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4.100. Out of 60 applicants to a university, 40 are from the East. If 20 applicants are to be selected at random, find the probability that (a) 10, (b) not more than 2, will be from the East.

(a). $P = \dfrac{\binom{40}{10}\binom{20}{10}}{\binom{60}{20}}$

(b) $P = \dfrac{1 + \binom{20}{19}\binom{40}{1} + \binom{20}{18}\binom{40}{2}}{\binom{60}{20}}$

4.110. Find the values of χ^2 for which the area of the right-hand tail of the χ^2 distribution is 0.05, if the number of degrees of freedom v is equal to (a) 8, (b) 19, (c) 28, (d) 40.

(a) 15.507. (b) 30.144 (c) 41.337. (d) 55.758

4.113. If the variable U is chi-square distributed with $v = 7$, find χ_1^2 and χ_2^2 such that (a) $P(U > \chi_2^2) = 0.025$, (b) $P(U < \chi_1^2) = 0.50$, (c) $P(\chi_1^2 \le U \le \chi_2^2) = 0.90$.

(a). 16.013 (b) 6.35. (c) 2.167, 14.067.

4.119. If a variable U has a Student's distribution with $v = 10$, find the constant c such that (a) $P(U > c) = 0.05$, (b) $P(-c \le U \le c) = 0.98$, (c) $P(U \le c) = 0.20$, (d) $P(U \ge c) = 0.90$.

(a) 1.812 (b) 2.764 (c) −0.879 (d). −1.372.

5.49. A population consists of the four numbers 3, 7, 11, 15. Consider all possible samples of size two that can be drawn with replacement from this population. Find (a) the population mean, (b) the population standard deviation, (c) the mean of the sampling distribution of means, (d) the standard deviation of the sampling distribution of means. Verify (c) and (d) directly from (a) and (b) by use of suitable formulas.

(a). $\mu = \frac{1}{4}(3+7+11+15) = 9.$

(b). $\sigma = \sqrt{\frac{1}{4}\left((9-3)^2+(9-7)^2+(11-9)^2+(15-9)^2\right)} = 2\sqrt{5}$

(C). $\mu_{\bar{X}} = \dfrac{3+7+11+15+3+7+3+11+3+15+7+11+7+15+11+15}{4\times 4} = 9$

(d) $\sigma_{\bar{X}} = \sqrt{\frac{1}{16}(36+4+4+36+16\times 2+4\times 2+4\times 2+16\times 2)} = \sqrt{10}$

5.50. Solve Problem 5.49 if sampling is without replacement.

(a). $\mu = 9.$ (b) $= 2\sqrt{5}.$

(c). $\mu_{\bar{X}} = \frac{1}{6}\left(\frac{3+7}{2} + \frac{3+11}{2} + \frac{3+15}{2} + \frac{7+11}{2} + \frac{7+15}{2} + \frac{11+15}{2}\right) = 9$

(d) $\sigma_{\bar{X}} = \sqrt{\frac{1}{6}(16+4+4+16)} = \frac{2\sqrt{15}}{3}$

5.58. Out of 1000 samples of 200 children each, in how many would you expect to find that (a) less than 40% are boys, (b) between 40% and 60% are girls, (c) 53% or more are girls?

(a) $P = \sum_{k=0}^{80} \binom{200}{k} \cdot \frac{1}{2^{200}}.$ $\bar{X} = 1000 \cdot P = 1000 \sum_{k=0}^{80} \binom{200}{k} \cdot \frac{1}{2^{200}}.$

(b). $\bar{X} = 1000 \cdot \sum_{k=80}^{120} \binom{200}{k} \cdot \frac{1}{2^{200}}.$

(c) $\bar{X} = 1000 \cdot \sum_{k=53}^{200} \binom{200}{k} \cdot \frac{1}{2^{200}}.$

5.74. A normal population has a variance of 15. If samples of size 5 are drawn from this population, what percentage can be expected to have variances (a) less than 10, (b) more than 20, (c) between 5 and 10?

(a). $\frac{n}{\sigma^2} S^2 = \frac{1}{3} S^2$ 服从 $v=4$的卡方分布.

$S^2 \le 10.$ $P \approx 50\%.$

(b). $S^2 \ge 20.$ $P \approx 15\%$

(c). $5 \le S^2 \le 10$ $P \approx 80\% - 50\% = 30\%.$

5.76. According to the table of Student's t distribution for 1 degree of freedom (Appendix D), we have $P(-1 \le T \le 1) = 0.50$. Check whether the results of Problem 5.1 are confirmed by this value, and explain any difference.

$t_{0.5} = 1.$ $\varepsilon = \frac{s}{\sqrt{n}} t_{0.5} = \frac{2.32}{\sqrt{25}} = 0.464.$

$P(6-0.464 < \bar{X} < 6+0.464) \approx \frac{5}{25} < 0.5.$

所以样本均值在该范围内的置信概率不是50%.