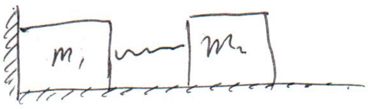
~~能~~动量

1. $dp = dm(u+v) = F dt$

$F = (u+v)\dfrac{dm}{dt}$

2. $dp = dm\cdot v = F\cdot dt$

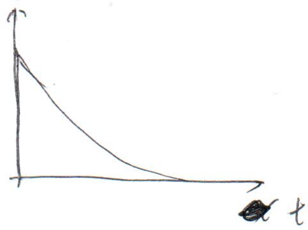
$F = \dfrac{dm}{dt}\cdot v$

3. 

$U_0 = \frac{1}{2}k(\Delta x)^2 = \frac{1}{2}k(\frac{L}{2})^2$ $F_{ext} = (m_1+m_2)\cdot a_{cm}$

m_1离开墙后 $F_{ext} = 0 = (m_1+m_2)\cdot a_{cm}$ $a_{cm} = 0$.

$T = k\cdot\Delta x$

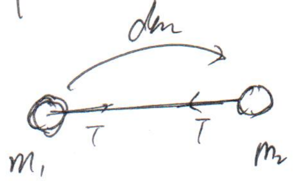


a_{max}发生在 $t=0$. V_{max}在 $a=0$ 时

$\dfrac{kL}{2} = (M+m)\,a_{max}$ $a_{max} = \dfrac{Lk}{2}\cdot\dfrac{1}{M+m}$

$U_0 = \frac{1}{2}m_2 v^2$ $V_{max} = \dfrac{m_2 V}{m_1+m_2} = \dfrac{m_2}{m_1+m_2}\cdot\sqrt{\dfrac{k}{m_2}}\cdot\dfrac{L}{2}$.

4. 作业题

5. 

$dm = S\cdot u\cdot dt\cdot\rho_水$.

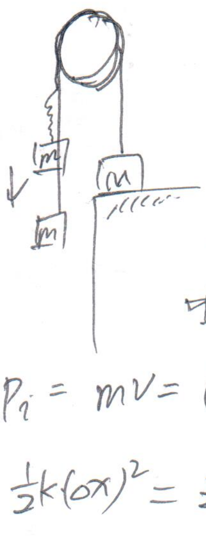
$m_L = m_1 + m - Su\rho_水 t$ $(Su\rho_水 t < m)$.

S 

$a = \dfrac{F}{m_1+m_2+m} = \dfrac{dv}{dt}$

$T\cdot dt - dm\cdot u = m_L\cdot dv$ $T = u\cdot\dfrac{dm}{dt} + m_L\cdot\dfrac{dv}{dt}$

$= Su^2\rho_水 + (m_1+m-Su\rho_水 t)\cdot\dfrac{F}{m_1+m_2+m}$ (for $t<\dfrac{m}{Su\rho}$)

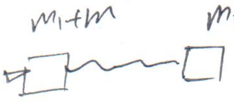
6. 

$$\begin{cases} \frac{1}{2}mv^2 = mgh \\ mv = (M+m)\cdot v' \\ (M-m)\cdot g\Delta h = \frac{1}{2}(M+m)\cdot v'^2 \end{cases}$$

$v = \sqrt{2gh}$

$v' = \dfrac{mv}{M+m}$

$\Delta h = \dfrac{m^2}{(M-m)\cdot(M+m)}\cdot h$



弹簧最短时/最长时速度一样.

