4.8

(a). $\phi = a_0 + b_0 \ln\rho + \sum_{n=1}^{\infty}\left[a_n\rho^n \sin(n\phi+\alpha_n) + b_n\rho^{-n}\sin(n\phi+\beta_n)\right]$

$r>a$. $\rho\to 0$ 时 ϕ 有限. $b_n=b_0=0$. 设 $\rho=0$ 时 $\phi=0$.

$\phi = a_0 + \sum_{n=1}^{\infty} a_n\rho^n \sin(n\phi+\alpha_n)$.

对称性: $\phi(\phi)=\phi(-\phi)$. $\to \sin(n\phi+\alpha_n)=\sin(-n\phi+\alpha_n)$. $\alpha_n=\frac{\pi}{2}$.

$\phi_1 = \sum_{n=1}^{\infty} a_{1n}\rho^n \cos n\phi$.

$a<r<b$. 对称性: $\phi(\phi)=\phi(-\phi)$. $\to \alpha_n=\beta_n=\frac{\pi}{2}$　设 $\rho=0$ 时 $\phi=0$.

$\phi_2 = b_{20}\ln\rho + \sum_{n=1}^{\infty} a_{2n}\rho^n \cos n\phi + b_{2n}\rho^{-n}\cos n\phi$

$r>b$. $\rho\to\infty$ 时 ϕ 有限. $a_n=0$. $b_0=0$. 设 $\rho=0$ 时 $\phi=0$.

对称性: $\alpha_n+\beta_n=\frac{\pi}{2}$.

$\phi_3 = \sum_{n=1}^{\infty} b_{3n}\rho^{-n}\cos n\phi - E_0\rho\cos\phi$.

边界条件 $-\varepsilon_0\frac{\partial\phi_1}{\partial\rho}\Big|_{\rho=a} = -\varepsilon\frac{\partial\phi_2}{\partial\rho}\Big|_{\rho=a}$.　$\varepsilon_0\sum_{n=1}^{\infty} n a_{1n} a^{n-1}\cos n\phi = \frac{\varepsilon b_{20}}{a} + \varepsilon\sum_{n=1}^{\infty} n a_{2n} a^{n-1}\cos n\phi - \varepsilon\sum_{n=1}^{\infty} n b_{2n} a^{-n-1}\cos n\phi$.

$\varepsilon_0 a_{11} = \varepsilon a_{21} - \varepsilon b_{21} a^{-2}$.

$-\varepsilon\frac{\partial\phi_2}{\partial\rho}\Big|_{\rho=b} = -\varepsilon_0\frac{\partial\phi_3}{\partial\rho}\Big|_{\rho=b}$.　$\frac{\varepsilon b_{20}}{b} + \varepsilon\sum_{n=1}^{\infty} n a_{2n} b^{n-1}\cos n\phi - \varepsilon\sum_{n=1}^{\infty} n b_{2n} b^{-n-1}\cos n\phi$

$= -\varepsilon_0\sum_{n=1}^{\infty} n\, b_{3n} b^{-n-1}\cos n\phi - \varepsilon_0 E_0\cos\phi$.

$\varepsilon a_{21} - \varepsilon b_{21} b^{-2} = -\varepsilon_0 b_{31} b^{-2}$

$-\frac{1}{a}\frac{\partial\phi_1}{\partial\phi}\Big|_{\rho=a} = -\frac{1}{a}\frac{\partial\phi_2}{\partial\phi}\Big|_{\rho=a}$.　$\sum_{n=1}^{\infty} n a_{1n} a^{n-1}\sin n\phi = \sum_{n=1}^{\infty} n a_{2n} a^{n-1}\sin n\phi + n b_{2n} a^{-n-1}\sin n\phi$.

