第十次习题课

- 要点
  - 钟慢、尺缩、特别是同时的相对性
  - 洛伦兹变换与其几何对应的闵可夫斯基空间
  - 光的多普勒效应

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1. kk-11.2

作业答案。

首先假设Michelson干涉仪的arm1沿着ether以速度v运动，则

$$T_1=\frac{2l_1}{c}(1+\beta^2),T_2=\frac{2l_2}{c}(1+\frac{1}{2}\beta^2)$$

如果旋转90度，则相当于两臂互换，有

$$T_1^{'}=\frac{2l_1}{c}(1+\frac{1}{2}\beta^2),T_2^{'}=\frac{2l_2}{c}(1+\beta^2)$$

进而，有

$$\Delta t=T_1-T_2,\Delta t'=T_1^{'}-T_2^{'};$$

$$\delta t=\Delta t-\Delta t'=\frac{l_1+l_2}{c}\beta^2;$$

$$N=\frac{\omega\cdot\delta t}{2\pi}=\frac{l_1+l_2}{\lambda}\beta^2=\frac{l_1+l_2}{\lambda}\frac{v^2}{c^2}$$

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2. Kk-11.5

设水中光速为$c'=\frac{c}{n}$，其中n为折射率。则顺时针与逆时针的光在M处干涉时候的时间差为

$$\Delta t=2l\left(\frac{1}{c'-v}-\frac{1}{c'+v}\right)$$

由于

$$N=\frac{\Delta\Phi}{2\pi}=\frac{\omega\Delta t}{2\pi}=\frac{c}{\lambda}\Delta t$$

进而可得

$$N=\frac{c}{\lambda}\frac{4lv}{c'^2-v^2}\approx\frac{c}{\lambda}\frac{4lv}{c'^2}=4n^2\frac{l}{\lambda c}v$$

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3. Hw11-3

- (1) 激光先击中B
- (2) 结果不变。但是有同时的相对性
- (3) 因果律不会被违反。

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4. Hw11-4

(a) 同地同时事件，所有惯性系看起来其同时性不会改变

(b)题目中给出的都是固有长度，隧道对于地面站而言是 L，火车发生尺缩，其长度为 L/γ，即$\frac{4}{5}L$，所以时间为

$$t=\frac{L+\frac{4L}{5}}{\frac{3c}{5}}=\frac{3L}{c}$$

(c) 对于地面观察者而言，该人在$t=\frac{3L}{c}$的时间内，经过了L的长度，故其速度为$v=\frac{c}{3}$

(d)把时间转换为该人的固有时间，有

$$t'=\frac{t}{\gamma},\gamma=\frac{1}{\sqrt{1-1/9}},t'=\frac{2\sqrt{2}L}{c}$$

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5. Hw11-6

对于A，B发射光子时间为 $t_1=\gamma T$，到达A所需时间为 $\frac{\gamma Tv}{c}$，故A此时的时刻为 $t_2=\gamma T(1+\beta)=T\sqrt{\frac{1+\beta}{1-\beta}}$。

对于B，B发射光子时间为$t_1'=T$，到达A所需时间为$\delta t=\frac{vT}{c-v}$，故到达A的时刻为 $t_2'=t_1'+\delta t=\frac{T}{1-\beta}$，根据钟慢效应，A的固有时间此时为$t_2=\frac{t_2'}{\gamma}=T\sqrt{\frac{1+\beta}{1-\beta}}$。

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6. Hw11-7

对于A，B下车时的时间为$t_A=\frac{L}{v}$，根据钟慢效应，B的时间为固有时间，即$t_B=\frac{L}{v}\frac{1}{\gamma}$。二者相差 $\Delta t=t_A-t_B=\frac{L}{v}\left(1-\frac{1}{\gamma}\right)\approx\frac{L}{2}\frac{v}{c^2}$，$v\to 0,\Delta t\to 0$。

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7. Hw11-8

如果要使得棒的前后光同时到达人眼，则需要后端提前发射光子。令前端到达观察者眼前时刻为0，则后端发射光子提前时间应为 $\Delta t=\frac{L/\gamma}{c-v}=\frac{L}{c}\sqrt{\frac{1+\beta}{1-\beta}}$。相应的，棒的长度变为（目测）$l=L\sqrt{\frac{1+\beta}{1-\beta}}$
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8. Hw12-4

a）以隧道或者火车为参考系，把光发射时刻记为0。使得光线到达隧道末端小于火车头部到达炸弹的时间。$r<\sqrt{\frac{1-\beta}{1+\beta}}$.

b)以隧道为参考系会简单一些。注意闵氏图的重要数学关系：$tan\theta=\beta$，1'=$\sqrt{\frac{1+\beta^2}{1-\beta^2}}$.

