电磁学
Electromagnetism

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清华大学物理系
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电磁学

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电磁学与电动力学（上册）

(科学出版社，北京，**2008)**

电磁学创立的例子

(1) 发现新现象	Thales of Miletus (Around 600BC) Shen Kuo (沈括，1031-1095) William Gilbert (1544-1603) Hans Christian Östed (1777-1851)	好奇/机敏/明眼
(2) 总结新规律	Charles Coulomb (1736-1806) André-Marie Ampère (1775-1836) Biot-Savart-Laplace Michael Faraday (1791-1867)	洞察/猜想/数学
(3) 创造新理论	James Clerk Maxwell (1831-1879)	简化/猜想/数学/慧眼
(4) 预言新现象	James Clerk Maxwell (1831-1879)	洞察/猜想/数学/法眼
(5) 验证新理论	Heinrich Hertz	好奇/机敏/数学
(6) 创造新应用	Thomas Edison (1847-1931) Nikola Tesla (1856-1943)	好玩/机敏

Pre Maxwell　静电学

库伦定律 ✚ 叠加原理

$$\vec{E}(\vec{X}) = \int_q \frac{dq}{4\pi\varepsilon_0} \cdot \frac{\vec{X}-\vec{X}'}{\left|\vec{X}-\vec{X}'\right|^3}$$

高斯定理 ✚ 环路定理

$$\oiint_S \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon_0}\iiint_{V\ inside\ S} \rho dV$$

$$\oint_L \vec{E}\cdot d\vec{r} = 0$$

$$\nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla\times\vec{E} = 0$$

Pre Maxwell

静磁场

静磁学的全部内容

B-S-L定律 ＋ 叠加原理

$$\vec{B}(\vec{X}) = \int_{\vec{j}} \frac{\mu_0 d^3X'}{4\pi} \cdot \frac{\vec{j}(\vec{X}')\times(\vec{X}-\vec{X}')}{\left|\vec{X}-\vec{X}'\right|^3}$$

矢量场的基本方程

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\oiint \vec{B} \cdot d\vec{S} = 0 \qquad \oint \vec{B} \cdot d\vec{r} = \mu_0 I$$

Pre Maxwell, Statics

$$\nabla \cdot \vec{D} = \rho_0$$

$$\nabla \times \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J}_0$$

$$\frac{\partial \rho_0}{\partial t} + \nabla \cdot \vec{J}_0 = 0$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

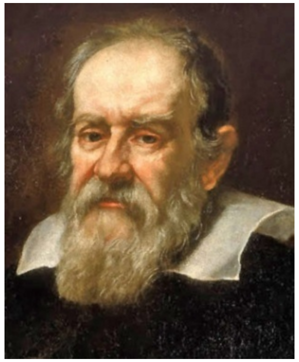
$$\vec{H} = \frac{\vec{B}}{\mu}$$

$$\vec{J}_0 = \sigma \vec{E}$$

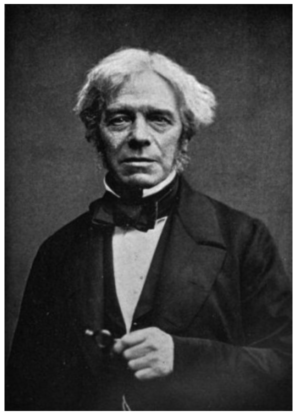
"Physicist's History of Physics"

"Physics has a history of synthesizing many phenomena into a few theories" [R. P. Feynman]

Classical Physics

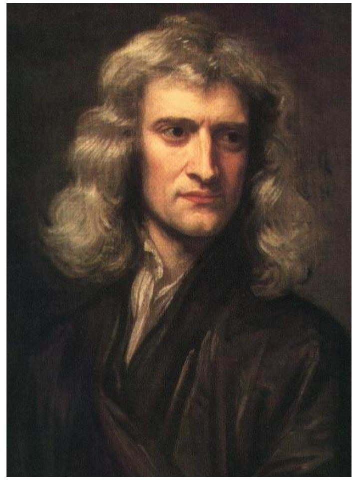


Phenomena of



- Motion
- Heat
- Sound

→ Isaac Newton's Laws of Motion

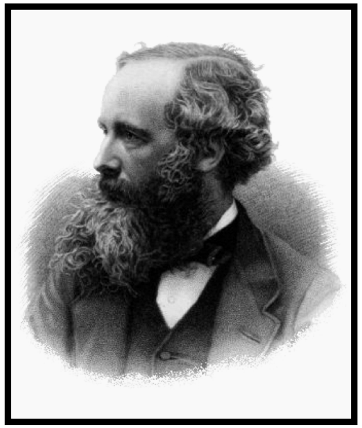


- Gravity

→ Laws of Gravity

- Light
- Optics
- Electricity
- Magnetism

→ Maxwell's Laws of EM



Maxwell的两个大胆推广

$$\nabla \cdot \vec{D} = \rho_0 \qquad \nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

普适成立

$$\nabla \cdot \vec{B} = 0$$

普适成立

Maxwell的两个大胆假设

1. 涡旋电场假设

$$\mathcal{E} = \oint \vec{E}_{涡旋} \cdot d\vec{l} = -\frac{d\Phi}{dt} = -\iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S}$$

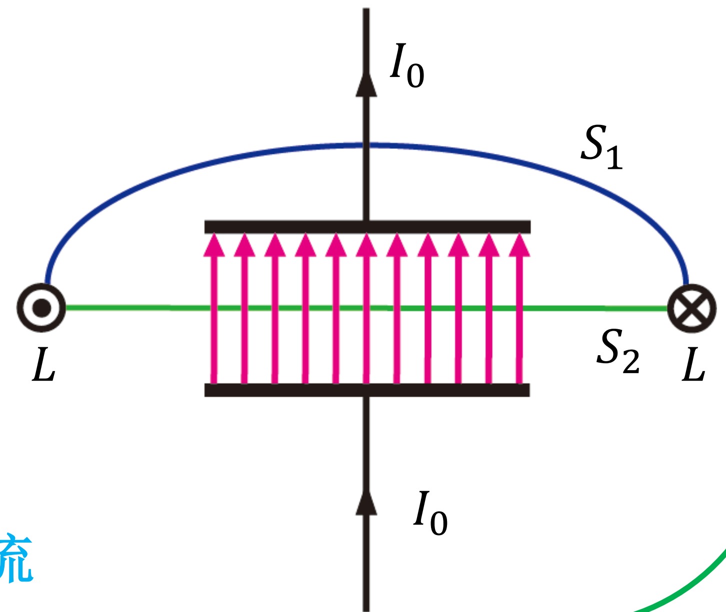
$$\nabla \times \vec{E}_{涡旋} = -\frac{\partial \vec{B}}{\partial t} \qquad \nabla \times \vec{E}_{势} = 0$$

$$\vec{E} = \vec{E}_{涡旋} + \vec{E}_{势} \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

