2.4

第一类边界：直接代入.

差分：$A_1+A_2+A_3+A_4-4A_0=-\mu_0 h^2 J_0$

积分：$\frac{1}{2}(\gamma_{i+1,j}+\gamma_{i+1,j+1})A_{i+1,j}+\frac{1}{2}(\gamma_{i+1,j+1}+\gamma_{i,j+1})A_{i,j+1}+\frac{1}{2}(\gamma_{i,j+1}+\gamma_{i,j})A_{i-1,j}$

$+\frac{1}{2}(\gamma_{i,j}+\gamma_{i+1,j})A_{i,j-1}-(\gamma_{i,j}+\gamma_{i,j+1}+\gamma_{i+1,j+1}+\gamma_{i+1,j})A_{i,j}$

$=-\frac{1}{4}h^2(J_{i,j}+J_{i,j+1}+J_{i+1,j}+J_{i+1,j+1})$. 当 γ, J 为常数时 两者相同.

第二类.

差分：$\frac{\partial A}{\partial n}\Big|_{AB}=C=\frac{A(x_0+h,y_0)-A(x_0-h,y_0)}{2h} \Rightarrow A_{i+1,j}-A_{i-1,j}=2hc$

$\Rightarrow 2A_{i+1,j}-2hc+A_{i,j+1}+A_{i,j-1}-4A_{i,j}=-\frac{1}{2}h^2\mu J_{i,j}$.

积分：$\oint_L \gamma\frac{\partial A}{\partial n}dl=\frac{A_{i+1,j}-A_{i,j}}{h}\frac{1}{2}(\gamma_{i+1,j}h+\gamma_{i+1,j+1}h)+\frac{A_{i,j+1}-A_{i,j}}{h}\frac{1}{2}(\gamma_{i+1,j+1}h+\gamma_{i,j+1}h)$

$+\frac{A_{i-1,j}-A_{i,j}}{h}\frac{1}{2}(\gamma_{i,j+1}h+\gamma_{i,j}h)+\frac{A_{i,j-1}-A_{i,j}}{h}\frac{1}{2}(\gamma_{i,j}h+\gamma_{i+1,j}h)$

$=\iint_{G_{i,j}}J\,dx\,dy=\frac{1}{4}(2\times Jh^2)=-\frac{1}{2}h^2J$

代入 $A_{i+1,j}-A_{i-1,j}=2hc$，若 γ, $J=const.$ 则 $2A_{i+1,j}+A_{i,j+1}+A_{i,j-1}-4A_{i,j}-2hc=-\frac{1}{2}h^2\mu J_{i,j}$.

两者相同.

媒质交界面：

差分：$A_{am}=A_{bm}=A_m \quad m=0,2,4$

$\frac{1}{\mu_a}\left(\frac{\partial A_a}{\partial n}\right)=\frac{1}{\mu_b}\left(\frac{\partial A_b}{\partial n}\right)\rightarrow\frac{1}{\mu_a}(A_{a1}-A_{a3})=\frac{1}{\mu_b}(A_{b1}-A_{b3})$.

$A_{a1}+A_{a2}+A_{a3}+A_{a4}-4A_{a0}=-h^2\mu_a J$

$A_{b1}+A_{b2}+A_{b3}+A_{b4}-4A_{b0}=0$. 联立可得：

$\Rightarrow 2A_{b\,i+1,j}+\left(1+\frac{\mu_b}{\mu_a}\right)A_{i,j+1}+2\frac{\mu_b}{\mu_a}A_{a\,i-1,j}+\left(1+\frac{\mu_b}{\mu_a}\right)A_{i,j-1}+\mu_b h^2 J-4\left(1+\frac{\mu_b}{\mu_a}\right)A_{i,j}=0$

积分：$\oint_L \gamma\frac{\partial A}{\partial n}dl=\frac{1}{\mu_b}A_{b\,i+1,j}+\frac{1}{2}\left(\frac{1}{\mu_b}+\frac{1}{\mu_a}\right)A_{i,j+1}+\frac{1}{2}\left(\frac{1}{\mu_b}+\frac{1}{\mu_a}\right)A_{i,j-1}+\frac{1}{\mu_a}A_{a\,i-1,j}-2\left(\frac{1}{\mu_b}+\frac{1}{\mu_a}\right)A_{i,j}$

$=-\iint J\,dx\,dy=-\frac{h^2}{4}(2\times J)=-\frac{1}{2}h^2J$.

