$$A := \sum_{\substack{n_1+n_2+n_3+n_4=15,\\ n_1\geq 1, n_2\geq 1, n_3\geq 1, n_4\geq 1}} \frac{15!}{n_1!n_2!n_3!n_4!}\left(\frac{1}{4}\right)^{15}.$$

$$= \left(\frac{1}{4}\right)^{15} \sum_{\substack{n_1+n_2+\cdots+n_4=15\\ n_1,n_2,n_3,n_4\geq 1}} \frac{15!}{n_1!\,n_2!\,n_3!\,n_4!}$$

$$= \left(\frac{1}{4}\right)^{15}\left(\sum_{n_1+\cdots+n_4=15} \frac{15!}{n_1!n_2!n_3!n_4!} - 4\sum_{n_2+n_3+n_4=15} \frac{15!}{n_2!\,n_3!\,n_4!} + 6\sum_{n_3+n_4=15} \frac{15!}{n_3!\,n_4!} - 4\,\frac{15!}{15!}\right)$$

$$= \left(\frac{1}{4}\right)^{15}\left(4^{15} - 4\times 3^{15} + 6\cdot 2^{15} - 4\right)$$

$$= 1 - \frac{3^{15}}{4^{14}} + \frac{6}{2^{15}} - \frac{1}{4^{14}}.$$