7. $$\begin{cases} P_i = mv = (m_1+m_2+m)\cdot v_f \\ \frac{1}{2}k(\Delta x)^2 = \underbrace{\quad}_{E_{Ki}} - \frac{1}{2}(m_1+m_2+m)\cdot v_f^2 = \dfrac{P_i^2}{2(m_1+m)} - \dfrac{P_i^2}{2(m_1+m_2+m)} = \dfrac{m^2v^2}{2}\left(\dfrac{1}{m_1+m} - \dfrac{1}{m_1+m_2+m}\right) \end{cases}$$

$E_{Ki} = \frac{1}{2}(m_1+m)\cdot v_i^2 = \dfrac{P_i^2}{2(m_1+m)}$ $(m_1+m)\cdot v_i = mv$

$$\begin{cases} mu + MV = 0 & u = -\frac{MV}{m} \\ \frac{1}{2}mu^2 + \frac{1}{2}MV^2 = E & \frac{1}{2}\frac{M^2V^2}{m} + \frac{1}{2}MV^2 = E \end{cases}$$

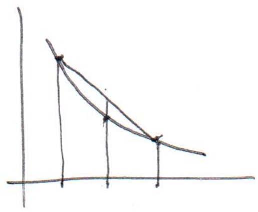
$$V = \sqrt{\frac{2Em}{(M+m)\cdot M}} = \sqrt{2E}\cdot\sqrt{\frac{1}{M} - \frac{1}{M+m}}$$

a) $m \to 10m$ $\quad E \to 10E$ $\quad V = \sqrt{20E}\cdot\sqrt{\frac{1}{M} - \frac{1}{M+10m}}$

b) $M \to (M + (n-1)m)$

$$\Delta V_n = \sqrt{2E}\cdot\sqrt{\frac{1}{M+(n-1)m} - \frac{1}{M+nm}}$$

琴生不等式



凹: $f\left(\frac{\sum_i^N x_i}{N}\right) \leq \frac{\sum_i f(x_i)}{N}$

凸: $f\left(\frac{\sum_i^N x_i}{N}\right) \geq \frac{\sum_i f(x_i)}{n}$

$f(x)=x^2$ 凹 $\quad \left(\frac{\sum_i^N x_i}{N}\right)^2 \leq \frac{\sum_i x_i^2}{N} \qquad \sqrt{\frac{\sum_i x_i^2}{N}} \geq \frac{\sum_i^N x_i}{N}$

$$\therefore \frac{\sum_{n=1}^{10} \Delta V_n}{10} = \frac{\Delta V_{tot}}{10} \leq \sqrt{\frac{1}{10}\cdot 2E\left(\frac{1}{M+(n-1)m} - \frac{1}{M+nm}\right)\cdots}$$

$$= \sqrt{\frac{2E}{10}}\cdot\sqrt{\frac{1}{M} - \frac{1}{M+10m}}$$

$$\Delta V_{tot} \leq \sqrt{20E}\cdot\sqrt{\frac{1}{M} - \frac{1}{M+10m}} = V$$

$$\begin{cases} (m-dm)(v+dv) + dm\,u = mv \\ \frac{1}{2}(m-dm)(v+dv)^2 + \frac{1}{2}dm\,u^2 = \frac{1}{2}mv^2 + c\,dm \end{cases}$$

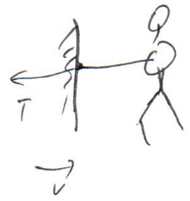
$$\frac{1}{2}u^2 = c + \frac{1}{2}v^2 - mv\cdot\frac{dv}{dm}$$

$$m\cdot dv - v\cdot dm + dm\,u = 0$$

$$\frac{1}{2}m\cdot 2v\cdot dv - \frac{1}{2}dm\cdot v^2 + \frac{1}{2}dm\,u^2 = c\cdot dm$$

$$m\cdot\frac{dv}{dm} - v + \sqrt{2C + v^2 - 2mv\cdot\frac{dv}{dm}} = mv.$$

9. 岸：不作　　水：作



2. 都不作

10. 地面：$\frac{m}{2}(u+v)^2 - \frac{m}{2}u^2$　　$u, v > 0.$

$\frac{m}{2}(u-v)^2 - \frac{m}{2}u^2$

一次作正功，另一次不一定作负功.

船：均为正功

两种可能 看这个人会不会爬 ☺　参考答案有误？

11.

