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6.30. A sample of 10 television tubes produced by a company showed a mean lifetime of 1200 hours and a standard deviation of 100 hours. Estimate (a) the mean, (b) the standard deviation of the population of all television tubes produced by this company.

(a) $\mu = \bar{X} = 1200\ h.$ (b) $\sigma = \sqrt{\frac{n}{n-1}}\, s^2 = \frac{10\sqrt{3}}{3}\ h.$

6.33. The mean and standard deviation of the diameters of a sample of 250 rivet heads manufactured by a company are 0.72642 inch and 0.00058 inch, respectively (see Problem 5.99). Find (a) 99%, (b) 98%, (c) 95%, (d) 90% confidence limits for the mean diameter of all the rivet heads manufactured by the company.

(a). $\phi(z) = 1 - \frac{1-0.99}{2} = 0.995. \rightarrow z = 2.58.$

$-2.58 \le \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le 2.58 \rightarrow \bar{X} - 2.58\,\sigma/\sqrt{n} \le \mu \le \bar{X} + 2.58\,\sigma/\sqrt{n}$

$0.72633 \le \mu \le 0.72651$ inch.

(b). $\phi(z) = 0.99. \rightarrow z = 2.33$

$0.72633 \le \mu \le 0.72650.$ inch

(c) $\phi(z) = 0.975. \rightarrow z = 1.96.$

$0.72635 \le \mu \le 0.72649.$ inch

(d) $\phi(z) = 0.95 \rightarrow z = 1.64.$

$0.72636 \le \mu \le 0.72648.$ inch

6.35. If the standard deviation of the lifetimes of television tubes is estimated as 100 hours, how large a sample must we take in order to be (a) 95%, (b) 90%, (c) 99%, (d) 99.73% confident that the error in the estimated mean lifetime will not exceed 20 hours.

(a) $\phi(z) = 0.975. \rightarrow z = 1.96.$

$\bar{X} - z\cdot\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{X} + z\cdot\frac{\sigma}{\sqrt{n}}. \rightarrow 2z\frac{\sigma}{\sqrt{n}} \le 20. \quad n \ge \left(\frac{z\sigma}{10}\right)^2$

$n \ge 385$

(b). $\phi(z) = 0.95.\ z = 1.64$

$n \ge 269$

(c) $\phi(z) = 0.995.\ z = 2.58.$

$n \ge 666$

(d). $\phi(z) = 0.99865.\ z = 2.998.$

$n \ge 899$

6.39. Five measurements of the reaction time of an individual to certain stimuli were recorded as 0.28, 0.30, 0.27, 0.33, 0.31 second. Find (a) 95%, (b) 99% confidence limits for the actual mean reaction time.

$n = 5. \quad \bar{X} = \frac{1}{5}(0.28 + 0.3 + \cdots + 0.31) = 0.298.$ s

$\hat{S}^2 = \frac{1}{4}(0.018^2 + 0.002^2 + \cdots) = 0.00057$

(a). $F_4(c) = 0.975. \quad c = 2.2$

$\bar{X} - \frac{c\hat{S}}{\sqrt{n}} \le \mu \le \bar{X} + \frac{c\hat{S}}{\sqrt{n}}.$

$\rightarrow 0.2745 \le \mu \le 0.3215$ s

6.41. How large a sample of marbles should one take in Problem 6.40 in order to be (a) 95%, (b) 99%, (c) 99.73% confident that the true and sample proportions do not differ more than 5%?

(a). $\phi(z) = 0.975.\ z = 1.96.$

$-1.96 \le \frac{X_n - np}{\sqrt{npq}} \le 1.96, \quad X_n = 60 \times 70\% = 42.\ n = 60$

$\left(\frac{X_n}{n} - p\right)^2 \le 5\%^2 = 0.0025$

$\rightarrow \frac{\left(\frac{X_n}{n} - p\right)^2}{\frac{p(1-p)}{n}} \le 1.96^2 \le \frac{0.0025n}{p(1-p)}$

$n \ge 7318.$

(b) $\phi(z) = 0.995.\ z = 2.58$

$n \ge 12679$

(c) $\phi(z) = 0.99865\ z = 2.998$

$n \ge 17120.$

6.46. The standard deviation of the breaking strengths of 100 cables tested by a company was 1800 lb. Find (a) 95%, (b) 99%, (c) 99.73% confidence limits for the standard deviation of all cables produced by the company.

(a) $\frac{0.05}{2} = F_{99}(X_1), \quad X_1 = 73.361$

$1 - \frac{0.05}{2} = 0.975 = F_{99}(X_2), X_2 = 128.422, \quad \hat{S}\sqrt{\frac{(n-1)}{X_2}} \le \sigma \le \hat{S}\sqrt{\frac{n-1}{X_1}}.$

$1580.412 \le \sigma \le 2091.017.$ lb.

(b). $X_1 = 66.51, \quad X_2 = 138.987.$

$1519.158 \le \sigma \le 2196.072.$ lb

(c). $X_1 = 62.057 \quad X_2 = 146.581. \quad 1479.283 \le \sigma \le 2273.499$ lb

6.57. A population has a density function given by

$$f(x) = \begin{cases} (k+1)x^k & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

For n observations $X_1, \ldots, X_n$ made from this population, find the maximum likelihood estimate of k.

$L = f(x_1, k) f(x_2, k) \cdots f(x_3, k) = (k+1)^n x^{nk}$, $0 \le x \le 1$.

$\frac{dL}{dk} = n(k+1)^{n-1} x^{nk} + (k+1)^n \ln x \cdot x^{nk} \cdot n = 0$.

$k = \frac{1}{\ln x} - 1 \quad 0 < x \le 1$.