Homework 4 GPI

1. KK 3.3

   3.3 Suppose that a system consists of several bodies, and that the position of the center of mass of each body is known. Prove that the center of mass of the system can be found by treating each body as a particle concentrated at its center of mass.

2. KK 3.4 （4.4）
3. KK 3.5 (4.5)
4. KK 3.7 (4.7)
5. KK 3.11 (4.12)
6. KK 3.14 (4.15)
7. KK 3.15 (4.16)
8. KK 3.18 (4.24)
9. KK 3.19 (4.25)
10. KK 3.20

    3.20 A rocket ascends from rest in a uniform gravitational field by ejecting exhaust with constant speed u. Assume that the rate at which mass is expelled is given by $dm/dt = \gamma m$, where m is the instantaneous mass of the rocket and γ is a constant, and that the rocket is retarded by air resistance with a force mbv, where b is a constant. Find the velocity of the rocket as a function of time.

    Ans. clue. The terminal velocity is $(\gamma u - g)/b$.

11. A rain drop initially with very small size and starts to fall under gravity. As it falls, the vapor will condensate on the droplet and the droplet will increase in size (and mass). Suppose the change of the volume of the droplet (always keeps the spherical shape) is proportional to the volume sweeps by it, i.e. $dV/dt = k\pi r^2 v$, where V is the spherical volume, r the radius and v the velocity of the droplet and k a constant. Please calculate the acceleration of the droplet during this process, if you work out the problem completely you will find the acceleration is surprisingly a constant (g/7), independent of k.
    (Hint: it is probably not too hard to write out the relation using impulse theorem. I hope you can complete this at least (both mass and v can change here). The trick of the problem is the spherical shape will also give you a relation on the variation of mass vs. time. The calculation may involve solving differential equations of which the answer can be guessed to take some function form. Of course there are possibly other methods to find the answer)

12. Balloon Rocket: In this problem I designed a simpler model to explain when you puncture a hole on a filled balloon, it will shoot out as a rocket. Consider a balloon as a closed box:

There are number N molecules inside volume V so that number density is: $\rho = N/V$; I simplify the molecular velocity (or momentum) distribution of the system: let's assume that the molecule only moves along x direction, with a constant averaged velocity v (statistically such v depend on

temperature T, since the balloon is emerged in atmosphere, we can assume constant T thus constant v), and it has equal chance to move along +x and -x direction. The collision with the wall (here only considering the left and right wall of the box) is elastic so that the after each collision molecule will have same magnitude v but reversed in direction. Such collision would also not change the equal probability for molecule moving along +x and -x. Let p be the magnitude of the momentum of each molecule. In short, we can treat the molecules inside the box has equal number moving along x with +v and -v or +p,-p in momentum.

1) Write the expression for the pressure P (force per area) exerted on the left and right walls by the molecule, express in terms of ρ, v and p.

2) Now suppose I open a small hole on the right wall of the box (puncture a hole on a balloon) with area A; gas will leak out the box with velocity +v (the leaked out gas has no collision with atmosphere molecule in short time interval). The box (balloon) will shrink and to make life simple, let's assume it shrinks but keeping the number density ρ constant. What is the force F felt by the box now? Express F in terms of pressure P and area A