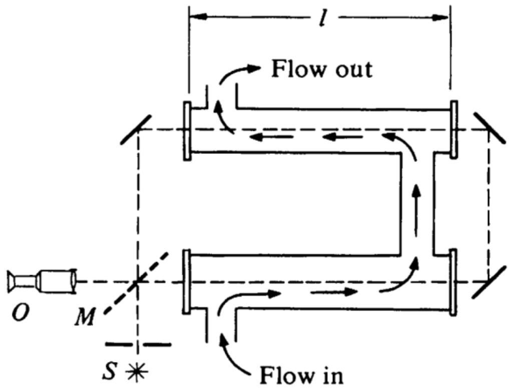
第十次习题课材料

1. KK-11.2

If the two arms of a Michelson interferometer have lengths l_1 and l_2, show that the fringe shift when the interferometer is rotated by 90^o with respect to the velocity v through the ether is $N=\frac{l_1+l_2}{\lambda}\frac{v^2}{c^2}$,where λ is the wavelength of the light.

2. KK-11.5

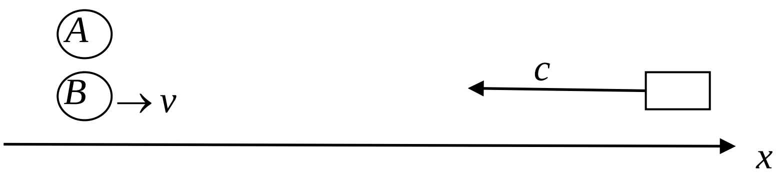


图表 1 习题 2

In 1851 H. L. Fizeau investigated the velocity of light through a moving medium using the interferometer shown in Fig.1. Light of wavelength λ from a source S is split into two beams by the mirror M. The beams travel around the interferometer in opposite directions and are combined at the telescope of the observer, O, who sees a fringe pattern. Two arms of the interferometer pass through water-filled tubes of length l with flat glass end plates. The water runs through the tubes, so that one of the light beams travels downstream while the other goes upstream. The velocity of light in water at rest is c/n, where n is the refractive index of water. If we assume that the velocity of the water is added to the velocity of light in the downstream direction, and subtracted in the upstream, direction, show that the fringe shift which occurs when the water flows with velocity v is $N=4n^2\frac{l}{\lambda c}v$.(The actual fringe shift measured by Fizeau was $N=4n^2\frac{l}{\lambda c}fv$, where f=1-1/n^2. f , known as the Fresnel drag coefficient, was postulated in1818, but it was not satisfactorily explained until the advent of relativity.)

3. HW11-3

As the figure below shows. Two particles A,B: A is stationary in lab and B is moving with velocity v. A laser is L (in lab frame) away from A. Viewed in lab frame, as A,B overlap, a light is simultaneously shoots out of the laser.



a) In lab frame what is the time difference between particle B and particle A receives the light signal?

b) If we are traveling with B with same speed v and in such frame, what is the time difference again? (Reminder: be careful of simultaneity here) Could the time order receiving signals by A,B reversed?

c) If the laser is not shooting light but bullets with velocity $V< c$, of course B will be hit before A in lab frame. Could the time order of A,B hit by bullets be reversed for observers in another inertial frames? You may not have learned the materials from the lecture yet, (you will need velocity transformation or causality) ***take a guess*** for the present if you have no logical reason.

4. HW11-4

A train of proper length L and speed of 3/5 c approaches a tunnel of length L (proper length). When the head of the train enters the tunnel, a person leaves the head and starts running towards back. He arrives at the back at exactly same moment as the back of the train leaves the tunnel.

(a) Do the simultaneity of the head (of the train) enters the tunnel and person starts running and the simultaneity of his arrival to the back and train leaves the tunnel depend on the frames of observation?
(b) How much time does the process described above take in ground frame?
(c) What is the speed of the person from the ground point of view?
(d) How much time elapsed on the running person's watch?

