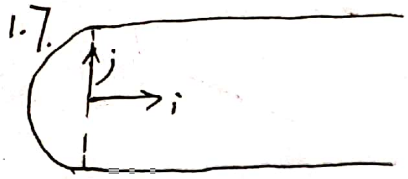
1.7.




$$\vec{E_1}=\frac{1}{4\pi\varepsilon_0}\int_0 \frac{(-\vec{X'})\cdot dq}{R^3}=\frac{1}{4\pi\varepsilon_0 R^3}\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}(-R\cos\theta,-R\sin\theta)\cdot\lambda\cdot R\cdot d\theta.$$

$$=\frac{\lambda}{4\pi\varepsilon_0 R}\cdot\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}(-\cos\theta,-\sin\theta)\cdot d\theta=\frac{\lambda}{2\pi\varepsilon_0 R}(1,0).$$

$$\vec{E_2}=\frac{1}{4\pi\varepsilon_0}\int\frac{(-\vec{X'})\cdot dq}{|\vec{X'}|^3}=\frac{1}{4\pi\varepsilon_0}\int\frac{(-x,-y)\cdot dq}{(x^2+y^2)^{3/2}}=\frac{1}{4\pi\varepsilon_0}\int_0^{\infty}\frac{(-x,-R)\cdot\lambda\cdot dx}{(x^2+R^2)^{3/2}}.$$

$$\vec{E_3}=\frac{1}{4\pi\varepsilon_0}\int_0^{\infty}\frac{(-x,R)\cdot\lambda\cdot dx}{(x^2+R^2)^{3/2}}.\quad \vec{E_2}+\vec{E_3}=\frac{\lambda}{2\pi\varepsilon_0}\int_0^{\infty}\frac{(-x,0)dx}{(x^2+R^2)^{3/2}}=\frac{\lambda}{2\pi\varepsilon_0 R}(-1,0).$$

$$\vec{E}=\vec{E_1}+\vec{E_2}+\vec{E_3}=\vec{0}.$$

1.8.(1). $\vec{E}_-=\frac{1}{4\pi\varepsilon_0}\frac{Q}{(D-\frac{l}{2})^2}(-1,0)$. $\vec{F}_-=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{(D-\frac{l}{2})^2}(-1,0)$.

$\vec{E}_+=\frac{1}{4\pi\varepsilon_0}\frac{Q}{(D+\frac{l}{2})^2}(1,0)$. $\vec{F}_+=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{(D+\frac{l}{2})^2}(1,0)$.

$$\vec{F}\approx\frac{1}{4\pi\varepsilon_0}\frac{Qq}{D^2}\left[(1+\frac{l}{D})-(1-\frac{l}{D})\right](-1,0)=\frac{Qql}{2\pi\varepsilon_0 D^3}(-1,0)=\frac{-Q\vec{P}}{2\pi\varepsilon_0 D^3}$$

$$\vec{M}=\vec{l}\times\vec{F}=\vec{0}.$$

(2). ~~$\vec{F}=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{D^2+(\frac{l}{2})^2}$~~ 电偶极子在Q处产生的电场为 $\vec{E}\approx\frac{1}{4\pi\varepsilon_0}\cdot\frac{-\vec{P}}{r^3}$

$\vec{F_0}\approx\frac{Q}{4\pi\varepsilon_0}\cdot\frac{-\vec{P}}{r^3}$. 由相互作用力：

$$\vec{F}\approx\frac{Q}{4\pi\varepsilon_0}\cdot\frac{\vec{P}}{r^3}.$$

$\vec{F}_-=\frac{1}{4\pi\varepsilon_0}\frac{Qq\cdot(\vec{r}+\frac{\vec{l}}{2})}{(D^2+(\frac{l}{2})^2)^{3/2}}$. 其中 $\vec{r}=(-D,0)$. $\vec{M}_-=\frac{\vec{l}}{2}\times\vec{F}_-=(0,\frac{l}{2})\times\vec{F}_-$

$$\vec{M}_-=\frac{Qq}{4\pi\varepsilon_0(D^2+(\frac{l}{2})^2)^{3/2}}(0,\frac{l}{2})\times(-D,\frac{l}{2})=\frac{QqD}{8\pi\varepsilon_0(D^2+(\frac{l}{2})^2)^{3/2}}(0,0,l).$$

同理 $\vec{M}_+=\frac{QqD}{8\pi\varepsilon_0(D^2+(\frac{l}{2})^2)^{3/2}}(0,0,l)$.

$$\vec{M}=\frac{QqD}{4\pi\varepsilon_0(D^2+(\frac{l}{2})^2)^{3/2}}(0,0,l)=\frac{Q\vec{P}\times\vec{D}}{4\pi\varepsilon_0(D^2+(\frac{l}{2})^2)^{3/2}}.$$

1.9. (1). 在球外取一个高斯面: $\oiint \vec{E}\cdot d\vec{S} = \frac{Q}{\varepsilon_0} = 0$. $Q=0$.