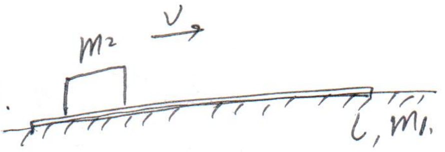
$\{ w = mgb$ （不会爬的）

轻质弹簧，任意点拉力相同

$\begin{cases} w = mgb - \frac{1}{2}k(b-a)^2 \\ k(b-a) = mg \quad k = \frac{mg}{b-a} \end{cases}$　　$w = mgb - \frac{1}{2}\cdot mg(b-a)$

$= \frac{mg}{2}(a+b).$

12.



参考答案有误？

1). $m_2 v = (m_1+m_2)v_f$

$\frac{1}{2}m_2v^2 - \frac{1}{2}(m_1+m_2)v_f^2 = f\cdot l$　　$f = \frac{1}{2l}\left(m_2v^2 - \frac{m_2^2v^2}{m_1+m_2}\right) = \frac{1}{2l}\cdot\frac{m_2^2v^2\cdot m_1}{m_2(m_1+m_2)}$

2). $\cancel{f = m_1 a \quad a = \frac{f}{m_1}}$　　$f_1\cdot\Delta l_1 = \frac{1}{2}m_1\cdot v_f^2$

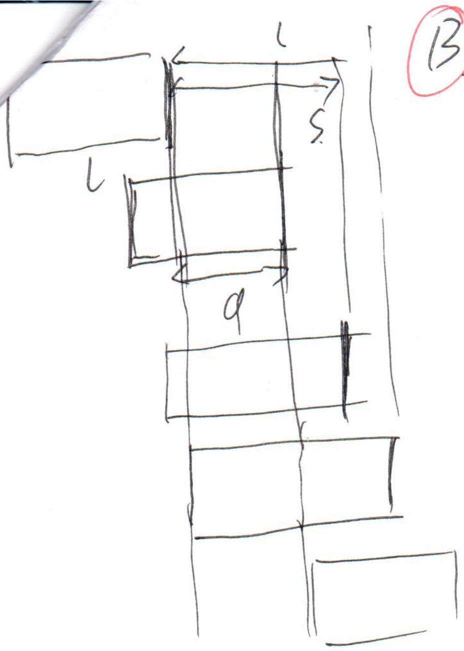
$\Delta l_1 = \frac{1}{2}\cancel{m_1}\cdot\frac{\cancel{m_2^2v^2}}{(m_1+m_2)^2}\cdot 2l\cdot\frac{\cancel{(m_1+m_2)}m_2}{\cancel{m_2^2v^2}\cdot\cancel{m_1}} = \frac{m_2}{m_1+m_2}\cdot l$

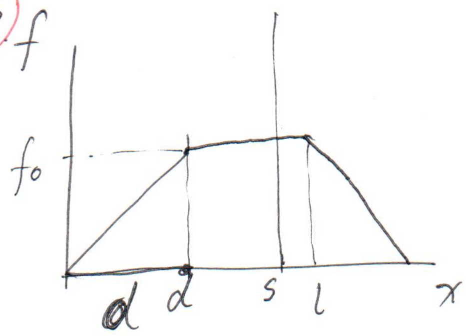
3.

$f\Delta l_2 = \frac{1}{2}m_2v^2 - \frac{1}{2}m_2\left(\frac{m_2}{m_1+m_2}\right)^2v^2$

