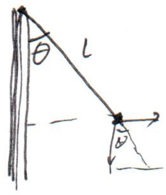
习题课5

$L_{径} = mv\times r = m\cdot l_1\sin\theta_1\cdot v_1 = L_0$ (Z方向无力矩)

1.



$\tan\theta = \dfrac{\frac{mv_1^2}{l_1\sin\theta_1}}{mg} = \dfrac{v_1^2}{gl_1\sin\theta_1} = \dfrac{\sin\theta_1}{\cos\theta_1}$ 对于θ, l, $v_{合}$ 均有上面的

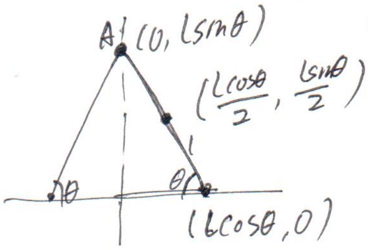
$$\begin{cases} V\cdot\sin\theta = \dfrac{L_0}{ml} \\ V^2\cos\theta = gl\sin^2\theta \end{cases}$$ ~~解得~~

令 $\frac{L_0}{ml} = C$ $V^2 = x$. 上式化为 $\begin{cases} \sin\theta = \frac{C}{V} \\ x\cdot\sqrt{1-\frac{C^2}{x}} = gl\frac{C^2}{x} \end{cases}$ 如此,解巨丑无比,其实答案也没解,只是说变化趋势.

2. 设碰地速度 v_f

~~平动动能: $v_{cm} = \frac{v_f}{2}$ $E_{k1} = \frac{1}{2}\cdot 2m\cdot\left(\frac{v_f}{2}\right)^2 = \frac{1}{4}mv_f^2$~~

~~转动动能~~ 只看一个杆.



$V_{cm} = \left(-\frac{l}{2}\sin\theta\cdot\dot\theta,\ \frac{l}{2}\cos\theta\,\dot\theta\right)$

$E_k = \frac{1}{2}m\left[\left(-\frac{l}{2}\sin\theta\cdot\dot\theta\right)^2 + \left(\frac{l}{2}\cos\theta\cdot\dot\theta\right)^2\right] + \frac{1}{2}\cdot I\cdot\dot\theta^2$

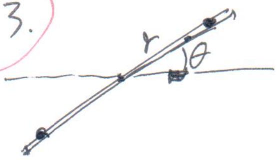
其中 $I = \frac{ml^2}{12}$ 代入

$E_k = \frac{ml^2}{8}\dot\theta^2 + \frac{ml^2}{24}\dot\theta^2 = \frac{1}{6}ml^2\cdot\dot\theta^2$ $E_{kf} = mg\cdot\frac{l\sin\theta_0}{2}$ $\theta_0 = 30°$

~~θ_f~~ $\frac{1}{6}ml^2\cdot\dot\theta_f^2 = mg\cdot\frac{l}{4}$ $\dot\theta_f = \sqrt{\frac{3g}{2l}}$ $\theta_f = 0$

$V_A = (0,\ l\cos\theta\cdot\dot\theta)$ $V_f = l\cdot\cos\theta_f\cdot\dot\theta_f = l\sqrt{\frac{3g}{2l}} = \sqrt{\frac{3gl}{2}}$

3.



$q = (r\cos\theta,\ r\sin\theta)$

$\dot q = (\dot r\cos\theta + \ -r\sin\theta\cdot\dot\theta,\ \dot r\sin\theta + r\cos\theta\cdot\dot\theta)$

$E_k = \frac{1}{2}\cdot(2m)\cdot\dot r^2 + \frac{1}{2}\cdot(2m)\cdot(r\cdot\dot\theta)^2$ $E_{k0} = \frac{1}{2}(2m)(a\cdot w_0)^2 = ma^2w_0^2$