所以在内球面的电荷量为 $-\sigma\cdot 4\pi R_2^2$.

$$\sigma_1 = \frac{-\sigma\cdot 4\pi R_2^2}{4\pi R_1^2} = -\sigma\frac{R_2^2}{R_1^2}.$$

(2). $\oiint \vec{E}\cdot d\vec{S} = \frac{Q_1}{\varepsilon_0} = 4\pi r^2\cdot E = \frac{\sigma_1\cdot 4\pi R_1^2}{\varepsilon_0} = -\frac{\sigma 4\pi R_2^2}{\varepsilon_0}$.

$E = \frac{-\sigma R_2^2}{\varepsilon_0 r^2}$. 方向朝球心.

(3). $\oiint \vec{E}\cdot d\vec{S} = 0$. $\vec{E}=\vec{0}$.

1.11. $\cancel{\int_a^\infty \rho(r) dE = \iiint_a^\infty \frac{q e^{-2r/a}}{\pi a^3} dr}$

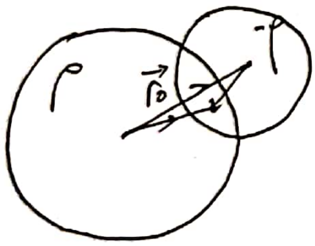
(1). $\int_0^{4\pi}\int_0^\infty \rho(r)\cdot r^2 d\Omega\cdot dr = \int_0^\infty \rho(r)4\pi r^2\cdot dr = \int_0^\infty -\frac{4r^2 q e^{-\frac{2r}{a}}}{a^3}\cdot dr = \frac{4q}{a^3}\cdot\frac{2a^3}{8} = -q$.

(2). $\oiint \vec{E}\cdot d\vec{S} = \iiint_V \frac{\rho\cdot dV}{\varepsilon_0} = 4\pi r^2 E = \frac{\int_0^r \rho(r)4\pi r^2\cdot dr}{\varepsilon_0}$

$$= -\frac{4q}{\varepsilon_0 a^3}\left(-\frac{ar^2}{2}e^{-\frac{2r}{a}} + \int_0^r e^{-\frac{2r}{a}}\cdot ar\cdot dr\right) = -\frac{4q}{\varepsilon_0 a^3}\left(-\frac{a}{2}r^2 e^{-\frac{2r}{a}} - \frac{a^2 r}{2}e^{-\frac{2r}{a}} + \frac{a^3}{4} - \frac{a^3}{4}e^{-\frac{2r}{a}}\right)$$

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 r^2}\left(\left(\frac{2r^2}{a^2}+\frac{2r}{a}+1\right)e^{-\frac{2r}{a}} - 1\right)\cdot\hat{r}$$

1.12.




$E\cdot 4\pi r^2 = \frac{\rho\cdot\frac{4\pi}{3}r^3}{\varepsilon_0} \rightarrow \vec{E_1} = \frac{\rho r_1}{3\varepsilon_0}\cdot\hat{r_1}$.

$\vec{E_2} = \frac{-\rho r_2}{3\varepsilon_0}\cdot\hat{r_2}$.

$\vec{E} = \vec{E_1}+\vec{E_2} = \frac{\rho}{3\varepsilon_0}(r_1\cdot\hat{r_1} - r_2\cdot\hat{r_2}) = \frac{\rho}{3\varepsilon_0}\vec{r_0}$. 为匀强电场.

