1. (1) $I = \frac{ml^2}{3} + \frac{MR^2}{2} + Ml^2$.

(2) $l_c = \frac{m \cdot \frac{l}{2} + Ml}{m+M}$

$I_c = I - (m+M) l_c^2 = \frac{ml^2}{3} + \frac{MR^2}{2} + Ml^2 - \frac{(m\frac{l}{2} + Ml)^2}{(m+M)}$.

(3) $E_p = (m+M) g l_c (1-\cos\theta) \approx \frac{l}{2}(\frac{m}{2}+M) g\theta^2$

刚性连接

$E_k = \frac{1}{2} I \dot{\theta}^2$

$= \frac{1}{2}(\frac{ml^2}{3} + \frac{MR^2}{2} + Ml^2)\dot{\theta}^2$

$k_0 = (\frac{m}{2}+M) g l$ $\quad$ $m_0 = (\frac{ml^2}{3} + \frac{MR^2}{2} + Ml^2)$

$\therefore T = 2\pi\sqrt{\frac{m_0}{k_0}} = 2\pi\sqrt{\frac{\frac{ml^2}{3} + \frac{MR^2}{2} + Ml^2}{(\frac{m}{2}+M) g l}}$.

自由转动：

$E_k' = \frac{1}{2}(\frac{ml^2}{3} + Ml^2)\dot{\theta}^2$

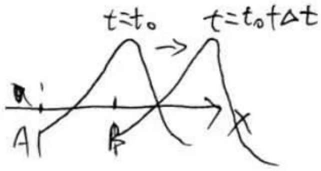
$\therefore m_0' \to (\frac{ml^2}{3} + Ml^2)$

$T' = 2\pi\sqrt{\frac{m_0'}{k_0}} = 2\pi\sqrt{\frac{\frac{ml^2}{3} + Ml^2}{(\frac{m}{2}+M) g l}}$

$T > T'$

$\therefore$ 周期变短

2. 波向右传播时



A在t_0时的相位经Δt后传播给B.

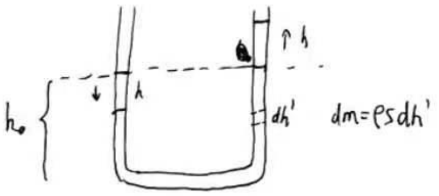
即B在$t_0+\Delta t$时的相位与A在t_0时相同

∴此时A相位超前于B.

∴ A超前　　向右传播

B超前　　向左传播

3. 重力势能:



dh: $dm=\rho s\,dh$

$$E_p=\int gh'dm=\int_0^{h_0-h}\rho sgh'dh'+\int_0^{h_0+h}\rho sgh'dh'=\frac{\rho sg}{2}\left[(h_0-h)^2+(h_0+h)^2\right]=\rho sg(h^2+h_0^2)$$

动能:

$$E_k=\frac{m}{2}\dot{h}^2=\frac{\rho sL}{2}\dot{h}^2$$　(m为液柱总质量, L为总长度)

$$\therefore \omega=\sqrt{\frac{2\rho sg}{\rho sL}}=\sqrt{\frac{2g}{L}}\qquad T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{L}{2g}}$$

4. (1) 设 x_0 时重力与弹力平衡了.

∴ $mg = kx_0$

∵ 最低位置在 $x' = 10cm$ 处.

∴ $mgx' = \frac{1}{2}kx'^2 \Rightarrow x_0 = \frac{mg}{k} = \frac{x'}{2} = 5cm$

$m\ddot{x} = mg - kx = k(x_0 - x)$

设 $y = x - x_0$ ∴ $m\ddot{y} + ky = 0$

∴ $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{5cm}} \approx 14 rad/s$

(2) $y = x - x_0 = A\cos(\omega t + \phi)$

$t=0$时. $x=0$ $v=0$

∴ $\begin{cases} -x_0 = A\cos\phi \\ 0 = -A\omega\sin\phi \end{cases}$

∴ $\begin{cases} \phi = 0 \\ A = -x_0 \end{cases}$

$x = x_0 - x_0\cos\omega t$

$x = 8cm$时.