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9. 利用LT变换的矩阵形式，令变换矩阵为L，则

$$S'=L_1S, S''=L_2S' \Rightarrow S''=L_2L_1S; S''=LS$$

对比 L与L_2L_1的矩阵元素可以找到 v_2与v_1, v_2'的关系

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10. Hw12-6

在洞的坐标系下，由于尺缩效应，棍子短于洞，因此可以穿过。

在棍子坐标系下，洞的两端不是同时提起的，而是右端先，经过$\frac{v}{c^2}\gamma L$ 时间后，左端提起。此时左端位置已经到达γL。

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11. Kk-12.7

a) 利用多普勒公式

$$\lambda=\sqrt{\frac{1+\beta}{1-\beta}}\lambda_0=662.7\times 10^{-9}m$$

注意星体在远离。或者利用 $\frac{\Delta\lambda}{\lambda}\approx\frac{v}{c}$.

b) 太阳自传，赤道上两点，一个在远离，一个在接近。则

$$\Delta\lambda=\frac{v_1-v_2}{c}\lambda=\frac{2v}{c}\lambda=9\times 10^{-12}m\Rightarrow v=2.1\times 10^3m/s\Rightarrow T=\frac{\pi D}{v}=24.3d$$

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12. Kk.12-8

详见作业答案。简单理解，对于镜子来说，光源向自己以速度v接近，对于观察者，镜子反射的光以速度v向自己接近。故(设多普勒频移公式中 $A=\sqrt{\frac{1+\beta}{1-\beta}}$)

$$\nu=A_{mirror}\nu_{mirror}=A_{mirror}A_{source}\nu_0,\ A_{mirror}=A_{source}$$

故有

$$\nu=\frac{1+\beta}{1-\beta}\nu_0$$

如果看作光源向观察者靠近，令上式系数等于A, 计算可得

$$v'=\frac{2v}{1+\frac{v^2}{c^2}}$$

11.7. **Seeing behind the stick**

The first reasoning is correct. You will be able to see a mark on the ruler that is less than L units from the wall. You will actually be able to see a mark even closer to the wall than L/γ, as we'll show below.

The error in the second reasoning (in the stick's frame) is that the second picture in Fig. 11.35 is *not* what you see. This second picture shows where things are at simultaneous times in the stick's frame, which are not simultaneous times in your frame. Alternatively, the error is the implicit assumption that signals travel instantaneously. But in fact the back of the stick cannot know that the front of the stick has been hit by the wall until a finite time has passed. During this time, the ruler (and the wall and you) travels farther to the left, allowing you to see more of the ruler. Let's be quantitative about this and calculate (in both frames) the closest mark to the wall that you can see.

Consider your reference frame. The stick has length L/γ. Therefore, when the stick hits the wall, you can see a mark a distance L/γ from the wall. You will, however, be able to see a mark even closer to the wall, because the back end of the stick will keep moving forward, since it doesn't know yet that the front end has hit the wall. The stopping signal (shock wave, etc.) takes time to travel.

Let's assume that the stopping signal travels along the stick at speed c. (We could instead work with a general speed u, but the speed c is simpler, and it yields an upper bound on the closest mark you can see.) Where will the signal reach the back end? Starting from the time the stick hits the wall, the signal travels backward from the wall at speed c, while the back end of the stick travels forward at speed v (from a point L/γ away from the wall). So the relative speed (as viewed by you) of the signal and the back end is $c+v$. Therefore, the signal hits the back end after a time $(L/\gamma)/(c+v)$. During this time, the signal has traveled a distance $c(L/\gamma)/(c+v)$ from the wall. The closest point to the wall that you can see on the ruler is therefore the mark with the value

$$\frac{L}{\gamma(1+\beta)} = L\sqrt{\frac{1-\beta}{1+\beta}}. \tag{11.81}$$

Now consider the stick's reference frame. The wall is moving to the left toward the stick at speed v. After the wall hits the right end of the stick, the signal moves to the left with speed c, while the wall keeps moving to the left with speed v. Where is the wall when the signal reaches the left end? The wall travels v/c as fast as the signal, so it travels a distance Lv/c in the time that the signal travels the distance L. This means that the wall is $L(1-v/c)$ away from the left end of the stick. In the stick's frame, this corresponds to a distance $\gamma L(1-v/c)$ on the ruler, because the ruler is length contracted. So the left end of the stick is at the mark with the value

$$L\gamma(1-\beta) = L\sqrt{\frac{1-\beta}{1+\beta}}, \tag{11.82}$$

in agreement with Eq. (11.81).