$l_2 = \frac{(m_1+2m_2)l}{m_1+m_2}$

13





$\because s<l$

$$f_0=\mu mg\frac{d}{l}$$

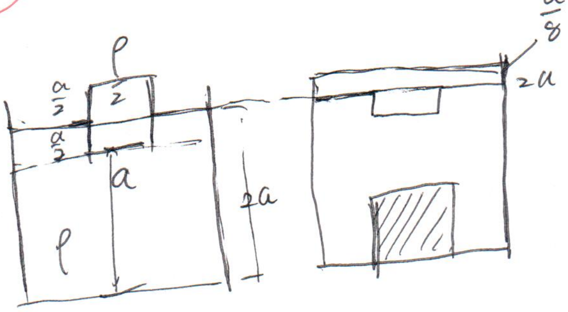
$$\frac{1}{2}f_0 d+(s-d)\cdot f_0=E_K$$

$$\left(s-\frac{d}{2}\right)f_0=\frac{1}{2}mv^2$$

$$\left(s-\frac{d}{2}\right)\cdot\mu mg\cdot\frac{d}{l}=\frac{1}{2}mv^2$$

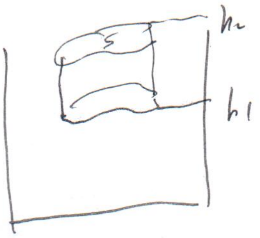
$$\mu=\frac{\frac{v^2}{2}l}{gd\left(s-\frac{d}{2}\right)}=\frac{lv^2}{gd(2s-d)}$$

14.



$$\rho g V_{排}=\frac{\rho}{2}\cdot V\cdot g \quad V_{排}=\frac{V}{2}$$

$$\int_{h_1}^{h_2}dm\,gh=\int_{h_1}^{h_2}\rho s\,dh\cdot gh=\rho sg\frac{h_2^2-h_1^2}{2}$$ (质心)



$$U_{wi}=\rho\cdot a^2\cdot g\cdot\frac{a^2}{2}=\frac{1}{2}\rho g a^4 \qquad \Delta U_{wood}=\frac{3a}{2}\cdot\frac{\rho}{2}a^3\cdot g=\frac{3}{4}\rho g a^4$$

$$U'_{wf}=\rho\cdot a^2\cdot g\cdot\frac{(2a)^2-\left(\frac{3a}{2}\right)^2}{2}=\frac{7}{8}\rho g a^4$$

$$\frac{a}{2}\cdot a^2=\Delta h\cdot 4a^2 \quad \Delta h=\frac{a}{8}$$

$$U^2_{wf}=\rho\cdot 4a^2\cdot g\cdot\frac{\left(2a+\frac{a}{8}\right)^2-(2a)^2}{2}=\frac{33}{32}\rho g a^4$$

$$W=\left(\frac{33}{32}+\frac{7}{8}-\frac{1}{2}-\frac{3}{4}\right)\rho g a^4=\frac{21}{32}\rho g a^4$$

15.

$$\Delta E=\cancel{mgh}\ wgh$$

广义坐标的动量守衡

$$wv=(2p+w)u=P$$
$$\frac{1}{2}wv^2+\frac{1}{2}(2p+w)u^2=\Delta E$$

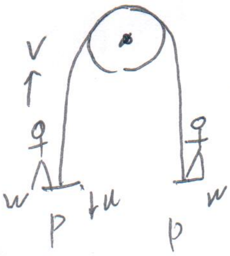
$$\frac{P^2}{2w}+\frac{P^2}{2(2p+w)}=\Delta E$$

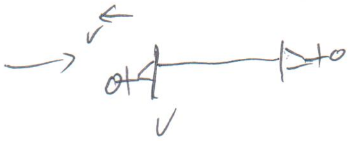
$$\left(E_K+\frac{w}{2p+w}\cdot E_K\right)=\Delta E$$

$$E_K=\frac{2p+w}{2p+2w}\cdot\Delta E$$

$$\therefore h'=\frac{2p+w}{2p+2w}h$$

(能达到吗)
动量守衡. 不能

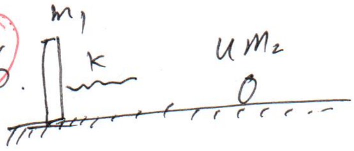




吊 $\downarrow g$ $\quad (2p+w)a = wg \quad a = \dfrac{w}{2p+w} g$

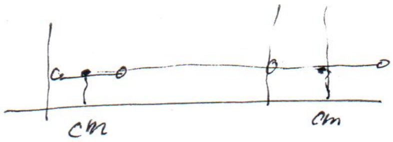
$\dfrac{V}{u} = \dfrac{2p+w}{w} \quad u = \dfrac{w}{2p+w} V$，同时达到静止。

16.



