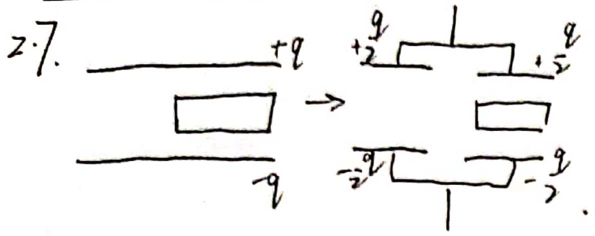
2.7.



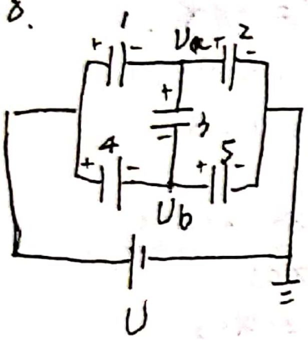
$E_1 = \frac{\sigma}{\varepsilon_0}$. $U_1 = \frac{\sigma}{\varepsilon_0} d$. $C_1 = \frac{q}{2} / U_1 = \frac{q\varepsilon_0}{2\sigma d}$.

$E_2 = \frac{\sigma}{\varepsilon_0}$. $U_2 = \frac{\sigma}{\varepsilon_0}(d-t)$. $C_2 = \frac{q}{2} / U_2 = \frac{q\varepsilon_0}{2\sigma(d-t)}$

$$C = C_1 + C_2 = \frac{q\varepsilon_0}{2\sigma d} + \frac{q\varepsilon_0}{2\sigma(d-t)}$$

$$= \frac{S\varepsilon_0}{2d} + \frac{S\varepsilon_0}{2(d-t)}$$

2.8.



$q_1 = (U - U_a)C_1$.

$q_2 = U_a C_2$

$q_3 = (U_a - U_b)C_3$

$q_4 = (U - U_b)C_4$

$q_5 = U_b C_5$.

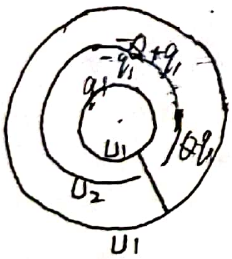
$q_1 = q_2 + q_3 \rightarrow 2U - 2U_a = 2U_a - U_b$

$q_5 = q_3 + q_4 \rightarrow 2U_b = U_a + U - 2U_b$.

$\Rightarrow \begin{cases} 4U_a - U_b = 2U \\ U_a - 4U_b = -U. \end{cases}$ $\begin{cases} U_a = \frac{3}{5}U = 360V \\ U_b = \frac{2}{5}U = 240V. \end{cases}$

$\rightarrow U_1 = 240V$. $U_2 = 360V$. $U_3 = 120V$. $U_4 = 360V$. $U_5 = 240V$.

2.9.



$E_1 = \frac{q_1}{2\pi r l \varepsilon_0}$

$U_2 - U_1 = \int_a^b -E_1 \cdot dr = \frac{q_1}{2\pi\varepsilon_0 l} \ln\frac{a}{b}$

$E_2 = \frac{-Q+q_1}{2\pi r l \varepsilon_0}$

$U_1 - U_2 = \int_b^d -E_2 \cdot dr = \frac{(-Q+q_1)}{2\pi\varepsilon_0 l} \ln\frac{b}{d}$

$\rightarrow q_1 \ln\frac{b}{a} = (-Q+q_1)\ln\frac{b}{d}$

$Q = \frac{q_1(\ln\frac{b}{d} + \ln\frac{a}{b})}{\ln\frac{b}{d}} = \frac{q_1(\ln\frac{a}{d})}{\ln\frac{b}{d}}$

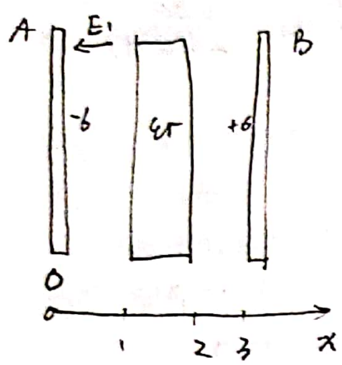
$$C = \frac{Q}{U_1 - U_2} = \frac{q_1(\ln\frac{a}{d})}{q_1 \ln\frac{b}{a} \ln\frac{d}{b}} 2\pi\varepsilon_0 l = 2\pi\varepsilon_0 l \frac{\ln\frac{d}{a}}{(\ln\frac{b}{a})\ln(\frac{d}{b})}.$$

2.10. $|\vec{p}| = 6.2\times10^{-30}\ \mathrm{C\cdot m}$

$n = \frac{598}{18}\times N_a = 1.99\times10^{25}$ 个$/m^3$

$|\vec{P}| = n|\vec{p}| = 1.24\times10^{-4}\ \mathrm{C\,m^{-2}}$.

2.13.



(1). 0~1: $\vec{E}_1 = \frac{-\sigma}{\varepsilon_0}\hat{x}$.