11.19. **Modified twin paradox**

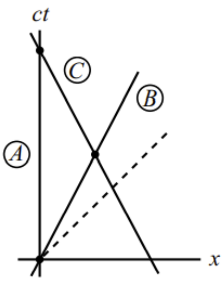

Fig. 11.59

(a) In A's reference frame, the worldlines of A, B, and C are shown in Fig. 11.59. B's clock runs slow by a factor $1/\gamma$. Therefore, if A's clock reads t when B meets C, then B's clock reads t/γ when he meets C. So the time he gives to C is t/γ.

In A's reference frame, the time between this event and the event where C meets A is again t, because B and C travel at the same speed. But A sees C's clock run slow by a factor $1/\gamma$, so A sees C's clock increase by t/γ. Therefore, when A and C meet, A's clock reads $2t$, and C's clock reads $2t/\gamma$. In other words, $T_C = T_A/\gamma$.

(b) Let's now look at things in B's frame. The worldlines of A, B, and C are shown in Fig. 11.60. From B's point of view, there are two competing effects that lead to the relation $T_C = T_A/\gamma$. The first is that B sees A's clock run slow, so the time he hands off to C is *larger* than the time A's clock reads at this moment. The second effect is that from this point on, B sees C's clock run *slower* than A's (because the relative speed of C and B is greater than the relative speed of A and B). It turns out that this slowness wins out over the head start that C's clock had over A's. So in the end, C's clock reads a smaller time than A's. Let's be quantitative about this.

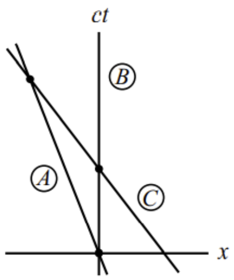

Fig. 11.60

Let B's clock read t_B when he meets C. Then when B hands off this time to C, A's clock reads only t_B/γ. We'll find all relevant times below in terms of t_B. We must determine how much additional time elapses on A's clock and C's clock, by the time they meet. From the velocity-addition formula, B sees C move to the left at speed $2v/(1+v^2)$. He also sees A move to the left at speed v. But A had a head start of vt_B in front of C, so if t is the time (as viewed from B) between the meeting of B and C and the meeting of A and C, then

$$\frac{2vt}{1+v^2} = vt + vt_B \quad \Longrightarrow \quad t = t_B\left(\frac{1+v^2}{1-v^2}\right). \tag{11.115}$$

During this time, B sees A's and C's clocks increase by t divided by the relevant time-dilation factor. For A, this factor is $\gamma = 1/\sqrt{1-v^2/c^2}$. For C, it is

$$\frac{1}{\sqrt{1-\left(\frac{2v}{1+v^2}\right)^2}} = \frac{1+v^2}{1-v^2}. \tag{11.116}$$

Therefore, the total time shown on A's clock when A and C meet is

$$\begin{aligned} T_A &= \frac{t_B}{\gamma} + t\sqrt{1-v^2} = t_B\sqrt{1-v^2} + t_B\left(\frac{1+v^2}{1-v^2}\right)\sqrt{1-v^2} \\ &= \frac{2t_B}{\sqrt{1-v^2}}. \end{aligned} \tag{11.117}$$

And the total time shown on C's clock when A and C meet is

$$T_C = t_B + t\left(\frac{1-v^2}{1+v^2}\right) = t_B + t_B\left(\frac{1+v^2}{1-v^2}\right)\left(\frac{1-v^2}{1+v^2}\right) = 2t_B. \tag{11.118}$$

Therefore, $T_C = T_A\sqrt{1-v^2} \equiv T_A/\gamma$.

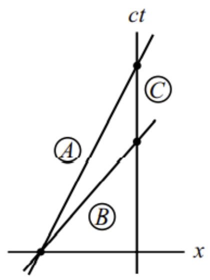

Fig. 11.61

(c) Let's now work in C's frame. The worldlines of A, B, and C are shown in Fig. 11.61. As in part (b), the relative speed of B and C is $2v/(1+v^2)$, and the time-dilation factor between B and C is $(1+v^2)/(1-v^2)$. Also, as in part (b), let B and C meet when B's clock reads t_B. So this is the time that B hands off to C. We'll find all relevant times below in terms of t_B.

