1. $\nabla\cdot\vec{E} = \frac{1}{r}\frac{\partial(r^2E_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta E_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial E_\phi}{\partial\phi}.$

$= \frac{1}{r}\cdot\frac{\partial(Ar)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\frac{B\sin\theta\cos\varphi}{r})}{\partial\varphi} = \frac{A}{r} - \frac{B\sin\varphi}{r^2}.$

$\rho = \frac{\nabla\cdot\vec{E}}{\varepsilon_0} = \frac{1}{\varepsilon_0}\left(\frac{A}{r} - \frac{B\sin\varphi}{r^2}\right).$

2.

(1). conducting.

$\vec{E}_{in} = \vec{0}.$

$\vec{E}_{out}$: $4\pi r^2\cdot E = Q/\varepsilon_0.$ $\vec{E} = \frac{Q}{4\pi r^2\varepsilon_0}\cdot\hat{r}.$

$$\vec{E}_1 = \begin{cases}\vec{0}. & r<R.\\ \frac{Q}{4\pi r^2\varepsilon_0}\cdot\hat{r} & r>R.\end{cases}$$

(2). $\vec{E}_{in}$: $4\pi r^2\cdot E = \frac{r^3}{R^3}Q/\varepsilon_0.$ $\vec{E} = \frac{Qr}{4\pi R^3\varepsilon_0}\cdot\hat{r}.$

$\vec{E}_{out}$: $4\pi r^2\cdot E = Q/\varepsilon_0.$ $\vec{E} = \frac{Q}{4\pi r^2\varepsilon_0}\cdot\hat{r}.$

$$\vec{E}_2 = \begin{cases}\frac{Qr}{4\pi R^3\varepsilon_0}\hat{r}. & r<R.\\ \frac{Q}{4\pi r^2\varepsilon_0}\cdot\hat{r}. & r>R.\end{cases}$$

(3). $\vec{E}_{in}$: $4\pi r^2\cdot E = \int_0^r\frac{\rho(r)}{\varepsilon_0}\cdot dr.$

$\rho(r) = K\cdot r^n.$ $\int_0^R\rho(r)\cdot dr = \frac{K}{n+1}R^{n+1} = Q.$

$K = \frac{(n+1)Q}{R^{n+1}}.$ $\vec{E} = \frac{\cancel{(n+1)}Q\, r^{n-1}}{4\pi\varepsilon_0 R^{n+1}}\cdot\hat{r}.$

$\vec{E}_{out}$: $4\pi r^2\cdot E = Q/\varepsilon_0$ $\vec{E} = \frac{Q}{4\pi r^2\varepsilon_0}\cdot\hat{r}.$

$$n=-2:\ \vec{E}_{3_1} = \begin{cases}\frac{QR}{4\pi\varepsilon_0 r^3}\cdot\hat{r}. & r<R.\\ \frac{Q}{4\pi r^2\varepsilon_0}\cdot\hat{r}. & r>R.\end{cases}$$

$$n=2:\ \vec{E}_{3_2} = \begin{cases}\frac{Qr}{4\pi\varepsilon_0 R^3}\cdot\hat{r}. & r<R\\ \frac{Q}{4\pi r^2\varepsilon_0}\cdot\hat{r}. & r>R\end{cases}$$

1.14.

$$\int_V (\phi\nabla^2\psi - \psi\nabla^2\phi)\, d^3y = \oint_S \left[\phi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\phi}{\partial n}\right] da$$

$$\int_V \left[G(x,y)\nabla^2 G(x',y) - G(x',y)\nabla^2 G(x,y)\right] d^3y = \oint_S \left[G(x,y)\frac{\partial G(x',y)}{\partial n} - G(x',y)\frac{\partial G(x,y)}{\partial n}\right] da$$

$$\to \int_V \left[-4\pi\delta(x'-y)G(x,y) + 4\pi\delta(x-y)G(x',y)\right] d^3y$$

$$= -4\pi\left[G(x,x') - G(x',x)\right]$$

$$-G(x,x') + G(x',x) = \frac{1}{4\pi}\oint_S \left[G(x,y)\frac{\partial G(x',y)}{\partial n} - G(x',y)\frac{\partial G(x,y)}{\partial n}\right] da.$$

(a). $G_D(x,y) = 0$ for y on S.

$\to G_D(x,x') - G_D(x',x) = 0. \Rightarrow$ symmetric.

(b). $\frac{\partial G_N(x,y)}{\partial n} = -\frac{4\pi}{S}$.

$$\to \frac{1}{4\pi}\oint_S \left[-G(x,y)\cdot\frac{4\pi}{S} + G(x',y)\frac{4\pi}{S}\right] da = \oint_S \frac{G(x',y) - G(x,y)}{S} da.$$

不一定为0，所以不一定会对称.

移项后 $-G_N(x,x') + \frac{1}{S}\oint_S G_N(x,y)\,da = -G_N(x',x) + \frac{1}{S}\oint_S G_N(x',y)\,da.$

所以 $G_N(x,x') - F(x)$ 对称.

(c). $$\Phi(x) = \langle\Phi\rangle_S + \frac{1}{4\pi\varepsilon_0}\int_V \rho(x')\left[G_N(x,x') - F(x)\right] d^3x' + \frac{1}{4\pi}\oint_S \frac{\partial\Phi}{\partial n'}\left[G_N(x,x') - F(x)\right] da'$$

$$= \langle\Phi_S\rangle + \frac{1}{4\pi\varepsilon_0}\int_V \rho(x')G_N(x,x')\,d^3x' + \frac{1}{4\pi}\oint_S \frac{\partial\Phi}{\partial n'}G_N(x,x')\,da'$$

$$\underbrace{+\frac{1}{4\pi\varepsilon_0}F(x)\int_V \rho(x')\,d^3x' - \frac{1}{4\pi}F(x)\oint_S \frac{\partial\Phi}{\partial n'}\,da'}_{} \Rightarrow \frac{F(x)}{4\pi\varepsilon_0}\Big[\int_V \rho(x')\,d^3x' - \oint_S \ldots\Big] = 0$$

(高斯定理)

所以与 $F(x)$ 无关.

扫描全能王 创建