3.1. $L=\int_a^b (1+y_x^2)^{\frac{1}{2}}dx$. 即求L的极值，即L的变分为0. $\delta L=\int_a^b \delta(1+y_x^2)^{1/2}dx=0$.

令 $f=(1+y_x^2)^{1/2}$. $\delta L=\int_a^b\left(\frac{\partial f}{\partial y}\delta y+\frac{\partial f}{\partial y_x}\delta y_x\right)dx=\int_a^b\left(\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y_x}\right)\right)\delta y\,dx=0$

$\rightarrow \frac{\partial f}{\partial y}-\frac{d}{dx}\frac{\partial f}{\partial y_x}=0$.

$\rightarrow \frac{d}{dx}\frac{y_x}{\sqrt{1+y_x^2}}=0$. $=\frac{dy_x}{dx}\cdot\frac{d}{dy_x}\frac{y_x}{\sqrt{1+y_x^2}}=\frac{1}{(1+y_x^2)^{3/2}}\frac{dy_x}{dx}=0$.

$\rightarrow \frac{dy_x}{dx}=0$. 即$y_x$不随$x$变化，$y_x=const.=\frac{dy}{dx}=c$

$\rightarrow y=c_1x+c_2$. 为直线，且过a, b两点.

所以两点间的最短距离为连接两点的直线.

补6.

$\left|\frac{\partial(N_i,N_j)}{\partial(x,y)}\right|=\begin{vmatrix}\frac{\partial N_i}{\partial x}&\frac{\partial N_i}{\partial y}\\\frac{\partial N_j}{\partial x}&\frac{\partial N_j}{\partial y}\end{vmatrix}$ 中 $N_i(x,y)=(a_i+b_ix+c_iy)/2\Delta$ $\quad b_i=-\begin{vmatrix}1&y_j\\1&y_m\end{vmatrix}$ $\quad c_i=\begin{vmatrix}1&x_j\\1&x_m\end{vmatrix}$

$N_j(x,y)=(a_j+b_jx+c_jy)/2\Delta$. $\quad b_j=\begin{vmatrix}1&y_i\\1&y_m\end{vmatrix}$ $\quad c_j=-\begin{vmatrix}1&x_i\\1&x_m\end{vmatrix}$.

$\rightarrow \left|\frac{\partial(N_i,N_j)}{\partial(x,y)}\right|=\frac{1}{2\Delta}$, $\left|\frac{\partial(x,y)}{\partial(N_i,N_j)}\right|=2\Delta$

$\iint_e dx\,dy=\int_0^1\int_0^{1-N_i}dN_i\,dN_j\cdot\left|\frac{\partial(x,y)}{\partial(N_i,N_j)}\right|=2\Delta\int_0^1\int_0^{1-N_i}dN_i\,dN_j=\Delta$.

$\iint_e N_i\,dx\,dy=\int_0^1\int_0^{1-N_i}N_i\,dN_i\,dN_j\cdot\left|\frac{\partial(x,y)}{\partial(N_i,N_j)}\right|=2\Delta\int_0^1\int_0^{1-N_i}N_i\,dN_i\,dN_j$

$=2\Delta\int_0^1 N_i(1-N_i)\,dN_i=2\Delta\cdot\frac{1}{6}=\frac{1}{3}\Delta$

$\iint_e N_i^2\,dx\,dy=\int_0^1\int_0^{1-N_i}N_i^2\,dN_i\,dN_j\cdot\left|\frac{\partial(x,y)}{\partial(N_i,N_j)}\right|=2\Delta\int_0^1\int_0^{1-N_i}N_i^2\,dN_i\,dN_j$

$=2\Delta\int_0^1 N_i^2(1-N_i)\,dN_i=2\Delta\cdot\frac{1}{12}=\frac{1}{6}\Delta$

$\iint_e N_iN_j\,dx\,dy=\int_0^1\int_0^{1-N_i}N_iN_j\,dN_i\,dN_j\cdot\left|\frac{\partial(x,y)}{\partial(N_i,N_j)}\right|=2\Delta\int_0^1\int_0^{1-N_i}N_iN_j\,dN_i\,dN_j$

$=2\Delta\int_0^1 N_i\cdot\frac{1}{2}(1-N_i)^2\,dN_i=2\Delta\cdot\frac{1}{24}=\frac{1}{12}\Delta$ ，证毕

补7. $\Delta_i=\frac{1}{2}\begin{vmatrix}1&x&y\\1&x_j&y_j\\1&x_m&y_m\end{vmatrix}$. $\Delta_j=\frac{1}{2}\begin{vmatrix}1&x_i&y_i\\1&x&y\\1&x_m&y_m\end{vmatrix}$. $\Delta_m=\frac{1}{2}\begin{vmatrix}1&x_i&y_i\\1&x_j&y_j\\1&x&y\end{vmatrix}$. $\Delta=\frac{1}{2}\begin{vmatrix}1&x_i&y_i\\1&x_j&y_j\\1&x_m&y_m\end{vmatrix}$

$\Delta_i=\frac{1}{2}\left(\begin{vmatrix}x_j&y_j\\x_m&y_m\end{vmatrix}-x\begin{vmatrix}1&y_j\\1&y_m\end{vmatrix}+y\begin{vmatrix}1&x_j\\1&x_m\end{vmatrix}\right)=\frac{1}{2}(a_i+b_ix+c_iy)$.

同理: $\Delta_j=\frac{1}{2}(a_j+b_jx+c_jy)$. $\Delta_m=\frac{1}{2}(a_m+b_mx+c_my)$

所以 $\frac{\Delta_i}{\Delta}=\frac{a_i+b_ix+c_iy}{2\Delta}=N_i$. $\frac{\Delta_j}{\Delta}=\frac{a_j+b_jx+c_jy}{2\Delta}=N_j$, $\frac{\Delta_m}{\Delta}=\frac{a_m+b_mx+c_my}{2\Delta}=N_m$.