Answers for Homework VI

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1 KK's 6.1

Ans:

a).

$$\vec{J} = \sum_i (\vec{r}_i \times \vec{p}_i)$$

Now we use another origin, so that $\vec{r}'_i = \vec{r}_i + \vec{R}$

$$\begin{aligned}
\vec{J}' &= \sum_i (\vec{r}'_i \times \vec{p}_i) \\
&= \sum_i \{\vec{r}_i \times \vec{p}_i + \vec{R} \times \vec{p}_i\} \\
&= \sum_i \vec{r}_i \times \vec{p}_i + \vec{R} \times (\sum_i \vec{p}_i) \\
&= \vec{J} + 0 = \vec{J}
\end{aligned}$$

b).

$$\vec{M} = \sum_i \vec{r}_i \times \vec{F}_i$$

Changing the origin, $\vec{r}'_i = \vec{r}_i + \vec{R}$

$$\begin{aligned}
\vec{M}' &= \sum_i \vec{r}'_i \times \vec{F}_i \\
&= \sum_i \{\vec{r}_i \times \vec{F}_i + \vec{R} \times \vec{F}_i\} \\
&= \vec{M} + \vec{R} \times (\sum_i \vec{F}_i) \\
&= \vec{M}
\end{aligned}$$

2 KK's 6.3

Ans:

a). The moment of inertia of the ring to the pivot is $I = MR^2 + MR^2 = 2MR^2$. The total angular momentum about the pivot is zero. So when the bug is half way around:

$$0 = m(v - \omega 2R)2R - 2MR^2\omega$$

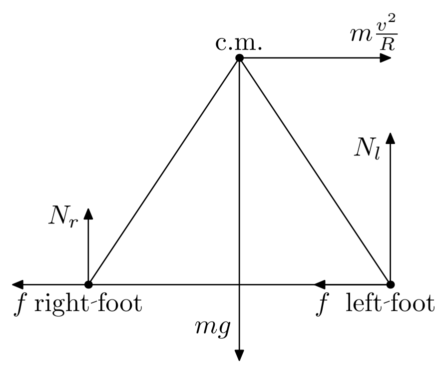
$$\omega = \frac{m}{M+2m}\frac{v}{R}$$

b).When the bug is back to the pivot, the angular momentum of the bug about the pivot is zero, so the angular momentum of the ring would also be zero.

$$\omega = 0$$

3 KK's 6.6

Ans:



$$m\frac{v^2}{R}L + N_r d = mg\frac{d}{2}$$

$$N_r = \frac{mg}{2} - m\frac{v^2}{R}\frac{L}{d}$$

$$N_r + N_l = mg$$

$$N_l = \frac{mg}{2} + m\frac{v^2}{R}\frac{L}{d}$$

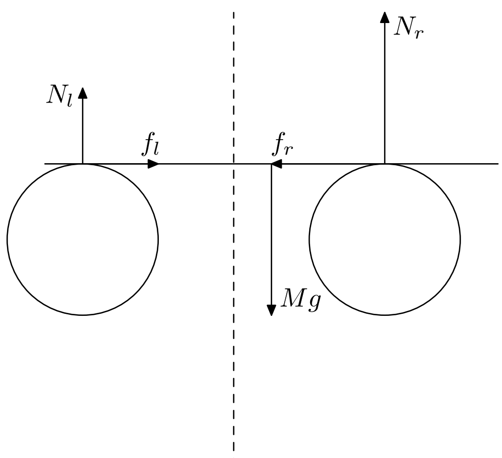
4 KK's 6.8

Ans:

$$
\begin{aligned}
I = \int r^2 dm &= \int r^2 \sin^2\varphi \rho dr r d\varphi r \sin\varphi d\theta \\
&= \rho \int_0^R r^4 dr \int_0^\pi \sin^3\varphi d\varphi \int_0^{2\pi} d\theta \\
&= \rho \frac{1}{5} R^5 \cdot \frac{4}{3} \cdot 2\pi \\
&= \frac{4}{3}\pi R^3 \rho \frac{2}{5} R^2 \\
&= \frac{2}{5} M R^2
\end{aligned}
$$

5 KK's 6.9

Ans:



$$
\begin{gathered}
N_r 2l = Mg(l+x) \\
N_r = \frac{Mg}{2}(1+\frac{x}{l}) \\
N_l = Mg - N_r = \frac{Mg}{2}(1-\frac{x}{l}) \\
F_x = f_l - f_r = \mu(N_l - N_r) = -\mu\frac{Mg}{l}x
\end{gathered}
$$

It is proportional to the distance x, so the motion of the bar is like a simple harmonic oscillator:

$$
x = x_0\cos(\omega t) \qquad \omega = \sqrt{\frac{k}{M}} = \sqrt{\frac{\mu g}{l}}
$$

6 KK's 6.13

Ans:

a). The angular momentum is conserved.

$$mv_0r = mvR$$

$$v = \frac{r}{R}v_0$$

b).The energy is conserved. $v \equiv v_0$

7 KK's 6.14

Ans:

a).

$$M_B = Mg\frac{l}{2} = \frac{Mgl}{2}$$

b). The moment of inertia about point B is:

$$I_B = \frac{1}{3}Ml^2$$

$$M_B = I_B\beta_B$$

$$\beta_B = \frac{3}{2}\frac{g}{l}$$

c).

