1. Suppose that a book of 300 pages contains 200 misprints. Use Poisson approximation to write down the probability that there is more than one misprint on a particular page.

$\alpha = \frac{200}{300} = \frac{2}{3}$.

$P(X=k) = \frac{\alpha^k}{k!}e^{-\alpha}$.

$P(X>1) = \sum_{k=2}^{\infty} \frac{\alpha^k}{k!}e^{-\alpha} = 1-\frac{5}{3}e^{-\frac{2}{3}}$.

2. In a school where 4% of the children write with their left hands, what is the probability that there are no left-handed children in a class of 25?

二项分布. $P(X=0) = \binom{25}{0}\cdot(1-4\%)^{25}$
$= (96\%)^{25}$.

3. Six dice are thrown 200 times by the players. Estimate the probability of obtaining "six different faces" k times, where $k = 0, 1, 2, 3, 4, 5$.

$p = \frac{6!}{6^6}$

$P(X=k) = \binom{200}{k}\cdot p^k\cdot(1-p)^{200-k}$

$P(X=0) = (1-\frac{6!}{6^6})^{200} \approx 0.0446$.

$P(X=1) = 200\,\frac{6!}{6^6}\cdot(1-\frac{6!}{6^6})^{199} \approx 0.140$

$P(X=2) = 19900\times(\frac{6!}{6^6})^2(1-\frac{6!}{6^6})^{198} \approx 0.218$.

$P(X=3) = 1313400\times(\frac{6!}{6^6})^3(1-\frac{6!}{6^6})^{197} \approx 0.222$.

$P(X=4) = 64684950\times(\frac{6!}{6^6})^4(1-\frac{6!}{6^6})^{196} \approx 0.174$.

$P(X=5) = 2535650040\times p^5(1-p)^{195} \approx 0.107$.

6. Find the maximum term or terms in the binomial distribution $B_k(n;p)$, $0 \le k \le n$. Show that the terms increase up to the maximum and then decrease. [Hint: take ratios of consecutive terms.]

$P(X=k) = \binom{n}{k}p^k(1-p)^{n-k}$.

$\frac{P(X=k+1)}{P(X=k)} = \frac{n-k}{k+1}\,\frac{p}{1-p}$

设 $F(k) = \frac{n-k}{k+1}\,\frac{p}{1-p}$, $k=0,1,2,\ldots$

若 $F(k)>1$, 则 $k < np+p-1$.

若 $F(k)<1$, 则 $k > np+p-1$.

所以当 $0 \le k < np+p-1$ 时 $P(X=k)$ 单增.

当 $k > np+p-1$ 时, $P(X=k)$ 单减.

二项分布最大项为 $P(X=\lceil np+p-1\rceil)$ 或 $P(X=\lfloor np+p\rfloor)$.

7. Find the maximum term or terms in the Poisson distribution $\pi_k(\alpha)$, $0 \le k < \infty$. Show the same behavior of the terms as in No. 6.

$\pi_k(\alpha) = \frac{\alpha^k}{k!}e^{-\alpha}$.

$\frac{\pi_{k+1}(\alpha)}{\pi_k(\alpha)} = \frac{\alpha}{k+1}$. $k=0,1,2,\ldots$

当 $k<\alpha-1$ 时 $\frac{\alpha}{k+1}>1$. 单增.

当 $k>\alpha-1$ 时. $\frac{\alpha}{k+1}<1$. 单减

所以最大项为 $P(X=\lceil\alpha-1\rceil)$ 或 $P(X=\lfloor\alpha\rfloor)$.

8. Let X be a random variable such that $P(X = c+kh) = \pi_k(\alpha)$, where c is a real and h is a positive number. Find the Laplace transform of X.

$E(e^{-\lambda X}) = \sum_{k=0}^{\infty} P(X=c+kh)\cdot e^{-\lambda(c+kh)}$

$= \sum_{k=0}^{\infty}\frac{\alpha^k}{k!}e^{-\alpha}\cdot e^{-\lambda(c+kh)}$.

$= e^{-(\alpha+\lambda c)}\sum_{k=0}^{\infty}\frac{(\alpha e^{-\lambda h})^k}{k!} = e^{-\alpha-\lambda c+\alpha e^{-\lambda h}}$

9. Find the convolution of two sequences given by Poisson distributions $\{\pi_k(\alpha)\}$ and $\{\pi_k(\beta)\}$.

$P(X*Y=m) = \sum_{k=0}^{\infty}\pi_k(\alpha)\,\pi_{m-k}(\beta) = \sum_{k=0}^{\infty}\frac{\alpha^k}{k!}e^{-\alpha}\,\frac{\beta^k}{(m-k)!}e^{-\beta}$

$= e^{-(\alpha+\beta)}\cdot\sum_{k=0}^{\infty}\frac{(\alpha\beta)^k}{k!(m-k)!} = \frac{e^{-(\alpha+\beta)}}{m!}\sum_{k=0}^{\infty}\binom{m}{k}(\alpha\beta)^k$

$= \frac{e^{-(\alpha+\beta)}}{m!}(1+\alpha\beta)^m$