

## 第 1 次作业题

1. 求下列集合  $\Omega$  的内部, 外部, 边界, 闭包.

(1)  $\Omega = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ ,

(2)  $\Omega = \{(x, y, z) \in \mathbb{R}^3 \mid 1 \leq x^2 + y^2 + z^2 < 4\}$ .

2. 若  $P_1, P_2, \dots, P_k \in \mathbb{R}^n$ , 求证:  $\{P_1, P_2, \dots, P_k\}$  为闭集.

3. 当  $(x, y) \rightarrow (0, 0)$  时, 下列函数的极限是否存在? 若存在, 求出该极限.

(1)  $(x^2 + y^2)e^{-x-y}$ , (2)  $\frac{x+y}{|x|+|y|}$ , (3)  $\frac{x^4 y^4}{(x^2 + y^4)^3}$ , (4)  $\frac{\sin(x^2 y) - \arcsin(x^2 y)}{x^6 y^3}$ .

4. 求下列函数极限

(1)  $\lim_{\substack{x \rightarrow 3 \\ y \rightarrow 0}} \frac{\ln(x + \sin y)}{\sqrt{x^2 + y^2}}$ , (2)  $\lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} \frac{x+y}{x^2 + xy + y^2}$ ,

(3)  $\lim_{\substack{x \rightarrow +\infty \\ y \rightarrow -\infty}} (x^2 + y^2)e^{y-x}$ , (4)  $\lim_{\substack{x \rightarrow \infty \\ y \rightarrow \infty}} \left(\frac{|xy|}{x^2 + y^2}\right)^{x^2}$ .

5. 讨论下列累次极限与二重极限是否存在, 若存在, 求其值:

$$\lim_{x \rightarrow +\infty} \lim_{y \rightarrow 0^+} \frac{x^y}{1 + x^y}, \quad \lim_{y \rightarrow 0^+} \lim_{x \rightarrow +\infty} \frac{x^y}{1 + x^y}, \quad \lim_{\substack{x \rightarrow +\infty \\ y \rightarrow 0^+}} \frac{x^y}{1 + x^y}.$$

6. 判断下列函数在原点  $(0, 0)$  的连续性

(1)  $f(x, y) = \begin{cases} \frac{\sin(x^3 + y^3)}{x^2 + y^2}, & x^2 + y^2 \neq 0, \\ 0, & x^2 + y^2 = 0. \end{cases}$

(2)  $f(x, y) = \begin{cases} \frac{xy^2}{x^2 + y^4}, & x^2 + y^2 \neq 0, \\ 0, & x^2 + y^2 = 0. \end{cases}$

7. 当  $(x, y) \rightarrow (0, 0)$  时, 讨论下列无穷小量的阶 (如果有阶, 则计算出该阶; 若无阶, 则需说明理由):

(1)  $\ln(1 + \sqrt{x^2 + y^2})$ , (2)  $(x^2 + y^2) \sin \frac{1}{\sqrt{x^2 + y^2}}$ .

8. 求下列函数的偏导数:

(1)  $z = \ln(x + \sqrt{x^2 - y^2})$ , (2)  $z = \cos(1 + 2^{xy})$ .

9. 考察下列函数在坐标原点的可微性:

(1)  $f(x, y) = \sqrt{|x|} \cos y$ , (2)  $f(x, y) = \begin{cases} \frac{2xy}{\sqrt{x^2 + y^2}}, & x^2 + y^2 \neq 0 \\ 0, & x^2 + y^2 = 0 \end{cases}$ ,

(3)  $f(x, y) = \begin{cases} \frac{x^2 y^2}{(x^2 + y^2)^{\frac{3}{2}}}, & x^2 + y^2 \neq 0 \\ 0, & x^2 + y^2 = 0 \end{cases}$ ,

(4)  $f(x, y) = |x - y|\varphi(x, y)$ , 其中  $\varphi$  在原点的某邻域内连续且  $\varphi(0, 0) = 0$ .

10. 求下列函数的全微分:

$$(1) u = \sqrt{1 + x^2 + y^2 + z^2}, \quad (2) z = \frac{x-y}{x+y}.$$