Homework 15 for GP1

By SJ

1. KK 12.4

Answer: a) 





b) As, , so .

From the problem, as 



(Taylor expansion)



1. KK 12.6

Answer: In S, u

In S’,  







1. A train with proper length L, runs at velocity v1 w.r.t. the ground, a person runs from back of the train to the front at v2 w.r.t. the train. How much time this process takes as viewed by a person on the ground.

Answer:

First Method: Using L-Transform

It is first easy to work in train’s frame S’

Event 1: Person leaves back of train 

Event 2: Person arrives at the front 

Then the time interval between the events measured by the ground observer is:



Second method: Use the velocity transform to find the velocity of the person with respect to ground, and the rest will be like a chasing problem:

The velocity of the person to ground is specified as u:



Chasing problem:





Same result. (I only listed two simplest solutions)

1. The same train above and this time the person walks with very low velocity u (u<<v1) from back to front. The person’s clock is synchronized with that of train’s at the back, and since the speed u is very small so that the time-dilation between the two clock’s (the train and the person’s) can be neglected, so when the person reaches the front, his clock will agree with the clock at the front of the train. Now considering observing this on the ground frame: for fixed time on the ground, the clocks on the train would appear reading differently. ( This is exactly like the figure I showed in notes (in section of *simultaneity is relative*), but here is with ground clock’s all read one value while the train’s are not) the train’s clock at the back will read Lv1/c2 more than that at the front (as viewed by the ground observer), so in view of the previous statement, the ground observer will conclude that the time elapsed by the person’s clock during the process (the person walks slowly from back to front of the train)must be Lv1/c2 less than that by the front clock. please show that the ground observer’s conclusion is correct.

Answer: This problem also at least could be solved in two ways:

1. By method of transform;



In the frame S (ground), the events are:

Event 1: Person leaves tail 

Event 2: The clock at this moment (simultaneous in ground frame S) at the front of train 

Event 3: The person reaches front of train (x3,t3) (Unknown at present)

What the problem asked is the time elapsed on clock at front i.e.  and the time elapsed on the person’s clock:  (we may call the person’s frame is S” and I do not have room to draw that in the figure), prove that 

Using the relation between event 1 and 2, easy to see that:



This is what stated in the problem that the back clock will lead front by this much (as shown in the figure above left).

The time difference between event 3 and 1 is actually easiest to compute in the S’ (trains frame), it is just: 

Then: 

For the time elapsed in person’s frame:

, but since u is very small as stated in the problem, so 

Thus indeed we have relation  as concluded by the ground observer.

1. By method of velocity transformation and proper time



Let the velocity of person w.r.t to ground is V:





(Of course you can get this result directly from last problem by letting v2 as u)

The time elapsed for front clock (which is proper time) can be calculated by:





This is exactly same as method 1.

(If you use the result in last problem and a clear mind on proper time, this problem can be solved in 3 lines)

1. In my notes I derived formula of gamma factor in 1-D motion case, i.e.

 or . Now extend this to 2-D motion case, use transformation between velocities to prove that:



Here, v is the velocity between frames and is along x direction. In frame S’, the particle is moving with velocity , and , where 

Answer: This is a straightforward (but maybe messy) calculation

From definition:



Using velocity transform relation:





Then we get the needed relation.

1. Show that with the relativity momentum formula, if the momentum is conserved in the frame of ground then it is conserved in the frame travelling with A, in the example of elastic collision at the beginning of my notes section 13.1.

Answer: As the figure in my notes



In the frames S: ,, rest mass (these are the given conditions). In S, clearly the initial momentum along x direction is:





The momentum of B has same magnitude and reversed direction so that the initial total momentum is zero (for both x and y directions). Similarly the final total momentum is zero too. The momentum is conserved in frame S (this is trivial given the conditions)

In the frames S’ that moves w.r.t. S with ux:

Use the velocity transform formula ( as I showed some results in the notes)

, since v=ux



After collision the velocities are:





The initial momentum along x direction (only has contribution from B):



And final momentum is:

. The x component velocity of B are same before and after, and the , which is because  (here you do not need to calculate this gamma factor, but if you do, you can use the formula derived in last problem which will give us:

 which can be further simplified if you plug in the values for v, ux ), so the momentum along x is conserved in S’.

For the momentum along y direction:



 and  (certainly you can calculate thefrom definition, but since we already did that kind of calculation before, I just used the result)

,

 and  so that 

Exactly same calculation for the final y component of momentum:



 so that final momentum along y is also 0

Thus the momentum is conserved in S’ too.

1. What is the velocity of the particle with rest mass m if its kinetic energy equals to the rest energy? What is the velocity if the kinetic energy is n times that of rest energy?

Answer:

, if , then, so 

If ,

1. KK 13.1

Answer: 





 In proton’s frame, Universe travel with c, due to length contraction 

.

1. KK 13.3

Answer: 

Or directly from , 

Assume  is not changing (which is also the assumption in the formula of Newtonian mechanics in the problem) for simplicity.

















1. KK 13.4

Answer: In frame of A(S’)



 

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1. KK 13.5

Answer: Initial  

 

Find  