$$a_c = \beta_B r_{CB} = \frac{3}{2}\frac{g}{l}\frac{l}{2} = \frac{3}{4}g$$

d).

$$Mg - N_B = Ma_c$$

$$N_B = \frac{1}{4}Mg$$

8 KK's 6.18

Ans:

i).Fixed Disk: The total motion of the disk could decompose into two part: the translation and the rotation around the center of the disk. In general, we could write down the equation of motion of the rod and the disk respectively. However the angular velocity of the disk is the same with that of the rod, so we could make these two equations into one equation:(You can try to write down these two equations, and make them into one equation. Or you could use the parallel axis theorem, and treat the motion of disk as the rotation around the origin instead of the center of the disk, and write down the equation of motion directly.)

$$\tau = I\beta$$

$$\tau = mg\frac{l}{2}\sin\theta + Mgl\sin\theta$$

$$I = \frac{1}{3}ml^2 + \frac{1}{2}MR^2 + Ml^2$$

$$\beta = \frac{mg\frac{l}{2} + Mgl}{\frac{1}{3}ml^2 + \frac{1}{2}MR^2 + Ml^2}\sin\theta \approx \frac{mg\frac{l}{2} + Mgl}{\frac{1}{3}ml^2 + \frac{1}{2}MR^2 + Ml^2}\theta$$

$$\omega = \sqrt{\frac{mg\frac{l}{2} + Mgl}{\frac{1}{3}ml^2 + \frac{1}{2}MR^2 + Ml^2}}$$

ii). Free Disk: Now the motion of the disk is only the translation. We could also write down two equations for the rod and the disk respectively, and make them into one:

$$\tau = I'\beta'$$

$$\tau = mg\frac{l}{2}\sin\theta + Mgl\sin\theta$$

$$I' = \frac{1}{3}ml^2 + Ml^2$$

$$\beta' = \frac{\frac{m}{2} + M}{\frac{1}{3}m + M}\frac{g}{l}\sin\theta \approx \frac{\frac{m}{2} + M}{\frac{1}{3}m + M}\frac{g}{l}\theta$$

$$\omega' = \sqrt{\frac{\frac{m}{2} + M}{\frac{1}{3}m + M}\frac{g}{l}}$$

9 KK's 6.20

Ans:

The conservation law of energy:

$$Mg\frac{l}{2}\sin\theta = \frac{1}{2}I\omega^2$$

$$I = \frac{1}{3}Ml^2$$

$$\omega = \sqrt{\frac{3g}{2l}}$$

And:

$$\tau = Mg\frac{l}{2} = I\beta$$

$$\beta = \frac{3g}{2l}$$

$$Mg - F_{\uparrow} = Ma_{c\perp} = M\beta\frac{l}{2}$$

$$F_{\uparrow} = \frac{1}{4}Mg$$

$$F_{\leftarrow} = M\omega^2\frac{l}{2} = \frac{3}{4}Mg$$

10 KK's 6.26

Ans:

For the sphere:

$$MgS\sin\theta = \frac{1}{2}I_s\omega_s^2$$

$$I_s = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

For the cylinder:

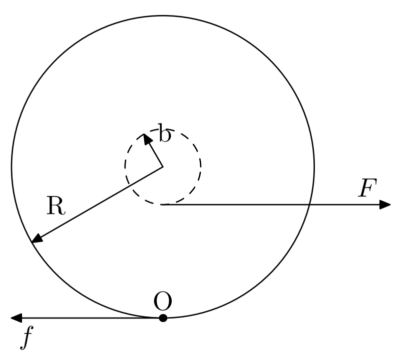
$$MgS\sin\theta = \frac{1}{2}I_c\omega_c^2$$

$$I_c = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

As $I_s < I_c$,so after same distance, $\omega_s > \omega_c$, so the sphere move faster than the cylinder. $T_s < T_c$.

11 KK's 6.27

Ans:



The force is towards the right, so the torque about point O is pointing into the paper. Thus the Yo-Yo will always move to the right! We need to find the maximum F for no slipping, so the friction force is also maximum.

For the c.m.:

$$F - \mu M g = M a_c$$

$$a_c = \frac{F - \mu M g}{M}$$

About point O:

$$F(R-b) = I\beta$$

$$I = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$

$$\beta = \frac{a_c}{R}$$

$$F(R-b) = \frac{3}{2}MR^2\frac{F - \mu M g}{MR}$$

$$F_{max} = \frac{2\mu M g R}{R + 2b}$$

12 KK's 6.30

Ans:

Conservation law of angular momentum about the point contact with ground. When the ball rolls without sliding, we can chose the instantaneous axis at the point contact with ground.

$$M v_0 R = I'\omega$$

$$I' = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$$

$$\omega = \frac{v}{R}$$

$$v = \frac{5}{7}v_0$$

(Or you may use the synthesis of angular momentum:

$$M v_0 R = I\omega + M v R$$

$I = \frac{2}{5}MR^2$ is the moment of inertia about the axis of the ball.

$$\omega = \frac{v}{R}$$

$$v = \frac{5}{7}v_0$$

we have the same result)