$J = 2m\cdot r^2\dot\theta = J_0 = 2m\cdot a^2\cdot w_0$

动能守恒，角动量守恒

$$\begin{cases} 2mr^2\dot{\theta} = J_0 \qquad \dot{\theta} = \dfrac{J_0}{2mr^2} \\ m[\dot{r}^2 + (r\dot{\theta})^2] = E_{k0} \end{cases}$$

$$m\left[\dot{r}^2 + \left(\frac{J_0}{2mr}\right)^2\right] = E_{k0}$$

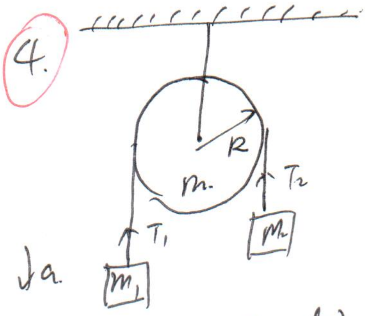
$$\left(\frac{dr}{dt}\right)^2 + \frac{J_0^2}{4m^2}\frac{1}{r^2} - \frac{E_{k0}}{m} = 0$$ 令 $c_1 = \frac{E_{k0}}{m}$ $\quad c_2 = \frac{J_0^2}{4m^2}$

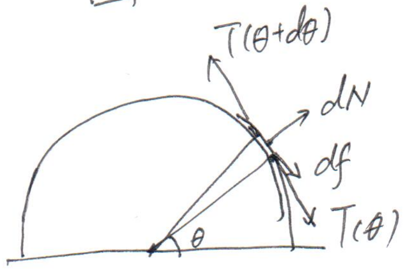
$$\left(\frac{dr}{dt}\right)^2 = c_1 - \frac{c_2}{r^2} \qquad \int \frac{r}{\sqrt{c_1 r^2 - c_2}}\,dr = t$$

$$\therefore\ t = \frac{1}{2}\int \frac{1}{\sqrt{c_1 r^2 - c_2}}\,dr^2 = \frac{1}{c_1}\sqrt{c_1 r^2 - c_2} + C_3$$ 代入始条件 $r(0) = a$

$$r = a\cdot\sqrt{\omega_0^2 t^2 + 1}$$

4.





$$dN = [T(\theta+d\theta) + T(\theta)]\cdot\sin\left(\frac{d\theta}{2}\right)$$ 保留一阶

$$= 2T(\theta)\frac{d\theta}{2} = T(\theta)\cdot d\theta$$

$$df = T(\theta+d\theta) - T(\theta) = \mu\cdot dN$$

$$dT = \mu\cdot T\cdot d\theta \qquad \frac{dT}{d\theta} = \mu T \qquad T = C\cdot e^{\mu\theta}$$

$$\therefore\ \cancel{T_1 = T_2 = e^{\mu\pi}} \qquad T_1 = e^{\mu\pi}\cdot T_2$$

$$f = T_1 - T_2 = (e^{\mu\pi} - 1)T_2$$

$$\begin{cases} m_1 g - T_1 = m_1 a \\ T_1 = T_2 e^{\mu\pi} \\ T_2 - m_2 g = m_2 a \end{cases}$$

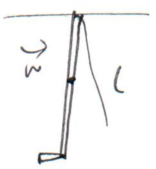
$$(m_1 - m_2\cdot e^{\mu\pi})g = (m_1 + m_2\cdot e^{\mu\pi})a$$

$$a = \frac{(m_1 - m_2\cdot e^{\mu\pi})g}{m_2 e^{\mu\pi} + m_1}$$

$$dm = df\cdot R \quad \therefore\ m = fR = (e^{\mu\pi} - 1)\cdot T_2\cdot R = \frac{MR^2}{2}\cdot\beta$$

$$\beta = \frac{2}{MR}\cdot(e^{\mu\pi} - 1)\cdot(m_2 g + m_2 a)$$

5.



$$l m v_0 = \left(\frac{m_0 l^2}{3} + m l^2\right) w.$$

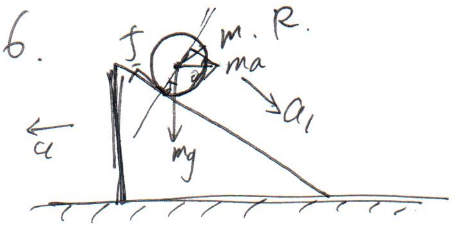
$$w = \frac{m v_0}{\frac{m_0 l}{3} + m l}$$

$$E_K = \frac{1}{2} \cdot I \cdot w = \frac{1}{2}\left(\frac{m_0 l^2}{3} + m l^2\right) \cdot w^2 = \frac{1}{2} \cdot \frac{m^2 v_0^2}{\frac{m_0 l}{3} + m l}$$

$$= m_0 g \cdot \frac{l}{2}(1-\cos\theta) + m g l(1-\cos\theta).$$

$$(1-\cos\theta) = \frac{E_K}{m_0 g \cdot \frac{l}{2} + m g l} = \frac{m^2 v_0^2}{(m_0 g l + 2 m g l)\left(\frac{m_0 l}{3} + m l\right)}$$

6.



