Answers for Homework III

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1 KK's 2.4

Ans:

Assume that the rotational radiuses for particles of mass m and M are r_m and r_M respectively. Then we have:

$$F = m\omega^2 r_m \qquad F = M\omega^2 r_M$$

So we get the rotational radiuses for each particle:

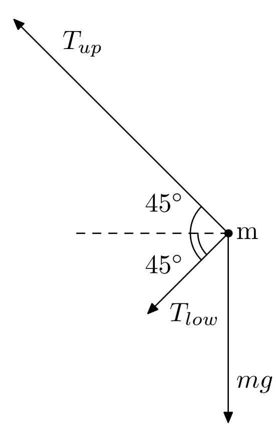
$$r_m = \frac{F}{\omega^2 m} \qquad r_M = \frac{F}{\omega^2 M}$$

So the separation of two particles is:

$$R = r_m + r_M = \frac{F}{\omega^2}(\frac{1}{m} + \frac{1}{M})$$

2 KK's 2.11

Ans:



$$mg + \frac{1}{\sqrt{2}} T_{low} = \frac{1}{\sqrt{2}} T_{up}$$

$$\frac{1}{\sqrt{2}} T_{up} + \frac{1}{\sqrt{2}} T_{low} = m\omega^2 \frac{l}{\sqrt{2}}$$

So

$$\frac{2}{\sqrt{2}} T_{low} = m\omega^2 \frac{l}{\sqrt{2}} - mg$$

$$\frac{2}{\sqrt{2}} T_{up} = m\omega^2 \frac{l}{\sqrt{2}} + mg$$

Finally we have:

$$T_{low} = m\omega^2 \frac{l}{2} - \frac{\sqrt{2}mg}{2}$$

$$T_{up} = m\omega^2 \frac{l}{2} + \frac{\sqrt{2}mg}{2}$$

3 KK's 2.12

Ans:

When pulling out the cloth, the grass does not move appreciably.

$$\mu mg\Delta t = \Delta p = mv$$

$$v = \mu g \Delta t$$

To get the longest time, the grass should move the longest distance.

$$\frac{1}{2} mv^2 = \mu mgl$$

where $l = 6in(1in = 2.54cm = 0.0254m)$

$$\frac{1}{2} \mu g (\Delta t)^2 = l$$

$$\Delta t = \sqrt{\frac{2l}{\mu g}} \approx 0.1s$$

4 KK's 2.14

Ans:

The force in the same rope is uniform. Assume that the force in the rope connect A and B is F_1, so the force in the rope connect C is $F_2 = 2F_1$. So we have:

$$m_C g - 2F_1 = m_C a_c$$

$$F_1 = m_A a_A$$
$$F_1 = m_B a_B$$

And there is a connected condition:

$$a_C = \frac{a_A + a_B}{2}$$

So we get:

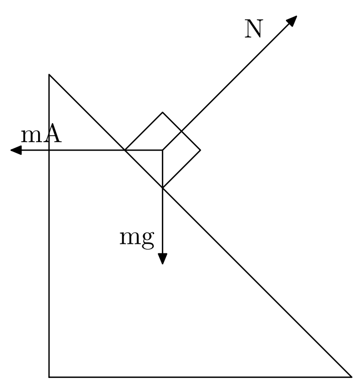
$$g - \frac{2F_1}{m_C} = \frac{F_1}{2}\left(\frac{1}{m_A} + \frac{1}{m_B}\right)$$
$$g = F_1\left(\frac{2}{m_C} + \frac{1}{2m_A} + \frac{1}{2m_B}\right)$$
$$F_1 = \frac{2m_A m_B m_C g}{4m_A m_B + m_C m_B + m_C m_A}$$

Finally:

$$a_C = g\left(1 - \frac{4m_A m_B}{4m_A m_B + m_C m_B + m_C m_A}\right) = \frac{m_C m_B + m_C m_A}{4m_A m_B + m_C m_B + m_C m_A} g$$
$$a_A = \frac{2m_B m_C g}{4m_A m_B + m_C m_B + m_C m_A}$$
$$a_B = \frac{2m_A m_C g}{4m_A m_B + m_C m_B + m_C m_A}$$

5 KK's 2.16

Ans:



