1. $\frac{\partial f}{\partial t}+\vec{v}\cdot\nabla_r f+\frac{\vec{F}}{m}\cdot\nabla_v f=0.$

$\rightarrow \int\left(\frac{\partial f}{\partial t}+\vec{v}\cdot\nabla_r f+\frac{\vec{F}}{m}\cdot\nabla_v f\right)\cdot dv=0$

$n(r,t)=\int f(r,v,t)\,dv. \rightarrow \frac{\partial n}{\partial t}=\frac{\partial}{\partial t}\int f\,dv=\int\frac{\partial f}{\partial t}\,dv.$

$\vec{u}(r,t)=\frac{1}{n}\int\vec{v}f(r,v,t)\,dv.$

$\int\vec{v}\cdot\nabla_r f\,dv=\int\left(\nabla_r\cdot(\vec{v}f)-f\,\nabla_r\cdot\vec{v}\right)dv=\nabla_r\cdot\int(\vec{v}f)\,dv-\int f\,\nabla_r\cdot\vec{v}\,dv.$

若 v_x 与 x 无关，则 $\nabla_r\cdot\vec{v}=0.$

$\rightarrow\int\vec{v}\cdot\nabla_r f\,dv=\nabla_r\cdot\int(\vec{v}f)\,dv=\nabla_r\cdot(n\vec{u}).$

$\int\frac{\vec{F}}{m}\cdot\nabla_v f\,dv=\int\left(\nabla_v\cdot\left(\frac{\vec{F}}{m}f\right)-f\left(\nabla_v\cdot\frac{\vec{F}}{m}\right)\right)dv=\int\nabla_v\cdot\left(\frac{\vec{F}}{m}f\right)dv-\int f\left(\nabla_v\cdot\frac{\vec{F}}{m}\right)dv.$

由于 $\int_\Omega\nabla_v\cdot\left(\frac{\vec{F}}{m}f\right)dv=\int_{\partial\Omega}f\frac{\vec{F}}{m}\,d\vec{S}$，且 $\frac{\vec{F}}{m}$ 不依赖于 $\vec{v}$，即 $\nabla_v\cdot\frac{\vec{F}}{m}=0.$

$\rightarrow\int_\Omega\frac{\vec{F}}{m}\cdot\nabla_v f\,dv=\int_{\partial\Omega}\frac{\vec{F}}{m}f\,d\vec{S}.$

由于能量不发散，取高斯面为无穷大球面，则 $\int_{\partial\Omega}\frac{\vec{F}}{m}f\,d\vec{S}=0.$

所以 $\int\left(\frac{\partial f}{\partial t}+\vec{v}\cdot\nabla_r f+\frac{\vec{F}}{m}\cdot\nabla_v f\right)dv=\frac{\partial n}{\partial t}+\nabla_r\cdot(n\vec{u})=0.$

2. $m_e n_e\left[\frac{\partial u_e}{\partial t}+u_e\cdot\nabla u_e\right]=-en_eE-\nabla p-m_en_e\nu_m u_e$

已知 $\frac{\partial u_e}{\partial t}=0$，$u_e=0$. $\nabla T=0$

$\rightarrow en_eE+\nabla p=0.$

由于 $p=n_ekT$，则 $\nabla p=\nabla n_e kT$. 因为 $\nabla T=0$，所以 $\nabla p=kT\cdot\nabla n_e.$

$\rightarrow n_e+\frac{kT}{eE}\cdot\nabla n_e=0.$

方程解为 $n_e=n_0\exp\left(-\frac{e\vec{E}\cdot\vec{r}}{kT}\right)=0$

$\rightarrow n_e=n_0\exp\left(\frac{e\phi(r)}{kT}\right)=0.$