设滑块向左加速度 a，换参至滑块

$$\begin{cases} m a_1 = m g \sin\theta + m a \cos\theta - f \\ N - m g \cos\theta + m a \sin\theta = 0. \\ \frac{1}{2} m r^2 \cdot \frac{a_1}{r} = f r \\ M a = N \cdot \sin\theta - f \cos\theta \end{cases}$$

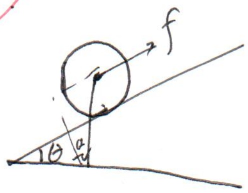
得到 $\begin{cases} M a = m a \sin^2\theta - m a \cos^2\theta + m a_1 \cos\theta \\ m g \sin\theta + m a \cos\theta = \frac{3}{2} m a_1 \end{cases}$

$$M a = m a(\sin^2\theta - \cos^2\theta) + \cdot \cos\theta \cdot \frac{2}{3}(m g \sin\theta + m a \cos\theta)$$

$$M a = -m a \cos(2\theta) + \frac{1}{3} m g \sin 2\theta + \frac{1}{3} m a \cdot (1 + \cos 2x)$$

$$\therefore a = \frac{\frac{1}{3} m g \sin 2\theta}{M - \frac{1}{3} m + \frac{2}{3} m \cos(2\theta)}$$

7.



$$f - mg\sin\theta = ma. \qquad f = \mu mg\cos\theta$$

$$\frac{2}{5}mR^2\cdot\frac{dw}{dt} = -fR \qquad \frac{dw}{dt} = -\frac{5\mu g\cos\theta}{2R}$$

$$\mu g\cos\theta - g\cdot\sin\theta = a \qquad V = at = (\mu g\cos\theta - g\sin\theta)t$$

$$w = w_0 - \frac{5\mu g\cos\theta}{2R}t$$

当 $v = wR$ $\qquad w_0R - \frac{5\mu g\cos\theta t}{2} = (\mu g\cos\theta - g\sin\theta)t$

$$\left(\frac{7}{2}\mu g\cos\theta - g\sin\theta\right)t = w_0R \qquad t = \frac{w_0R}{\frac{7}{2}\mu g\cos\theta - g\sin\theta}$$

$$L_0 = \frac{1}{2}at^2 = \frac{1}{5}(\mu g\cos\theta - g\sin\theta)\frac{w_0^2R^2}{\left(\frac{7}{2}\mu g\cos\theta - g\sin\theta\right)^2}$$

$$w_t = w_0 - \frac{5\mu g\cos\theta\, w_0}{7\mu g\cos\theta - 2g\sin\theta} \qquad V_t = \frac{w_0R\cdot(\mu g\cos\theta - g\sin\theta)}{\frac{7}{2}\mu g\cos\theta - g\sin\theta}$$

$$E_k = \frac{1}{5}mV_t^2 + \frac{1}{2}\cdot\frac{2}{5}mR^2\cdot w_t^2 = mgL_2$$

$$L_{tot} = L_0 + L_2$$

8. case I.

$$RmV_0 = \left(mR^2 + \frac{1}{2}m_0R^2\right)w \qquad w = \frac{2mV_0}{(2m+m_0)R} \qquad E = \frac{m^2V_0^2}{2m+m_0}$$

case II.

$$\begin{cases} mV_0 = mV_f + m_0\cdot V_{cm} & ① \\ mV_0R = m(V_f)R + \frac{1}{2}m_0R^2\cdot w & ② \\ V_f - V_{cm} = w\cdot R & ③ \end{cases}$$

$$(m+m_0)V_f = \left[\frac{m_0}{m+\frac{1}{2}m_0} + 1\right]mV_0 \qquad V_f = \frac{(2m+3m_0)mV_0}{(2m+m_0)(m+m_0)}$$

$$V_{cm} = \frac{m_0^2\cdot m}{2m^2+3m_0m+m_0^2}\cdot\frac{1}{m_0} = \frac{mm_0}{2m^2+3mm_0+m_0^2}$$

$$E_k = \frac{1}{2}mV_f^2 + \frac{1}{2}m_0V_{cm}^2 + \frac{1}{2}\cdot\frac{1}{2}mR^2\cdot w^2 = \frac{3}{4}mV_f^2 + \left(\frac{1}{2}m_0 + \frac{1}{4}m\right)V_{cm}^2 - \frac{m}{2}V_fV_{cm}$$

