Homework 10 for GP1

1. KK 9.2   (10.2)

2. KK 9.4   (10.4)

3. KK 9.5   (10.5)

4. Rutherford scattering:

The electrons [1] are coming from a distance (can be taken as infinity, i.e. potential energy of interaction is zero initially) towards a positively charged nuclei (see the figure below). The initial electrons will have same velocity v_0, but different values of impact parameter b relative to the nuclei. First consider a 2-D model, where the flux density of electrons per unit length is constant n ,(this is the number of electrons passing an unit length per unit time) i.e. the number of incoming electrons within Δb per time is $n\Delta b$. The electrons will be deflected due to interaction with nuclei and the deflection angle is ϕ, defined as angle between incoming and detected electron after deflection (See the figure below; the detector is placed also infinitely distant relative to nuclei). The interaction potential (Coulomb) is in form of $U(r) = -\dfrac{C}{r}$, where C is a constant given the charges of electron and nuclei, r is the distance between them. The reduced mass can be taken as that of electron $\mu = m_e$.

1）Please express the relation between deflection angle ϕ and impact parameter b. (answer:

$$\phi = \pi - 2\tan^{-1}(\frac{m_e v_0^2}{C}b) \ \text{ or } \ \cot\frac{\phi}{2} = \frac{m_e v_0^2}{C}b)$$

2） In 2-D model: From the detector's record, we can get angular number density ρ of the electron, i.e. $\rho = $(number of scattered electrons/unit angle), find the expression of this ρ. (The relation $d(\cot x)/dx = -1/(\sin x)^2$ may be useful) (answer: $\rho = n\dfrac{C}{m_e v_0^2}\dfrac{1}{2\sin^2(\phi/2)}$)

3) Now consider the realistic 3-D model, the n here will be flux density per area per unit time; please give the expression of detected solid angular density ρ of the electron, that is the number of electrons detected per unit steradian（单位立体角）(Hint: You shall need

$nd\sigma = \rho d\Omega$, where $d\sigma$ is small area of the incoming, $d\Omega$ is the solid angle of the scattering, thus the key is to get what is called differential cross section: $d\sigma/d\Omega$,which is derived in lecture $d\sigma/d\Omega = \dfrac{b}{\sin\phi}|\dfrac{db}{d\phi}|$, and this will lead to answer: $\rho = n(\dfrac{C}{m_e v_0^2})^2\dfrac{1}{4\sin^4\dfrac{\phi}{2}}$)

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[1] In real experiment, Rutherford used Helium nuclei (+2 charge) and measured its scattering by gold (Au) nuclei, so the central field is repulsive. Here to make calculation be exactly same as attractive central field, I changed it to electron scattering with nuclei.
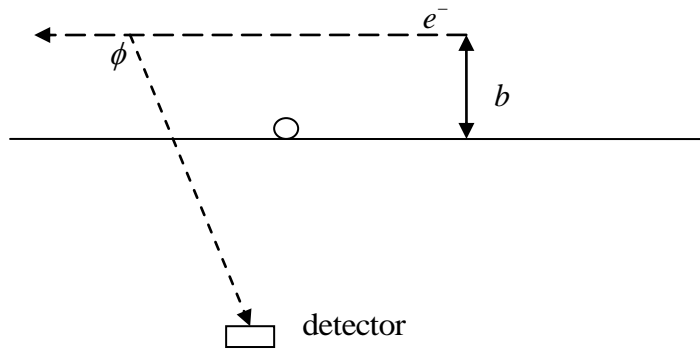
5. KK9.8 (10.8)   6. KK 9.12 (10.12)   7. KK 10.3 (11.4)

8. KK10.5 (11.7)   9. KK 10.11 (11.13)   10. KK 10.12 (11.14)

11. A mass m is on a frictionless table and is attached to two identical springs as shown in

the figure. The spring has constant k and natural length l_0. The stretched length l is

much larger than l_0. The m can have horizontal (along x) and vertical displacement from

equilibrium.
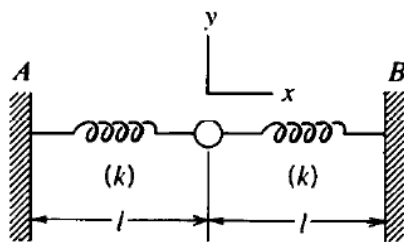
a) For motion along x, y directions (the displacement along y is much smaller than l),
    write out the equation of motion for the x and y motion.

b) In terms of values provided (k,l,m,l_0), calculate the ratio of periods of oscillations

    along x and y.

c) If the initial conditions are: at t=0, x=y=A with zero velocity, what are x,y at later t?

12. Given a transverse wave traveling along a string with density of 1kg/m, the wave is in

form of: $y = 0.5\cos\pi(0.005x - 50t)$, x, t in SI units.

(a) Find the amplitude, wavelength, wavenumber, frequency, period and phase velocity.

(b) Find the tension on the string

(c) Find the power (of energy) carried by the wave

(d) Find the maximum transverse speed of any particle on the string.

13. Knowing a wave with frequency of 20Hz and velocity of 100m/sec:

(a) What is the spatial distance between two points whose phase difference is 30 degree?

(b) For a fixed point in space, but with time difference of 0.01 sec., what is the phase

    difference between these two moments?

14. We are observing two points on a string. The two points are separated by 1 m, so

$x_1=0, x_2=1m$. The transverse oscillation at these two points are measured to be:

$$y_1 = 0.2\cos 3\pi t; \quad y_2 = 0.2\cos(3\pi t + \frac{\pi}{8})$$

(a) What is the frequency of the wave?

(b) And wavelength? And phase velocity?

(c) Along which direction the wave propagates?

(Hint: The provided information may not sufficiently determine one unique wave)

15. Consider the figure below:
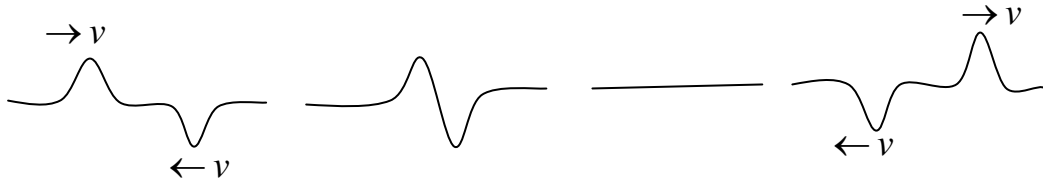
Suppose two men hold the ends of a rope and create two wave packets with inversed shape travelling against each other. The wave packets will meet somewhere between the rope and due to the superposition of reversed phase will finally cancel each other resulting in a flat string as shown in the third figure. But moment later the two wave packets will appear again and travel along the original direction. Describe this phenomenon qualitatively, especially from energy point of view, i.e. when the wave packets disappear where does the energy go?