$\Rightarrow 2A_{b\,i+1,j}+\left(1+\frac{\mu_b}{\mu_a}\right)A_{i,j+1}+2\frac{\mu_b}{\mu_a}A_{a\,i-1,j}+\left(1+\frac{\mu_b}{\mu_a}\right)A_{i,j-1}+\mu_b h^2 J-4\left(1+\frac{\mu_b}{\mu_a}\right)A_{i,j}=0$.

两者相同.

2.5.

(1). 空气区: $A_1+A_2+A_3+A_4-4A_0=0$

$$A_{i+1,j}+A_{i,j+1}+A_{i-1,j}+A_{i,j-1}-4A_{ij}=0$$

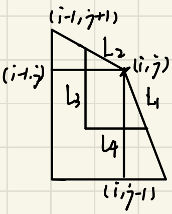
(2) 电流区: $A_1+A_2+A_3+A_4-4A_0=-h^2\mu J_0$

$$A_{i+1,j}+A_{i,j+1}+A_{i-1,j}+A_{i,j-1}-4A_{ij}=-\frac{1}{4}\mu h^2(J_{i,j}+J_{i,j+1}+J_{i+1,j}+J_{i+1,j+1}).$$

(3) 铁区: $A_1+A_2+A_3+A_4-4A_0-\frac{1}{4\mu_0}((A_1-A_3)(\mu_1-\mu_3)+(A_2-A_4)(\mu_2-\mu_4))=0$

$$\frac{1}{2}(\gamma_{i+1,j}+\gamma_{i+1,j+1})A_{i+1,j}+\frac{1}{2}(\gamma_{i+1,j+1}+\gamma_{i,j+1})A_{i,j+1}+\frac{1}{2}(\gamma_{i,j+1}+\gamma_{i,j})A_{i-1,j}$$
$$+\frac{1}{2}(\gamma_{i,j}+\gamma_{i+1,j})A_{i,j-1}-(\gamma_{ij}+\gamma_{i,j+1}+\gamma_{i+1,j}+\gamma_{i+1,j+1})A_{ij}=0$$

2.6



$$\oint_L \gamma\frac{\partial A}{\partial n}dl=\int_{L_1}\gamma\frac{\partial A}{\partial n}dl+\int_{L_2}\gamma\frac{\partial A}{\partial n}dl+\int_{L_3}\gamma\frac{\partial A}{\partial n}dl+\int_{L_4}\gamma\frac{\partial A}{\partial n}dl$$

$$=qL_1+qL_2+\frac{A_{i-1,j}-A_{i,j}}{h_i}\left(\gamma_2\times\frac{1}{2}h_{j+1}+\gamma_{i,j}\times\frac{1}{2}h_j\right)$$
$$+\frac{A_{i,j-1}-A_{i,j}}{h_j}\left(\gamma_{i,j}\times\frac{1}{2}h_i+\gamma_1\times\frac{1}{2}\times h_{i+1}\right).$$

$$L_1=\frac{1}{2}\sqrt{h_j^2+h_{i+1}^2},\quad L_2=\frac{1}{2}\sqrt{h_i^2+h_{j+1}^2}.$$

$$\iint_{G_{i,j}}J\,dx\,dy=\frac{1}{8}h_jh_{i+1}J_1+\frac{1}{8}h_ih_{j+1}J_2+\frac{1}{4}h_ih_jJ_{i,j}.$$

$$\Rightarrow \frac{1}{2}q\left(\sqrt{h_j^2+h_{i+1}^2}+\sqrt{h_i^2+h_{j+1}^2}\right)+\frac{A_{i-1,j}-A_{i,j}}{h_i}\left(\frac{1}{2}\gamma_2h_{j+1}+\frac{1}{2}\gamma_{ij}h_j\right)+\frac{A_{i,j-1}-A_{i,j}}{h_j}\left(\frac{1}{2}\gamma_{ij}h_i+\frac{1}{2}\gamma_1h_{i+1}\right)$$
$$=-\frac{1}{8}h_jh_{i+1}J_1-\frac{1}{8}h_ih_{j+1}J_2-\frac{1}{4}h_ih_jJ_{i,j}.$$