12.6.

$dx' = \gamma(dx - v\,dt)$. $dt' = \gamma(dt - \frac{v}{c^2}dx)$

$$u' = \frac{dx'}{dt'} = \frac{dx/dt - v}{1 - \frac{v}{c^2}\frac{dx}{dt}} = \frac{u-v}{1-\frac{uv}{c^2}}.$$

$$\rightarrow v_{AC} = \frac{v_{AB} - (-v_{BC})}{1 - \frac{v_{AB}(-v_{BC})}{c^2}} = \frac{v_{AB} + v_{BC}}{1 + \frac{v_{AB}v_{BC}}{c^2}}$$

12.34.

$\bar{E} = \gamma' mc^2$.

$V' = \frac{2V}{1+V^2/c^2}$. $\rightarrow \beta' = \frac{2\beta}{1+\beta^2}$.

$\bar{E} = \frac{1}{\sqrt{1-\beta'}} mc^2 = \frac{1+\beta^2}{1-\beta^2} mc^2$.

$E = \frac{1}{\sqrt{1-\beta}} mc^2$.

$\rightarrow \bar{E} = \frac{2E^2}{mc^2} - mc^2$.

若 $mc^2 = 1\,GeV$. $E = 30\,GeV$

$\rightarrow \bar{E} = 1799\,GeV >> 4E$.

12.45.

C: $\vec{E} = -\frac{q}{4\pi\varepsilon_0 d^2}\hat{y}$. $\vec{B} = 0$. $\vec{F} = -\frac{q^2}{4\pi\varepsilon_0 d^2}\hat{y}$.

A: $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$. $\vec{E} = -\frac{\gamma q}{4\pi\varepsilon_0 d}\hat{y}$. $\vec{B} = -\gamma\frac{v}{c^2}\frac{q}{4\pi\varepsilon_0 d^2}\hat{z}$.

$\vec{F} = -\gamma c(1+\beta^2)\frac{q^2}{4\pi\varepsilon_0 d^2}\hat{y}$.

B: $V = \frac{2v}{1+v^2/c^2}$. $\rightarrow \beta' = \frac{2\beta}{1+\beta^2}$. $\rightarrow \gamma' = \frac{1+\beta^2}{1-\beta^2}$.

$\vec{E} = -\frac{\gamma' q}{4\pi\varepsilon_0 d}\hat{y}$. $\vec{B} = -\gamma\beta'\frac{1}{c}\frac{q}{4\pi\varepsilon_0 d^2}\hat{z}$. $\vec{F} = -\gamma'\frac{q^2}{4\pi\varepsilon_0 d^2}\hat{y}$.

9.1. (补交第四周作业缺的一题)

(a). $q_{lm}=\int Y_{lm}^{*}(\theta,\phi)r^{l}\rho(r,\theta,\varphi)d^{3}x$. $\quad \rho(r,\theta,\varphi)=\rho(r,\theta,\phi-\omega t)$

$=\int Y_{lm}^{*}(\theta,\varphi+\omega t)r^{l}\rho(r,\theta,\varphi)d^{3}x$.

$=\left(\int Y_{lm}^{*}(\theta,\varphi)r^{l}\rho(r,\theta,\varphi)d^{3}x\right)e^{-im\omega t}=\bar{q}_{lm}e^{-im\omega t}$

$q_{l-m}(t)Y_{l-m}(\theta,\phi)+q_{lm}(t)Y_{lm}(\theta,\phi)=(q_{lm}(t)Y_{lm}(\theta,\phi))^{*}+q_{lm}(t)Y_{lm}(\theta,\phi)$

$=Re\left[2\left(\int Y_{lm}^{*}(\theta,\phi)r^{l}\rho(r,\theta,\varphi)d^{3}x\right)Y_{lm}(\theta,\phi)e^{-im\omega t}\right]$

(b) $\rho_{-n}(\vec{x})=\rho_{n}^{*}(\vec{x})$.

$\rho(x,t)=\sum_{n=-\infty}^{\infty}\rho_{n}(x)e^{-in\omega_{0}t}$

(c). $\rho(x,t)=\frac{q}{R^{2}}\delta(r-R)\delta(\cos\theta)\delta(\phi-\omega_{0}t)$.

a: $\bar{q}_{00}=\sqrt{\frac{1}{4\pi}}q$. $\quad \bar{q}_{00}\text{eff}=\sqrt{\frac{1}{4\pi}}qR$.

$\bar{q}_{11}=-\sqrt{\frac{3}{8\pi}}qR$ $\quad \bar{q}_{11}\text{eff}=-\sqrt{\frac{3}{2\pi}}qR$.

b: $\rho_{n}(x)=\frac{1}{T}\int_{0}^{T}\rho(x,t)e^{in\omega_{0}t}dt=\frac{q}{2\pi R^{2}}\delta(r-R)\delta(\cos\theta)e^{in\phi}$.

$q_{lm}(\rho_{n}(x))=\int r^{l}Y_{lm}^{*}(\theta,\phi)\rho_{n}(r,\theta,\phi)r^{2}\sin\theta\, dr\, d\theta\, d\phi=\delta_{mn}qR^{l}Y_{lm}^{*}(\frac{\pi}{2},0)$.

当$l-m=2k$时，辐射频率为$m\omega_{0}$.