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（科目：　　）清华大学数学作业纸　4120238

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3.13.

$$G(\vec{x},\vec{x}') = 4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{Y^*_{lm}(\theta',\varphi')Y_{lm}(\theta,\varphi)}{(2l+1)\left(1-\left(\frac{a}{b}\right)^{2l+1}\right)}\left(r_<^l-\frac{a^{2l+1}}{r_<^{l+1}}\right)\left(\frac{1}{r_>^{l+1}}-\frac{r_>^l}{b^{2l+1}}\right)$$

$$= 4\pi\sum_{l=0}^{\infty}\frac{\sqrt{\frac{2l+1}{4\pi}}P_l(\cos\theta')\sqrt{\frac{2l+1}{4\pi}}P_l(\cos\theta)}{(2l+1)\left(1-\left(\frac{a}{b}\right)^{2l+1}\right)}\left(r_<^l-\frac{a^{2l+1}}{r_<^{l+1}}\right)\left(\frac{1}{r_>^{l+1}}-\frac{r_>^l}{b^{2l+1}}\right)$$

$$= \sum_{l=0}^{\infty}\frac{P_l(\cos\theta')P_l(\cos\theta)}{1-\left(\frac{a}{b}\right)^{2l+1}}\left(r_<^l-\frac{a^{2l+1}}{r_<^{l+1}}\right)\left(\frac{1}{r_>^{l+1}}-\frac{r_>^l}{b^{2l+1}}\right)$$

$$\left.\frac{\partial G}{\partial r'}\right|_{r'=b} = -\sum_{l=0}^{\infty}\frac{(2l+1)P_l(\cos\theta')P_l(\cos\theta)}{b^{l+2}\left(1-\left(\frac{a}{b}\right)^{2l+1}\right)}\left(r^l-\frac{a^{2l+1}}{r^{l+1}}\right)\quad (r_>=r';\ r_<=r)$$

$$\left.\frac{\partial G}{\partial r'}\right|_{r'=a} = \sum_{l=0}^{\infty}\frac{(2l+1)P_l(\cos\theta')P_l(\cos\theta)}{a^{-l+1}\left(1-\left(\frac{a}{b}\right)^{2l+1}\right)}\left(\frac{1}{r^{l+1}}-\frac{r^l}{b^{2l+1}}\right)\quad (r_>=r,\ r_<=r')$$

$$\phi(r,\theta) = -\frac{1}{4\pi}\left(\int_0^{2\pi}\int_0^{\frac{\pi}{2}}V\left.\frac{\partial G}{\partial r'}\right|_{r'=b}b^2\sin\theta'\,d\theta'\,d\varphi' - \int_0^{2\pi}\int_0^{\frac{\pi}{2}}V\left.\frac{\partial G}{\partial r'}\right|_{r'=a}a^2\sin\theta'\,d\theta'\,d\varphi'\right)$$

$$= \frac{Va(b-r)}{2\left(1-\frac{a}{b}\right)rb} + \frac{3P_1(\cos\theta)Va^2(b^3-r^3)}{4\left(1-\left(\frac{a}{b}\right)^3\right)r^3b^3} + \sum_{n=1}^{\infty}\frac{(-1)^nVa^{2n+2}(4n+3)(b^{4n+3}-r^{4n+3})}{2\left(1-\left(\frac{a}{b}\right)^{4n+3}\right)r^{4n+3}b^{4n+3}}P_{2n+1}(\cos\theta)\frac{(2n-1)!!}{(2n+2)!!}$$

$$+\frac{V(r-a)}{2\left(1-\frac{a}{b}\right)r} + \frac{3P_1(\cos\theta)V(r^3-a^3)}{4\left(1-\left(\frac{a}{b}\right)^3\right)r^2b} + \sum_{n=1}^{\infty}\frac{(-1)^{n+1}(4n+3)V(r^{4n+3}-a^{4n+3})}{2\left(1-\left(\frac{a}{b}\right)^{4n+3}\right)r^{2n+2}b^{2n-1}}P_{2n+1}(\cos\theta)\frac{(2n-1)!!}{(2n+2)!!}$$

