Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
A		**2106614**

Analysis and prediction of fungal decomposition of litter

Summary

Fungi, as the main decomposers of organic matter in terrestrial ecosystems, are key factors in the global carbon cycle. It plays a key role in the process of lignin decomposition. It will be of great significance to study the decomposition characteristics of wood fiber and litter.

First, we established a model of colony area growth by studying the mechanism of lig-nin decomposition by fungi. We take leaf litter as the research object, consider the above fungi growth and reproduction process is affected by its own conditions and abiological factors, and draw out the Logistic model of colony area through the correlation between colony area and mycelium length.

Secondly, we studied the interaction relationship of colonies and divided it into two functions for discussion. In the early stage, it can be simply considered that the colony area is increasing due to the superposition of the decomposition effect of various colonies on lit-ter. The competition model among colonies was introduced to analyze the number of colo-nies. The results showed that under the action of multiple fungi, the growth of colonies with weak competitiveness would be restricted by the strong, and the strong would tend to the maximum capacity.

In addition, in the model of colony interaction, we changed the initial value of the number of fungal species to analyze the influence of the change in the number of fungal species on the decomposition efficiency, ensuring that other conditions were the same. The results showed that the litter decomposition rate increased with the increase of fungal di-versity.

Finally, for the climate of different species, we learned the value range of temperature and humidity in each climate by consulting data, and summarized the ranking of the influ-ence of each climate on the decomposition rate of fungi: tropical rain forests > arid > semi-arid > temperate > arboreal. Based on this, we made predictions for different envi-ronments, and found that the decomposition rate of litter was the fastest in the tropical rain forest climate, and the decomposition rate was the lowest in the arboreal area.

It is worth mentioning that in sensitivity analysis, the initial area is set as the inde-pendent variables, we will change its size, the results show that the area of the litter effect on fungi, the existence of the decomposition rate of litter: when the litter area is larger, faster colonies of litter decomposition, when dropped with the increase of leaf area can cause the increase of colony types, consistent with our study of colony diversity, shows that our model is reliable.

Keywords: fungi; differential equation; litter decomposition

Contents

1 Introduction

1.1 Problem Background

The carbon cycle describes the exchange of carbon throughout the geochemical cycle, in which a key component is the decomposition of plant materials and wood fibers. The key factor in the decomposition of wood fiber is fungi. A study on the decomposition of fungal wood identified fungal traits that determine the rate of decomposition: mycelial expansion rate and humidity tolerance. Slow-growing fungal strains tend to be better able to survive and grow water and temperature under environmental changes, while faster-growing strains tend to be less robust to the same changes.

1.2 Restatement of the Problem

We need to simulate the decomposition of wood fiber in a given land. Given the background information and constraints identified in the problem statement, we need to address the following issues:

- Establish a mathematical model to describe the decomposition of leaf litter and wood fiber in the presence of a variety of fungi.
- Combine the interactions of different species of fungi with different growth rates and different moisture tolerance in your model.
- Provide a model to describe the interaction between different types of fungi. A dy-namic description of the interaction should include both short - and long-term trends. Your analysis should also examine its sensitivity to fluctuations in environmental change and determine what the trend of constant change in the atmospheric envi-ronment is to aid in assessing the impact of climate change on the model.
- Also include predictions of the relative strengths and weaknesses of 'each species' or' combinations of species likely to persist over a period of time 'and for different environments such as arid, semi-arid, temperate, arboreal, tropical rain forest, etc.
- Describe how the diversity of fungal communities affects the overall decomposition efficiency of litter. The importance and role of predicting biodiversity in the context of varying degrees of variability in the local environment.

1.3 Our Work

This is a fungal biological problem. Firstly, relevant data were collected to understand the research status of fungi decomposition of organic matter. Combined with the knowledge of biology and ecology, I studied and analyzed the mechanism and process of fungi decomposition of organic matter and the relationship between population diver-sity. Searching the previous research experimental data, the data are screened and ana-lyzed.

After making these preparations, we began to study the decomposition mechanism of litter. Considering the two environmental factors of temperature and humidity, and considering that the change of the mass of falling matter is essentially the process of lignin decomposition by fungi, we took the mass of litters as the research object to build a decomposition model of litters. The model is based on ideal conditions. In view of the shortcomings of this model, we analyzed the microscopic process of fungal growth, took the litter as a flat sheet with a certain shape, and set up a growth model of fungal colony area with the area of fungal colony as the research object. Then the two models are considered comprehensively to get our final mixed model of litter decom-position.

Based on the above analysis, we continue to study the interaction between differ-ent colonies. Assuming that they don't interact with each other in the early stage and compete for resources in the later stage, we finally get the short - and long-term effects of colony interaction. Finally, we did the influence factor analysis, and obtained the in-fluence of abiotic influence factors temperature and humidity on the growth rate of fungi. The superiority of fungal community diversity to ecosystem was expounded, and the environment of different regions was predicted. Discuss the important role of bio-diversity in ecosystems.