1. $m_2 u = (m_1+m_2) V_{cm}$

$V_{cm} = \dfrac{m_2 u}{m_1+m_2}$



$\frac{1}{2} k \Delta x^2 = \frac{1}{2} m_2 u^2 - \frac{1}{2}(m_1+m_2) V_{cm}^2 = \frac{1}{2} m_2 u^2 \left(1 - \dfrac{m_2}{m_1+m_2}\right)$

$\Delta x^2 = \dfrac{2}{k} \cdot \frac{1}{2} m_2 u^2 \dfrac{m_1}{m_1+m_2} = \dfrac{m_1 m_2 u^2}{k(m_1+m_2)}$

2. $\mu = \dfrac{m_1 m_2}{m_1+m_2} \quad t = \dfrac{T}{4} = \dfrac{\pi}{2} \sqrt{\dfrac{1}{k} \dfrac{m_1 m_2}{m_1+m_2}}$

3. 质心在接触后移动：

$S_{cm} = V_{cm} t = \dfrac{m_2 u}{m_1+m_2} \cdot \dfrac{\pi}{2} \sqrt{\dfrac{1}{k} \dfrac{m_1 m_2}{m_1+m_2}}$

由于被压缩，左球移动小于上面。

$S_1 = S_{cm} - \Delta x \dfrac{m_2}{m_1+m_2}$

代入即得

$S_1 = \left(\dfrac{\pi}{2} - 1\right) \dfrac{m_2 u}{m_1+m_2} \sqrt{\dfrac{1}{k} \dfrac{m_1 m_2}{m_1+m_2}}$

17. $Mv + dm\, u = M(v+dv) + dm \cdot u_f$

$v - u = -\left((v+dv) - u_f\right)$

$u_f = 2v - u + dv \qquad dm = \left(\dfrac{u-v}{u}\right) 6 . dt$

$u\, dm = M dv + dm(2v - u)$

$M \cdot \dfrac{dv}{dt} = 2(u-v) \dfrac{dm}{dt} \qquad \dfrac{1}{(u-v)^2} dv = \dfrac{26}{Mu} dt \qquad \dfrac{1}{u-v} = \dfrac{26t}{mu} + C$

$u - v = \dfrac{1}{\frac{26t}{mu} + C} \quad v = u - \dfrac{1}{\frac{26t}{mu} + C} \qquad v(0) = u - \dfrac{1}{C} = 0 \quad C = \dfrac{1}{u}$

$v = u\left(1 - \dfrac{1}{\frac{26t}{M} + 1}\right) \qquad S(t) = \int_0^t v(t')\, dt'$

$\frac{1}{2} m v^2 = mgr\sin\theta \quad v = \sqrt{2gr\sin\theta}$

$T - mg\sin\theta = 2mg\sin\theta \quad T = 3mg\sin\theta$

$3mg\sin\theta\cos\theta$ ~~$= f < T_2$~~ $= f < T_2$

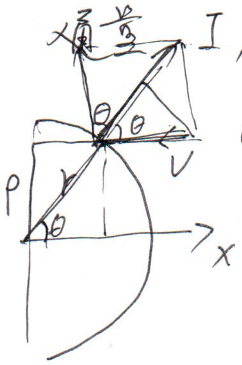
$T_2 = Mg + 3mg\sin^2\theta$

$\frac{3}{2} mg\sin(2\theta) \le Mg + 3mg \dfrac{1-\cos 2\theta}{2} \qquad \dfrac{2M}{3m} + 1 \geq \sin(2\theta) + \cos(2\theta)$

$\theta \in [0, \pi]$

$\frac{2M}{3m}+1 \geqslant \sqrt{2} \quad \frac{M}{m} \geqslant \frac{3\sqrt{2}}{2}-\frac{3}{2}$

19. 刚球散射



$dN = I\cdot dt ds = ds\cdot v\cdot dt\cdot n \quad I = vn$

$ds = 2\pi\rho\, d\rho \quad \rho = a\cdot \sin\theta \quad d\rho = a\cos\theta\cdot d\theta$

$dp = dN\cdot m\cdot \Delta V$.