5. HW11-6

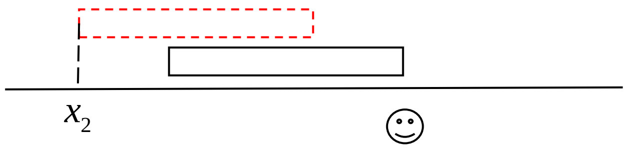
Alice and Bob start at same location at time 0 (both their watches read 0 when they meet), Alice travels right and bob towards left with relative speed v between them. When Bob's watch read T, he sends out a photon to Alice; when Alice receives the photon, what is time reading on her watch? Try this problems in both Bob's and Alice's frame.

6. HW11-7

Let Alice and Bob synchronize their watches at Starbucks and Bob immediately jumps on a bus with velocity v and travels with bus distance L (L in ground frame). Bob jumps off the bus and the reading of his watch will be different from proper synchronization (the watches of Alice and Bob will have different reading), calculate this difference and show that as v is small, the difference approaches 0.

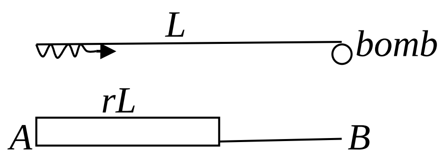
7. HW11-8

This problem is to illustrate the difference of measuring a moving rod length and seeing (really see with eyes) the length of the same rod: A rod with proper length L travels with v along the x direction (the rod is also aligned along x direction). The measured length for the lab frame is of course L/γ. Now imagine you are at the right of the whole rod and the rod is moving towards you from left, you are also standing close to the track of the motion of the rod. a) Explain that the lights which reaches your eyes at one time (the time your eyes record the picture, think your eye as a camera) must leaves the two ends of the rod at different times. b) from there, calculate the length as seeing by your own eyes, and show that it is different than L/γ, actually in this situation, it is even larger than L.(Here is quite analogous to Doppler effect we shall talk about).



8. HW12-4

Refer to the figure below:



A tunnel (proper length L)and a train (proper length is rL), the train is running with velocity v through the tunnel. A bomb was planted at the far-end (B) of the tunnel and it is going to explode when the head of train reaches B. As the tail of the train first enters the tunnel (A), a deactivation light signal is sending out to disarm the bomb. Find the requirement of length of the train, i.e. the value of r that can safely pass this tunnel. Work this A) using algebra method. B) using the Minkowski diagram. (You can pick any frame of your preference, for example I worked this problem using train frame in algebra method; and using tunnel frame as the orthogonal one in Minkowski diagram).

9. HW12-3

Use the LT to derive the velocity relations: Suppose 3 inertial frames: S, S',S''. S' is moving with v_1 relative to S; S" is moving with v_2' relative to S'. From the LT between S-S' and S'-S", we can work out the transform relation between S-S" (In matrix representation, will be product of matrix). Find out the transform between S-S" throughS-S' and S'-S" and show the v_2, the relative velocity between S-S" from the transformation you just calculated in terms of v_1,v_2'.

10. HW12-6

Another popular paradox (very similar to the above):

A hole and a rod(both are very thin in thickness), the proper length of the rod and radius of the hole are both L. The rod is moving with v and at the moment the rod head reaches the ring of the hole (as in the figure right above), the hole is lifted and question is: will the rod go under the hole? Answer this in both hole's frame and rod's frame.

11. KK-12.7

One of the most prominent spectral lines of hydrogen is the H_α line, a bright red line with a wavelength of $656.1X10^{-9}$ m.

a) What is the expected wavelength of the H_α line from a star receding with a speed of 3,000 km/s ? Ans. $662.7\ X10^{-9}$ m

b) The H_α line measured on earth from opposite ends of the sun's equator differ in wavelength by $9X10^{-12}$ m. Assuming that the effect is caused by rotation of the sun, find the period of rotation. The diameter of the sun is $1.4X10^{6}$ km. Ans. 25 d

