补8.

$$f_i^{(e)} = K_{ii}^{(e)} A_i + K_{ij}^{(e)} A_j + K_{im}^{(e)} A_m - R_i^{(e)}.$$

$$\frac{\partial f_i^{(e)}}{\partial A_j} = K_{ij}^{(e)} + \frac{\partial K_{ii}^{(e)}}{\partial A_j} A_i + \frac{\partial K_{ij}^{(e)}}{\partial A_j} A_j + \frac{\partial K_{im}^{(e)}}{\partial A_j} A_m.$$

由于 $K_{st}^{(e)} = \frac{\gamma}{4\Delta}(b_t b_s + c_t c_s)$. $\rightarrow \frac{\partial K_{st}^{(e)}}{\partial A_j} = \frac{1}{\gamma}\frac{\partial \gamma}{\partial A_j} K_{st}^{(e)}$

$$\rightarrow \frac{\partial f_i^{(e)}}{\partial A_j} = K_{ij}^{(e)} + \frac{1}{\gamma}\frac{\partial \gamma}{\partial A_j}(K_{ii}^{(e)} A_i + K_{ij}^{(e)} A_j + K_{im}^{(e)} A_m).$$

$$A = N_i A_i + N_j A_j + N_m A_m = \frac{1}{2\Delta}[(a_i + b_i x + c_i y) A_i + (a_j + b_j x + c_j y) A_j + (a_m + b_m x + c_m y) A_m].$$

$$B = \sqrt{\left(\frac{\partial A}{\partial x}\right)^2 + \left(\frac{\partial A}{\partial y}\right)^2}.$$

$$\frac{\partial \gamma}{\partial A_j} = \frac{\partial \gamma}{\partial B}\frac{\partial B}{\partial A_j} = \frac{\partial \gamma}{\partial B}\frac{1}{2B}\left[2\left(\frac{\partial A}{\partial x}\right)\frac{\partial}{\partial A_j}\left(\frac{\partial A}{\partial x}\right) + 2\left(\frac{\partial A}{\partial y}\right)\frac{\partial}{\partial A_j}\left(\frac{\partial A}{\partial y}\right)\right]$$

$$= \frac{\partial \gamma}{\partial B}\frac{1}{B}\frac{1}{\Delta\gamma}\frac{\gamma}{4\Delta}[(b_j^2 + c_j^2) A_j + (b_i b_j + c_i c_j) A_i + (b_j b_m + c_j c_m) A_m]$$

$$= \frac{\partial \gamma}{\partial B}\frac{1}{B}\frac{1}{\Delta\gamma} g_j^{(e)}$$

其中 $g_j^{(e)} = K_{jj}^{(e)} A_j + K_{ij}^{(e)} A_i + K_{jm}^{(e)} A_m.$

$$g_i^{(e)} = K_{ii}^{(e)} A_i + K_{ij}^{(e)} A_j + K_{im}^{(e)} A_m.$$

所以 $\frac{\partial f_i^{(e)}}{\partial A_j} = K_{ij}^{(e)} + \frac{g_i^{(e)} g_j^{(e)}}{\gamma^2 B\Delta}\frac{\partial \gamma}{\partial B}.$

由对称性: $\frac{\partial f_j^{(e)}}{\partial A_i} = K_{ji}^{(e)} + \frac{g_i^{(e)} g_j^{(e)}}{\gamma^2 B\Delta}\frac{\partial \gamma}{\partial B}.$

由于 $K_{ij}^{(e)} = K_{ji}^{(e)}$.

所以 $\frac{\partial f_i^{(e)}}{\partial A_j} = K_{ij}^{(e)} + \frac{g_i^{(e)} g_j^{(e)}}{\gamma^2 B\Delta}\frac{\partial \gamma}{\partial B} = \frac{\partial f_j^{(e)}}{\partial A_i}$

同理: $\frac{\partial f_i^{(e)}}{\partial A_m} = K_{im}^{(e)} + \frac{g_i^{(e)} g_m^{(e)}}{\gamma^2 B\Delta}\frac{\partial \gamma}{\partial B} = \frac{\partial f_m^{(e)}}{\partial A_i}.$