Homework 6 for GPI

1. KK 4.23 （6.5）
2. KK.4.28 (6.13)
3. KK 4.29 (6.14)
4. KK 4.30 (6.16)
5. Some practice on partial derivatives:

   (a) A function called Lagrange is given by: $L = \frac{1}{2}(v_x^2 + v_y^2) - 3\cos t$, it is a function explicitly depend on v_x,v_y and t.
   Find the following derivatives: 1) The partial derivative of L vs. t; vs. v_x.
   2) Suppose we know that the velocities are related to time:
   $v_x = a\cos t, v_y = a\sin t$. The total derivative (i.e. as t changes by small amount, how much L will change): dL/dt. Understand the difference between this vs. $(\frac{\partial L}{\partial t})_{v_x,v_y}$ in 1).

   (b) $G(x,y) = \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, an equation represents a ellipse, use the partial derivative method to find dy/dx (the tangent line on the ellipse)

   (c) For a function of $f(x,y) = xy^2 + y\cos x$

   1) Find $(\frac{\partial f}{\partial x})_y, (\frac{\partial f}{\partial y})_x$
   2) Now I make a transformation, using s,t as variable instead of x,y and they are related by: s=x, t=x+y, now find $(\frac{\partial f}{\partial s})_t$, is it same as $(\frac{\partial f}{\partial x})_y$?

Following are the problems on gradient, line integral and Green (Stokes) Theorem:

6. KK 5.1

   5.1 Find the forces for the following potential energies.
   a. $U = Ax^2 + By^2 + Cz^2$
   b. $U = A \ln(x^2 + y^2 + z^2)$ $(\ln = \log_e)$
   c. $U = A \cos\theta/r^2$ (plane polar coordinates)

7. KK 5.4

5.4 Determine whether each of the following forces is conservative. Find the potential energy function if it exists. A, α, β are constants.

a. $\mathbf{F} = A(3\hat{\mathbf{i}} + z\hat{\mathbf{j}} + y\hat{\mathbf{k}})$

b. $\mathbf{F} = Axyz(\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$

c. $F_x = 3Ax^2y^5e^{\alpha z}$, $F_y = 5Ax^3y^4e^{\alpha z}$, $F_z = \alpha Ax^3y^5e^{\alpha z}$

d. $F_x = A \sin(\alpha y) \cos(\beta z)$, $F_y = -Ax\alpha \cos(\alpha y) \cos(\beta z)$, and $F_z = Ax \sin(\alpha y) \sin(\beta z)$

8. KK 5.5 (Try U=C, U=2C and you will see what the contours(equal potential lines) look alike)

5.5 The potential energy function for a particular two dimensional force field is given by $U = Cxe^{-y}$, where C is a constant.

a. Sketch the constant energy lines.

b. Show that if a point is displaced by a short distance dx along a constant energy line, then its total displacement must be $d\mathbf{r} = dx(\hat{\mathbf{i}} + \hat{\mathbf{j}}/x)$.

c. Using the result of b, show explicitly that ∇U is perpendicular to the constant energy line.

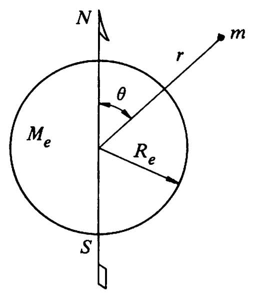
9. KK 5.7

5.7 When the flattening of the earth at the poles is taken into account, it is found that the gravitational potential energy of a mass m a distance r from the center of the earth is approximately

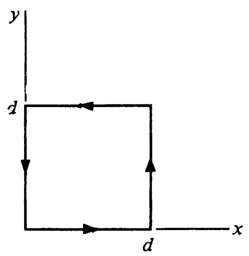
$$U = -\frac{GM_e m}{r}\left[1 - 5.4 \times 10^{-4}\left(\frac{R_e}{r}\right)^2 (3\cos^2\theta - 1)\right],$$

where θ is measured from the pole.

Show that there is a small tangential gravitational force on m except above the poles or the equator. Find the ratio of this force to $GM_e m/r^2$ for $\theta = 45°$ and $r = R_e$.



10. KK 5.8



5.8 How much work is done around the path that is shown by the force $\mathbf{F} = A(y^2\hat{\mathbf{i}} + 2x^2\hat{\mathbf{j}})$, where A is a constant and x and y are in meters? Find the answer by evaluating the line integral, and also by using Stokes' theorem.

Ans. $W = Ad^3$

If you have spare time, try to derive the gradient formula in spherical coordinate system.