12. KK-12.8

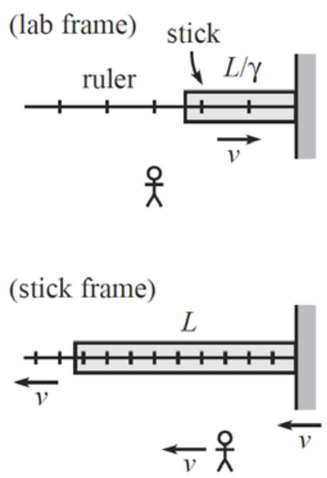
The frequency of light reflected from a moving mirror undergoes a Doppler shift because of the motion of the image. Find the Doppler shift of light reflected directly back from a mirror which is approaching the observer with speed v, and show that it is the same as if the image were moving toward the observer at speed $2v/(1+v^2/c^2)$.

13. Morin-11.7

A ruler is positioned perpendicular to a wall, and you stand at rest with respect to the ruler and the wall. A stick of length L flies by at speed v. It travels in front of the ruler, so that it obscures part of the ruler from your view. When the stick hits the wall it stops. Which of the following two reasonings is correct (and what is wrong with the incorrect one)?

In your reference frame, the stick is shorter than L. Therefore, right before it hits the wall, you are able to see a mark on the ruler that is closer than L units to the wall (see Fig. 2).

But in the stick's frame, the marks on the ruler are closer together. Therefore, when the wall hits the stick, the closest mark to the wall that you can see on the ruler is greater than L units (see Fig. 2).



图表 2 习题 13

14. Morin-11.19

Consider the following variation of the twin paradox. A , B, and C each have a clock. In A's reference frame, B flies past A with speed v to the right. When B passes A, they both set their clocks to zero. Also, in A's reference frame, C starts far to the right and moves to the left with speed v. When B and C pass each other, C sets his clock to read the same as B's. Finally, when C passes A, they compare the readings on their clocks. At this moment, let A's clock read T_A, and let C's clock read T_C.

a) Working in A's frame, show that $T_C = T_A/\gamma$, where $\gamma = 1/\sqrt{1-v^2/c^2}$.

b) Working in B's frame, show again that $T_C = T_A/\gamma$.

c) Working in C's frame, show again that $T_C = T_A/\gamma$.

15. Morin-11.35

A train of proper length L has clocks at the front and back. A photon is fired from the back of the train to the front. Working in the train frame, we can easily say that if the photon leaves the back of the train when the clock there reads zero, then it arrives at the front when the clock there reads L/c.

Now consider this setup in the ground frame, where the train travels by at speed v. Rederive the above result (that the difference in the readings of the two clocks is L/c) by working only in the ground frame.

16. Morin-11.38

Two trains of proper length L move toward each other in opposite directions on parallel tracks. They both move at speed v with respect to the ground. Both trains have clocks at the front and back, and these clocks are synchronized as usual in the frame of the train they are in. A tree is located on the ground at the place where the fronts of the trains pass each other. The clocks at the fronts of the trains both read zero when they pass. Find the reading on the clocks at the backs of the trains when they (the backs) pass each other at the tree. Do this in three different ways:

(a) Imagine that you stand next to the tree on the ground, and you observe what one of the rear clocks is doing.

(b) Imagine that you are on one of the trains, and you observe what your own rear clock is doing during the time the tree travels the relevant distance.

(c) Imagine that you are on one of the trains, and you observe what the other train's rear clock is doing during the time the tree travels the relevant distance. (You'll need to use velocity addition.)

17. Morin-11.67

Twin A stays on the earth, and twin B flies at speed v to a distant star and back. The star is a distance L from the earth in the earth – star frame. Use the Doppler effect to show that B is younger by a factor γ when she returns (don't use any time dilation or length contraction results). Do this in the following two ways; both are doable by working in either A's frame or B's frame(s), so take your pick.

(a) A sends out flashes at intervals of t seconds (as measured in his frame). By considering the numbers of redshifted and blueshifted flashes that B receives, show that $T_B = T_A/\gamma$.

(b) B sends out flashes at intervals of t seconds (as measured in her frame). By considering the numbers of redshifted and blueshifted flashes that A receives, show that $T_B = T_A/\gamma$.