2. $g_{X+Y}(z) = g_X(z)\, g_Y(z)$

$$g(z) = \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} z^k = \sum_{k=0}^{n} \binom{n}{k} (zp)^k (1-p)^{n-k}$$

$$= (1-p+zp)^n.$$

$$g_{X+Y}(z) = (1-p+zp)^n (1-p+zp)^m$$

$$= (1-p+zp)^{n+m}.$$ 得证.

$$f(k; n, p) = \binom{n}{k} p^k (1-p)^{n-k}. \quad k = 0, 1, \dots, n.$$

$E(S_n) = np.\quad \sigma^2(X) = npq$

$$g(z) = \sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} z^k = (1-p+zp)^n$$

$$f(k, \lambda) = \frac{\lambda^k}{k!} e^{-\lambda}. \quad k = 0, 1, 2, \dots.$$

$E(X) = \lambda + \lambda^2. \quad \sigma^2(X) = \lambda.$

$$g(z) = \sum_{k=0}^{\infty} \frac{e^{-\lambda}}{k!} \lambda^k z^k = e^{\lambda(z-1)}$$

$$\lim_{n\to\infty} \frac{n!\, e^n}{n^{n+1/2}} = \sqrt{2\pi}.$$

$$\lim_{n\to\infty} P\left(a < \frac{S_n - nm}{\sqrt{n}\,\sigma} \le b\right) = \frac{1}{\sqrt{2\pi}} \int_a^b e^{-x^2/2}\, dx$$

条件: $P(S_n = k) = \binom{n}{k} p^k q^{n-k}$. $E(X_j) = m.\ \sigma^2(X_j) = \sigma^2.$

$-\infty < a < b < +\infty$

$$\varphi(x) = \frac{1}{\sigma} \varphi\left(\frac{x+1}{\sqrt{2}}\right) = \frac{1}{2\sqrt{\pi}} \exp\left(-\frac{(x+1)^2}{4}\right). \quad -\infty < x < +\infty.$$

$$M(\theta) = E\left(e^{\theta(-1+\sqrt{2}X^*)}\right) = e^{-\theta+\theta^2}.$$

$m = -1.\ \sigma^2 = 2.$