Homework 12 for GP1

By SJ

1. KK 12.2

Answer: 



 or 

The two frame are stationary to each other, surely you can do this also by LT:





Replace 





1. Considering simultaneous events in S frame, i.e. t=0, x1=0 for event 1; t=0,x2=A. (A>0) Now for the S’ frame which travels with v along +x direction, what is the time difference and order of the event (which event happens first)? For another observer in S” frame which is traveling –v with respect to S (along –x), what is the time difference and order? First base your argument by LT, then use Minkowski diagram to qualitatively show the time ordering.

Answer:

Event 1 (0,0), Event 2 (A,0) in S,

For S’ (+v), LT is: 

Then , event 2 happens earlier (tail event happens earlier).

For S” (-v), LT is: 

, event 1 happens earlier (also tail event earlier)

Above using LT is much simpler than the methods I used in the previous homework and you should use it whenever you can.



The black lines are S, red are S’ and blue are S”, the time order of event 2 is marked by the dashed lines intercept with the ct’ and ct” axes. Even the quantitative calculation based on geometry can be done as I showed in the class.

1. Use the LT to derive the velocity relations: Suppose 3 inertial frames: S, S’,S’’. S’ is moving with v1 relative to S; S” is moving with v2’ relative to S’. From the LT between S-S’ and S’-S”, we can work out the transform relation between S-S” (In matrix representation, will be product of matrix). Find out the transform between S-S” through S-S’ and S’-S” and show the v2, the relative velocity between S-S” from the transformation you just calculated in terms of v1, v2’.

Answer:

Define: 

The transform between S and S’, expressed in matrix form (I shall put ct first as convention):





But the transformation between S” and S should also be just using v2:

 Compare the two relation, then:

 This is the velocity transform for vx as in (12-19) in the note.

1. Refer to the figure below:



A tunnel (proper length L)and a train (proper length is rL), the train is running with velocity v through the tunnel. A bomb was planted at the far-end (B) of the tunnel and it is going to explode when the head of train reaches B. As the tail of the train first enters the tunnel (A), a deactivation light signal is sending out to disarm the bomb. Find the requirement of length of the train, i.e. the value of r that can safely pass this tunnel. Work this A) using algebra method. B) using the Minkowski diagram. (You can pick any frame of your preference, for example I worked this problem using train frame in algebra method; and using tunnel frame as the orthogonal one in Minkowski diagram)

Answer:

As I stated in the problem, I will use train’s frame to work out the algebra method (It is actually much easier using tunnel frame)

Event 1. Left tunnel end overlaps with tail of train: (0,0)

Event 2. The light emitted reaches the bomb:

In the train’s frame, the tunnel is moving with –v and its length appears to be , so the time of this event is:



Event 3: the right end of tunnel reaches the head of train:



To save the train, it requires t3>t2 (event 2 happens first, at earlier time)

You may try the tunnel frame (easier) and find the same result as should be.

Now use geometric method---Minkowski diagram



The blue line is the world line of the tunnel’s right end (bomb), the green line is the light signal. O origin taken as event 1 defined above. Red lines are the train’s frame. The world line of the head of train is represented by the red dashed line. The limiting case is shown in the figure, the OB is the maximum length. The question is reduced to find the length OB (a pure geometrical problem, but need to keep in mind the unit length between the two frames are different, I shall solve the OB in units of the tunnel frame and then convert it to the trains frame, since rL is the length in train’s frame)



I forgot the cosine formula, but can derive it as:



So 

 do not forget

 This needs to be converted to units in S’, which is shown in the relation (12-10) in the note.  Then:



1. Here is one interesting paradox (chopping of snake, adapted from example 15.3 in Taylor’s book): A snake with proper length of 100 cm is moving with . You are holding two clippers 100cm apart and chop them down simultaneously when the snake’s tail coincides with the left clipper (the downward distance is so small that the time taken by the clippers to come down is negligible). The figure below is the reasoning in your frame, and it will do no harm to the snake.



1. Is the reasoning in the figure above correct?
2. What happened in the snake’s point of view? Will it be harmed? Solve this by doing calculation using LT transform, i.e. what are the events (clippers come down) viewed in snake’s frame.