2. 位移电流假设

$$\oint_L \vec{H} \cdot d\vec{l} = \iint_{S_1} \vec{J}_0 \cdot d\vec{S}_1 = I_0$$

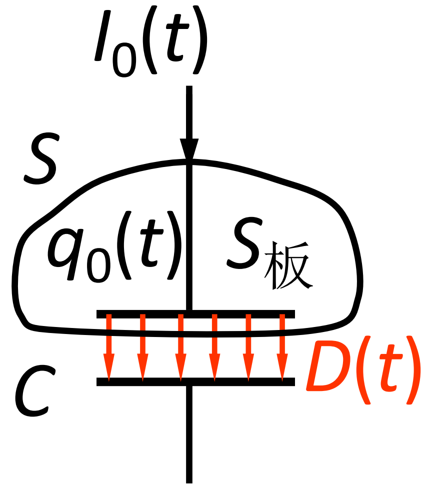
$$\oint_L \vec{H} \cdot d\vec{l} = \iint_{S_2} \vec{J}_0 \cdot d\vec{S}_2 = 0$$



$\vec{J} = \vec{J}_0 + \vec{J}_d$ 　 $\vec{J}_d$为位移电流

位移电流（Displacement Current）

$\vec{J} = \vec{J}_0 + \vec{J}_d$　　$\vec{J}_d$为位移电流



$$\oiint_S \vec{J} \cdot d\vec{S} = 0 \Rightarrow \oiint_S (\vec{J}_0 + \vec{J}_d) \cdot d\vec{S} = 0$$

$$\Rightarrow \oiint_S \vec{J}_d \cdot d\vec{S} = -\oiint_S \vec{J}_0 \cdot d\vec{S}$$

$$\frac{\partial \rho_0}{\partial t} + \nabla \cdot \vec{J}_0 = 0 \qquad \nabla \cdot \vec{D} = \rho_0$$

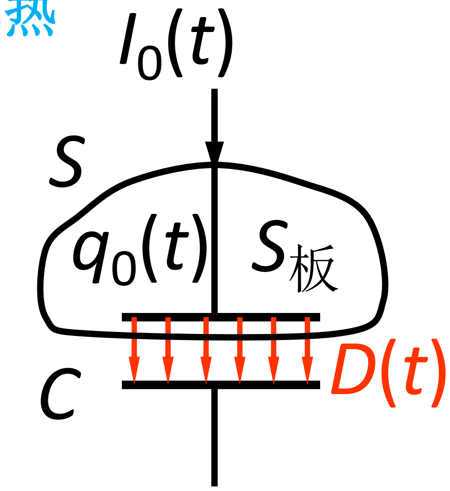
$$\Rightarrow \frac{\partial (\nabla \cdot \vec{D})}{\partial t} + \nabla \cdot \vec{J}_0 = 0 \Rightarrow \nabla \cdot \left(\frac{\partial \vec{D}}{\partial t}\right) + \nabla \cdot \vec{J}_0 = 0$$

$$\Rightarrow \oiint_S \vec{J}_0 \cdot d\vec{S} = -\oiint_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S}$$

$$\oiint_S \vec{J}_d \cdot d\vec{S} = -\oiint_S \vec{J}_0 \cdot d\vec{S} = \oiint_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{S} \qquad \vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$

位移电流（Displacement Current）

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t}$$ $\vec{J}_d$为位移电流，无焦耳热



$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{J}_d = \frac{\partial \vec{D}}{\partial t} = \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t}$$

$$\vec{J}_p = \frac{\partial \vec{P}}{\partial t}$$ $\vec{J}_p$为极化电流

对安培环路定律的修正 $$\nabla \times \vec{H} = \vec{J}_0 + \frac{\partial \vec{D}}{\partial t}$$

$$\nabla \times \vec{B} = \mu_0\left(\vec{J}_0 + \frac{\partial \vec{D}}{\partial t} + \vec{J}'\right) = \mu_0\left(\vec{J}_0 + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} + \frac{\partial \vec{P}}{\partial t} + \nabla \times M\right)$$

产生磁场的电流源：自由电流、变化电场、极化电流、磁化电流

除自由电流外，后面三项都无焦耳热

Maxwell Equations

Static

$$\nabla \cdot \vec{D} = \rho_0$$

$$\nabla \times \vec{E} = 0$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J}_0$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\frac{\partial \rho_0}{\partial t} + \nabla \cdot \vec{J}_0 = 0$$

Dynamic

$$\nabla \cdot \vec{D} = \rho_0$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J}_0 + \frac{\partial \vec{D}}{\partial t}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{f} = \rho \vec{E} + \vec{J} \times \vec{B})$$

