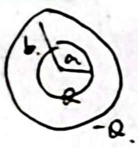
1.6. (a).

$Q = A\sigma.$

$E = \frac{\sigma}{\varepsilon_0} \quad U = Ed = \frac{\sigma d}{\varepsilon_0}.$

$C = \frac{Q}{U} = \frac{A\sigma\varepsilon_0}{\sigma d} = \frac{A\varepsilon_0}{d}.$

(b).



$E = \frac{Q}{4\pi r^2\varepsilon_0}. \quad U = \int_a^b E\cdot dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) = \frac{Q(b-a)}{4\pi\varepsilon_0 ab}.$

$C = \frac{Q}{U} = \frac{4\pi\varepsilon_0 ab}{b-a}.$

(c).

$E = \frac{Q}{2\pi r L\varepsilon_0}. \quad U = \int_a^b E\cdot dr = \frac{Q}{2\pi L\varepsilon_0}\ln\frac{b}{a}.$

$C = \frac{2\pi L\varepsilon_0}{\ln\frac{b}{a}}.$

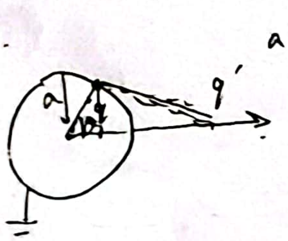
(d).
air : $\varepsilon = 1.$

$C_l = \frac{2\pi\varepsilon_0}{\ln\frac{b}{a}} = 3\times10^{-11}\ F/m.$

$b = a\cdot e^{\frac{3\times10^{-11}}{2\pi}} \approx 1mm.$

2.2.

a.



$\phi = \frac{q}{4\pi\varepsilon_0\sqrt{(x-x_1)^2+y^2+z^2}} + \frac{q'}{4\pi\varepsilon_0\sqrt{(x-x_1')^2+y^2+z^2}}. \quad x_1' = \frac{a^2}{x_1}.\ q' = -\frac{a}{x_1}q$

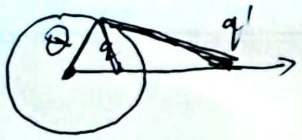
$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{\sqrt{(x-x_1)^2+y^2+z^2}} - \frac{a}{x_1}\cdot\frac{1}{\sqrt{(x-\frac{a^2}{x_1})^2+y^2+z^2}}\right)$

b. $\sigma = -\varepsilon_0\frac{\partial\phi}{\partial n}. \quad \phi\big|_{r=a} = V.$

$= \frac{q}{4\pi}\cdot\frac{a}{x_1}\cdot\frac{a^2 - \frac{a^4}{x_1^2}}{\left(a^2 + \frac{a^4}{x_1^2} - 2\frac{a^3}{x_1}\cos\theta\right)^{3/2}}$

(c). $F = \frac{\frac{a}{x_1}q^2}{4\pi\varepsilon_0(x_1'-x)^2} = \frac{\frac{a}{x_1}q^2}{4\pi\varepsilon_0(\frac{a^2}{x_1}-x_1)^2}$. 方向指向镜像电荷

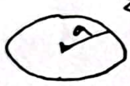
(d).



2.

(a) $G(\vec{x},\vec{x}') = \frac{1}{|\vec{x}-\vec{y}'|} + \frac{1}{|\vec{x}-\vec{x}'|} = \frac{1}{\sqrt{\rho^2+\rho'^2+(z+z')^2-2\rho\rho'\cos(\varphi-\varphi')}} + \frac{1}{\sqrt{\rho^2+\rho'^2+(z-z')^2-2\rho\rho'\cos(\varphi-\varphi')}}$

(b)



$z'=0$ 时 $\frac{\partial G}{\partial n'} = \frac{-2z}{(\rho^2+\rho'^2+z^2-2\rho\rho'\cos(\varphi-\varphi'))^{3/2}}$

$$\Phi = -\frac{1}{4\pi}\oint_s \Phi(\vec{x}')\frac{\partial G}{\partial n'}da'$$

$$= -\frac{V}{4\pi}\oint_s \frac{\partial G}{\partial n'}\cdot\rho' d\rho' d\varphi = \frac{V}{2\pi}\oint_s \frac{z\rho' d\rho' d\varphi}{(\rho^2+\rho'^2+z^2-2\rho\rho'\cos(\varphi-\varphi'))^{3/2}}$$

(c) $\rho=0$.

$$\Phi = \frac{V}{4\pi}\iint \frac{2z}{(\rho'^2+z^2)^{3/2}}\rho' d\rho' d\varphi = \frac{V}{4\pi}\cdot 2\pi\cdot\int_0^a \frac{z}{(\rho'^2+z^2)^{3/2}}d(\rho'^2+z^2)$$

$$= -Vz\frac{1}{\sqrt{\rho'^2+z^2}}\Big|_0^a = V\left(1-\frac{1}{\sqrt{a^2+z^2}}\right).$$

(d). $\rho^2+z^2 \gg a^2$

$$\Phi = \frac{V}{4\pi}\oint_s \frac{2z\rho' d\rho' d\varphi}{(\rho^2+z^2)^{3/2}}\cdot\frac{1}{\left(1+\frac{\rho'^2-2\rho\rho'\cos(\varphi-\varphi')}{\rho^2+z^2}\right)^{3/2}}$$

$$= \frac{Vz\cdot 2\pi}{4\pi(\rho^2+z^2)^{3/2}}\int_0^a \rho' d\rho'\left(1-\frac{3}{2}\frac{\rho'^2-2\rho\rho'\cos(\varphi-\varphi')}{\rho^2+z^2}+\frac{15}{8}(\cdots)^2+\cdots\right)$$

$$= \frac{Vz}{2(\rho^2+z^2)^{3/2}}\left(a^2-\frac{3a^4}{4(z^2+\rho^2)}+\frac{5(a^4+3\rho^2a^6)}{8(\rho^2+z^2)^2}-\cdots\right)$$

当$\rho=0$时. $\Phi = \frac{V}{2z^2}\left(a^2-\frac{3a^4}{4z^2}+\frac{5a^6}{8z^4}\cdots\right)$

$$= V\left(\frac{a^2}{2z^2}-\frac{3a^4}{8z^4}+\frac{5a^6}{16z^6}\cdots\right) = V\left(1-\frac{1}{\sqrt{a^2+z^2}}\right)$$