③代入①:　$mV_0 = m(\omega R + V_{cm}) + m_0 V_{cm}$

$\frac{②}{R} - ①$:　$\frac{1}{2} m_0 R\omega = m_0 V_{cm}$

解得　$V_{cm} = \frac{mV_0}{m_0 + 3m}$　$\omega = \frac{2V_{cm}}{R} = \frac{2mV_0}{(m_0+3m)R}$　$V_f = 3V_{cm}$

$$E_2 = \frac{1}{2} m V_f^2 + \frac{1}{2} m_0 V_{cm}^2 + \frac{1}{2}\cdot\frac{1}{2} m_0 R^2 \cdot \omega^2$$

$$= \frac{1}{2} m\cdot(3V_{cm})^2 + \frac{1}{2} m_0 V_{cm}^2 + \frac{1}{4} m_0 R^2 \cdot \frac{4V_{cm}^2}{R^2}$$

$$= \frac{3}{2} m_0 V_{cm}^2 + \frac{9}{2} m V_{cm}^2 = 6mV_{cm}^2 = 6m\cdot\left(\frac{mV_0}{m_0+3m}\right)^2$$

⑨ $fR = \frac{2}{5} mR^2 \cdot \frac{d\omega}{dt}$

$f = \mu mg$

$\therefore \frac{d\omega}{dt} = \frac{5\mu g}{2R}$

在碰撞中

$\Delta p = 2mV_0 = \int N\cdot dt$

$\int Rf dt = \mu R \cdot \int N dt = \int \tau dt = \Delta J = \frac{2}{5} mR^2 \cdot \Delta\omega$.

$\therefore \Delta\omega = \frac{2mV_0 \mu R}{mR^2} \cdot \frac{5}{2} = \frac{5\mu V_0}{R}$

注意到 $\omega_0 + \Delta\omega < 0$（摩擦最多使球停转）

其中: $\omega_0 = -\frac{V_0}{R}$　$\therefore$　$5\mu - 1 < 0$　$\mu < \frac{1}{5}$

$$\begin{cases} \omega(t) = -\frac{V_0}{R} + \frac{5\mu g}{2R} t + \frac{5\mu V_0}{R} \\ v(t) = V_0 - \mu g t \end{cases}$$

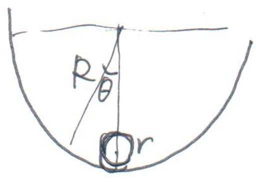
当 t_0 时达到纯滚动

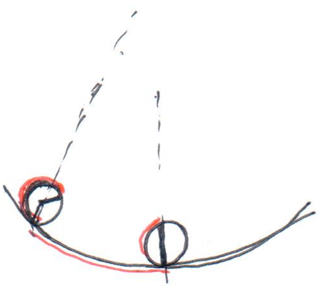
$R\omega(t_0) = v(t_0)$ 解得 $t_0 = \frac{4 - 10\mu}{7\mu g} V_0$　$V(t_0) = V_0 - \mu g t_0$　$(\mu < \frac{1}{5})$.

(2) $N = \frac{2mV_0}{\Delta t}$　$f = \mu N < mg$　$\mu < \frac{g\Delta t}{2V_0}$

10. 见 KK 7.6

11.





设小球走过弧度θ，相对于瞬时的法线转过φ，由纯滚动

$R\theta = \varphi r$

小球相对于地面转过角度 $\theta+\varphi$. 设小球自转角速度

$$w = \frac{d(\theta+\varphi)}{dt} = \frac{d}{dt}\left(\frac{R}{r}\theta+\theta\right) = \frac{R+r}{r}\frac{d\theta}{dt}$$

$$E_{k_1} = \frac{1}{2}m\cdot V_{cm}^2 = \frac{1}{2}m\left[(R-r)\frac{d\theta}{dt}\right]^2 = \frac{1}{2}m(R-r)^2\left(\frac{d\theta}{dt}\right)^2$$

$$E_{k_2} = \frac{1}{2}Iw^2 = \frac{1}{2}\cdot\frac{2}{5}mr^2\left(\frac{R+r}{r}\right)^2\left(\frac{d\theta}{dt}\right)^2 = \frac{1}{5}m\cdot(R+r)^2\left(\frac{d\theta}{dt}\right)^2$$