In the frame of the wedge, the force diagram is as showed. Then in this frame we have:

$$\frac{mA}{\sqrt{2}} + \frac{mg}{\sqrt{2}} = N$$

$$ma'_{\parallel} = \frac{mA}{\sqrt{2}} - \frac{mg}{\sqrt{2}}$$

$$a'_{\|} = \frac{\sqrt{2}}{2}(A-g)$$

Back to the lab frame:

$$a_x = A - \frac{1}{2}(A-g) = \frac{1}{2}(A+g)$$

$$a_y = \frac{1}{2}(A-g)$$

6 KK's 2.19

Ans:

When M_3 is rest, then the whole acceleration is:

$$a = \frac{F}{M_1 + M_2 + M + 3}$$

In the frame of M_3, for M_2:

$$M_2 a = M_3 g$$

So:

$$F = \frac{M_3}{M_2}(M_1 + M_2 + M_3)g$$

7 KK's 2.22

Ans:

For the whole rope, it is under 3 forces: mg, T_{end1}, T_{end2}. Under these forces the rope is at rest:

$$T_{end1} \sin\theta = T_{end2} \sin\theta$$

$$T_{end1} \cos\theta + T_{end2} \cos\theta = mg$$

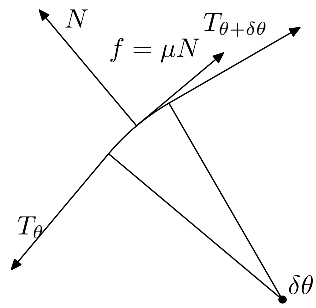
So we get:

$$T_{end1} = T_{end2} = \frac{mg}{\cos\theta}$$

In the middle of the rope, let's consider about the left(or right) half side of the rope:

$$T_{mid} = Tend1 \sin\theta$$

$$T_{mid} = \tan\theta mg$$



8 KK's 2.24

Ans:
From the figure, we have:

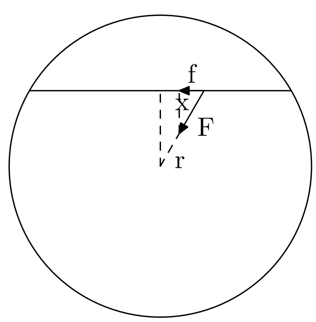
$$N = T_\theta \sin\frac{\delta\theta}{2} + T_{\theta+\delta\theta}\sin\frac{\delta\theta}{2} \approx T_\theta \delta\theta$$
$$T_\theta = T_{\theta+\delta\theta} + \mu N = T_{\theta+\delta\theta} + \mu T_\theta \delta\theta$$

So:

$$-dT = \mu T d\theta$$
$$\frac{dT}{T} = -\mu d\theta$$
$$\ln\frac{T}{T_A} = -\mu\theta$$
$$T = T_A e^{-\mu\theta}$$

9 KK's 2.26

Ans:



When we are at radius r, we are under force F:

$$F = G\frac{M_r m}{r^2} = G\frac{\frac{4\pi r^3}{3}\rho m}{r^2} = \frac{4\pi G\rho m r}{3}$$

$$f = \frac{x}{r}F = \frac{4\pi G\rho m}{3}x$$

The force is proportional to the distance x, so it is a simple harmonic motion.

$$T = 2\pi\sqrt{\frac{3}{4\pi G\rho}}$$

The time needed for satellite:

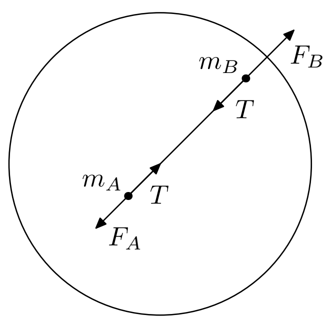
$$F = G\frac{\frac{4\pi R^3}{3}\rho m}{R^2} = m(\frac{2\pi}{T})^2 R$$

$$(\frac{T}{2\pi})^2 = \frac{3}{4\pi G\rho}$$

$$T = 2\pi\sqrt{\frac{3}{4\pi G\rho}}$$

10 KK's 2.30

Ans:



In the frame of the disk, the masses are under another forces:

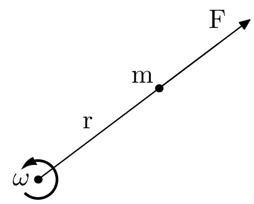
$$F_A = m_A\omega^2 r_A \qquad F_B = m_B\omega^2(l - r_A)$$

$$m_A a_A = T - m_A\omega^2 r_A \qquad m_B a_B = m_B\omega^2(l - r_A) - T$$

As the two masses are in the same rope, we have the connectional condition:

$$a_A = a_B$$

$$a_A = \frac{m_B(l - r_A) - m_A r_A}{m_A + m_B}\omega^2$$



11 KK's 2.33

Ans:

$$F = m\omega^2 r$$
$$\ddot{r} = \omega^2 r$$

The generate solution is:

$$r = Ae^{-\gamma t} + Be^{\gamma t}$$

where $\gamma = \omega$ and A B are two constants decided by initial condition.