C sees B's clock running slow, so from C's point of view, B travels for a time $t_B(1 + v^2)/(1 - v^2)$ after his meeting with A. In this time, B covers a distance in C's frame equal to

$$d = t_B \left(\frac{1 + v^2}{1 - v^2} \right) \frac{2v}{1 + v^2} = \frac{2vt_B}{1 - v^2} \,. \tag{11.119}$$

A must travel this same distance (from where he met B) to meet up with C. We can now find T_A. The time (as viewed by C) that it takes A to travel the distance d to reach C is $d/v = 2t_B/(1 - v^2)$. But since C sees A's clock running slow by a factor $\sqrt{1 - v^2}$, A's clock will read only

$$T_A = \frac{2t_B}{\sqrt{1 - v^2}} \,. \tag{11.120}$$

Now let's find T_C. To find T_C, we must take t_B and add to it the extra time it takes A to reach C, compared with the time it takes B to reach C. From above, this extra time is $2t_B/(1 - v^2) - t_B(1 + v^2)/(1 - v^2) = t_B$. Therefore, C's clock reads

$$T_C = 2t_B. \tag{11.121}$$

Hence, $T_C = T_A\sqrt{1 - v^2} \equiv T_A/\gamma$.

15. 在火车参考系中，火车前面的时钟在$(L,0)$指向零。此事件在地面参考系中，发生于$(\gamma L,\frac{v}{c^2}\gamma L)$。故光子到达前面的时钟所需时间为

$$\delta t=\frac{\gamma L-\beta\gamma L}{c-v}=\frac{L\gamma}{c}$$

此为地面参考系观察到时间间隔，在火车上即为 $\tau=\frac{\delta t}{\gamma}=\frac{L}{c}$。

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16. 注意是时钟读数，注意分清固有时间

a) 地面参考系中。右面火车末尾时钟在车头相遇时指向0，坐标$(L,0)$，LT变换到地面参考系，为$(\gamma L,-\frac{v}{c^2}\gamma L)$，则车尾经过树时，地面时间经过了

$$\Delta t=\frac{\gamma L}{v}-\frac{v}{c^2}\gamma L=\frac{L/\gamma}{v}$$

地面时间在同一地点测量，为固有时，则火车上时钟读数为 $\frac{L}{v}$。

b) 火车参考系中。树以速度大小v，向车尾运动，时钟读数 $\frac{L}{v}$。

c) A火车参考系中观察另一个火车B，车尾时钟运动速度为

$$u=\frac{2v}{1+\beta^2}$$

B火车车尾时钟指向零的坐标（B中）为$(L(1+\frac{1}{\gamma'}),0)$，变换到A火车中，为

$$x=L(1+\gamma'),\ t=-\frac{u}{c^2}L(1+\gamma')$$

则两火车末尾重合时，经过的时间为 $\frac{x}{u}+t=\frac{L/\gamma'}{v}$。

变换到B火车中，为 $\frac{L}{v}$。

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17. 设红移变换因子 $d_r=\sqrt{\frac{1+\beta}{1-\beta}}$，蓝移变换因子 $d_b=\sqrt{\frac{1-\beta}{1+\beta}}$。B远离阶段与接近阶段时间一样，设为T。

a) A向B发送闪光，B远离时，二者发射与接收到的闪烁次数是一样的，满足

$$\frac{T_{A1}}{t}=\frac{T}{d_r t}$$

B接近时，同理，有

$$\frac{T_{A2}}{t}=\frac{T}{d_b t}$$

故有

$$T_A=T_{A1}+T_{A2}=(d_b+d_r)T=2\gamma T=\gamma T_B\Rightarrow T_B=\frac{T_A}{\gamma}$$

b) B向A发送闪光，同理，满足

$$\begin{aligned}\frac{T_{A1}}{d_r t}&=\frac{T}{t}\\ \frac{T_{A2}}{d_b t}&=\frac{T}{t}\\ \Rightarrow\ T_A&=T_{A1}+T_{A2}\\ &=(d_r+d_b)T=2\gamma T=\gamma T_B\end{aligned}$$

即

$$T_B=\frac{T_A}{\gamma}$$