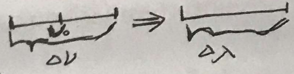
(1) 相速度 $v=\omega/k=\sqrt{\omega_p^2+c^2k^2}/k$

群速度 $v_g=d\omega/dk=\frac{d\omega}{d\omega^2}\frac{d\omega^2}{dk}=\frac{1}{d\omega^2/d\omega}2c^2k=\frac{1}{2\omega}2c^2k=c^2\frac{k}{\omega}=c^2\frac{k}{\sqrt{\omega_p^2+c^2k^2}}$

$\therefore v v_g=\frac{\sqrt{\omega_p^2+c^2k^2}}{k}\cdot\frac{c^2k}{\sqrt{\omega_p^2+c^2k^2}}=c^2$

(2) 飞秒激光脉冲，估算频谱宽度即频域宽度。光谱分布的中心波长500nm

$\Delta t\Delta\nu\sim 1$，$\Delta t\sim 10^{-15}s \Rightarrow \Delta\nu\sim 10^{15}Hz$



频谱分布的中心频率 $\nu_0=\frac{c}{\lambda_0}=\frac{3\times10^8}{500\times10^{-9}}Hz=6\times10^{14}Hz$

波长的光谱宽度 $\Delta\lambda=\frac{c}{\nu_0-\Delta\nu/2}-\frac{c}{\nu_0+\Delta\nu/2}=\frac{3\times10^8}{1\times10^{14}}-\frac{3\times10^8}{11\times10^{14}}=2730nm$

$\downarrow$ 3000nm　　$\downarrow$ 270nm

$\therefore$ 能激发580nm附近的钠转变

系统方法：首先在观察屏上表示来自每个源的场分布，然后把所有贡献叠加得总场，然后平方得场强分布。

双波干涉：复数法 $\tilde{U}_1=A_1(p)e^{i\phi_1(p)}$　$\tilde{U}_2=A_2(p)e^{i\phi_2(p)}$

合成 $\tilde{U}(p)=\tilde{U}_1(p)+\tilde{U}_2(p)$

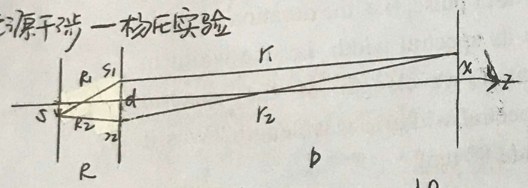
强度 $I(p)=\tilde{U}(p)\tilde{U}^*(p)=(\tilde{U}_1(p)+\tilde{U}_2(p))(\tilde{U}_1^*(p)+\tilde{U}_2^*(p))=I_1(p)+I_2(p)+2\sqrt{I_1(p)I_2(p)}\cos\delta(p)$

$=A_1^2(p)+A_2^2(p)+2A_1(p)A_2(p)\cos\delta(p)$

$\delta(p)=\phi_1(p)-\phi_2(p)$ 为两波在场点p的位相差

$=k\Delta L$

点光源干涉 — 杨氏实验



$\Delta L=(R_1+r_1)-(R_2+r_2)$

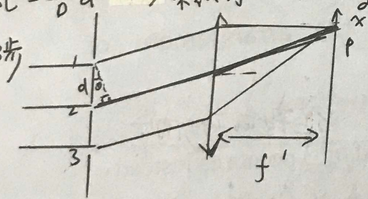
$=(R_1-R_2)+(r_1-r_2)$

$=\frac{s}{R}d-\frac{x}{D}d$

$\Delta x\frac{d}{D}=\Delta s\frac{d}{R}\Rightarrow\Delta x=\Delta s\cdot\frac{D}{R}$ 条纹移动

$s=0$ 时 $\Delta L=\frac{x}{D}d \Rightarrow$ 条纹间距 $\Delta x=\frac{\lambda D}{d}$

多光束干涉



$\tilde{U}=U_0+U_0e^{i\delta}+U_0e^{i2\delta}+\cdots+U_0e^{i(N-1)\delta}$

$(\delta=d\sin\theta\frac{2\pi}{\lambda})$

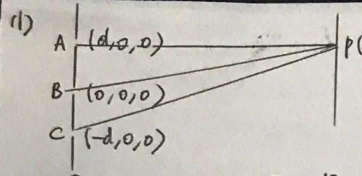
$=\begin{cases}U_0\frac{1-e^{iN\delta}}{1-e^{i\delta}}, & \delta\neq2\pi m\\ NU_0, & \delta=2\pi m\rightarrow I=N^2I_0\end{cases}$

