补4:

$$A_1=A_0+\left(\frac{\partial A}{\partial x}\right)_0 h_1+\frac{1}{2!}\left(\frac{\partial^2 A}{\partial x^2}\right)_0 h_1^2+\frac{1}{3!}\left(\frac{\partial^3 A}{\partial x^3}\right)_0 h_1^3+\cdots \quad ①$$

$$A_3=A_0-\left(\frac{\partial A}{\partial x}\right)_0 h_3+\frac{1}{2!}\left(\frac{\partial^2 A}{\partial x^2}\right)_0 h_3^2-\frac{1}{3!}\left(\frac{\partial^3 A}{\partial x^3}\right)_0 h_3^3+\cdots \quad ②$$

$$A_4=A_0+\left(\frac{\partial A}{\partial y}\right)_0 h_2+\frac{1}{2!}\left(\frac{\partial^2 A}{\partial y^2}\right)_0 h_2^2+\frac{1}{3!}\left(\frac{\partial^3 A}{\partial y^3}\right)_0 h_2^3+\cdots \quad ③$$

$$A_4=A_0-\left(\frac{\partial A}{\partial y}\right)_0 h_4+\frac{1}{2!}\left(\frac{\partial^2 A}{\partial y^2}\right) h_4^2-\frac{1}{3!}\left(\frac{\partial^3 A}{\partial y^3}\right)_0 h_4^3+\cdots \quad ④$$

略去h^4高阶项，由①②式联立可得:

$$\left(\frac{\partial^2 A}{\partial x^2}\right)_0=\frac{2A_1}{h_1^2+h_1h_3}+\frac{2A_3}{h_1h_3+h_3^2}-\frac{2A_0}{h_1h_3}$$

同理，由③、④式联立可得:

$$\left(\frac{\partial^2 A}{\partial y^2}\right)_0=\frac{2A_2}{h_2^2+h_2h_4}+\frac{2A_4}{h_2h_4+h_4^2}-\frac{2A_0}{h_2h_4}$$

由泊松方程: $\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}=-\mu J$.

则 $$\frac{A_1}{h_1^2+h_1h_3}+\frac{A_2}{h_2^2+h_2h_4}+\frac{A_3}{h_1h_3+h_3^2}+\frac{A_4}{h_2h_4+h_4^2}-\left(\frac{1}{h_1h_3}+\frac{1}{h_2h_4}\right)A_0=-\frac{1}{2}\mu J.$$

由于 $\alpha=\frac{h_3}{h}$, $\beta=\frac{h_2}{h}$, $h_1=h_4=h$. 则上式可化为:

$$\frac{A_1}{1+\alpha}+\frac{A_2}{\beta^2+\beta}+\frac{A_3}{\alpha+\alpha^2}+\frac{A_4}{\beta+1}-\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)A_0=-\frac{1}{2}h^2\mu J.$$

补5:

将媒质a向外延拓: $\varphi_{a1}+\varphi_{a2}+\varphi_{a3}+\varphi_{a4}-4\varphi_0=-h^2\frac{\rho_a}{\varepsilon_a}$.

将媒质b向外延拓: $\varphi_{b1}+\varphi_{b2}+\varphi_{b3}+\varphi_{b4}-4\varphi_0=0$.

$\varphi_{aM}=\frac{\varphi_{a1}+\varphi_{a4}}{2}$, $\varphi_{bN}=\frac{\varphi_{b2}+\varphi_{b3}}{3}$. 虚设.

$\varphi_{aN}=\frac{\varphi_{a2}+\varphi_{a3}}{2}$, $\varphi_{bM}=\frac{\varphi_{b1}+\varphi_{b4}}{2}$. 实际.

在交界面，由于 $\sigma_s=0$，则:

$$\varepsilon_a\frac{\varphi_{aM}-\varphi_{aN}}{\sqrt{2}h}=\varepsilon_b\frac{\varphi_{bM}-\varphi_{bN}}{\sqrt{2}h}$$

$$\varphi_0=\frac{1}{4}\left(\frac{2\varepsilon_b}{\varepsilon_b+\varepsilon_a}(\varphi_{b1}+\varphi_{b4})+\frac{2\varepsilon_a}{\varepsilon_b+\varepsilon_a}(\varphi_{a2}+\varphi_{a3})+\frac{h^2\rho_a}{\varepsilon_a+\varepsilon_b}\right)$$

令 $K=\frac{\varepsilon_a}{\varepsilon_b}$.

$$\Rightarrow \varphi_0=\frac{1}{4}\left(\frac{2}{1+K}(\varphi_{b1}+\varphi_{b4})+\frac{2K}{1+K}(\varphi_{a2}+\varphi_{a3})+\frac{K}{1+K}h^2\frac{\rho_a}{\varepsilon_a}\right)$$