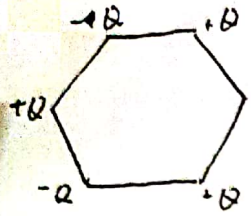
3.3. (1).



$W=\frac{1}{2}\sum_{i=1}^{6}\sum_{\substack{j=1\\ j\neq i}}^{6}Q_iU_{ji}$

$=\frac{1}{2}\sum_{i=1}^{6}Q^2\cdot\frac{1}{4\pi\varepsilon_0}\left(-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{3}a}+\frac{1}{\sqrt{3}a}-\frac{1}{2a}\right)$

$=\frac{3Q^2}{4\pi\varepsilon_0 a}\left(-2+\frac{2\sqrt{3}}{3}-\frac{1}{2}\right)=\frac{Q^2(4\sqrt{3}-15)}{8\pi\varepsilon_0 a}$

(2). $W'=\frac{1}{2}\sum_{i=1}^{4}\sum_{\substack{j=1\\ j\neq i}}^{4}Q_iU_{ji}=\frac{1}{2}\cdot\frac{2Q^2}{4\pi\varepsilon_0 a}\left[\left(-\frac{1}{2a}+\frac{1}{\sqrt{3}a}-\frac{1}{a}\right)+\left(-\frac{1}{a}-\frac{1}{a}+\frac{1}{\sqrt{3}a}\right)\right]$

$=\frac{Q^2}{4\pi\varepsilon_0 a}\left(\frac{-3+2\sqrt{3}-6-6-6+2\sqrt{3}}{6}\right)=\frac{Q^2}{8\pi\varepsilon_0 a}\left(\frac{4\sqrt{3}-21}{3}\right)$

$W''=-\frac{Q^2}{4\pi\varepsilon_0 a}$

$W_F=W'+W''-W=\frac{Q^2}{8\pi\varepsilon_0 a}\left(\frac{4\sqrt{3}}{3}-7-4\sqrt{3}+15-2\right)$

$=\frac{Q^2}{12\pi\varepsilon_0 a}(9-4\sqrt{3})$.

3.4.

(1). $W=\frac{1}{2}U(e)$. $U=\frac{-e}{4\pi\varepsilon_0 r_0}$.

$=\frac{e^2}{8\pi\varepsilon_0 r_0}=mc^2$. $\rightarrow r_0=\frac{e^2}{8\pi\varepsilon_0 mc^2}$

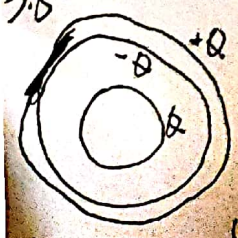
(2). $U_r=\frac{1}{4\pi\varepsilon_0 r}\cdot\rho\cdot\frac{4\pi}{3}r^3$

$W=\iiint\frac{1}{2}\rho\cdot U_r\cdot dV=\frac{1}{2}\iiint\left(\frac{-e}{\frac{4\pi}{3}r_0^3}\right)^2\cdot\frac{\frac{4\pi}{3}r^3}{4\pi\varepsilon_0 r}\cdot r^2\sin\theta\cdot d\theta\cdot d\varphi\cdot dr=\frac{4\pi\rho^2}{15\varepsilon_0}=\frac{3e^2}{20\pi\varepsilon_0 r_0}=mc^2$

$\rightarrow r_0=\frac{3e^2}{20\pi\varepsilon_0 mc^2}$

(3) $r_0=\frac{e^2}{4\pi\varepsilon_0 mc^2}\approx 2.81\times10^{-15}$ m.

3.5.



(1). $W=\frac{1}{2}\left(\frac{Q^2}{4\pi\varepsilon_0 R_3}+\frac{-Q^2}{4\pi\varepsilon_0 R_3}+\frac{Q^2}{4\pi\varepsilon_0 R_3}+\frac{Q^2}{4\pi\varepsilon_0}\int_{R_2}^{R_1}\frac{1}{r^2}dr\right)$

$=\frac{1}{2}\cdot\frac{Q^2}{4\pi\varepsilon_0}\left(\frac{1}{R_3}+\frac{1}{R_1}-\frac{1}{R_2}\right)=\frac{(3\times10^{-8})^2}{8\pi\times8.85\times10^{-12}}\times\left(\frac{1}{0.05}+\frac{1}{0.02}-\frac{1}{0.04}\right)$

≈ 1.8

(2). $U=\frac{Q}{4\pi\varepsilon_0 R_3}$

$W=\frac{1}{2}UQ=\frac{1}{2}\cdot\frac{Q}{4\pi\varepsilon_0}\cdot\frac{1}{R_3}=\frac{3\times10^{-8}}{8\pi\times8.85\times10^{-12}\times0.05}\approx 8\times10^{-5}$ J.

3.6. (1) 由3.4:

(1) $W = \frac{3Q^2}{20\pi\varepsilon_0 r}$

$= \frac{3\times(9\times1.6\times10^{-19})^2}{20\pi\times8.85\times10^{-12}\times9.2\times10^{-15}} \approx 1.3\times10^{-10}\,J.$

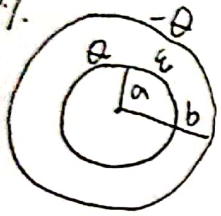
(2) $r_1 = \left(\frac{r^3}{2}\right)^{\frac{1}{3}} = \frac{r}{\sqrt[3]{2}}$

$\Delta W = \frac{3Q^2}{20\pi\varepsilon_0 r} - 2\frac{3Q^2}{20\pi\varepsilon_0 \frac{r}{\sqrt[3]{2}}} \approx 5\times10^{-11}\,J.$

(3). $\frac{1000}{235}\times6.02\times10^{23} \approx 2.6\times10^{24}$ 个.

$\Delta W = 2.6\times10^{24}\times5\times10^{-11} \approx 1.3\times10^{14}\,J.$

3.7.



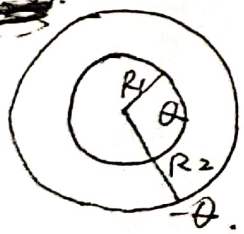
(1). $\vec{E} = \frac{Q}{2\pi r l\varepsilon}$. $\vec{D} = \frac{Q}{2\pi r l}$.

$we = \frac{1}{2}\vec{E}\cdot\vec{D} = \frac{Q^2}{8\pi^2 r^2 l^2 \varepsilon}$.

(2). $W = \iiint we\cdot dV = \frac{Q^2}{8\pi^2 r^4 \varepsilon}\cdot 2\pi r l\cdot dr = \frac{Q^2}{4\pi l\varepsilon}\int_a^b \frac{1}{r}dr = \frac{Q^2}{4\pi l\varepsilon}\ln\frac{b}{a}$.

(3). $U = \int_a^b \vec{E}\cdot d\vec{r} = \frac{Q}{2\pi l\varepsilon}\ln\frac{b}{a}$. $C = \frac{Q}{U} = 2\pi l\varepsilon / \ln\frac{b}{a}$. 所以 $W = \frac{Q^2}{2C}$ 得证

3.10.



(1). $W_{1自} = \frac{Q^2}{8\pi\varepsilon R_1}$ $W_{2自} = \frac{Q^2}{8\pi\varepsilon R_2}$

(2) $W_{互} = \frac{-Q^2}{4\pi\varepsilon R_2}$

(3). $W_{总} = W_{自} + W_{互} = \frac{Q^2}{8\pi\varepsilon}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.