$\vec{P} = 0$.

$\vec{D} = \varepsilon_0\vec{E} = -\sigma\hat{x}$.

1~2: $\vec{P} = \chi_e\varepsilon_0\vec{E}_2 = (\varepsilon_r-1)\varepsilon_0\vec{E} = \varepsilon_0\vec{E}_2$

$\vec{D} = 2\varepsilon_0\vec{E}_2 = -\sigma\hat{x}$.

$\rightarrow \vec{E}_2 = \frac{-\sigma}{2\varepsilon_0}\hat{x}$.

$\vec{P} = -\frac{\sigma}{2}\hat{x}$

$\vec{D} = -\sigma\hat{x}$.

2~3:

$\vec{E}_3 = \frac{-\sigma}{\varepsilon_0}\hat{x}$.

$\vec{P} = 0$.

$\vec{D} = -\sigma\hat{x}$.

$\therefore \vec{E} = \begin{cases} -\frac{\sigma}{\varepsilon_0}\hat{x}. & 0<x<1.\ 2<x<3 \\ -\frac{\sigma}{2\varepsilon_0}\hat{x}. & 1<x<2 \end{cases}$

$\vec{P} = \begin{cases} 0. & 0<x<1.\ 2<x<3 \\ -\frac{\sigma}{2}\hat{x}. & 1<x<2 \end{cases}$

$\vec{D} = -\sigma\hat{x}.\ 0<x<3.$

(2). 0~1:

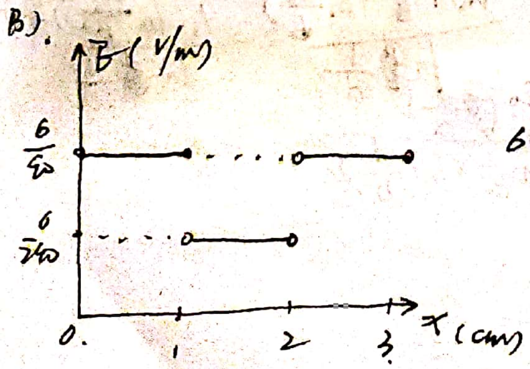
$U_1 = \int_0^{x} \vec{E}_1\cdot dx = \frac{\sigma}{\varepsilon_0}x$.

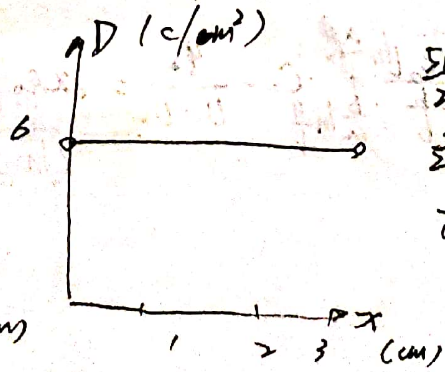
1~2: $U_2 = U_1 - \int_1^x \vec{E}_2\cdot dx = \frac{\sigma}{\varepsilon_0}\cdot 0.01 + \frac{\sigma}{2\varepsilon_0}(x-0.01)$.

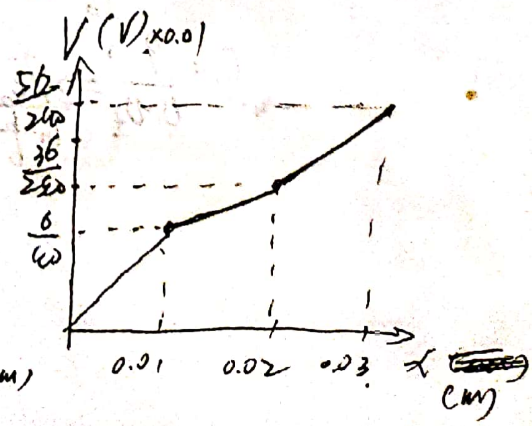
2~3: $U_3 = U_2 - \int_2^x \vec{E}_3\cdot dx = \frac{3\sigma}{2\varepsilon_0}\cdot 0.01 + \frac{\sigma}{\varepsilon_0}(x-0.02)$.

$\rightarrow U = \begin{cases} \frac{\sigma}{\varepsilon_0}x. & 0<x<0.01\,m. \\ (\frac{\sigma}{2\varepsilon_0}\cdot 0.01 + \frac{\sigma}{2\varepsilon_0}x)\ V. & 0.01<x<0.02\,m. \\ (\frac{\sigma}{\varepsilon_0}x - \frac{\sigma}{2\varepsilon_0}\cdot 0.01)\ V. & 0.02<x<0.03\,m. \end{cases}$

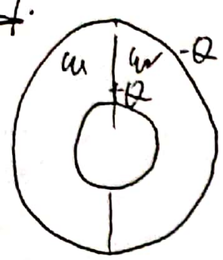
(3).







2.17.