由能量守恒.

$$E_{k_1}+E_{k_2}+mg(R-r)\cdot(1-\cos\theta) = E_0.$$

$$\left[\frac{1}{5}m(R+r)^2+\frac{1}{2}m(R-r)^2\right]\cdot\left(\frac{d\theta}{dt}\right)^2 + mg(R-r)\cdot\frac{\theta^2}{2} = E_0.$$

$c_1 = \frac{1}{5}m(R+r)^2+\frac{1}{2}m(R-r)^2 \quad c_2 = \frac{mg(R-r)}{2}$ 对上式两边同时求导

$$2c_1\cdot\left(\frac{d\theta}{dt}\right)\cdot\frac{d^2\theta}{dt^2} + 2c_2\theta\cdot\frac{d\theta}{dt} = 0$$

$$\frac{d\theta}{dt}\cdot\left(c_1\frac{d^2\theta}{dt^2}+c_2\cdot\theta\right) = 0$$

得到 $w = \sqrt{\frac{c_2}{c_1}} = \sqrt{\frac{\frac{mgR}{2}\left(1-\frac{r}{R}\right)}{\frac{1}{5}mR^2\left(1+\frac{r}{R}\right)^2+\frac{1}{2}mR^2\left(1-\frac{r}{R}\right)^2}}$ $\qquad y = \frac{r}{R}$. 展开至一阶

$$= \sqrt{\frac{\frac{mgR}{2}(1-y)}{\frac{1}{10}mR^2(7-6y)}} = \sqrt{\frac{5g}{R}}\cdot\sqrt{\frac{1-y}{7-6y}}$$

12. 方程与1类似

$$\begin{cases} v\cdot\sin\theta = \dfrac{L_0}{ml} \\ v^2\cos\theta = gl\sin^2\theta \end{cases}$$

消去 v. $\begin{cases} \left(\dfrac{L_0}{ml\sin\theta}\right)^2\cdot\cos\theta = gl\sin^2\theta \\ \sin\theta = \dfrac{r}{l} \end{cases} \Rightarrow \left(\dfrac{L_0}{mr}\right)^2\cos\theta = gl\left(\dfrac{r}{l}\right)^2$

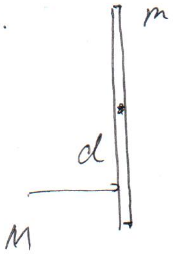
当 $\theta\sim 0$ 时 $\sin\theta\approx\theta$ $\cos\theta\sim 1$ (泰勒展开至1阶)

$$\left(\frac{L_0}{mr}\right)^2 = g\cdot l\cdot\left(\frac{r}{l}\right)^2 \qquad L_0^2 = m^2 g\frac{r^4}{l}$$

当 $\theta\sim\frac{\pi}{2}$ 时 $\cos\theta = 0-d\theta$ $\sin\theta = 1$ $\frac{r}{l} = 1$ $\frac{gl}{d\theta} = \left(\frac{L_0}{ml}\right)^2$ $m^2gl^3 = L_0^2\cdot d\theta$

$l = r\sim 0.$

13.



$$\begin{cases} M v_0 = M v_f + m v_{cm} \\ M v_0 d = M v_f d + I \cdot w + m \cdot v_{cm} \times 0 \\ \frac{1}{2} M v_0^2 = \frac{1}{2} M v_f^2 + \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I w^2 \end{cases}$$

$$\frac{1}{2} M v_0^2 = \frac{1}{2} M v_f^2 + \frac{M^2 (v_0 - v_f)^2}{2m} + \frac{M^2 (v_0 - v_f)^2 d^2}{2I}$$

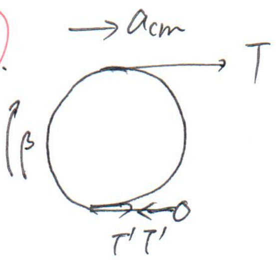
$$\frac{M}{2} (v_0 - v_f)(v_0 + v_f) = \left[\frac{M^2}{2m} + \frac{M^2 d^2}{2I} \right] (v_0 - v_f)^2$$ $\because v_0 \neq v_f$.（否则解为未碰上）

$$\frac{M}{2} (v_0 + v_f) = \left(\frac{M^2}{2m} + \frac{M^2 d^2}{2I} \right) (v_0 - v_f)$$

$$\therefore v_f = \frac{\frac{M^2}{2m} + \frac{M^2 d^2}{2I} - \frac{M}{2}}{\frac{M^2}{2m} + \frac{M^2 d^2}{2I} + \frac{M}{2}} v_0 \qquad w = \frac{M (v_0 - v_f) d}{I}$$

$$v_{cm} = \frac{M (v_0 - v_f)}{m}.$$

14.