1.20

(1) $U=\int_r^\infty \vec{E}\cdot d\vec{r}=\int_r^\infty \frac{q}{4\pi\varepsilon_0 r^2}\left(\frac{2r^2}{a^2}e^{-\frac{2r}{a}}+\frac{2r}{a}e^{-\frac{2r}{a}}+e^{-\frac{2r}{a}}-1\right)\cdot\hat{r}\cdot d\vec{r}$

$=\frac{q}{4\pi\varepsilon_0}\left(\int_r^\infty \frac{2}{a^2}e^{-\frac{2r}{a}}\,dr+\int_r^\infty \frac{2}{ar}e^{-\frac{2r}{a}}\,dr+\int_r^\infty \frac{1}{r^2}e^{-\frac{2r}{a}}\,dr-\int_r^\infty \frac{1}{r^2}\,dr\right)$

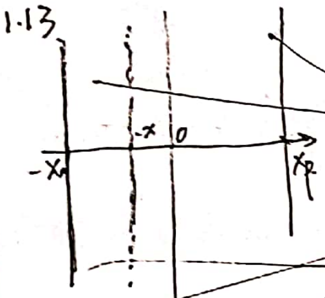
$=\frac{q}{4\pi\varepsilon_0}\left(-\frac{a}{2}\frac{2}{a^2}e^{-\frac{2r}{a}}-\frac{1}{r}e^{-\frac{2r}{a}}\right)$

$=\frac{-q}{4\pi\varepsilon_0 r}+\frac{q\cdot e^{-\frac{2r}{a}}}{a4\pi\varepsilon_0}+\frac{q}{4\pi\varepsilon_0 r}e^{-\frac{2r}{a}}$

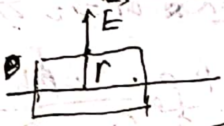
(2). $U=U_{电}+U_{核}=U+\frac{q}{4\pi\varepsilon_0 r}=\frac{q}{4\pi\varepsilon_0 a}\cdot e^{-\frac{2r}{a}}+\frac{1}{4\pi\varepsilon_0 r}\cdot e^{-\frac{2r}{a}}$.

$\cancel{(2). U=U_{电}+U_{核}=U+\frac{q}{4\pi\varepsilon_0 r}=}$

1.13



无限长直线的电场:



$\vec{E}=\frac{\lambda}{2\pi\varepsilon_0 r}$. $\because dl\cdot\lambda=dl\cdot dx\cdot\rho$.

n区: $\vec{E_1}=\int_0^x \frac{\rho\cdot dx'}{2\pi\varepsilon_0(x-x')}=\frac{-\rho}{2\pi\varepsilon_0}\ln(x-x')\Big|_0^x(-1,0)$

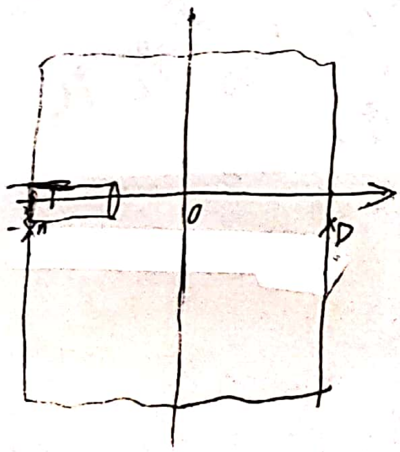
$\vec{E_2}=\int_x^{x_n}\frac{\rho\cdot dx'}{2\pi\varepsilon_0(x'-x)}=\frac{\rho}{2\pi\varepsilon_0}\ln(x'-x)\Big|_x^{x_n}(1,0)$

$\Big\}\ \frac{\rho e}{2\pi\varepsilon_0}\ln\frac{x_n-x}{x}\ (1,0)$.

$\vec{E_3}=\int_0^{x_p}\frac{\rho\,dx'}{2\pi\varepsilon_0(x'+x)}=\frac{N_D e}{2\pi\varepsilon_0}\ln\frac{x_p+x}{x}\ (1,0)$

$\vec{E}=$

1.13. $N_A x_p=N_D x_n$.



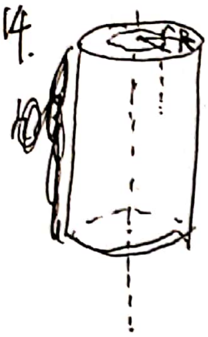
(1) $E_n S=\frac{\rho_e(x)(x-(-x_n))\cdot S}{\varepsilon_0}$

$E_n=\frac{N_D e(x+x_n)}{\varepsilon_0}$

(2). $E\cdot S=\frac{\rho_e(x)(x_p-x)\cdot S}{\varepsilon_0}$

$E_p=\frac{N_A e(x_p-x)}{\varepsilon_0}$.

1.14.



