杨雨函. 20200112119. 作业2.

1.一组含^{24}Na和^{32}P的混合物，^{24}Na的活度占98%，^{32}P的活度占2%，问多长时间后二者的活度相同？

343.2h.

$A_1 = A_{10}e^{-\lambda_1 t} = A_2 = A_{20}e^{-\lambda_2 t} \rightarrow 49e^{-\lambda_1 t} = e^{-\lambda_2 t} \rightarrow t = \frac{\ln 49}{\lambda_1 - \lambda_2}$, $T_1 = 14.6h$, $T_2 = 14.3d$.

$\rightarrow t \approx 85.62h$.

2.推导氡的平衡当量浓度：

$$C_{eq} = 0.104 \cdot C(^{218}Po) + 0.514 \cdot C(^{214}Pb) + 0.382 \cdot C(^{214}Bi)$$

式中: $C(^{218}Po)$、$C(^{214}Pb)$和$C(^{214}Bi)$分别代表氡的短寿命衰变产物^{218}Po、^{214}Pb、^{214}Bi的活度浓度。（提示：可列出平衡组和不平衡组α潜能浓度相等的算式，并利用活度和原子数之间的关系式）

平衡组: $C_p = \left(13.7 \cdot \frac{Ceq}{\lambda_{218Po}} + 7.7 \times \left(\frac{Ceq}{\lambda_{214Pb}} + \frac{Ceq}{\lambda_{214Bi}} + \frac{Ceq}{\lambda_{214Po}}\right)\right) \times 1.602 \times 10^{-13}$ J/m³.

不平衡组. $C_p = 13.7 \cdot \left(\frac{C(^{218}Po)}{\lambda_{218Po}} + 7.7 \times \left(\frac{C(^{214}Pb)}{\lambda_{214Pb}} + \frac{C(^{214}Bi)}{\lambda_{214Bi}} + \frac{C(^{214}Po)}{\lambda_{214Po}}\right)\right) \times 1.602 \times 10^{-13}$ J/m³.

$\rightarrow [13.7 \times 3.05 + 7.7 \times (26.8 + 19.9)] Ceq = 13.7 \times 3.05\, C(^{218}Po) + 7.7 \times 26.8\, C(^{214}Pb) + 7.7 \times 19.9\, C(^{214}Bi)$.

$\rightarrow 401.375\, Ceq = 41.785\, C(^{218}Po) + 206.36\, C(^{214}Pb) + 153.23\, C(^{214}Bi)$.

$\rightarrow Ceq = 0.104\, C(^{218}Po) + 0.514\, C(^{214}Pb) + 0.382\, C(^{214}Bi)$.

3.在黑火药中,硝酸钾(KNO_3)是主要成分。在天然钾中^{40}K含量0.0118%,它是放射性核素。因此通过放射性强度的测量,有可能对火药进行探测。试计算100克硝酸钾样品中的放射性强度。

$N = \frac{100g}{(39+14+48)g/mol} \cdot N_A \approx 5.96 \times 10^{23}$ 个.

$N_{40K} = N \cdot 0.0118\% \approx 7.03 \times 10^{19}$ 个

$A = \lambda N = N \cdot \frac{\ln 2}{T} \approx 7.03 \times 10^{19} \cdot \frac{\ln 2}{1.277 \times 10^9 y} \approx 3.82 \times 10^{10}\, y^{-1}$.

4. 求与1g Ra-226处于放射性平衡的氡气（Rn-222）体积（假设温度0°C和压强760mmHg情况下）

^{226}Ra与^{222}Rn处于长期平衡，即 $A_{226Ra} = A_{222Rn} \rightarrow \lambda_1 N_1 = \lambda_2 N_2$.

$N_{226Ra} = \frac{1g}{226g/mol} \cdot N_A \approx 2.664 \times 10^{21}$ 个.

$N_{222Rn} = \frac{\lambda_1}{\lambda_2} N_{226Ra} = \frac{T_2}{T_1} N_{226Ra} \approx 1.745 \times 10^{16}$ 个

$PV = NkT$.

$V = \frac{NkT}{P} = \frac{1.745 \times 10^{16} \times 1.38 \times 10^{-23} \times 273.15}{1.01 \times 10^5} \approx 6.51 \times 10^{-10}\, m^3$

5. ^{99}Mo-^{99m}Tc发生器（被形象地称为“钼锝母牛”）有如下的衰变规律：

$$^{99}Mo \xrightarrow[66.02\,h]{\beta} {}^{99m}Tc \xrightarrow[6.02\,h]{\gamma} {}^{99}Tc$$

问：在一次淋洗后再经过多长时间淋洗^{99m}Tc，可以使淋洗后得到的^{99m}Tc的活度为^{99}Mo活度的98%？

$N_1 = N_0 \cdot e^{-\lambda_1 t}$. $\rightarrow A_1 = \lambda_1 N_0 \cdot e^{-\lambda_1 t}$.

$dN_2 = \lambda_1 N_1 \cdot dt - \lambda_2 N_2 dt \longrightarrow N_2 = N_0 \cdot \frac{\lambda_1}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$

$\rightarrow A_2 = N_0 \cdot \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} (e^{-\lambda_1 t} - e^{-\lambda_2 t})$.

$A_2 = 0.98 A_1$

$\rightarrow t = \frac{\ln(0.02\lambda_2 + 0.98\lambda_1) - \ln \lambda_2}{\lambda_1 - \lambda_2} \approx 21.15\,h$.