电动力学 H1.

1. (1). $\nabla\cdot\vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} = \cos x\cosh y - \cos x\cdot\frac{e^{y}+e^{-y}}{2} = \cos x\cosh y - \cos x\cosh y$

$= 0.$

$$\nabla\times\vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \sin x\cosh y & -\cos x\sinh y & 0 \end{vmatrix} = \sin x\sinh y\,\hat{z} - \sin x\sinh y\,\hat{z} = \vec{0}$$

(2). $\nabla\cdot\vec{F} = 2x+2y.$

$$\nabla\times\vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & 2xy+z^2 & 2yz \end{vmatrix} = 2z\,\hat{x} + 2y\,\hat{z} - 2y\,\hat{z} - 2z\,\hat{x} = \vec{0}.$$

2. (1). $\nabla\cdot\vec{F} = y+2z+3x.$

$$\int_V(\nabla\cdot\vec{F})d\tau = \int_V (y+2z+3x)\,dx\,dy\,dz = \int_0^2\left(\int_0^2\left(\int_0^2 (y+2z+3x)dy\right)dz\right)dx.$$

$$=\int_0^2\left(\int_0^2(2+4z+6x)dz\right)dx = \int_0^2(4+12x+8)\,dx = 24+24 = 48.$$

$$\oint\vec{F}\cdot d\vec{a} = \int_1\vec{F}\cdot d\vec{a_1} + \int_2\vec{F}\cdot d\vec{a_2} + \cdots + \int_6\vec{F}\cdot d\vec{a_6}.$$

$$\int_1\vec{F}\cdot d\vec{a_1} = \int_{S_1}F_1\,dy\,dz + F_2\,dz\,dx + F_3\,dx\,dy = 0.$$

$$\int_2\vec{F}\cdot d\vec{a_2} = \int_0^2\left(\int_0^2 6x\,dx\right)dy = 24.$$

$$\int_3\vec{F}\cdot d\vec{a_3} = 0. \quad \int_4\vec{F}\cdot d\vec{a_4} = \int_0^2\left(\int_0^2 4z\,dz\right)dx = 16.$$

$$\int_5\vec{F}\cdot d\vec{a_5} = 0. \quad \int_6\vec{F}\cdot d\vec{a_6} = \int_0^2\left(\int_0^2 2y\,dy\right)dz = 8$$

$$\oint\vec{F}\cdot d\vec{a} = 24+16+8 = 48.$$

$$\Rightarrow \int_V(\nabla\cdot\vec{F})d\tau = \oint\vec{F}\cdot d\vec{a}.$$

(2). $\nabla\times\vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ xy & 2yz & 3xz \end{vmatrix} = -x\hat{z} - 2y\hat{x} - 3z\hat{y}$.

$$\int_s (\nabla\times\vec{F})\cdot d\vec{a} = \int -2y\cdot dydz - 3z\,dzdx - x\,dxdy$$

3A $$= \int_0^2 \left(\int_0^{-y+2} -2y\cdot dz\right) dy = \int_0^2 2y(y-2)\,dy = \int_0^2 (2y^2-4y)\,dy = \frac{2\times 8}{3} - 2\times 4 = -\frac{8}{3}.$$

3k $$\oint \vec{F}\cdot d\vec{l} = \int \vec{F}\cdot d\vec{l_1} + \int \vec{F}\cdot d\vec{l_2} + \int \vec{F}\cdot d\vec{l_3}$$

$$= \int_2^0 2y(-y+2)\cdot dy = -\frac{8}{3}.$$

$$\Rightarrow \int_s (\nabla\times\vec{F})\cdot d\vec{a} = \oint \vec{F}\cdot d\vec{l}.$$

3. $\nabla\times(f\vec{A}) = f(\nabla\times\vec{A}) - \vec{A}\times(\nabla f)$.

$$\int_s \nabla\times(f\vec{A})\,d\vec{a} = \int_s f(\nabla\times\vec{A})\,d\vec{a} - \int_s \vec{A}\times(\nabla f)\cdot d\vec{a}$$

$$\rightarrow \int_s f(\nabla\times\vec{A})\cdot d\vec{a} = \int_s [A\times(\nabla f)]\cdot d\vec{a} + \int \nabla\times(f\vec{A})\cdot d\vec{a}.$$

$$\int \nabla\times(f\vec{A})\cdot d\vec{a} = \oint_l f\vec{A}\cdot d\vec{l} \quad \text{(stokes)}$$

$$\rightarrow \int_s f(\nabla\times\vec{A})\,d\vec{a} = \int_s [\vec{A}\times(\nabla f)]\,d\vec{a} + \oint_l f\vec{A}\cdot d\vec{l}.$$