$8 = 5(1 - \cos\omega t)$

$\cos\omega t = -\frac{3}{5}$

∴ $\sin\omega t = \frac{4}{5}$

$v = x_0\omega\sin\omega t$

$= 5 \times 14 \times \frac{4}{5} = 56 cm/s$

(3) $\omega' = \sqrt{\frac{k}{m+300g}} = \frac{\omega}{2} = \frac{1}{2}\sqrt{\frac{k}{m}}$

∴ $m = 100g$

(4) $x_0' = \frac{(m+300g)g}{k} = \frac{4mg}{k} = 20cm$

5. 圆柱与地面间、平板与圆柱间均有 $v=\omega r$ 的相对速度，设 x 为 M 偏离平衡位置的距离

$\because v_m=\omega r \qquad v_M=2\omega r=\dot{x}$

$\therefore v_m=\frac{\dot{x}}{2} \qquad \omega=\frac{\dot{x}}{2r}$

动能 $E_k=\frac{M}{2}\dot{x}^2+2\left(\frac{m}{2}v_m^2+\frac{1}{2}I\omega^2\right)$

$=\frac{M}{2}\dot{x}^2+m\left(\frac{\dot{x}^2}{4}+\frac{1}{2}r^2\frac{\dot{x}^2}{4r^2}\right)$

$=\frac{1}{2}\left(M+\frac{3}{4}m\right)\dot{x}^2$

势能 $E_p=2\cdot\frac{1}{2}kx^2=kx^2$

$\therefore$ 简谐运动

$\omega=\cancel{\sqrt{\frac{M+\frac{3}{4}m}{}}}\sqrt{\frac{2k}{M+\frac{3}{4}m}} \qquad T=\frac{2\pi}{\omega}=2\pi\sqrt{\frac{4M+3m}{8k}}$

1、先求弹簧在自身重量下伸长的长度（l_0为原长）

取一段 dy，质量为 $dm=\rho dy=\frac{m}{l_0+x_0}dy$

重力势能 $dE=-dm\cdot g\cdot y=-\frac{mg}{l_0+x_0}y\,dy$

总势能 $E_p=\frac{1}{2}kx_0^2+\int dE=\frac{1}{2}kx_0^2-\frac{mg}{l_0+x_0}\int_0^{l_0+x_0}y\,dy$

$$=\frac{1}{2}kx_0^2-\frac{mg}{2}(l_0+x_0)$$

平衡时势能取极小值

则 $dE_p/dx_0=0 \Rightarrow kx_0=\frac{mg}{2}$　　$x_0=\frac{mg}{2k}$

所以弹簧在自身重量下的伸长等价于无质量的弹簧挂 $\frac{m}{2}$ 的物体.

此时势能为：

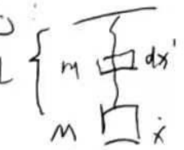
$E_p=\frac{1}{2}k(x_0+x)^2-(\frac{m}{2}+M)gx$

x_0为平衡时伸长长度，有 $kx_0=(\frac{m}{2}+M)g$

∴ $E_p=\frac{1}{2}kx^2+kx_0x-(\frac{m}{2}+M)gx$

$$=\frac{1}{2}kx^2$$

动能



$E_{KM}=\frac{M}{2}\dot{x}^2$

考虑 dx' 段，$dm=\frac{m}{L}dx'$　　$v=\frac{x'}{L}\dot{x}$

∴ $dE_{km}=\frac{1}{2}dm\,v^2=\frac{1}{2}(\frac{m}{L}dx')(\frac{x'}{L}\dot{x})^2$

$E_{km}=\int dE_{km}=\frac{m\dot{x}^2}{2L^3}\int_0^L x'^2dx'=\frac{m}{6}\dot{x}^2$

$E_K=E_{KM}+E_{km}=\frac{1}{2}(M+\frac{m}{3})\dot{x}^2$

∴ $T=2\pi\sqrt{\frac{M+\frac{m}{3}}{k}}$

7. $A=0.001m$ $\lambda=2\times0.1m=0.2m$ $v=330m/s$

$\cancel{\omega=}$ $k=\frac{2\pi}{\lambda}=10\pi\ rad/m$

$\omega=kv=3300\pi rad/s$

$u=A\cos(kx+\omega t+\varphi)$

$\because t=0$ $x=0$ 时 $u=0$ $\therefore\varphi=\frac{\pi}{2}$ $\therefore u=0.001\cos(10\pi x+3300\pi t+\frac{\pi}{2})$