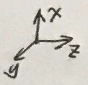
$\delta\neq2\pi m$ 时 $\tilde{U}=U_0\frac{\sin^2\frac{N\delta}{2}}{\sin^2\delta/2}=U_0\frac{\sin^2(\frac{N\pi d}{\lambda}\sin\theta)}{\sin^2(\frac{\pi d}{\lambda}\sin\theta)}$

主极大 $\delta=2\pi m$　$d\sin\theta=m\lambda$

次极大 $d\sin\theta=m\lambda/N$

(1)





物点和场点着满足傍轴条件

$\tilde{U}(x',y') = \frac{a}{z}\exp\left[ik\left(r_0+\frac{x^2+y^2}{2z}\right)\right]\exp\left[\frac{-ik}{z}(xx'+yy')\right]$

本题 $z=D$，$x=d,0,-d$，$y=0$，$r_0=\sqrt{X^2+Y^2+D^2}$

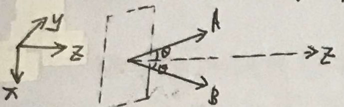
$\therefore \tilde{U}_A = A\exp\left[ik\left(r_0+\frac{d^2}{2D}-\frac{Xd}{D}\right)\right]$ $\quad \tilde{U}_B = A\exp[ikr_0]$ $\quad \tilde{U}_C = A\exp\left[ik\left(r_0+\frac{d^2}{2D}+\frac{Xd}{D}\right)\right]$

$$\tilde{U}_P = \tilde{U}_A+\tilde{U}_B+\tilde{U}_C = A\exp\left[ik\left(r_0+\frac{d^2}{2D}\right)\right]\times\left(e^{-ik\frac{d^2}{2D}}+e^{ik\frac{Xd}{D}}+e^{-ik\frac{Xd}{D}}\right)$$

$$= A\exp\left[ik\left(r_0+\frac{d^2}{2D}\right)\right]\left[e^{-ik\frac{d^2}{2D}}+2\cos\left(\frac{kXd}{D}\right)\right]$$

$$I_P = \tilde{U}_P^*\tilde{U}_P = A^2\left[1+4\cos^2\left(\frac{kd}{D}X\right)+4\cos\left(\frac{kd}{D}X\right)\cos\left(\frac{kd^2}{2D}\right)\right]$$

(2) (a)



波矢 $\vec{k}_A = (0, k\sin\theta, k\cos\theta)$

$\vec{k}_B = (0, -k\sin\theta, k\cos\theta)$

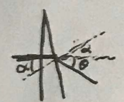
$E_A = A_0\exp[i(\vec{k}_A\cdot\vec{r}-\omega t)] = A_0\exp[i(k\sin\theta y+k\cos\theta z-\omega t)]$

$E_B = A_0\exp[i(\vec{k}_B\cdot\vec{r}-\omega t)] = A_0\exp[i(-k\sin\theta y+k\cos\theta z-\omega t)]$

(b) $z=0$，$E=E_A+E_B = A_0e^{-i\omega t}\left(e^{ik\sin\theta y}+e^{-ik\sin\theta y}\right) = 2A_0\cos(k\sin\theta y)e^{-i\omega t}$

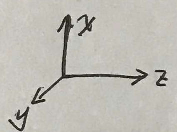
$I = 4A_0^2\cos^2(k\sin\theta y)$，$k\sin\theta y = 0,\pi$ 时 max，$\Delta y = \pi/k\sin\theta = \frac{\lambda}{2\sin\theta}$

(3) 菲涅尔双棱镜



$n\sin\alpha = \sin\beta$ 近似 $\beta = n\alpha$ $\quad$ 而 $\beta = \alpha+\theta$ $\quad \therefore \theta = (n-1)\alpha$

