$R(\rho)=a_0+b_0\ln\rho$

$\Phi(\phi)=A_0+B_0\phi$ $\Big\}\ \nu=0$

$R(\rho)=a\rho^{\nu}+b\rho^{-\nu}$

$\Phi(\phi)=A\cos(\nu\phi)+B\sin(\nu\phi)$ $\Big\}\ \nu=1,2,\dots$

$\begin{cases}\phi=0\ \&\ \rho\geq a,\ \Phi=0\\ \phi=\beta\ \&\ \rho\geq a:\ \Phi=0.\end{cases}$

$\rightarrow \begin{cases}A_0(a_0+b_0\ln\rho)=0\\ (A_0+B_0\phi)(a_0+b_0\ln\rho)=0\end{cases}$ $\begin{cases}A(a\rho^{\nu}+b\rho^{-\nu})=0\\ (A\cos(\beta\nu)+B\sin(\beta\nu))(a\rho^{\nu}+b\rho^{-\nu})=0\end{cases}$

$\rho\to\infty$时，$\Phi\to C.$

$\begin{cases}b_0=0\\ B_0=0.\end{cases}$ $\begin{cases}A=0.\\ B\sin(\beta\nu)(a\rho^{\nu}+b\rho^{-\nu}).\ \nu=\frac{m\pi}{\beta}.\end{cases}$

$$\Phi=a_0A_0+\sum_{m=1}^{\infty}b_m\rho^{-\frac{m\pi}{\beta}}\sin\left(\frac{m\pi}{\beta}\phi\right)+\sum_{m=1}^{\infty}a_m\rho^{\frac{m\pi}{\beta}}\sin\left(\frac{m\pi}{\beta}\phi\right).$$

$$=\sum_{m=1}^{\infty}C_m\sin\left(\frac{m\pi}{\beta}\phi\right).\left(a\rho^{\frac{m\pi}{\beta}}+b\rho^{-\frac{m\pi}{\beta}}\right).$$

(b) $\Phi=C_1\sin\left(\frac{\pi}{\beta}\phi\right)\left(a\rho^{\frac{\pi}{\beta}}+b\rho^{-\frac{\pi}{\beta}}\right)$

$$E_\rho=-\frac{\partial\Phi}{\partial\rho}=C_1\sin\left(\frac{\pi}{\beta}\phi\right)\left(\frac{\pi}{\beta}a\rho^{\frac{\pi}{\beta}-1}-b\frac{\pi}{\beta}\rho^{-\frac{\pi}{\beta}-1}\right)$$

$$E_\phi=-\frac{1}{\rho}\frac{\partial\Phi}{\partial\phi}=-\frac{C_1\pi}{\beta}\left(a\rho^{\frac{\pi}{\beta}-1}+b\rho^{-\frac{\pi}{\beta}-1}\right)\cos\left(\frac{\pi}{\beta}\phi\right)$$

$$\sigma_\rho=\varepsilon_0E_\phi(\rho,0)=-\frac{\varepsilon_0C_1\pi}{\beta}\left(a\rho^{\frac{\pi}{\beta}-1}+b\rho^{-\frac{\pi}{\beta}-1}\right)$$

$$\sigma(\rho,\beta)=\frac{\varepsilon_0C_1\pi}{\beta}\left(a\rho^{\frac{\pi}{\beta}-1}+b\rho^{-\frac{\pi}{\beta}-1}\right)$$

$$\sigma(a,\phi)=\varepsilon_0E_\rho=\frac{\varepsilon_0\pi C_1}{\beta}\sin\left(\frac{\pi}{\beta}\phi\right)\left(a^{\frac{\pi}{\beta}}-b\cdot a^{-\frac{\pi}{\beta}-1}\right)$$

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(c). $\beta=\pi$

$$\sigma(\rho,0)=-\varepsilon_0 C_1(1+b\rho^{-2})=-\varepsilon_0 C_1(\cancel{1}-a^2\rho^{-2})$$

$$\sigma(\rho,\pi)=\varepsilon_0 C_1(1-a^2\rho^{-2})$$

$$\sigma(a,\varphi)=2\varepsilon_0 C_1 a\sin\varphi$$

$$\begin{cases} q(a,\varphi)=\int_0^{\pi}\sigma\,d\varphi=-4\varepsilon_0 C_1 a \\ q_{平板}=2a(\varepsilon_0 C_1)=2a\varepsilon_0 C_1 \end{cases} \Rightarrow q(a,\varphi)=2q_{平板}$$

$$\begin{cases} -2a\varepsilon_0 C_1+\int_a^L\sigma(\rho,0)\,d\rho+\int_a^L\sigma(\beta,\beta)\,d\rho=-2\varepsilon_0 C_1 L \\ 2L(-\varepsilon_0 C_1)=2L\varepsilon_0 C_1 \end{cases}$$

$\Rightarrow$ 二者相等.

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2.29

1. $\int dx \int dy\, \varphi_{i,j}(x,y) = \left(2\int_0^h (1-\frac{y}{h})dy\right)^2 = h^2.$

2. $\int dx \int dy\, \nabla\varphi_{i,j} \cdot \nabla\varphi_{i,j} = \frac{4}{h^2}\int_0^h dx \int_0^h dy \left(2+\frac{x^2}{h^2}+\frac{y^2}{h^2}-2\frac{x}{h}-2\frac{y}{h}\right)$

$= \frac{4}{h^2}\int_0^h dx\left(2h+\frac{x^2}{h}+\frac{h}{3}-2x-h\right) = \frac{4}{h^2}\left(2h^2+\frac{h^2}{3}+\frac{h^2}{3}-h^2-h^2\right) = \frac{8}{3}.$

3. $\int dx \int dy\, \nabla\varphi_{i+1,j} \nabla\varphi_{i,j}$ $\cancel{= \frac{1}{h^2}\int}$ $\nabla\varphi_{i+1,j} = \nabla\left(\frac{x}{h}\right)(1-|y|/h)\quad 0<x<h$

$= 2\int_0^h dx \int_0^h dy \cdot \frac{1}{h^2}\left(\frac{x}{h}(1-\frac{x}{h}) - (1-\frac{y}{h})^2\right)$

$= \frac{2}{h^2}\int_0^h dx \left(x-\frac{x^2}{h}-1+2\frac{y}{h}-\frac{y^2}{h^2}\right) = \frac{2}{h^2}\left(\frac{h^2}{6}-\frac{h^2}{3}\right) = \frac{-1}{3}.$

4. 同理于上.

$\int dx \int dy\, \nabla\varphi_{i,j+1} \nabla\varphi_{i,j} = 2\int_0^h dy \int_0^h dx\, \frac{1}{h^2}\left(\frac{y}{h}(1-\frac{y}{h}) - (1-\frac{x}{h})^2\right)$

$= -\frac{1}{3}.$

5. $\int dx \int dy\, \nabla\varphi_{i+1,j+1} \nabla\varphi_{i,j} = \int_0^h dx \int_0^h dy\, \frac{1}{h^2}\left(\frac{-x}{h}(1-\frac{x}{h}) + \frac{-y}{h}(1-\frac{y}{h})\right)$

$\cancel{= \frac{1}{h^2}\left(\frac{h^2}{2}-\frac{h}{3}\right)} = \frac{1}{h^2}\left(\frac{-h^2}{2}+\frac{h^2}{3}-\frac{h^2}{2}+\frac{h^2}{3}\right) = -\frac{1}{3}.$

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