$a_{1n} = a_{2n} - \frac{b_{20}}{a^{2n}}$　$a_{11} = a_{21} + \frac{b_{21}}{a^2}$

$-\frac{1}{b}\frac{\partial\phi_2}{\partial\phi}\Big|_{\rho=b} = -\frac{1}{b}\frac{\partial\phi_3}{\partial\phi}\Big|_{\rho=b}$　$\sum_{n=1}^{\infty} n a_{2n} b^{n-1}\sin n\phi + n b_{2n} b^{-n-1}\sin n\phi = \sum_{n=1}^{\infty} n b_{3n} b^{-n-1}\sin n\phi - E_0\sin\phi$.

$a_{21} b^{2} + b_{21} = b_{31} - E_0 b^{2}$.

$$\begin{cases} \varepsilon_0 a_{11} = \varepsilon a_{21} - \dfrac{\varepsilon b_{21}}{a^2} \\ b^2 \varepsilon a_{21} - \varepsilon b_{21} = -\varepsilon_0 b_{31} \\ a_{11} = a_{21} + \dfrac{b_{21}}{a^2} \\ a_{21} b^2 + b_{21} = b_{31} - E_0 b^2 \end{cases} \Rightarrow$$

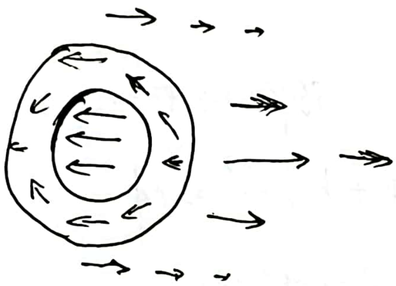
$$b_{21} = \frac{\varepsilon_0(\varepsilon-\varepsilon_0)E_0 b^2 a^2}{(\varepsilon-\varepsilon_0)^2 a^2 - (\varepsilon_0+\varepsilon)^2 b^2}.$$

$$a_{21} = \frac{\varepsilon_0(\varepsilon+\varepsilon_0)E_0 b^2}{(\varepsilon-\varepsilon_0)^2 a^2 - (\varepsilon_0+\varepsilon)^2 b^2}.$$

$$a_{11} = \frac{2\varepsilon^2 E_0 b^2}{(\varepsilon-\varepsilon_0)^2 a^2 - (\varepsilon_0+\varepsilon)^2 b^2}$$

$$b_{31} = \frac{\varepsilon_0(\varepsilon-\varepsilon_0)E_0 b^2 a^2 - \varepsilon(\varepsilon+\varepsilon_0)E_0 b^4}{(\varepsilon-\varepsilon_0)^2 a^2 - (\varepsilon_0+\varepsilon)^2 b^2}.$$

(b).



(c). $a \to 0$

$b_{21} \to 0$

$a_{21} \to -\dfrac{E_0 \varepsilon_0}{(\varepsilon_0+\varepsilon)}$

$b_{31} \to \dfrac{E_0 b^2 \varepsilon}{\varepsilon_0+\varepsilon}.$

$b \to \infty.$

$b_{21} \to -E_0 \dfrac{a^2 \varepsilon_0(\varepsilon-\varepsilon_0)}{(\varepsilon_0+\varepsilon)^2}.$

$a_{21} \to \dfrac{\varepsilon_0 E_0}{\varepsilon_0+\varepsilon}.$

$a_{11} \to \dfrac{2\varepsilon^2 E_0}{(\varepsilon_0+\varepsilon)^2}.$

4.10

(a) $2\pi r^2 \varepsilon E + 2\pi r^2 \varepsilon_0 E = Q \to E = \dfrac{Q}{2\pi r^2(\varepsilon+\varepsilon_0)}.$

(b). $\sigma = \hat{n}\cdot\vec{D}$ 左: $\dfrac{\varepsilon_0 Q}{2\pi a^2(\varepsilon+\varepsilon_0)}.$ 右. $\dfrac{\varepsilon Q}{2\pi a^2(\varepsilon+\varepsilon_0)}$

(c). $\vec{P} = (\varepsilon-\varepsilon_0)\vec{E}.$

左. : 0.   右: $\dfrac{(\varepsilon-\varepsilon_0)Q}{2\pi a^2(\varepsilon+\varepsilon_0)}$