4. (1). $\int_0^\infty A v^2 \cdot dv = \int_0^{v_0} A v^2 \cdot dv = \frac{A}{3} v_0^3 = 1.$

$A = \frac{3}{v_0^3}$

(2). $\bar{v} = \int_0^{v_0} A v^2 \cdot v \cdot dv = A \cdot \frac{v_0^4}{4} = \frac{3}{4} v_0.$

$\bar{v^2} = \int_0^{v_0} A v^2 \cdot v^2 \cdot dv = A \cdot \frac{v_0^5}{5} = \frac{3}{5} v_0^2.$

$\sqrt{\bar{v^2}} = \sqrt{\frac{3}{5}} v_0.$

1. (1). $\frac{n_1}{n_0} = \frac{P}{P_0}.$

$\sqrt{\bar{v^2}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3RT}{\mu}}.$

进入细管 n_0 分子数

$\frac{1}{dt}$ 时间

$dN = \frac{1}{4} n \bar{v} \cdot dt.$

$d(n \cdot V) = dN$

$dn \cdot V = \frac{1}{4} n \cdot \sqrt{\frac{8RT}{\pi \mu}} \cdot dt.$

$\int_{n_0}^{n_1} \frac{-dn}{n} = \int \frac{1}{4V} \sqrt{\frac{8RT}{\pi \mu}} \cdot dt.$

$= \ln \frac{n_0}{n_1} = \ln \frac{P}{P_0}.$

$t = 4V \cdot \sqrt{\frac{\pi \mu}{8RT}} \ln \frac{P}{P_0}.$

(3) 在压强为 P 时刻

$dN = \frac{1}{4} n_1 \bar{v} dt.$

$PV = n_1 V \cdot kT.$

$\rightarrow n_1 = \frac{P}{kT}$

$\Delta N = \frac{P}{4kT} \sqrt{\frac{8kT}{\pi m}} = \frac{P}{4} \sqrt{\frac{8}{\pi m kT}}$

$\Delta Q = \frac{1}{2} \Delta N \cdot \bar{v^2}$

$= \frac{1}{2} \cdot \frac{P}{2} \sqrt{\frac{2}{\pi m kT}} \cdot \frac{3kT}{m}$

$= \frac{3P}{4m} \sqrt{\frac{2kT}{\pi m}}$

2. (1). $PV = nkT$

$n = \dfrac{PV}{kT} = 7.25 \times 10^{22}$ 个.

(2). $\bar{v} = \sqrt{\dfrac{8kT}{\pi m}} = \sqrt{\dfrac{8RT}{\pi \mu}} = 12.6\, m/s.$

(3). $N = \dfrac{1}{4} \cdot n \cdot \bar{v} \cdot A = \dfrac{1}{4} \cdot \dfrac{n}{V} \cdot \bar{v} \cdot A = 2.28 \times 10^{7}$

(4). $dn = N \cdot dt = \dfrac{1}{4} \dfrac{n}{V} \cdot \bar{v} \cdot A \cdot dt$

$\displaystyle \int_{\frac{n_0}{e}}^{n_0} \dfrac{4V}{\bar{v} \cdot A} \cdot \dfrac{dn}{n} = \int dt$

$= \dfrac{4V}{\bar{v} \cdot A} \cdot \ln e = 3.17 \times 10^{5} s.$

(5). $Q = \dfrac{1}{2} n m \bar{v}^2 = \dfrac{1}{2} n \cdot m \cdot \dfrac{3kT}{m} = \dfrac{3nkT_0}{2} = \dfrac{3nk}{2} T.$

$dN = \dfrac{1}{4} \dfrac{n}{V} \sqrt{\dfrac{8RT}{\pi \mu}} \cdot A \cdot dt$

$T = \dfrac{n_0}{n} T_0.$

$dn = \dfrac{n}{4V} \sqrt{\dfrac{8RT_0 n_0}{\pi \mu n}} \cdot A \cdot dt.$

$= \dfrac{1}{4V} \sqrt{\dfrac{8RT_0 n_0 n}{\pi \mu}} A \cdot dt.$

$\displaystyle \int_{\frac{n_0}{e}}^{n_0} \dfrac{4V}{A} \sqrt{\dfrac{\pi \mu}{8RT_0 n_0}} \cdot \dfrac{dn}{\sqrt{n}} = \int dt.$

$= \dfrac{4V}{A} \sqrt{\dfrac{\pi \mu}{8RT_0 n_0}} \cdot 2(\sqrt{n_0} - \sqrt{\dfrac{n_0}{e}})$

$= \dfrac{8V}{A} \sqrt{\dfrac{\pi \mu}{8RT_0}} (1 - \sqrt{\dfrac{1}{e}}).$

$= \dfrac{8V}{A\bar{v}} (1 - \sqrt{\dfrac{1}{e}}) = 2.5 \times 10^{5} s.$