Material

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$$

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{H} = \frac{\vec{B}}{\mu}$$

$$\vec{J}_0 = \sigma \vec{E}$$

$$\vec{J}_0 = \sigma(\vec{E} + \vec{v} \times \vec{B})$$

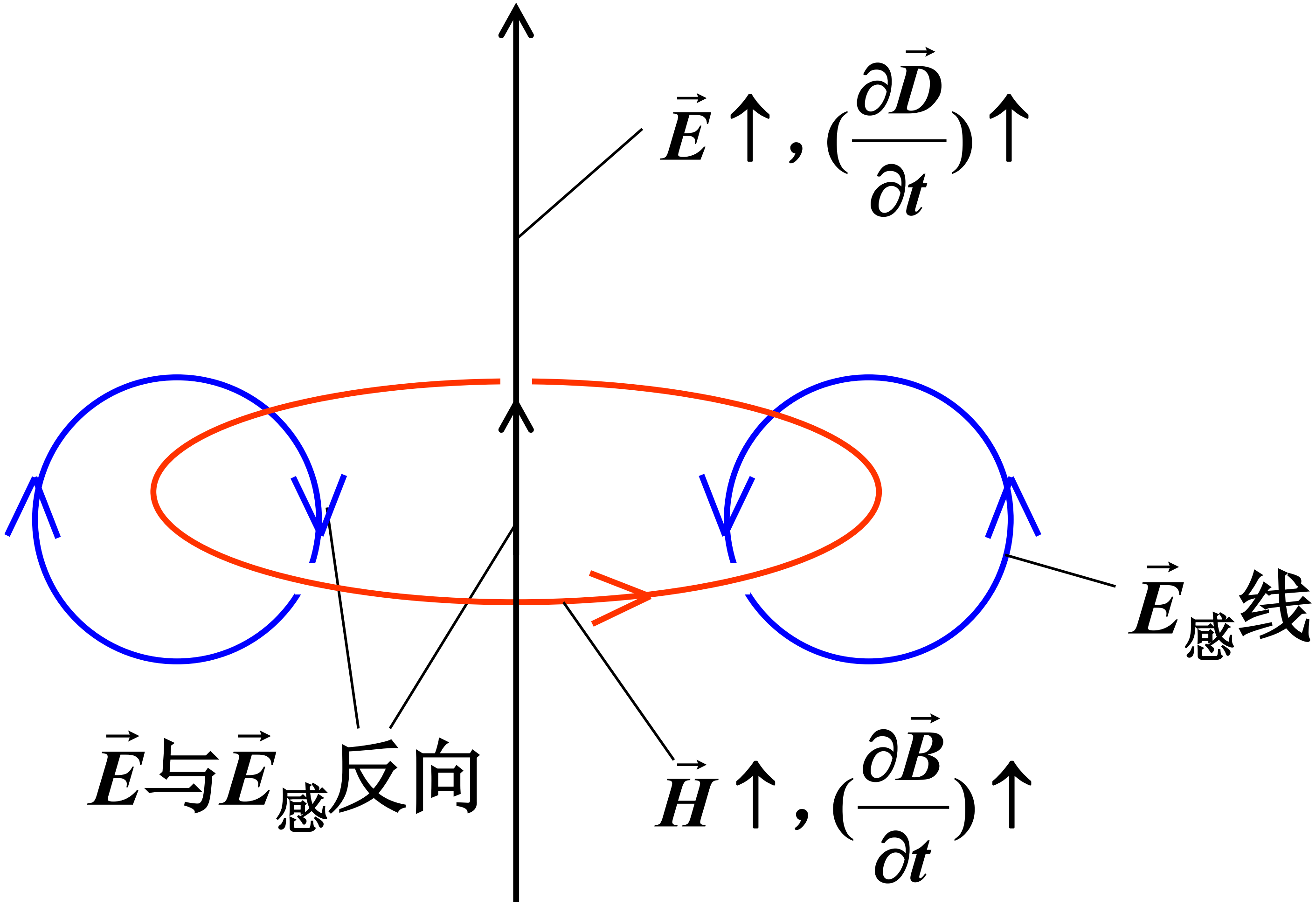
Boundary Conditions

$$\hat{n} \cdot \left(\vec{D}_2 - \vec{D}_1\right) = \sigma_0 \qquad \hat{n} \cdot \left(\vec{B}_2 - \vec{B}_1\right) = 0$$

$$\hat{n} \times \left(\vec{E}_2 - \vec{E}_1\right) = 0 \qquad \hat{n} \times \left(\vec{H}_2 - \vec{H}_1\right) = i_0$$

$$\nabla\times\vec{H}=\frac{\partial\vec{D}}{\partial t}\qquad \oint_L \vec{H}\cdot\mathrm{d}\vec{l}=\int_S\frac{\partial\vec{D}}{\partial t}\cdot\mathrm{d}\vec{s}=\frac{\mathrm{d}\Phi_D}{\mathrm{d}t}$$

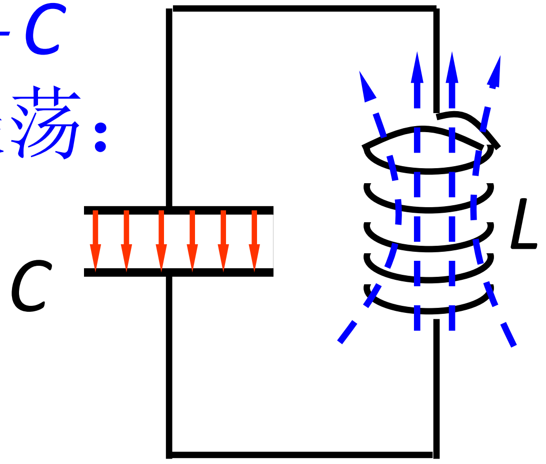
$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}\qquad \oint_L \vec{E}_{感}\cdot\mathrm{d}\vec{l}=-\int_S\frac{\partial\vec{B}}{\partial t}\cdot\mathrm{d}\vec{s}=-\frac{\mathrm{d}\Phi}{\mathrm{d}t}$$