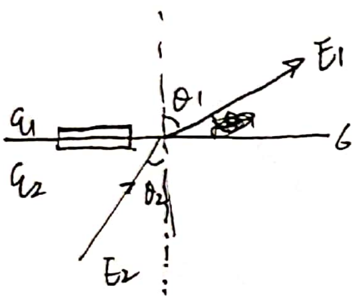
(1). $\oiint \vec{D}\cdot\vec{n}\,dS = Q = E\cdot 2\pi r^2\varepsilon_1 + E\cdot 2\pi r^2\varepsilon_2$

$\to \vec{E} = \frac{Q}{2\pi r^2(\varepsilon_1+\varepsilon_2)}\hat{r}$.

(2). $U = \int_a^b \vec{E}\cdot d\vec{r} = \frac{Q}{2\pi(\varepsilon_1+\varepsilon_2)}\left(\frac{1}{a}-\frac{1}{b}\right)$.

$C = \frac{Q}{U} = \frac{2\pi(\varepsilon_1+\varepsilon_2)ab}{b-a}$.

2.19.



$\nabla\times\vec{E}=0 \to E_{1t}=E_{2t} \to E_1\sin\theta_1 = E_2\sin\theta_2$.

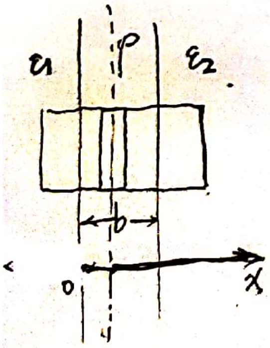
$\oiint \vec{D}\cdot\vec{n}\,ds = \Delta S\cdot\sigma \to D_{1n}-D_{2n}=\sigma$.

$\to \varepsilon_1E_1\cos\theta_1 - \varepsilon_2E_2\cos\theta_2 = \sigma$

$\to \varepsilon_2\cos\theta_2 = \frac{\varepsilon_1E_1\cos\theta_1-\sigma}{E_2} = \frac{\varepsilon_1E_1\cos\theta_1-\sigma}{E_1\sin\theta_1/\sin\theta_2}$

$\to \varepsilon_2\cot\theta_2 = \varepsilon_1\cot\theta_1 - \frac{\sigma}{E_1\sin\theta_1} = \varepsilon_1\left(1-\frac{\sigma}{\varepsilon_1E_1\cos\theta_1}\right)\cot\theta_1$. 得证.

2.20.



(1). 板内: $\sigma\cdot dS = \rho\cdot dS\,dx$. 设板内为$\varepsilon$.

无限大平板对某一点的电场 $\vec{E} = \frac{\sigma}{2\varepsilon}$

$\vec{E} = \int_0^x \frac{\rho\,dx}{2\varepsilon} - \int_x^b \frac{\rho\,dx}{2\varepsilon} = \frac{\rho}{2\varepsilon}(2x-b)\hat{x}$.

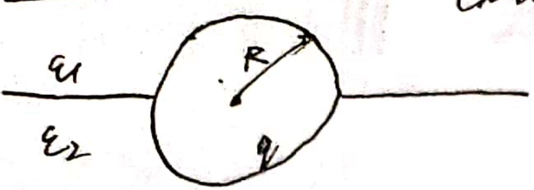
板外: $E_1\cdot\varepsilon_1 + E_2\cdot\varepsilon_2 = b\rho$.

$E_x\varepsilon + E_1\varepsilon_1 = x\rho \to \vec{E_1} = \frac{-\rho b}{2\varepsilon_1}\hat{x}$ $\vec{E_2} = \frac{\rho b}{2\varepsilon_2}\hat{x}$.

(2). $U_1 = \int_{-l}^0 \vec{E_1}\cdot d\vec{x} = \frac{-\rho b}{2\varepsilon_1}l$. $U_2 = \int_0^l \vec{E_2}\cdot d\vec{x} = \frac{\rho b}{2\varepsilon_2}l$. $U_中 = \int_0^b \vec{E}\cdot d\vec{x} = \frac{\rho}{2\varepsilon}(b^2-b^2)=0$

$U_{AB} = \frac{\rho bl}{2}\left(\frac{1}{\varepsilon_2}-\frac{1}{\varepsilon_1}\right)$.

2.21



Gauss:
$$\begin{cases} \vec{E} = \dfrac{q}{2\pi r^2(\varepsilon_1+\varepsilon_2)}\cdot\hat{r}. & r>R \\ \vec{E}=0. & r<R. \end{cases}$$

$$\cancel{2\pi r^2}\cdot\Omega\cdot r^2\cdot E = \cancel{\frac{\Omega\cdot R^2 q}{4\pi R^2\varepsilon_1}} = \frac{\sigma_1\cdot\Omega\cdot R^2}{\varepsilon_1}.$$

$$\rightarrow \sigma_1 = \frac{\varepsilon_1 q}{2\pi R^2(\varepsilon_1+\varepsilon_2)}.$$

$$\sigma_2 = \frac{\varepsilon_2 q}{2\pi R^2(\varepsilon_1+\varepsilon_2)}.$$