$\Delta\vec{V} = \cancel{V\cdot\sin\theta}\, 2V\cdot\cos\theta\cdot\vec{e}$

$\Delta V_x = 2V\cos^2\theta$

$\therefore dp = vn\cdot dt\cdot 2\pi \overset{a\sin\theta}{\rho}\, a\cos\theta\cdot d\theta\cdot 2V\cdot\cos^2\theta\cdot m$

$\frac{dp}{dt} = \cancel{v^2 n\cdot 4\pi\rho\cdot\cos^2\theta\, d\theta} \quad 4\pi v^2 n a^2 \sin\theta\cos^3\theta\, d\theta\cdot m$

$\frac{dP_{tot}}{dt} = m4\pi V^2 n a^2\int_0^{\frac{\pi}{2}}\sin\theta\cos^3\theta\, d\theta \quad \Big| \quad -\int_0^{\frac{\pi}{2}}\cos^3\theta\, d\cos\theta = \int_0^1 x^3\, dx = \frac{1}{4}$

$= m\pi V^2 n a^2$

20. 纪书题

1. 关于习题课中出现疑惑较大的一题：16

$$m_1\frac{d^2r_1}{dt^2}=m_1a=-k\Delta l=-k[l_0-(r_2-r_1)]$$

$$m_2\frac{d^2r_2}{dt^2}=m_2a=k\Delta l=k[l_0-(r_2-r_1)]$$ 令 $l_0+r_1=r_1'$ $\frac{d^2r_1'}{dt^2}=\frac{d^2r_1}{dt^2}$

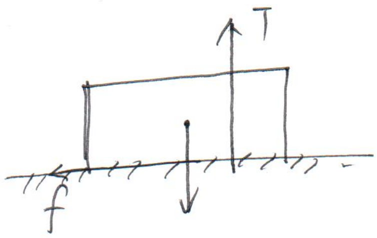
$$\begin{cases} m_1\frac{d^2r_1'}{dt^2}=-k(r_1'-r_2) & ① \\ m_2\frac{d^2r_2}{dt^2}=k(r_1'-r_2) & ② \end{cases}$$

$$\frac{①}{m_1}-\frac{②}{m_2}=\frac{d^2r_1'}{dt^2}-\frac{d^2r_2}{dt^2}=-\frac{k}{m_1}(r_1'-r_2)-\frac{k}{m_2}(r_1'-r_2)$$

$$\Rightarrow \frac{d^2(r_1'-r_2)}{dt^2}=-\left(\frac{k}{m_1}+\frac{k}{m_2}\right)(r_1'-r_2)$$

令 $r_1'-r_2=r$ $\frac{1}{m_1}+\frac{1}{m_2}=\frac{1}{\mu}$ 则 $\frac{d^2r}{dt^2}=-\frac{k}{\mu}r$ $T=2\pi\sqrt{\frac{\mu}{k}}$

2.



一个有趣的想法：讨论课后有同学提出了一个有趣的想法。在13题中，我们讨论了一个滑块减速的过程，现假设这个滑块有一定高度，我们来更仔细的处理一下这个问题，简单分析右图滑块相对于O点（或相对于质心）角动量，$L\equiv0$，为使合外力矩为0，T需偏离重心一点，导致我们在13题中的处理（压力均匀分布）出现问题。讨论这个压力的分布是一件较麻烦的问题（将滑块微元求解一系列方程）有兴趣的同学可以一试

作业题答案勘误

第五次：KK 4.10(2). $\Delta E=-\frac{1}{2}kA_0^2\cdot\left(\frac{m}{m+M}\right)$ 简单的量纲分析可知原来的答案不是能量量纲

第六次：第五题：题中椭圆方程给错了 $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ 而非0.

KK5.4 (a) $U=-3Ax-Ayz+C$

(c) $U=-Ax^3y^5e^{\alpha z}+C$