干涉：两平面波以 $\pm\theta$ 传播



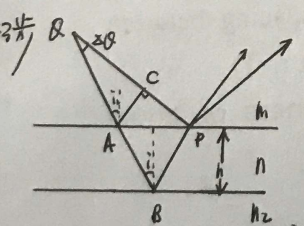
$E_+ = Ae^{i(k\cos\theta z+k\sin\theta x)}e^{-i\omega t}$ $\quad E_- = Ae^{i(k\cos\theta z-k\sin\theta x)}e^{-i\omega t}$

on the screen $z=z_0$：$E_{total}(z_0) = Ae^{i(k\cos\theta z_0-\omega t)}\left(e^{ik\sin\theta x}+e^{-ik\sin\theta x}\right)$

$= 2A\cos(k\sin\theta x)e^{i(k\cos\theta z_0-\omega t)}$

$\therefore I = 4A^2\cos^2(k\sin\theta x) = 2A^2(1+\cos(2k\sin\theta x))$ $\quad 2k\sin\theta\Delta x = 2\pi \rightarrow \Delta x = \frac{\lambda}{2\sin\theta}$

薄膜干涉



光程差 $\Delta L = (QA)+(ABP)-(QP)$

$\Delta\theta$ 很小，$QA-QP = -CP = -n_1AP\sin i_1 = -nAP\sin i_1$

$= -n(2h\tan i)\sin i = -2nh\sin^2 i/\cos i$

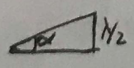
$(ABP) = 2AB = 2nh/\cos i$

$\therefore \Delta L = 2nh\cos i = \begin{cases} k\lambda & 亮纹 \\ \frac{2k+1}{2}\lambda & 暗纹 \end{cases}$

半波损 $n_1<n>n_2$ 或 $n_1>n<n_2$

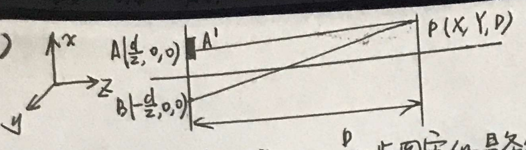
垂直光入射 $\Delta L = 2nh$ $\quad$ 相邻条纹 $\Delta L = \lambda$ $\quad \therefore$ 相邻等厚条纹对应厚度差 $\Delta h = \lambda/2n$

空气楔形薄膜



条纹间隔 $\Delta x = \frac{\lambda}{2\alpha}$

(1)



$D=2m$，$\lambda=600nm$，$d=1mm$

glass：$n=1.5$，$t=0.05mm$

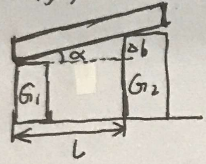
屏中心不再对应光程差为0，干涉图案仍是条纹但会移位

A路差光程差改变 $(n-1)t=0.025mm$，故光程差点上移如图所示

A、B之间光程差 $d\sin\theta \approx d\theta = 0.025mm$ $\therefore \theta = 0.025$

条纹移动 $D\theta = 0.05m = 5cm$

2.



(1) $\Delta h = \alpha L = \frac{\lambda}{2\Delta x}L = 29.47\mu m$

轻压T中部，条纹变密而一端长，条纹变疏而一端短 $\Delta x = \frac{\lambda}{2\alpha}$

$\alpha\uparrow$ $\alpha\downarrow$

(2) G_2上下两表面不平行，故使其上表面不严格平行G_1的上表面，造成两边空气层劈角不等

劈角差 $\Delta\alpha = \alpha_2 - \alpha_1 = \left(\frac{1}{\Delta x_2} - \frac{1}{\Delta x_1}\right)\frac{\lambda}{2} = \left(\frac{1}{0.3mm} - \frac{1}{0.5mm}\right)\frac{\lambda}{2} \approx 3.93\times10^{-4}rad = 1.35'$

6. 半波损 $2nh\cos i = (2k+1)\frac{\lambda}{2}$ 亮纹.

反射光相对入射光相位突变 π

令$k=0$得最小厚度 $h_0 = \frac{\lambda}{4n\cos i} = \frac{\lambda}{4n\sqrt{1-\sin^2 i}} = \frac{\lambda}{4\sqrt{n^2-\sin^2 35^\circ}}$

$= \frac{5000}{4\sqrt{1.33^2-\sin^2 35^\circ}} \approx 1042\,\mathring{A}$