_{\pi}^{+,*} = 1.52, \phi_{\pi}^{-,*} = 3$ : accommodative to inflation/expansion, but aggressive to deflation.

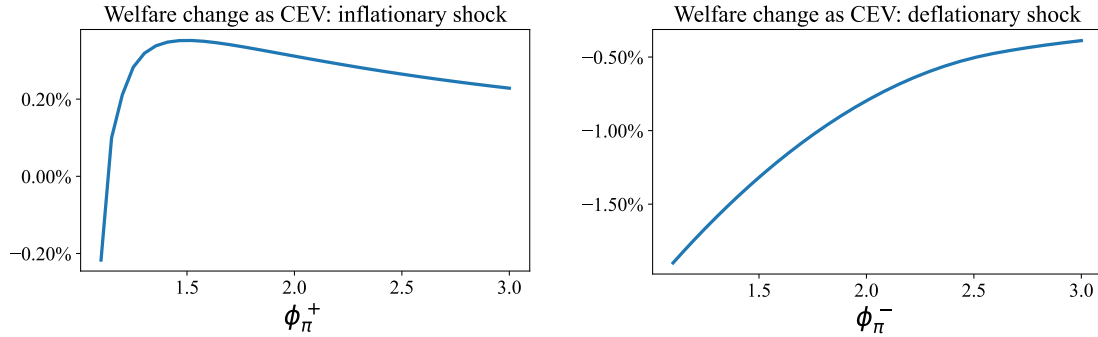


Figure 12: Welfare loss with  $\phi_{\pi}^{+}, \phi_{\pi}^{-}$ , measured as consumption loss relative to steady state

### C.2.2 Results with alternative calibration of inflation cost

Figure 13 plots social welfare with  $\chi = 1000$ , which implies a very high social cost of inflation in central bank's objective function (15). This means price stability is the top priority, such as the case of the ECB. In this case, the optimal policy in HANK is symmetric, like that in RANK.

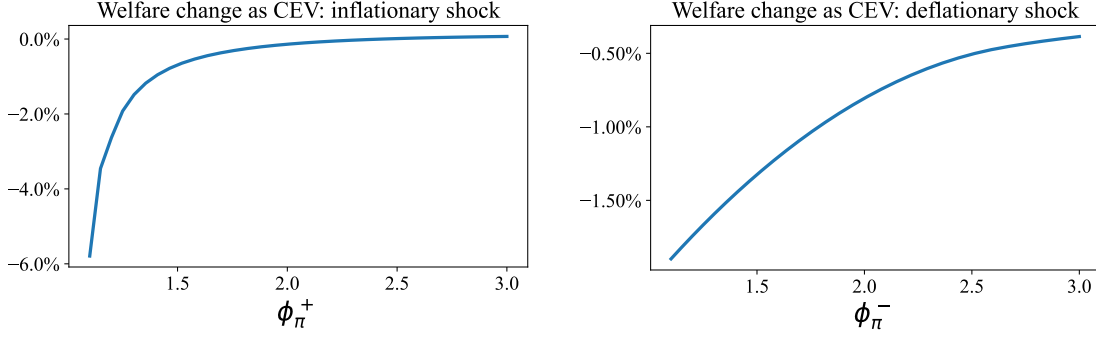


Figure 13: With  $\chi = 1000$ , welfare change with  $\phi_\pi^+, \phi_\pi^-$  as consumption equivalent variation (CEV) to steady state

### C.2.3 Results with alternative calibration as McKay and Wolf (2022)

In a linear-quadratic framework, McKay and Wolf (2022) solves the Ramsey problem by introducing a certain set of Pareto weights that make the deterministic steady state efficient. This implies that the Pareto weights should be higher for high-income people. In this setup, they find the concern for inequality only has a moderate effect on optimal monetary policy. I argue that their results crucially depend on this particular choice of Pareto weights that “hates the poor”. In Figure 14, I plot the social welfare with McKay and Wolf (2022) Pareto weights, and the results are similar to their findings.

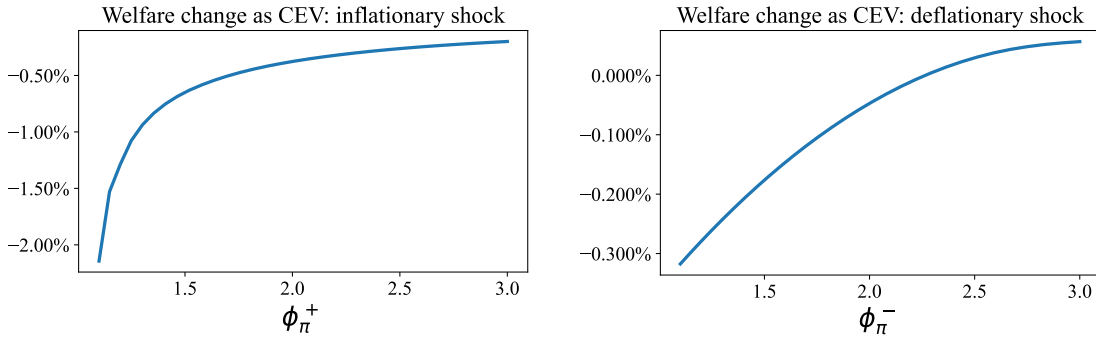


Figure 14: With Pareto weights in McKay and Wolf (2022), welfare change with  $\phi_\pi^+, \phi_\pi^-$  as consumption equivalent variation (CEV) to steady state

Comparing Figure 14 and Figure 5 with utilitarian Pareto weights, we can see that the main results of McKay and Wolf (2022) crucially depend on their choice of Pareto weights, which is quite different from the perspective of an “inclusive” central bank.

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