$$r_0 = A + B$$
$$v_0 = -A\omega + B\omega \qquad \frac{v_0}{\omega} = -A + B$$
$$B = \frac{1}{2}(r_0 + \frac{v_0}{\omega})$$
$$A = \frac{1}{2}(r_0 - \frac{v_0}{\omega})$$

when $B = 0$ the r will decrease continually.

$$v_0 = -r_0\omega$$

The minus sign means that the direction of the velocity points to the origin.

12 KK's 2.34

Ans:

As the force point to the origin, so the acceleration along the θ direction is zero:

$$0 = 2\dot{r}\omega + r\dot{\omega}$$
$$\dot{\omega} = \frac{2V\omega}{r}$$
$$\frac{d\omega}{\omega} = 2\frac{Vdt}{r_0 - Vt}$$

$$\ln\frac{\omega}{\omega_0} = \ln(\frac{r_0}{r})^2$$

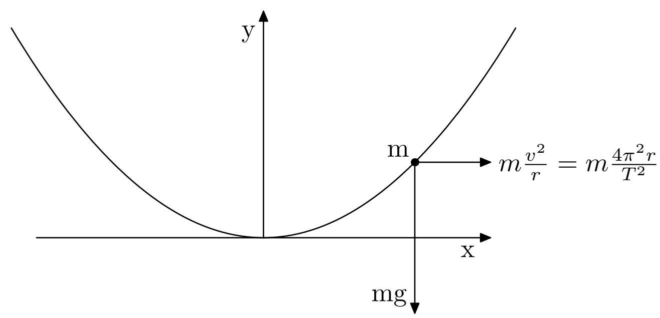
$$\omega = \omega_0(\frac{r_0}{r_0 - Vt})^2$$

The force along the radius is:

$$F = m\omega^2 r = m\omega_0^2\frac{r_0^4}{r^3}$$

13 KK's 2.37

Ans:



In the frame of the hovercraft, the force diagram is as show. In this frame, we have the condition:

$$\frac{dy}{dx} = \tan\theta = \frac{\frac{4\pi^2 r}{T^2}}{g} = \frac{4\pi^2 x}{gT^2}$$

$$dy = \frac{4\pi^2}{gT^2}xdx$$

$$y = \frac{2\pi^2}{gT^2}x^2$$

This is the equation of cross section of the bowl shaped track.

14 Additional

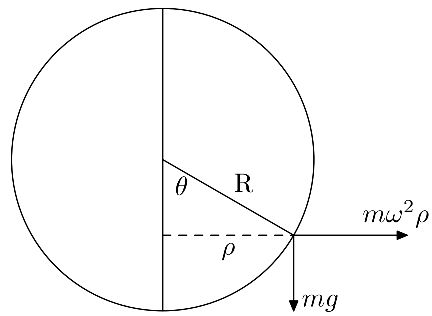
Ans:

We consider the potential energy:

$$E = -mgR\cos\theta - \frac{1}{2}m\omega^2 R^2\sin^2\theta$$

The equilibrium position has the extreme potential energy

$$\frac{dE}{d\theta} = 0 = mgR\sin\theta - m\omega^2 R^2\sin\theta\cos\theta$$



$$\cos\theta = \frac{g}{\omega^2 R}$$

whether the equilibrium is stable or unstable is decided by the 2nd derivative.

$$\frac{d^2E}{d\theta^2} = mgR\cos\theta - m\omega^2 R^2 \cos 2\theta$$

When $\cos\theta = \frac{g}{\omega^2 R}$

$$\frac{d^2E}{d\theta^2} = \frac{m}{\omega^2}(\omega^4 R^2 - g^2)$$

If the ring has a equilibrium position, $\cos\theta = \frac{g}{\omega^2 R}$ must have solution. That is:

$$g \leq \omega^2 R$$

Thus

$$\frac{d^2E}{d\theta^2} = \frac{m}{\omega^2}(\omega^4 R^2 - g^2) \geq 0$$

So it is a stable position.