$$= \frac{V}{2} + \frac{3P_1(\cos\theta)V(r^2a^2b^3+a^3b^2r^2-a^2r+b^2r)}{4(b^3-a^3)} + \sum_{n=1}^{\infty}\frac{(-1)^nV(4n+3)(a^{2n+2}b^{4n+3}-a^{2n+2}r^{4n+3}-b^{2n+2}r^{6n+6}-a^{4n+3}b^{2n+1}r^{\dots})}{2\left(1-\left(\frac{a}{b}\right)^{4n+3}\right)r^{4n+3}b^{4n+3}}\cdot P_{2n+1}(\cos\theta)\frac{(2n-1)!!}{(2n+2)!!}$$

△

4.1

(a) $q_{lm} = \int Y_{lm}^* r'^l \rho(x')\, d^3x' = a^l q\left[Y_{lm}^*(\frac{\pi}{2},\frac{\pi}{2}) - Y_{lm}^*(\frac{\pi}{2},\pi) + Y_{lm}^*(\frac{\pi}{2},0) - Y_{lm}^*(\frac{\pi}{2},\frac{3\pi}{2})\right]$

$q_{1,1} = -q_{1,-1}^* = -aq(1-i)\sqrt{\frac{3}{2\pi}}$.

$q_{3,3} = -\frac{1}{4}a^3 q(1+i)\sqrt{\frac{27}{\pi}}$

$q_{3,1} = \frac{1}{4}a^3 q(1-i)\sqrt{\frac{21}{\pi}}$.

(b) $q_{lm} = a^l q\left[Y_{lm}^*(0,\varphi) + Y_{lm}^*(\pi,\varphi)\right]$

$q_{2,0} = a^2 q\sqrt{\frac{5}{\pi}}$

$q_{4,0} = a^4 q\sqrt{\frac{9}{\pi}}$

(c) $\Phi(\vec r) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l}\frac{1}{4\pi\epsilon_0}\frac{4\pi}{2l+1} q_{lm}\frac{Y_{lm}(\theta,\varphi)}{r^{l+1}} = \frac{1}{\epsilon_0(2l+1)}\sum_{l=1}^{\infty} q_{l0}\frac{Y_{l0}(\theta,\varphi)}{r^{l+1}}$

$= \frac{q}{2\pi\epsilon_0}\sum_{l=1}^{\infty}\frac{a^{2l}P_{2l}(\cos\theta)}{r^{2l+1}}$

第一项：$\frac{q}{2\pi\epsilon_0}\cdot\frac{a^2 P_{2\cdot1}(\cos\theta)}{r^3} = \frac{qa^2}{2\pi\epsilon_0}\cdot\frac{3\cos^2\theta-1}{2r^3}$

x-y 平面：$\Phi(r) = \frac{qa^2}{4\pi\epsilon_0 r^3}$.

(d). $\Phi_0(r) = \frac{2q}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{\sqrt{r^2+a^2}}\right)$.

$\Phi_0(r) - \Phi(r) = \frac{qa^2}{4\pi\epsilon_0 r}\left(\frac{2}{a^2} - \frac{2}{a^2\sqrt{1+\frac{a^2}{r^2}}} - \frac{1}{r^2}\right)$

$r\to\infty$. $\simeq \frac{2q}{4\pi\epsilon_0 r}\left(1 - (1-\frac{a}{2r}) - \frac{a^2}{2r^2}\right) = \frac{2q}{4\pi\epsilon_0 r}\left(\frac{a}{2r} - \frac{a^2}{2r^2}\right)$.

$= \frac{qa}{4\pi\epsilon_0 r^2}\left(1 - \frac{a}{r}\right)$.