圆盘质心运动：

$$T + T' = m \cdot a_{cm}$$

在质心系下：

$$T' + m \cdot a_{cm} = m a$$

$$a = \beta r$$

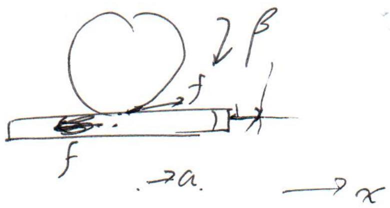
$$d\tau = (T(\theta + d\theta) - T(\theta)) r \quad \therefore \tau = r \int T(\theta + d\theta) - T(\theta)\, d\theta = r (T - T')$$

$$r (T - T') = \frac{1}{2} m r^2 \beta.$$

$$3T = 2ma \qquad a = \frac{3T}{2m} \qquad a_{cm} = \frac{5T}{4m}$$

小球相对地面：$\frac{3T}{2m} - \frac{5T}{4m} = \frac{6T - 5T}{4m} = \frac{T}{4m}$ 向左.

15. $\cancel{-kx - f = ma}$



$$\begin{cases} -kx - f = ma \\ f - ma = ma' \\ a' = \beta R \\ I\beta = -Rf \end{cases}$$

$\beta = -\dfrac{2f}{mR}$

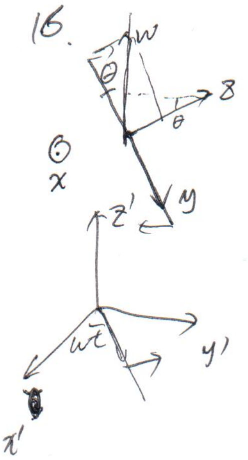
$a' = -\dfrac{2f}{m}$

$f - ma = -2f \qquad f = \dfrac{ma}{3}$

$-kx - \dfrac{ma}{3} - ma = 0$

$\dfrac{4m}{3}\dfrac{d^2x}{dt^2} + kx = 0. \qquad \dfrac{d^2x}{dt^2} + \dfrac{3k}{4m}x = 0 \qquad w = \sqrt{\dfrac{3k}{4m}}$

16.



$w = (0, -w\cos\theta, w\sin\theta)$

$J = Iw = (0, 0, \dfrac{ml^2}{12} w\sin\theta)$ 记 $J_0 = \dfrac{ml^2}{12} w \cdot \sin\theta$

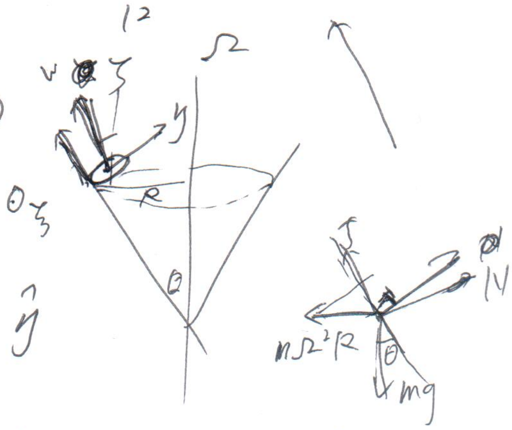
换回实验室系 $(J_0 \cancel{\sin\theta}\cos\theta \cdot \cos wt,\ J_0 \cancel{\sin\theta}\cos\theta \cdot \sin wt,\ J_0 \sin\theta)$.

$\dfrac{dJ}{dt} = (wJ_0\cos\theta \cdot -\sin wt,\ wJ_0\cos\theta \cdot \cos wt,\ 0)$.

$|\tau| = wJ_0\cos\theta = 2F \cdot \dfrac{l}{2}\cos\theta$ 答案不一样?

