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1. $\nabla p = \vec{j} \times \vec{B}$. $\nabla \times \vec{B} = \mu_0 \vec{j}$

$$\mu_0 \nabla p = (\nabla \times \vec{B}) \times \vec{B} = \mu_0 \frac{\partial p}{\partial r} \hat{r}$$

$$= \left(-\frac{\partial B_z}{\partial r}\hat{\theta} + \frac{\partial}{r\partial r}(rB_\theta)\cdot\hat{z}\right) \times (B_\theta\hat{\theta} + B_z\hat{z}) = -\frac{\partial B_z}{\partial r}\cdot B_z\hat{r} - \frac{B_\theta}{r}\left(B_\theta + r\frac{\partial B_\theta}{\partial r}\right)\hat{r}$$

$$= \left(-B_z\frac{\partial B_z}{\partial r} - \frac{B_\theta^2}{r} - B_\theta\frac{\partial B_\theta}{\partial r}\right)\cdot\hat{r}$$

$$\rightarrow \mu_0\frac{\partial p}{\partial r} + B_z\frac{\partial B_z}{\partial r} + B_\theta\frac{\partial B_\theta}{\partial r} + \frac{B_\theta^2}{r} = 0.$$

腊肠不稳定性

$$B_{\theta a} = \frac{\mu I}{2\pi a}$$

设半径减小 δa，极向磁压强增量为 $\frac{B_{\theta a}^2}{2\mu}\left(\frac{a^2}{(a-\delta a)^2} - 1\right) = \frac{B_{\theta a}^2 \delta a}{\mu a}$.

纵向磁场 B_z 通量不变. $\frac{B_z^2}{2\mu}\left(\frac{a^4}{(a-\delta a)^4} - 1\right) = \frac{2B_z^2\cdot\delta a}{\mu a}$

$$\rightarrow B_z^2 > B_{\theta a}^2/2.$$

扭曲不稳定性.

$$\frac{B_z^2}{2\mu}\pi a^2\cdot 2\sin\alpha \simeq \frac{B_z^2}{\mu}\pi a^2\alpha = \frac{B_z^2\pi a^2\lambda}{2\mu R}$$

$$2\sin\alpha\int_0^\lambda \frac{B_\theta^2}{2\mu}2\pi r\,dr = \frac{\lambda}{R}\int_a^\lambda \frac{B_\theta^2}{2\mu}2\pi r\,dr = \frac{B_\theta^2}{2\mu}\pi a^2\cdot\frac{\lambda}{R}\ln\frac{\lambda}{a}$$

$$\rightarrow \frac{B_z^2}{B_{\theta a}^2} > \ln\frac{\lambda}{a}.$$

2. $R = \frac{B_{max}}{B_{min}} = \frac{5}{3}$. $\rightarrow \theta_0 = \sin^{-1}\left(\sqrt{\frac{1}{R}}\right) \approx 50.77°$.

$$\ln\Lambda \approx 7 + 2.3\lg\left(\left(\frac{T_e}{e}\right)^{3/2}\Big/\left(\frac{n_e}{10^{20}}\right)^{1/2}\right) \approx 21.15$$

$$I_L = \frac{\ln\Lambda\, e^4 n^2}{(2\pi)^{1/2}\, 8\pi\varepsilon_0^2 m^{1/2}\, T^{3/2}}\cdot\frac{(3\ln(2^{1/2}+1) - 2^{1/2})}{(\ln(\mathrm{ctg}\frac{\theta_0}{2}) - \cos\theta_0)}\cos\theta_0$$

$$\approx 1.3\times10^{22}\ \mathrm{m^{-3}\,s^{-1}}$$