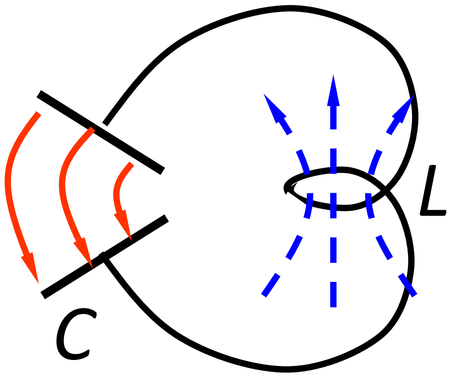
磁场的增加以电场的削弱为代价（能量守恒）。

电磁辐射（ electromagnetic radiation）

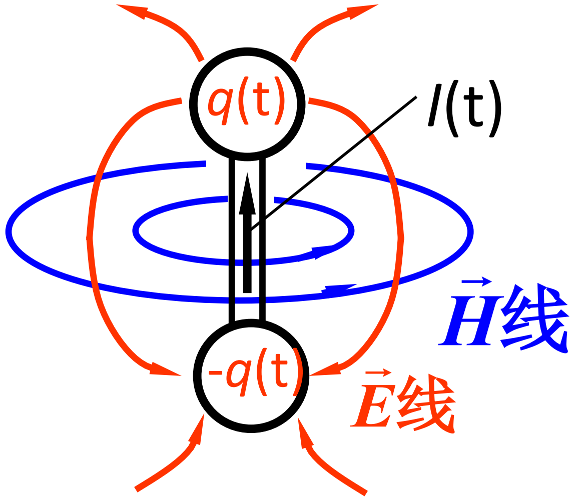
$L-C$ 振荡：



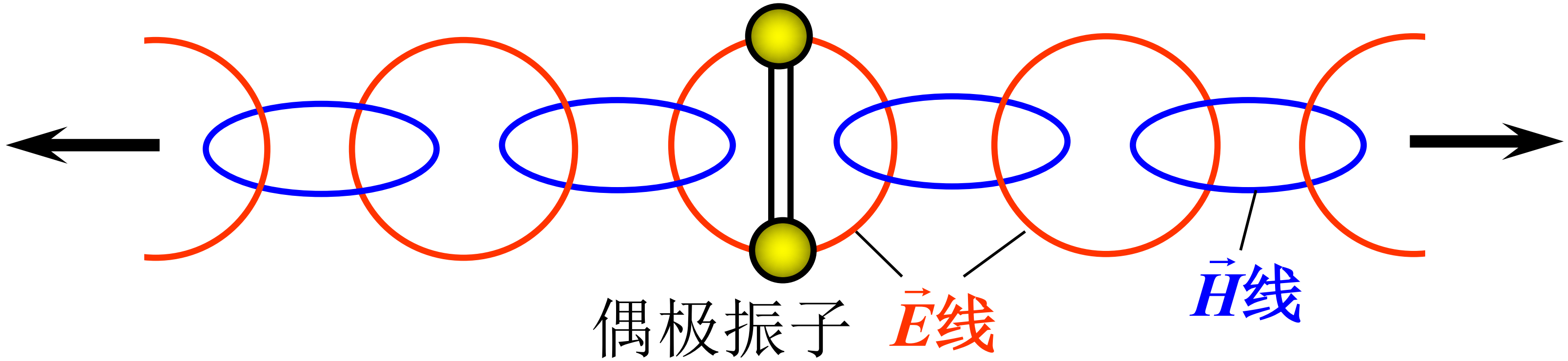
电磁场封闭
在 L、C 中



电磁场
较开放



电偶极振子天线
电磁场完全开放



演示　　电磁波的辐射和接收

胡 ·10.2.1 无限线性各向同性介质中的电磁波

场方程：$\nabla \cdot \vec{D} = \rho$ $\quad \nabla \cdot \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\quad \nabla \cdot \vec{B} = 0$ $\quad \nabla\times\vec{H} = \vec{j_0} + \frac{\partial \vec{D}}{\partial t}$

物质方程：$\vec{D} = \epsilon\vec{E}$ $\quad \vec{B} = \mu\vec{H}$

下面求解无自由电荷和自由电流的情况。 $\rho = 0$ $\quad \vec{j} = 0$

先化成只含有$\vec{E}$, $\vec{H}$的方程

$$\nabla \cdot \vec{E} = 0 \qquad \nabla\times\vec{E} = -\mu\frac{\partial \vec{H}}{\partial t} \qquad \nabla \cdot \vec{H} = 0 \qquad \nabla\times\vec{H} = \epsilon\frac{\partial E}{\partial t}$$

$$\left.\begin{array}{l} \nabla\times(\nabla\times\vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2\vec{E} \\ \nabla\times(\nabla\times\vec{E}) = -\mu\frac{\partial}{\partial t}(\nabla\times\vec{H}) = -\mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} \\ \nabla \cdot \vec{E} = 0 \end{array}\right\} \Rightarrow \nabla^2\vec{E} = \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\therefore \nabla^2\vec{E} - \mu\epsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0$$

$$\frac{\partial^2 \vec{E}}{\partial t^2} - \frac{1}{\mu\epsilon}\nabla^2 \vec{E} = 0 \qquad \text{or} \qquad \nabla^2 \vec{E} - \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

同样 $\frac{\partial^2 \vec{H}}{\partial t^2} - \frac{1}{\mu\epsilon}\nabla^2 \vec{H} = 0 \qquad \text{or} \qquad \nabla^2 \vec{H} - \mu\epsilon \frac{\partial^2 \vec{H}}{\partial t^2} = 0$

这就是波动方程　波速$\mu = \frac{1}{\sqrt{\mu\epsilon}}$　真空中波速 $C = \frac{1}{\sqrt{\mu_0\epsilon_0}}$

介质折射率$n = \frac{c}{\nu} = \sqrt{\frac{\mu\epsilon}{\mu_0\epsilon_0}}$

对大数介质(除铁磁质)　$\mu \sim \mu_0$　$n \approx \sqrt{\frac{\epsilon}{\epsilon_0}}$

并且通常μ, ϵ都是ω的函数(不同机制导致不同极化率χ_e，χ_m)

此时 $n \sim n(\omega)$ 色散现象

无限均匀线性各向同性介质中，$\rho_0 = 0 = j_0$

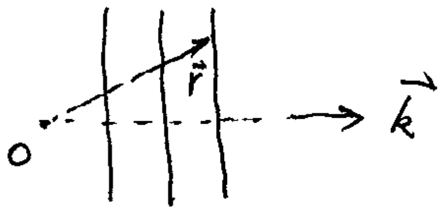
$$\nabla^2 \vec{E} - \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

$$\nabla^2 \vec{H} - \mu\epsilon \frac{\partial \vec{H}}{\partial t^2} = 0$$

一维通解　$f(x - ut) + g(x + ut)$　　　$u = \frac{1}{\sqrt{\mu\epsilon}}$

平面波单频波解：$\vec{E} = \overrightarrow{E_0} e^{-i(\omega t - \vec{k}\cdot\vec{r})}$

$$\vec{H} = \overrightarrow{H_0} e^{-i(\omega t - \vec{k}\cdot\vec{r})}$$



将平面单频解代入波动方程

$$\left(\nabla^2 \vec{E}\right)_i = \partial_j \partial_j E_i = \partial_j \partial_j E_{0i} e^{-i(\omega t - \vec{k}\cdot\vec{r})}$$

$$= E_{0i} \partial_j \partial_j e^{-i(\omega t - \vec{k}\cdot\vec{r})}$$

$$\partial_j e^{-i(\omega t-\vec{k}\cdot\vec{r})} = e^{-i(\omega t-\vec{k}\cdot\vec{r})}\partial_j(i\vec{k}\cdot\vec{r})$$

$$= ie^{-i(\omega t-\vec{k}\cdot\vec{r})}\partial_j(k_m r_m)$$

$$= ie^{-i(\omega t-\vec{k}\cdot\vec{r})}k_m\partial_j r_m \text{其中} \partial_j r_m = \delta_{jm}$$

$$= ie^{-i(\omega t-\vec{k}\cdot\vec{r})}k_j$$

$$\partial_j\partial_j e^{-i(\omega t-\vec{k}\cdot\vec{r})} = i^2 e^{-i(\omega t-\vec{k}\cdot\vec{r})}k_j k_j = -e^{-i(\omega t-\vec{k}\cdot\vec{r})}k_j k_j$$

$$\therefore \nabla^2\vec{E} = -(\vec{k}\cdot\vec{k})\vec{E}$$

$$\frac{\partial^2\vec{E}}{\partial t^2} = -\omega^2\vec{E}$$

代入波动方程 $\nabla^2\vec{E} - \frac{1}{u^2}\frac{\partial^2\vec{E}}{\partial t^2} = 0$

可得 $\left(-\vec{k}\cdot\vec{k} + \frac{\omega^2}{u^2}\right)\vec{E} = 0$

$\vec{E}$有非零解条件 $\frac{\omega^2}{u^2} = k^2$ $\frac{\omega}{k} = \pm u = \pm\frac{1}{\sqrt{\epsilon\mu}}$

由$\nabla \cdot \vec{E} = 0 \qquad \nabla \cdot \vec{H} = 0$可得

$$\nabla \cdot \vec{E} = \partial_j E_j = \partial_j \left(E_{0j} e^{-i(\omega t - \vec{k} \cdot \vec{r})} \right)$$

$$= E_{0j} \partial_j e^{-i(\omega t - \vec{k} \cdot \vec{r})} = i E_{0j} k_j e^{-i(\omega t - \vec{k} \cdot \vec{r})}$$

$$= i\vec{E} \cdot \vec{k} = 0 \qquad \therefore \vec{k} \perp \vec{E}$$

由样 $\quad \nabla \cdot \vec{H} = i\vec{H} \cdot \vec{k} = 0 \qquad \Rightarrow \qquad \vec{k} \perp \vec{H}$

