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1. (1) 汽油的质量能量密度 ρ_E 约为 46MJ/kg. 汽油密度 ρ 约为 0.73 g/ml

$$E = V\rho \cdot \rho_E = 1L \cdot 0.73\,kg/L \cdot 46MJ/kg$$
$$\simeq 34MJ$$

(2) 1个 ^{235}U 裂变能产生约 207 MeV 能量. 1kg ^{235}U 有 $N = \frac{10^3}{235} \cdot N_A \simeq 2.56\times10^{24}$ 个原子.

$$E = 207MeV \cdot 2.56\times10^{24}$$
$$\simeq 5.3\times10^{26}MeV \simeq 8.48\times10^{13}J$$

(3) 1L 海水中约有 0.03g D. 即有 $N = \frac{0.03g}{2} \cdot N_A \simeq 9.03\times10^{21}$ 个D原子.

催化DD聚变 1次放出 43.2 MeV 能量.

$$E = \frac{9.03\times10^{21}}{6} \times 43.2MeV$$
$$\simeq 6.5\times10^{22}MeV. \simeq 1.04\times10^{10}J.$$

(4) 0.5L 海水中约有 $N \simeq 4.52\times10^{21}$ 个D原子.

D-T 聚变 1次放出 17.6 MeV 能量

$$E = 4.52\times10^{21}\times17.6MeV$$
$$\simeq 7.9\times10^{22}MeV \simeq 1.26\times10^{10}J.$$

2. ① $D+T \rightarrow \alpha + n + 17.6MeV.$

动量守恒: $m_\alpha v_\alpha = m_n v_n.$ $\rightarrow K_{\alpha_1} = 3.52MeV.$

能量守恒: $\frac{1}{2}m_\alpha v_\alpha^2 + \frac{1}{2}m_n v_n^2 = 17.6MeV.$ $K_{n_1} = 14.08MeV.$

② $D+D \rightarrow {}^3He + n + 3.267MeV \rightarrow K_{n_2} = 2.45MeV$

③ $D+D \rightarrow T + p + 4.032MeV. \rightarrow K_{p_1} = 3.024MeV.$

④ $D + {}^3He \rightarrow \alpha + p + 18.3MeV \rightarrow K_{\alpha_2} = 3.66MeV$

$K_{\alpha_1} = 14.64MeV.$

中子所带能量共为 16.53MeV, 占总能量的 38%.. 带电粒子所带能量占总能的 62%.
因此可以考虑将能量直接转换为电能.

DT 聚变中, 中子携带了大部分能量, 只能通过慢化转化为热能, 受到热机效率限制.

3. $\langle \sigma v \rangle = \int f_1(v_1) f_2(v_2)\, \sigma v \, dv_1 dv_2$

(1) $f_j(v) = \left(\frac{m_j}{2\pi k_B T_j}\right)^{3/2} \exp\left(-\frac{m_j v^2}{2k_B T_j}\right), \quad j=1,2$

$$\rightarrow \langle \sigma v \rangle = \frac{(m_1 m_2)^{3/2}}{(2\pi kT)^3} \iint \exp\left(-\frac{m_1+m_2}{2kT}\left(V + \frac{1}{2}\frac{m_1-m_2}{m_1+m_2} v'\right)^2\right)$$
$$\times\, \sigma(v')\, v' \exp\left(-\frac{\mu v'}{2T}\right)^2 d^3v' \, d^3V$$

其中 $V = \frac{v_1+v_2}{2}$. $v' = v_1 - v_2$. $\mu = \frac{m_1 m_2}{m_1+m_2}$

$$\rightarrow \langle \sigma v \rangle = 4\pi \left(\frac{\mu}{2\pi kT}\right)^{3/2} \int \sigma(v') v'^3 \exp\left(-\frac{\mu v'}{2kT}\right)^2 dv'$$
$$= \left(\frac{8}{\pi}\right)^{1/2} \left(\frac{\mu}{kT}\right)^{3/2} \frac{1}{m_1^2} \int \sigma(\varepsilon)\, \varepsilon \exp\left(-\frac{\mu \varepsilon}{m_1 kT}\right) d\varepsilon, \qquad \varepsilon = \frac{1}{2} m v'^2.$$

(2) $f_2(v) = \delta(v - v_0)$

$$\langle \sigma v \rangle = \left(\frac{m_1}{2kT\pi}\right)^{1/2} \cdot \frac{1}{v_0} \int \sigma(\varepsilon) \cdot v'^2 \cdot \left[\exp\left(-\left(\frac{m_1}{2kT}\right)^2 (v_0 - v')^2\right) - \exp\left(-\left(\frac{m_1}{2kT}\right)^2 (v_0 + v')^2\right)\right] \cdot dv'$$

(3). $f_i(v) = \delta(v - v_i)$

$$\langle \sigma v \rangle = \sigma(v_1 - v_2)$$

(4). $f_1(v) = \delta(v - v_0)$.

$$\langle \sigma v \rangle = \left(\frac{m_2}{2kT\pi}\right)^{1/2} \cdot \frac{1}{v_0} \int \sigma(\varepsilon) \cdot v'^2 \cdot \left[\exp\left(-\left(\frac{m_2}{2kT}\right)^2 (v_0 - v')^2\right) - \exp\left(-\left(\frac{m_2}{2kT}\right)^2 (v_0 + v'^2)\right)\right] \cdot dv'.$$

画图:

(1).