$\therefore F = \dfrac{w}{l} \cdot J_0 = \dfrac{w}{l} \cdot \dfrac{ml^2}{12} w \cdot \sin\theta = \dfrac{mlw^2\sin\theta}{12}$

17. $\cancel{m\dfrac{\Omega^2}{R}\sin\theta +}$ $mg\sin\theta + m\Omega^2 R\cos\theta = N$ ①



$m\Omega^2 R\sin\theta + f = mg\cos\theta$ ② 无向内摩擦.

$\vec{\tau} = \cancel{\vec{r}\times\vec{f}} = \dfrac{d\vec{L}}{dt} \qquad \vec{\Omega} = \Omega\cos\theta \cdot \hat{\xi} + \Omega\sin\theta\,\hat{\eta}$

$$\vec{L} = \begin{pmatrix} mr^2 & & \\ & \frac{mr^2}{2} & \\ & & \frac{mr^2}{2} \end{pmatrix}\begin{pmatrix} w + \Omega\cos\theta \\ \Omega\sin\theta \\ 0 \end{pmatrix} = mr^2(w + \Omega\cos\theta)\hat{\xi} + \frac{mr^2}{2}(\Omega\sin\theta)\cdot\hat{\eta}$$

$\dfrac{d\hat{\xi}}{dt} = \vec{\Omega}\times\vec{\xi} = \Omega\sin\theta\ \hat{\eta}\times\hat{\xi} \qquad \dfrac{d\vec{\eta}}{dt} = \vec{\Omega}\times\vec{\eta} = \Omega\cos\theta\ \hat{\xi}\times\hat{\eta}$

$$\tau = (-r\cdot\hat{y}) \times (f\cdot\hat{\xi})$$

$$= -rf\cdot\hat{y}\times\hat{\xi} = \frac{dL}{dt} = mr^2(w+\Omega\cos\theta)\,\Omega\cdot\sin\theta\cdot\hat{y}\times\hat{\xi}$$

$$+\frac{mr^2}{2}(\Omega\sin\theta)\,\Omega\cos\theta\cdot\hat{\xi}\times\hat{y}$$

$$\therefore -rf = mr^2(w+\Omega\cos\theta)\cdot\Omega\sin\theta - \frac{mr^2}{2}(\Omega\sin\theta)\cdot\Omega\cos\theta.$$

纯滚动条件 $\vec{\Omega}\times\vec{R} + \vec{w}\times\vec{r} = 0.$ (已经做了一步近似).

$$\Omega R + wr = 0. \quad \therefore w = -\frac{R}{r}\Omega.$$

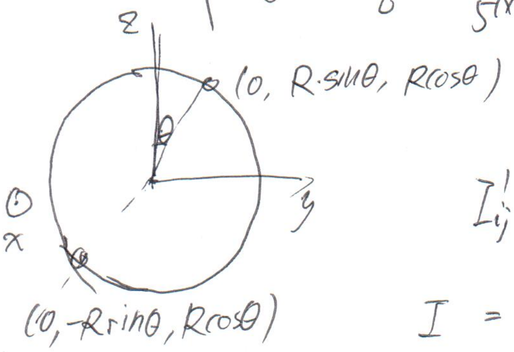
$$-rf = mr^2 w\cdot\Omega\cdot\sin\theta = m\cdot r^2\cdot -\frac{R}{r}\cdot\Omega^2\cdot\sin\theta$$

答案不一样?

$$f = mR\Omega^2\sin\theta$$

$$2m\Omega^2 R\sin\theta = mg\cos\theta \qquad \Omega = \sqrt{\frac{g\cos\theta}{2R\sin\theta}} = \sqrt{\frac{g}{2R\tan\theta}}$$

18. $$I_0 = \begin{pmatrix} \frac{2}{5}MR^2 & 0 & 0 \\ 0 & \frac{2}{5}MR^2 & 0 \\ 0 & 0 & \frac{2}{5}MR^2 \end{pmatrix} \qquad w_0 = \begin{pmatrix} 0 \\ 0 \\ w \end{pmatrix}.$$



$$I'_{ij} = \int (r^2\delta_{ij} - r_i r_j)\,dm$$

$$I = 2m\begin{pmatrix} R^2 & 0 & 0 \\ 0 & R^2 & 0 \\ 0 & 0 & R^2 \end{pmatrix} - 2m\begin{pmatrix} 0 \\ R\sin\theta \\ R\cos\theta \end{pmatrix}(0, R\cdot\sin\theta, R\cos\theta)$$