Answer:

1. The reasoning from the person’s frame is correct. I will list out events explicitly:

Event 0: tail of snake and left clipper overlaps, I choose this as origin (x,t)=(0,0)

Event 1: the head of snake simultaneous in person’s frame: (x1=80,t1=0)

The 80 comes from length contraction, , In the person’s frame the simultaneous measurement of head-tail will give: .

Event 2: The right clipper chops at (x2=100,t2=0)

That is indeed what is shown in the figure.

1. In the snakes frame, its length is 100 cm and the distance of two clippers would appear 80cm apart, but this cause no contradiction. The two clippers are not coming down simultaneously! So the snake won’t be hurt in his frame too.

The event 0 is still (0,0) in S’ frame (snake’s)

The head of snake will be at (100,t’) (from snakes point of view, its head is at 100 in all times)

The question is where and when the event 2 happened?





So in the S’, the right clipper comes down at earlier time (compare with event 0) and further distance than where the head is, no harms down.

This could also be argued from simultaneity is relative. Though the right clipper is 80 cm away from left is simultaneous event viewed by snake, but the right clipper chops at earlier time by , and since the speed of clippers to the snake is 0.6c, and that corresponds to a extra distance of 45 cm. (Of course this is just a explanation of the LT transform results)

1. Another popular paradox (very similar to the above):



A hole and a rod(both are very thin in thickness), the proper length of the rod and radius of the hole are both L. The rod is moving with v and at the moment the rod head reaches the ring of the hole (as in the figure right above), the hole is lifted and question is: will the rod go under the hole? Answer this in both hole’s frame and rod’s frame.

Answer :

From the hole’s point of view, the rod will go under the hole. Because the length of the rod is  measured in hole’s frame, and the hole’s diameter is bigger. No problem for the rod to pass.

In all these kind of questions, if you get correct answer in one frame, such answer should be explained in other frames too. i.e. in rod’s frame, the rod is still able to pass the hole, despite the fact that the hole’s diameter is measured as , shorter than rod’s length. The trick is of course simultaneity! In rod’s point of view, the hole is moving, as the person lifts the hole (simultaneous in hole’s frame), the two ends of hole are not lifted simultaneously in rod’s frame. The left end (tail of the hole in rod’s view) was lifted earlier than the right end. That is why the smaller hole can pass a longer rod from rod’s point of view.

Let’s work out this quantitatively by defining events first:

In hole’s frame (where the coordinates of events can be assigned easily):

Event 1: Right end lifted and rod’s right end overlap the right end of the hole (0,0)

Event 2: The left end of the rod at this time 0: (-,0) (This event is actually not important in this problem, you choose skip this event)

Event 3: Left end of the hole lifted: (-L,0)

Now we can use L-Transform to see how these events as viewed by the rod

Event 1 is (0,0) still of course.

Event 2 is (-L, t’) (you may use L-Transform to get one value of t’, but the rod is stationary in its own frame, so that end will be at –L all time, this is exactly same as last problem, actually I am a bit regretful to give you this problem because it is little different from the last one)

The key is event 3: use L-T to get: 

So the left end of hole is lifted in rod’s point of view at position  (longer than L) and at later time by . The rod will pass the hole too.

From the later time, you can also get the position of the left end of hole by:

 (same as the L-T result of course)

1. KK 12.7

Answer: (a) 



(b) 







1. KK 12.8

Answer: Without using Doppler formula, we can derive this. Taking the observer and light source frame as S:

Set t=0 when waveform 1 hits mirror at x1. It takes 

1

2

for event 1 (observer receives the light 1 from mirror).

cτ0

After an interval, the mirror meets 2 at x2.

1

v

The time for light 2 reaching observer is .

 this is period.

2

x1

We also have relations: 

x2



, compare with formula of moving source, has , is the velocity of moving source.











Of course the above argument could also follow from velocity transformation. In frame of mirror S’, the object and image approach mirror with velocity v, so in lab frame the velocity bet. Image and object (observer have also) will be , but the method here is from Doppler effect is called K-calculus after H. Bondi).

1. KK 12.9

Answer:  light in vacuum,  light in glass.

A

B

 c’ is velocity of light in glass in lab frame S.

L





Note the thickness of glass in lab frame is contracted.

 





If  

If  