因此$\vec{k} \perp \vec{E}$，$\vec{H}$

$$由\nabla\times\vec{E} = -\mu\frac{\partial\vec{H}}{\partial t}$$

$$\left(\nabla\times\vec{E}\right)_k = \mathcal{E}_{klm}\partial_l E_m = \mathcal{E}_{klm}\partial_l[E_{0m}e^{-i(\omega t-\vec{k}\cdot\vec{r})}]$$

$$= \mathcal{E}_{klm}\partial_l\left[e^{-i\left(\omega t-\vec{k}\cdot\vec{r}\right)}\right]E_{om}$$

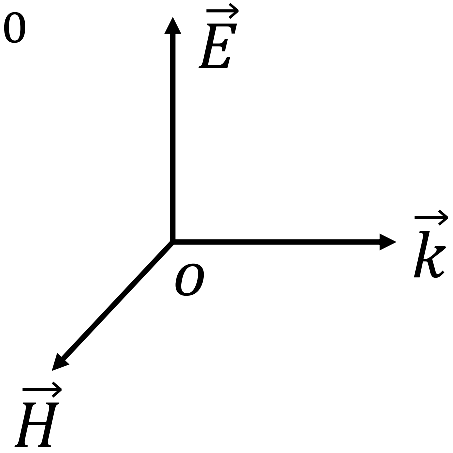
$$= i\mathcal{E}_{klm}k_l E_{0m}e^{-i(\omega t-\vec{k}\cdot\vec{r})}$$

$$\therefore\ \nabla\times\vec{E} = i\vec{k}\times\vec{E}\quad\Rightarrow i\vec{k}\times\vec{E} = i\omega\mu\vec{H}$$

$$\frac{\partial\vec{H}}{\partial t} = -i\omega\vec{H}$$

$$\therefore \vec{k}\times\vec{E} = \omega\mu\vec{H} \Rightarrow kE_0 = \omega\mu H_0 \Rightarrow \sqrt{\epsilon}E_0 = \sqrt{\mu}H_0$$

$$\Rightarrow B_0 = \frac{E_0}{u} \qquad \Rightarrow D_0E_0 = B_0H_0$$



同样：由$\nabla\times\vec{H} = \epsilon\dfrac{\partial\vec{E}}{\partial t}$ $\boxed{真空B = \dfrac{E}{c}}$

$$\vec{k}\times\vec{H} = -\epsilon\omega\vec{E}$$

$$\therefore (\vec{E} \perp \vec{H}, \qquad \vec{k} \perp \vec{E},\vec{H})$$

$\vec{k}$，$\vec{E}$，$\vec{H}$构成右手正交关系

$$\vec{E}\times\vec{H} = \frac{1}{\omega\mu}\vec{E}\times\left(\vec{k}\times\vec{E}\right) = \frac{1}{\omega\mu}\left(\vec{E}\cdot\vec{E}\right)\vec{k} - \left(\vec{E}\cdot\vec{k}\right)\vec{E}$$ 其中$\vec{E}\cdot\vec{k}$=0

