Homework for General PhysicsII-set8

1. AB's 4-23:

23. The longest wavelength in the Lyman series is 121.5 nm and the shortest wavelength in the Balmer series is 364.6 nm. Use the figures to find the longest wavelength of light that could ionize hydrogen.

(work out this only from the wavelengths provided and other physical constants such as Planck h and speed of light)

2. AB's 4-46:

46. A beam of 8.3-MeV alpha particles is directed at an aluminum foil. It is found that the Rutherford scattering formula ceases to be obeyed at scattering angles exceeding about 60°. If the alpha-particle radius is assumed small enough to neglect here, find the radius of the aluminum nucleus.

3. AB 3-8:

8. Find the kinetic energy of an electron whose de Broglie wavelength is the same as that of a 100-keV x-ray.

4. AB 3-10:

10. Show that the de Broglie wavelength of a particle of mass m and kinetic energy KE is given by

$$\lambda = \frac{hc}{\sqrt{\mathrm{KE}(\mathrm{KE} + 2mc^2)}}$$

In the following problems 5,6,7, you need group velocity and phase velocity we learned before, here is just a reminder: (of course you may read AB's section 3.4, but I bet you can work 5,6,7 out independently)

$phase\ v_\varphi = \frac{\omega}{k} = \frac{E}{p};\ group\ v_g = \frac{d\omega}{dk} = \frac{dE}{dp}$, Planck $E = \hbar\omega$, and de-Broglie $p = \hbar k$ are used; and E, p we use relativistic formula:

5. For a "particle" which has duality(a matter wave), its particle behavior can be described as a momentum p and velocity u, prove the following:

a) The associated matter wave generally has relation between phase and group velocity, the group velocity is just u. i.e. the group velocity of the matter wave corresponds to classical u

b) $v_\varphi v_g = c^2$

6. AB's 3-13

13. An electron and a proton have the same velocity. Compare the wavelengths and the phase and group velocities of their de Broglie waves.

7. AB's 3-19

19. Find the phase and group velocities of the de Broglie waves of an electron whose kinetic energy is 500 keV.

8. An electron microscope using electrons to study the micro-structure, for the electron at 1eV, using them to study the structure, what is the spatial resolution limit? (You may wonder whether you can directly use the results learned from light diffraction, the answer is you can. The reasoning is I only used superposition principle of waves in the treatment of light interference and diffraction, the same superposition principle also holds in Q.M, though the physical meaning of the wave itself is different from that of classical wave theory)
9. Using the uncertainly relation between position and momentum to derived the Fraunhoffer single slit diffraction patter, i.e. $a\Delta\theta \doteq \lambda$, where a is the width of slit and $\Delta\theta$ is the angular width between the maximum and first zero in the diffraction pattern.
10. Use the uncertainty relation to explain the stability of electron in hydrogen atom. (You may find discussion outlined in my notes， pg452-453; please work out the detail)
11. A problem of particle in an infinite deep potential. Of course you may solve this problem with Schrödinger equation and the answers are in all textbooks（e.g. Griffiths' section 2.2）. Here I urge you to use general properties of wave (please recall the discussion on standing wave, it is a superposition of two traveling wave against each other, and the particle here travels along one direction with no potential can be expressed as a plane wave, of course I did not prove it here) and simple quantum relation (p=h/lambda)to solve it. The potential is defined as: The potential energy $V(x)=0,$ when $|x|<L/2,$ and is ∞ elsewhere. A particle of mass m is inside such potential well

(1). Plot this one dimension potential, and also plot the ***two*** waveforms that can exist inside such potential with longest wavelengths. (one is the longest and the other is 2nd longest)

(2). What are the energies of the particle associated with this two waveform? (You are calculating lowest two energy levels in this case. The energy is still K+V, kinetic+potential, potential is 0 everywhere inside the well, the kinetic energy is still $p^2/2m$ in non-relativistic)

