杨雨函. 2020011219. 作业 6.

1. $\frac{d\,m\vec{v}}{dt} = m\frac{d(r\dot\theta \hat{i}_\theta + \dot r \hat{i}_r + \dot z \hat{i}_z)}{dt} = m\left[\ddot r \hat{i}_r + \frac{d(r\dot\theta)}{dt}\hat{i}_\theta + \ddot z \hat{i}_z + \dot r\dot\theta \hat{i}_\theta - r\dot\theta^2 \hat{i}_r\right]$

$q\vec{v}\times\vec{B}$, $\vec{v} = \frac{d\vec{r}}{dt} = \dot r \hat{i}_r + r\dot\theta \hat{i}_\theta + \dot z \hat{i}_z$. $\vec{B} = B_r \hat{i}_r + B_\theta \hat{i}_\theta + B_z \hat{i}_z$

$$\rightarrow \begin{cases} q(r\dot\theta B_z - r\dot\theta B_\theta) = m(\ddot r - r\dot\theta^2) \\ q(\dot z B_r - \dot r B_z) = m(r\ddot\theta + 2\dot r\dot\theta) \\ q(\dot r B_\theta - r\dot\theta B_r) = m\ddot z \end{cases} \xrightarrow{B_\theta = 0,\ \dot r = 0} \begin{cases} qr\dot\theta B_z = -mr\dot\theta^2 \\ q\dot z B_r = mr\ddot\theta \\ -qr\dot\theta B_r = m\ddot z \end{cases}$$

$$\Rightarrow \dot\theta = -\frac{qB_z}{m},\quad r = \frac{mr\dot\theta}{qB_z}.$$

$\dot z^2 + \dot r^2 + (r\dot\theta)^2 = \dot z^2 + (r\dot\theta)^2 = \text{constant}$.

$\nabla\cdot\vec{B} = 2\frac{B_r}{r} + \frac{dB_z}{dz} = 0$.

$\mu = IS = 2\pi r^2\cdot\left(\frac{-qB_z}{2\pi m}\right) = -\frac{m(r\dot\theta)^2}{qB_z}$

$$\frac{d\mu}{dt} = -\frac{m}{q}\cdot\frac{2r\dot\theta\frac{dr\dot\theta}{dt}B_z - (r\dot\theta)^2\frac{dB_z}{dt}}{B_z^2} = -\frac{mr\dot\theta}{qB_z^2}\cdot\left(2\frac{\dot z\ddot z}{r\dot\theta}B_z - (r\dot\theta)\frac{dB_z}{dz}\cdot\dot z\right)$$

$$= -\frac{mr\dot\theta}{qB_z^2}\left(2\frac{qB_z B_r\dot z}{m} - 2\frac{qB_z B_r\dot z}{m}\right) = 0.$$

$\rightarrow \mu$ 守恒.

2.