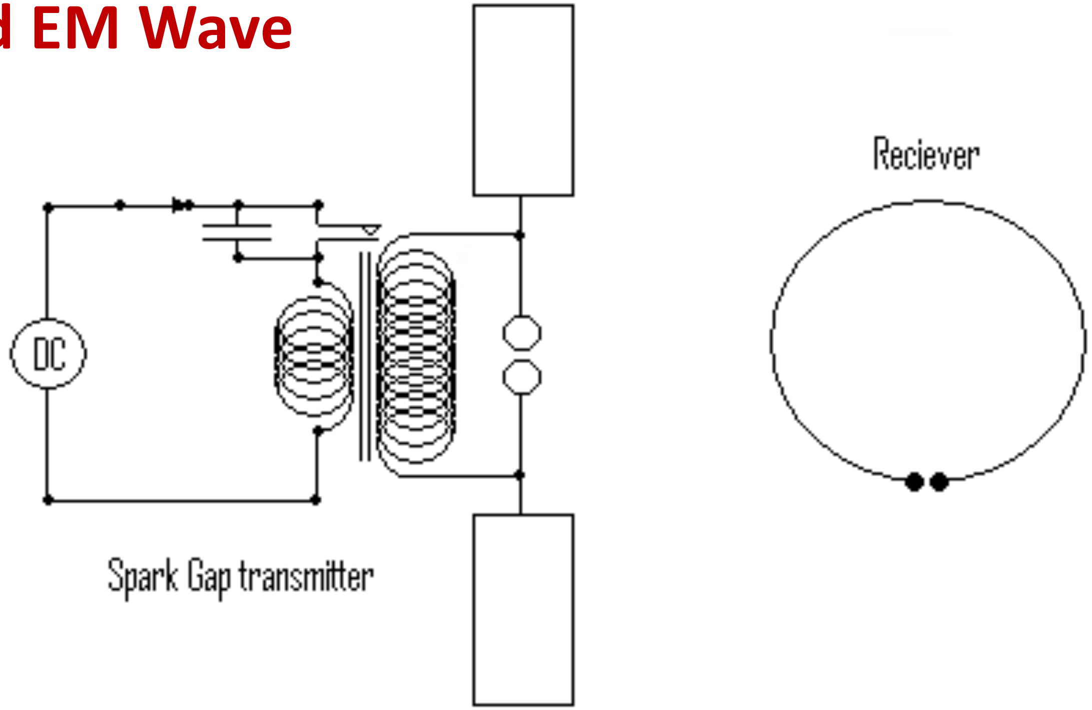
能流密度
Poynting Vector
坡印亭矢量

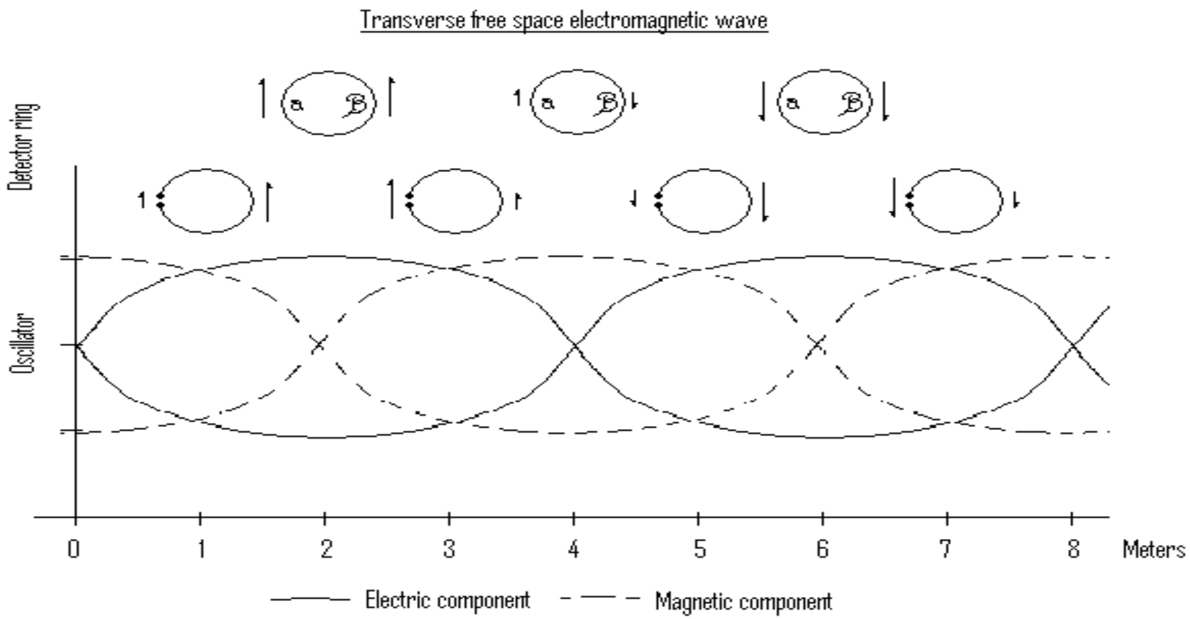
$$= \frac{\vec{E}\cdot\vec{E}}{\omega\mu}\vec{k} = \frac{\vec{E}\cdot\vec{E}}{ku\mu}\vec{k} = \left(\vec{D}\cdot\vec{E}\right)\cdot u\cdot\hat{k} \qquad u = \frac{1}{\sqrt{\epsilon\mu}}$$

能量密度 波速 $\vec{D} = \epsilon\vec{E}$

1886 Hertz Discovered EM Wave

Heinrich Rudolf Hertz
22 February 1857
Hamburg, German
Confederation





电磁波谱

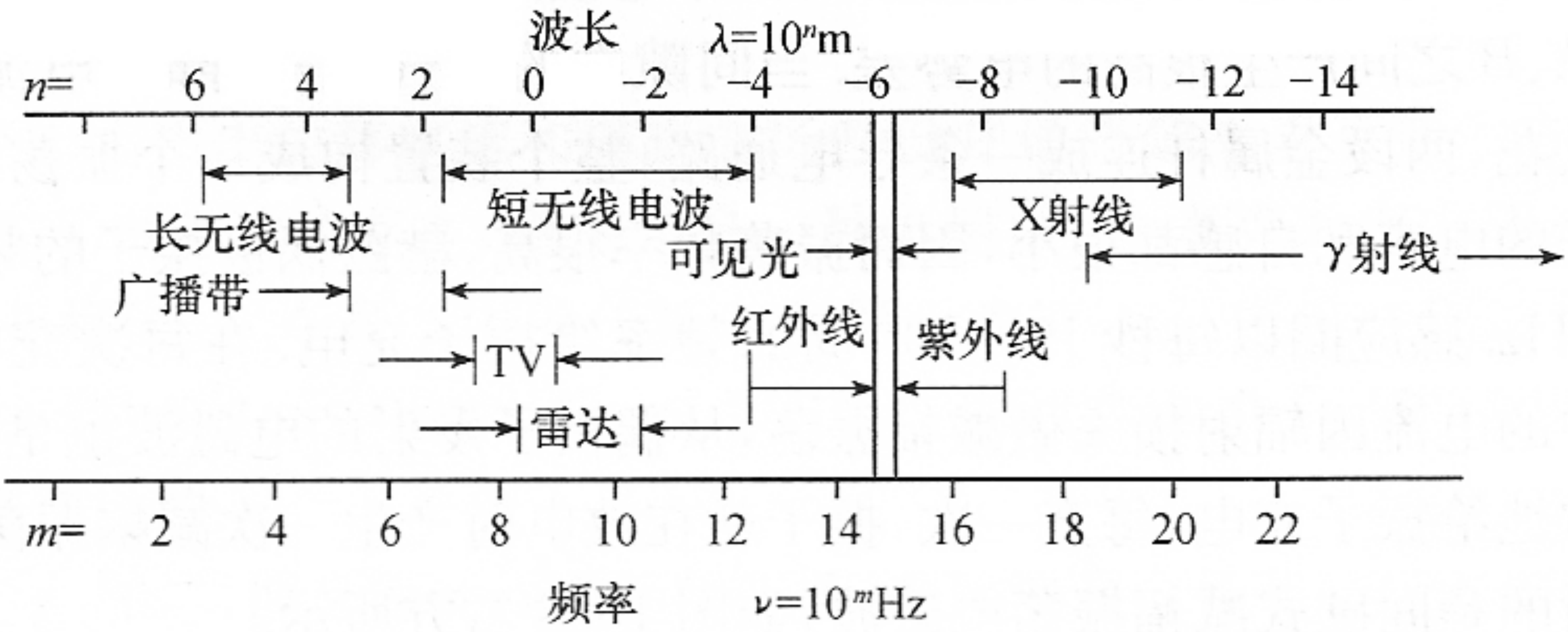


图 10.10　电磁波谱

电磁场的能量、动量和角动量

胡 10.3　　费chapter 27

Local Conservation

与相对论兼容的守恒定律：Local Conservation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0$$

电荷守恒

$$Q_1 + Q_2 = constant$$

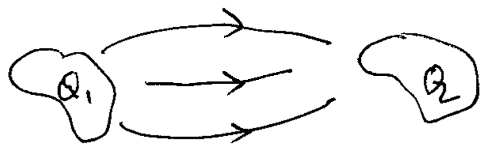
Energy Conservation

$$-\frac{\partial U}{\partial t} = \nabla \cdot \vec{S} + \vec{E} \cdot \vec{j}$$