(3). You should find that the lowest energy level for the system is actually not zero, and if you guess that is due to uncertainty relations, that is great. What is the energy of the system you estimated from uncertainty relations? (may not exactly as you get in question 2, but would be same order)

(4). The uncertainty of the momentum is defined as $(\Delta p)^2 = <(p-<p>)^2>$, (<>means average) which equals to $<p^2>-<p>^2$. Find the $<p^2>$ and $<p>$ for the lowest energy level (the longest wavelength). You really do not need to do calculation if you remember the standing wave. Use this to determine the uncertainty between x and p, taking $\Delta x = L/2$. Does it satisfy the limit set by the uncertainty relation?

(5). For a small macroscopic object with 1 ug (10^{-6}kg) inside a well with L=1 um (10^{-6}m); and a electron (m=10^{-30} kg) trapped inside a well with L=1nm. Calculate the ***energy difference*** between the lowest two levels for each case. If you use electron bombardment to excite the particle from the ground (the lowest) to its first excited state, what is the energy (in eV) needed for each case? If you use light to do the excitation, what are the frequencies of light? (I hope this convince you that for macroscopic object, the energy spacing is so small that its energy is almost continuous; while for microscopic object, quantization is more obvious)

12. Consider the double slit experiment with two slits labelled as A, B. Electrons pass the slits will hit detector screen. For each of the following cases draw a rough graph of on the relative number of detected electrons as a function of position on the screen and give a brief explanation. (no calculation is needed)

(a) Slit A open, B closed.

(b) Slit B open, A closed.

(c) Both slits open.

(d) Stern-Gerlach apparatus attached to the slits so that only electrons with spin up can pass A, and only electrons with spin down can pass B. Here you can just treat the so called spin-up and spin-down of electron same as two orthogonal polarization of light (we shall discuss it in detail later and you will see the reason)

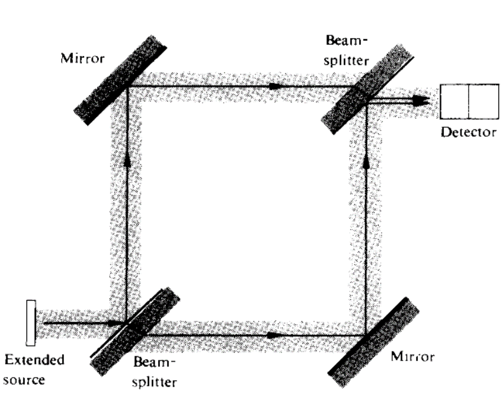
(e) Only select electrons with spin up pass through both A and B.

13. AB's 3-39 (hint: for kinetic energy, consider average $< p^2 >$

39. The frequency of oscillation of a harmonic oscillator of mass m and spring constant C is $\nu = \sqrt{C/m}/2\pi$. The energy of the oscillator is $E = p^2/2m + Cx^2/2$, where p is its momentum when its displacement from the equilibrium position is x. In classical physics the minimum energy of the oscillator is $E_{\min} = 0$. Use the uncertainty principle to find an expression for E in terms of x only and show that the minimum energy is actually $E_{\min} = h\nu/2$ by setting $dE/dx = 0$ and solving for $E_{\min}$.

14. Interference and Flitzur-Vaidman bomb:

Part I: Mach-Zehnder (MZ) Interferometer and Phase difference between reflected and transmitted light by beam splitter (BS):



The beam splitter is 50%-50%, i.e. 50% light intensity reflected，50% transmitted. In terms of reflection and transmission coefficients: $|r^2| = |t^2| = 1/2$,

1) Some light beams are missing in the figure, what are the missing light beams?

