第 12 次作业题

1. 计算下列广义积分:

(1) $\int_0^{+\infty} xe^{-x}\,\mathrm{d}x$, (2) $\int_{-\infty}^{+\infty} \frac{\mathrm{d}x}{x^2+2x+5}$,
(3) $\int_1^{+\infty} \frac{\mathrm{d}x}{x(1+x^2)}$, (4) $\int_0^{+\infty} \frac{\arctan x}{(1+x^2)^{\frac{3}{2}}}\,\mathrm{d}x$,
(5) $\int_{-\infty}^{-2} \frac{\mathrm{d}x}{x\sqrt{x^2-1}}$, (6) $\int_{-1}^{1} \sqrt{\frac{1-x}{1+x}}\,\mathrm{d}x$.

2. 判断下列广义积分的敛散性:

(1) $\int_1^{+\infty} \frac{\arctan x}{x^2}\,\mathrm{d}x$, (2) $\int_1^{+\infty} \frac{\cos(x^2)}{x}\,\mathrm{d}x$,
(3) $\int_1^{+\infty} \frac{\sqrt{1+x^{-1}}-1}{x^p\log(1+x^{-2})}\,\mathrm{d}x$, (4) $\int_1^2 \frac{\mathrm{d}x}{\log x}$,
(5) $\int_0^{+\infty} \frac{\log(1+x^2)}{x^p}\,\mathrm{d}x$, (6) $\int_0^{+\infty} \frac{\sin x}{\sqrt{x^3}}\,\mathrm{d}x$,
(7) $\int_1^{+\infty} \frac{1-\cos\sqrt{2x}-\sin x}{x^p}\,\mathrm{d}x$ $(p>\frac{1}{2})$, (8) $\int_0^{\frac{\pi}{2}} \frac{\mathrm{d}x}{(\sin^p x)(\cos^q x)}$.

3. 考察下列广义积分的绝对收敛性与条件收敛性:

(1) $\int_1^{+\infty} \frac{\sin x}{x^p}\,\mathrm{d}x$ $(p>0)$, (2) $\int_0^{+\infty} \sin(x^2)\,\mathrm{d}x$,
(3) $\int_1^{+\infty} x\big(\arctan\frac{2}{x}-\arctan\frac{1}{x}\big)\,\mathrm{d}x$, (4) $\int_0^{+\infty} \frac{\sqrt{x}\sin x}{x+1}\,\mathrm{d}x$.

4. 利用 Euler 积分计算下列积分:

(1) $\int_0^{+\infty} x^{\frac{3}{2}}e^{-4x}\,\mathrm{d}x$, (2) $\int_0^1 \frac{\mathrm{d}x}{\sqrt{1-x^{\frac{1}{3}}}}$, (3) $\int_0^1 \frac{\mathrm{d}x}{\sqrt[n]{1-x^n}}$, 其中 $n>1$ 为整数.