$\Uparrow$ 空间某点能量减少

$\Uparrow$ 流出的能量

$\Uparrow$ 对介质做的功



$\vec{j}$

$$\frac{\partial Q}{\partial t} + \nabla \cdot \vec{j} = 0$$

$$-\frac{\partial\mu}{\partial t} = \nabla\cdot\vec{S} + \vec{E}\cdot\vec{j}$$

$\Uparrow$ 空间某点能量减少 $\Uparrow$ 流出的能量 $\Uparrow$ 对介质做的功

$$\begin{cases}\nabla\cdot\vec{E} = \dfrac{\rho}{\epsilon_0}\\ \nabla\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}\\ \nabla\cdot\vec{B} = 0\\ \nabla\times\vec{B} = \mu_0(\vec{j}+\epsilon_0\dfrac{\partial\vec{E}}{\partial t})\end{cases}$$

$$\vec{E}\cdot\vec{j} = \vec{E}\cdot\left(\frac{\nabla\times\vec{B}}{\mu_0} - \epsilon_0\frac{\partial\vec{E}}{\partial t}\right)$$

$$= \frac{1}{\mu_0}\vec{E}\cdot\left(\nabla\times\vec{B}\right) - \epsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

$$\partial_i\epsilon_{ijk}E_jB_k$$
$$= \epsilon_{ijk}\partial_i(E_jB_k)$$
$$= \epsilon_{ijk}\left(\partial_iE_j\right)B_k + \epsilon_{ijk}E_j\partial_iB_k$$
$$= B_k\epsilon_{kij}\partial_iE_j + E_j(-\epsilon_{jik}\partial_iB_k)$$
$$= \vec{B}\cdot\left(\nabla\times\vec{E}\right) - \vec{E}\cdot(\nabla\times\vec{B})$$

$$\nabla\cdot\left(\vec{E}\times\vec{B}\right) = -\vec{E}\cdot\left(\nabla\times\vec{B}\right) + \vec{B}\cdot(\nabla\times\vec{E})$$

代入上式

$$\vec{E}\cdot\vec{j} = -\frac{1}{\mu_0}\nabla\cdot\left(\vec{E}\times\vec{B}\right) + \frac{1}{\mu_0}\vec{B}\cdot\left(\nabla\times\vec{E}\right) - \epsilon_0\vec{E}\cdot\frac{\partial\vec{E}}{\partial t}$$

$$= -\frac{1}{\mu_0}\nabla\cdot\left(\vec{E}\times\vec{B}\right) - \frac{1}{\mu_0}\vec{B}\cdot\frac{\partial\vec{B}}{\partial t} - \frac{\epsilon_0\vec{E}\cdot\vec{E}}{2}$$

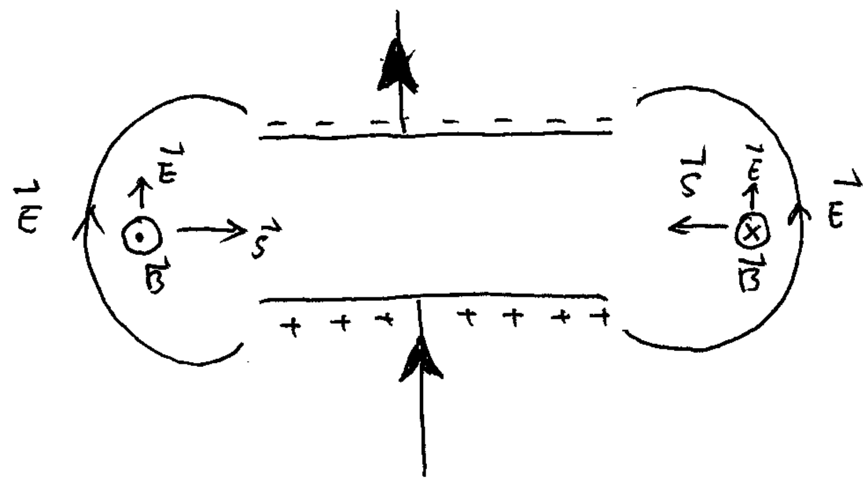
$$= -\nabla\cdot\left[\frac{\vec{E}\times\vec{B}}{\mu_0}\right] - \frac{\partial}{\partial t}[\frac{\vec{B}\cdot\vec{B}}{2\mu_0} + \frac{\epsilon_0\vec{E}\cdot\vec{E}}{2}]$$

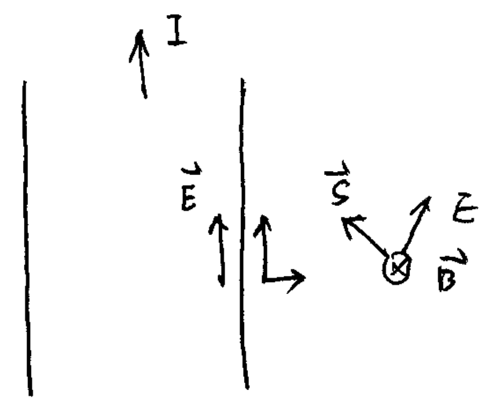
$$= -\nabla\cdot\vec{S} - \frac{\partial u}{\partial t}$$

$$\Rightarrow\begin{cases}u = \dfrac{\vec{B}\cdot\vec{B}}{2\mu_0} + \dfrac{\epsilon_0\vec{E}\cdot\vec{E}}{2} = \dfrac{1}{2}(\vec{E}\cdot\vec{D} + \vec{B}\cdot\vec{H})\\ \vec{S} = \dfrac{\vec{E}\times\vec{B}}{\mu_0} = \vec{E}\times\vec{H}\end{cases}$$

例：

1. 电容充电
2. 导线





电磁场的能量、动量、角动量

能量密度 $$w=\frac{1}{2}\vec{E}\cdot\vec{D}+\frac{1}{2}\vec{H}\cdot\vec{B}$$

能流密度 $$\vec{S}=\vec{E}\times\vec{H}$$

动量密度 $$\vec{g}=\vec{D}\times\vec{B}$$

角动量密度 $$\vec{l}=\vec{r}\times\vec{g}$$

平面电磁波的能量、动量、角动量

能量密度	$w = \varepsilon\vec{E}\cdot\vec{E} = \mu\vec{H}\cdot\vec{H}$
能流密度	$\vec{S} = w\vec{v}$
动量密度	$\vec{g} = \dfrac{\vec{S}}{v^2}$
角动量密度	$\vec{l} = \vec{r}\times\vec{g}$
光子的能量	$W = h\nu$
光子的动量	$W = \hbar\vec{k}$