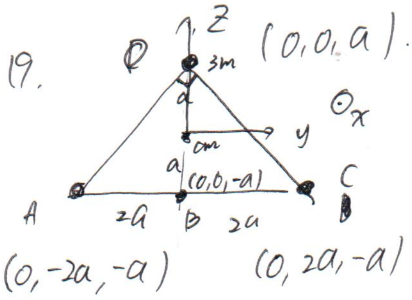
$$= 2m\begin{pmatrix} R^2 & 0 & 0 \\ 0 & R^2\cos^2\theta & -R^2\sin\theta\cos\theta \\ 0 & -R^2\sin\theta\cos\theta & R^2\sin^2\theta \end{pmatrix}$$

$$\therefore I_f = R^2\begin{pmatrix} \frac{2}{5}M - 2m & 0 & 0 \\ 0 & \frac{2M}{5} - 2m\cos^2\theta & -2m\sin\theta\cos\theta \\ 0 & -2m\sin\theta\cos\theta & \frac{2M}{5} - 2m\sin^2\theta \end{pmatrix}.$$

$$I_0 w_0 = I_f w_f \qquad \frac{2M}{5} - 2m\cos^2\theta\, w_y - 2m\sin\theta\cos\theta\cdot w_z = 0$$

$$\frac{w_y}{w_z} = \frac{2m\sin\theta\cos\theta}{\frac{2M}{5} - 2m\cos^2\theta}$$

19.



cm:

$$I_A = m\cdot\begin{pmatrix}5a^2 & 0 & 0\\ 0 & 5a^2 & 0\\ 0 & 0 & 5a^2\end{pmatrix} - m\cdot\begin{pmatrix}0\\ -2a\\ -a\end{pmatrix}(0,-2a,-a)$$

$$= ma^2\begin{pmatrix}5 & 0 & 0\\ 0 & 1 & -2\\ 0 & -2 & 4\end{pmatrix}$$

$$I_B = m\begin{pmatrix}a^2 & & \\ & a^2 & \\ & & a^2\end{pmatrix} - m\cdot\begin{pmatrix}0\\ 0\\ -a\end{pmatrix}(0,0,-a) = ma^2\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 0\end{pmatrix}$$

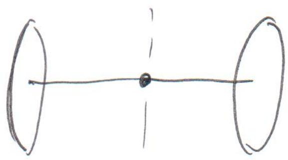
$I_C = I_A \qquad I_D = 3I_B \qquad \therefore\ I_{tot} = \sum_{i=A,B,C,D} I_i = ma^2\begin{pmatrix}14 & 0 & 0\\ 0 & 6 & -4\\ 0 & -4 & 8\end{pmatrix}.$

$$I_{tot}\cdot\vec{w} = \int\vec{\tau}\,dt \qquad \int\vec{\tau}\,dt = \begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ 0 & 2a & -a\\ -p & 0 & 0\end{vmatrix} = ap\,\hat{y} + 2ap\,\hat{z}$$

$$ma^2\cdot\begin{pmatrix}14 & 0 & 0\\ 0 & 6 & -4\\ 0 & -4 & 8\end{pmatrix}\begin{pmatrix}w_x\\ w_y\\ w_z\end{pmatrix} = a\begin{pmatrix}0\\ p\\ 2p\end{pmatrix} \qquad w_x = 0 \qquad \begin{pmatrix}6 & -4\\ -4 & 8\end{pmatrix}\begin{pmatrix}w_y\\ w_z\end{pmatrix} = \frac{p}{ma}\begin{pmatrix}1\\ 2\end{pmatrix}.$$

$$\begin{pmatrix}w_y\\ w_z\end{pmatrix} = \frac{p}{ma}\begin{pmatrix}0.25 & 0.125\\ 0.125 & 0.1875\end{pmatrix}\begin{pmatrix}1\\ 2\end{pmatrix} = \begin{pmatrix}0.5\\ 0.5\end{pmatrix} \qquad \therefore\ w_y = w_z = \frac{p}{2ma}$$

20.



由KK P298页结论

$$\frac{dL_z}{dt} = 2I\cdot w\cdot\Omega$$ (这里已经讨论了盘与分开时质心的平动角动量，但其对时间求导为0)

$$\tau_{max} = 2mg\cdot\frac{l}{2} = 2Iw\Omega \qquad \therefore\ \Omega = \frac{mgl}{2Iw}$$