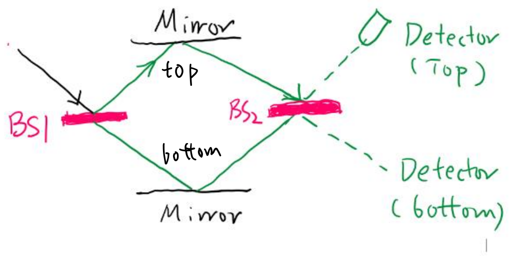
2) If we adjust the optical path so that no light is recorded by the detector shown in the figure (destructive interference), where does the energy go?

3) Considering the above effect, there must be a phase difference between the reflected light and

transmitted light at each beam splitter, i.e. $E_R/E_T \propto e^{i\phi}$ or equivalently (since $E_r = rE_{in}$) $\frac{r}{t} = e^{i\phi}$. What is the value for ϕ? (use convention to put its value between 0, π)

Part II: Single photon interference result and F-V bomb detection.

4）From results in Part I, if we set up an interference for single photon using MZ interferometer as shown in the figure below：

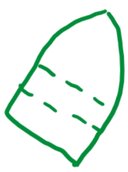


The incoming photon comes from top, the two BS are identical 50%-50%, the optical path difference between top and bottom path is adjusted to be zero $\Delta l = 0$, and the reflection mirror introduce no change in phase. What is the probability for the top detector and bottom detector records photon in each experiment, i.e. P(top) and P(bot)=?

5) If the optical path length between bottom and top by $\Delta l = \frac{\lambda}{2}$, then what is P(top) and P(bot)?

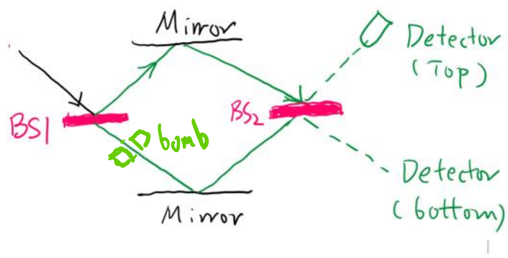
6) A fancy way to calculate 4) is by setting up matrix and vector: Let's define the light coming (or leaving) from top (as shown in figure) to be $\begin{pmatrix}1\\0\end{pmatrix}$, and light coming (or leaving) from bottom be $\begin{pmatrix}0\\1\end{pmatrix}$; Then a) What is the matrix expression for Beam Splitter in such base? (|r|=|t|=$\frac{\sqrt{2}}{2}$, and there is a phase difference between them you evaluated in 3)) b) If the input light only comes from top i.e.$|in>=\begin{pmatrix}1\\0\end{pmatrix}$, what will be the |out>? (you will find out exactly same result as in 4), the |out> will be $\begin{pmatrix}0\\1\end{pmatrix}$)

In 1990s, two Israeli physicists Elitzur and Vaidman raise an interesting model, called Elitzur-Vaidman bomb:



The part of the bomb between the dash lines is transparent embedded with photo-sensitive material which can absorb single photon of certain wavelength with 100% efficiency. Once the photon absorbed, the bomb will be detonated and explode. If this photo sensitive material is defective (bad egg), then all light can pass(just like vacuum) and the bomb cannot be detonated. Suppose the only way to know whether the photo-sensitive material works or not is by shining light of that wavelength, but this immediately raise a dilemma: If you shine the light directly, the good bomb will explode; only bad bombs left. The chance of successfully test, i.e. making sure the bomb works and not detonate it, is zero. Now with quantum and interference, we have certain probability to successfully test the bomb. Here is how:

In the following questions, the setup is let incoming single photon along the top path before BS1 as shown in the figure of question 4, and the bomb will always be placed at the bottom path:



7) We adjust the path length difference $\Delta l = 0$ as in 4), if the bomb is a bad egg, what is P(top) and P(bot)?

8) If the bomb is a good one, you do the single photon test once, what is the probability you determine it is a good egg without detonating it?

9) Now suppose we have many ZV bombs, there are N good ones. You can repeat the above test until you definitively pick out the good ones (or explosion), how many good eggs you successfully pick out? （ answer: 1/3 N）