演示实验　　光压

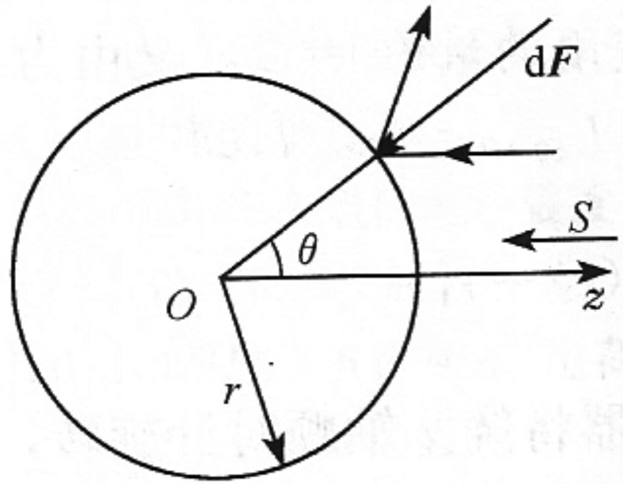
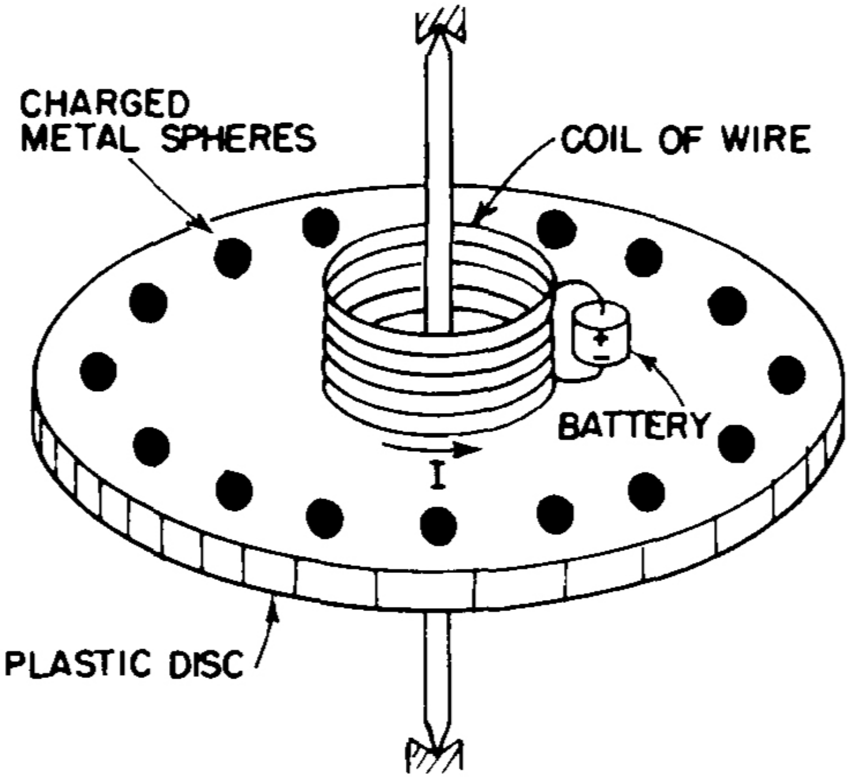


图 10.12　平行光束给球面的总压力

电磁场的角动量

$$\vec{L} = \vec{r}\times\vec{p}$$



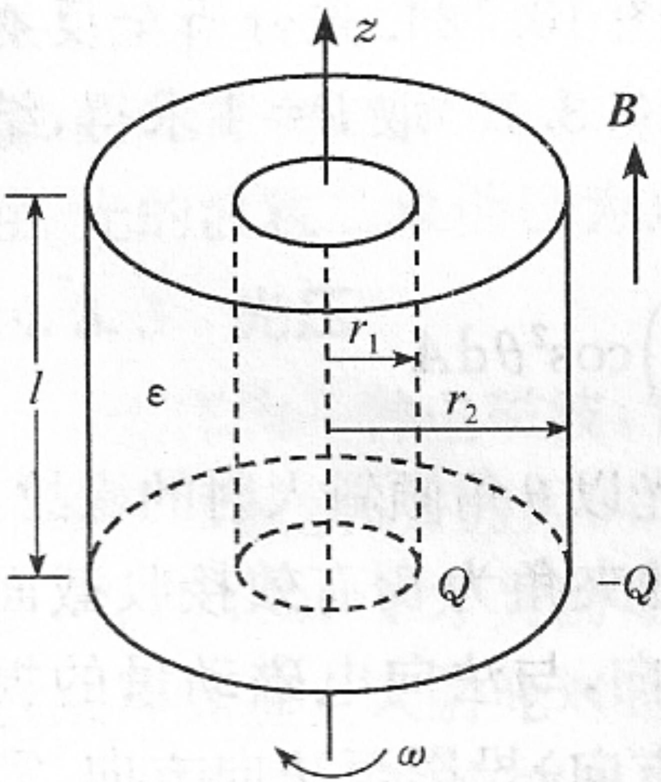


图 10.13　轴向均匀磁场中的圆柱电容器

演示实验　趋肤效应

$$\nabla\cdot\vec{D}=\rho_0 \qquad \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \qquad \nabla\cdot\vec{B}=0 \qquad \nabla\times\vec{H}=\vec{J}_0+\frac{\partial\vec{D}}{\partial t}$$

良导体中， $\rho_0\approx 0$ ➡ $\nabla\cdot\vec{D}=0$

各向同性线性介质，$\vec{D}=\varepsilon\vec{E}$ $\quad \vec{H}=\frac{\vec{B}}{\mu}$ $\quad \vec{J}_0=\sigma\vec{E}$

➡ $$\nabla\cdot\vec{E}=0 \qquad \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t} \qquad \nabla\cdot\vec{B}=0 \qquad \nabla\times\vec{B}=\mu\sigma\vec{E}+\mu\varepsilon\frac{\partial\vec{E}}{\partial t}$$

➡ $\nabla^2\vec{E}=\mu\sigma\frac{\partial\vec{E}}{\partial t}+\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$ 考虑一维平面波解 $\vec{E}(\vec{x},t)=\vec{E}_0e^{i(kx-\omega t)}$

$k^2=i\mu\sigma\omega+\mu\varepsilon\omega^2$ 　令 $k=k_1+ik_2$

$$k_{1,2}=\omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+\left(\frac{\sigma}{\varepsilon\omega}\right)^2}\pm 1\right]^{1/2} \qquad \vec{E}(\vec{x},t)=\vec{E}_0e^{-k_2x}e^{i(k_1x-\omega t)}$$

趋肤效应

$$\nabla^2\vec{E}=\mu\sigma\frac{\partial\vec{E}}{\partial t}+\mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$ 考虑一维平面波解 $\vec{E}(\vec{x},t)=\vec{E}_0e^{i(kx-\omega t)}$

$$k^2=i\mu\sigma\omega+\mu\varepsilon\omega^2$$ 令 $k=k_1+ik_2$

$$k_{1,2}=\omega\sqrt{\frac{\varepsilon\mu}{2}}\left[\sqrt{1+(\frac{\sigma}{\varepsilon\omega})^2}\pm1\right]^{1/2}$$ $\vec{E}(\vec{x},t)=\vec{E}_0e^{-k_2x}e^{i(k_1x-\omega t)}$

良导体 $\sigma\gg\varepsilon\omega$ $\quad k_1=k_2=\sqrt{\frac{\mu\sigma\omega}{2}}$

趋肤深度 $\quad\delta\equiv\frac{1}{k_2}=\sqrt{\frac{2}{\mu\sigma\omega}}$ $\quad$ Cu，10^{14}Hz，10nm