若 $r<R$: $\oiint \vec{E}\cdot d\vec{s} = \iiint \frac{\rho\cdot dv}{\varepsilon_0} = \int_0^r \frac{\rho(r) h 2\pi r\cdot dr}{\varepsilon_0} = E\cdot 2\pi r\cdot h$

$= \int_0^r \frac{\rho_0 2\pi h}{\varepsilon_0} \frac{r\cdot dr}{(1+(\frac{r}{a})^2)^2} = \int_0^r \frac{a^4\rho_0 \pi h \, d(r^2)}{\varepsilon_0 (r^2+a^2)^2} = -\frac{a^4\rho_0\pi h}{r^2+a^2}\Big|_0^r$

$= \frac{a^4\rho_0\pi h}{\varepsilon_0}\left(\frac{1}{a^2}-\frac{1}{a^2+r^2}\right) = E\cdot 2\pi r\cdot h.$

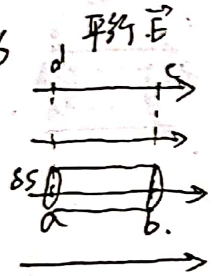
$E = \frac{a^4\rho_0}{2r\varepsilon_0}\left(\frac{1}{a^2}-\frac{1}{a^2+r^2}\right) = \frac{a^2\rho_0 r}{2\varepsilon_0(a^2+r^2)}$

若 $r>R$: $\oiint \vec{E}\cdot d\vec{s} = \int_0^R \frac{\rho(r)\cdot h\cdot 2\pi r\cdot dr}{\varepsilon_0} = E\cdot 2\pi r\cdot h.$

$2\pi h\int_0^R \frac{\rho(r) r\cdot dr}{\varepsilon_0} = \int_0^R \frac{\rho_0}{\varepsilon_0}\frac{2\pi r h}{(1+(\frac{r}{a})^2)^2}dr = \frac{a^4\rho_0 \pi h}{\varepsilon_0}\left(\frac{1}{a^2}-\frac{1}{a^2+R^2}\right) = E\cdot 2\pi r h.$

$E = \frac{a^4\rho_0}{2r\varepsilon_0}\left(\frac{1}{a^2}-\frac{1}{a^2+R^2}\right) = \frac{a^2\rho_0 R^2}{2r\varepsilon_0(a^2+R^2)}$

1.16




1). 对于 a、b 两点，选取高斯面使 $\delta S\to 0$.

$\iint \vec{E}\cdot d\vec{s} = \frac{Q}{\varepsilon_0} = 0 \approx \lim_{\delta S\to 0}(-E_a\cdot \delta S + E_b\cdot \delta S) = 0.$

$E_a = E_b$. 所以同一条电场线上电场大小相等.

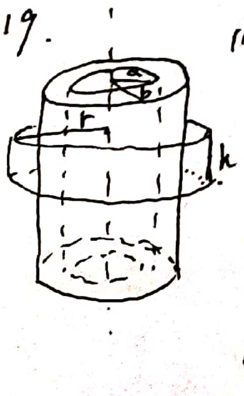
2) 对于 a,b,c,d. 环路积分 $\oint_L \vec{E}\cdot d\vec{r} = 0$.

由(1): 同电场线上电场大小相等. 所以 $\oint_L \vec{E}\cdot d\vec{r} = E_a\cdot L - E_d\cdot L = 0$. $E_a = E_d$.

所以不同电场线上电场大小相等.

得证.

1.19.




(1). $r<a$: $\oiint \vec{E}\cdot d\vec{s} = 0$. $\vec{E} = \vec{0}$.

$a<r<b$: $\oiint \vec{E}\cdot d\vec{s} = E\cdot 2\pi r\cdot h = \frac{\lambda_e h}{\varepsilon_0}$ $\vec{E} = \frac{\lambda_e}{2\pi r\varepsilon_0}$ 向外.

$r>b$: $\oiint \vec{E}\cdot d\vec{s} = E\cdot 2\pi r\cdot h = \frac{\lambda_e h - \lambda_e h}{\varepsilon_0} = 0$. $\vec{E} = \vec{0}$.

取无穷远处电势为0.

$r\geq b$: $U=0$.

$a<r<b$: $U = -\int_b^r E\cdot dr = -\frac{\lambda_e}{2\pi\varepsilon_0}\ln r\Big|_b^r = \frac{\lambda_e}{2\pi\varepsilon_0}\ln\frac{b}{r}.$

$r\leq a$: $U = -\int_b^a E\cdot dr = \frac{\lambda_e}{2\pi\varepsilon_0}\ln\frac{b}{a}.$

(2). $U_{ab} = \frac{\lambda_e}{2\pi\varepsilon_0}\ln\frac{b}{a}.$