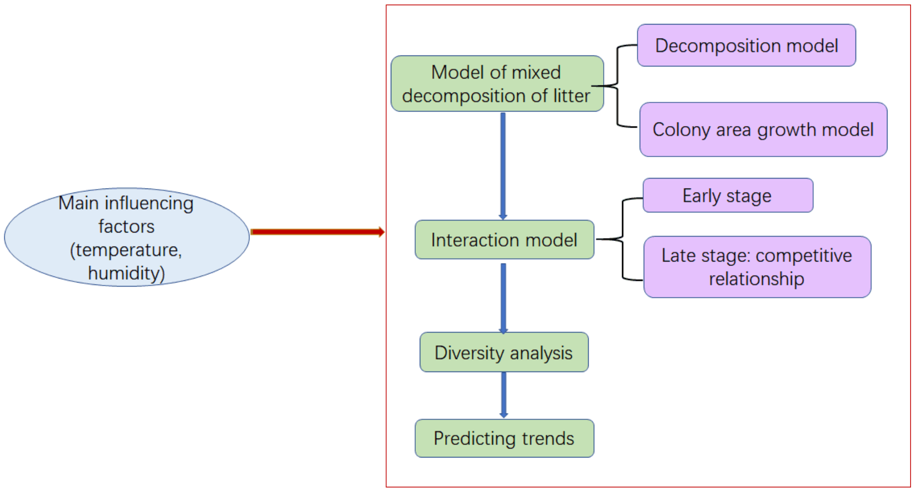

Figure 1: The structure of our paper

2 Model Preparation

2.1 Assumptions and justifications

To simplify the problem, we make the following basic assumptions, each of which is properly justified.

- All the fungi in a fungal colony are descended from the same fungus
- In the microscopic process of fungal hyphae elongation, the visible litters (leaves) are pla-nar flakes of a certain shape. Litter (leaves) acts as a medium, providing nutrients for fungal growth
- It is assumed that the colony formed is round and its diameter can be measured by precise instruments
- It is assumed that the waste generated by different types of fungi in the growth and repro-duction process will hinder the growth of other fungi to different degrees
- During the study of an ecosystem, no abnormal factors such as floods, volcanoes, and earthquakes are considered to damage the system

- It is assumed that the decomposition process of litters is not affected by abiotic factors (such as light radiation, CO2, N concentration) and other biological factors except tempera-ture and humidity

2.2 Notations

Important notations used in this paper are listed in Table 1,

Table 1: Notations

Symbols	description	Unit
t	time	day
K	Decomposition rate	mg/d
r	Growth rate (Hyphal Extension Rate)	mm/d
S	Area of the colony	mm^2
σ	The area ratio between colonies	
m_t	The remaining quality	mg
m_0	Initial leaf mass	mg
k_a	Principal decomposition rate	
S_m	Leaf area	mm^2
T	The environment temperature	

3 Mixing model of fall decomposition

The decomposition of organic matter is a long-term process, which cannot be revealed by short-term experimental means, and the model is one of the more ideal methods. The decomposition rate of litter is affected by many factors. These factors, including microorganisms, climate, nutrients and lignin concentrations, are interactive, variable, and even difficult to measure.

Because of the complexity of decomposition process, it is impossible for a single model to describe the decomposition process well. Therefore, we fully consider the decom-position mechanism, as well as various environmental factors and conditions, establish a specific set of equations to describe this complex decomposition process.

3.1 Litter decomposition model

Olson (1963) proposed to use the exponential decay model to estimate the decomposition rate of litters. Based on previous studies, we used the double exponential model to de-scribe the decomposition process of litters. Using double exponential model to simulate the process of litter decomposition, because the model decomposition process can be divided into two processes separately fitting, respectively characterized early quick decomposition of phase control (nutrition) and late phase (fiber length (control) the slow decomposition process, therefore better fitting with the actual decomposition process. The following for-mula is obtained:

$$\frac{m_t}{m_0} = ae^{(-k_a t)} + be^{(-k_b t)}$$

Where: m_t represents the residual mass of litters, and m_0 represents the initial mass of litters. The first term on the right side of the equation represents the principal exponent, and k_a is the principal decomposition rate, which is the determinant of the long-term decompo-sition process. The second term is the additive exponent, and k_b is the early decomposition rate, which represents the early decomposition process.

3.2 Colony Area Model

3.2.1 The establishment of model

Taking a leaf leaf as the research object, the leaf was abstracted into a flat sheet of a certain shape (the culture medium that provides nutrients for the fungus), and the leaf was randomly distributed with different sizes of colonies.

The area of leaf litter is fixed, and it can only provide the necessary conditions for a certain number of colonies. With the growth and reproduction of fungi, the colony area is increasing, and the nutrition of litter and the environmental conditions of the ecosystem have more and more obvious limiting effects on the growth of fungi. At the initial stage, we considered the growth rate R of the fungus as a constant. When the colony area increased to a certain value, it was seen that the growth rate R decreased with the increase of the colony area: that is, the growth rate R was a function of the colony area S and a decreasing function of S. Thus, we could derive the Logistic model of the colony area:

$$r\left(s\right) = r\left(1 - \frac{S}{S_m}\right)$$

Since the colony area is related to the rate of mycelium expansion, we assume that fungi expand with the mycelium length as the radius, so the area is S, which is a function of the mycelium length, namely:

$$S\left(L\right) = \pi R\left(t\right)^2$$

The fungal expansion rate (R) is approximately the growth rate of mycelium length (L), which is a function of time, namely:

$$\frac{dL}{dt} = r$$

Our initial conditions for the given colony area are:

$$S\left(0\right) = S_0$$

We obtained the litter decomposition rate – colony area model as follows:

$$\begin{cases} \frac{dS}{dt} = rS\left(1 - \frac{S}{S_m}\right) \\ S\left(0\right) = S_0 \\ \frac{dL}{dt} = r \\ S\left(t\right) = \pi L\left(t\right)^2 \end{cases}$$

3.2.2 The solution of the model

We found an experimental report [1] on the decomposition of decayed leaves by white rot fungi, and used his experimental data to verify the accuracy of our model. Through data screening, some abnormal data are filtered out. Put it into our model to solve the equation, and we get the relationship curve between the total area of colony and time, as shown in the figure below:

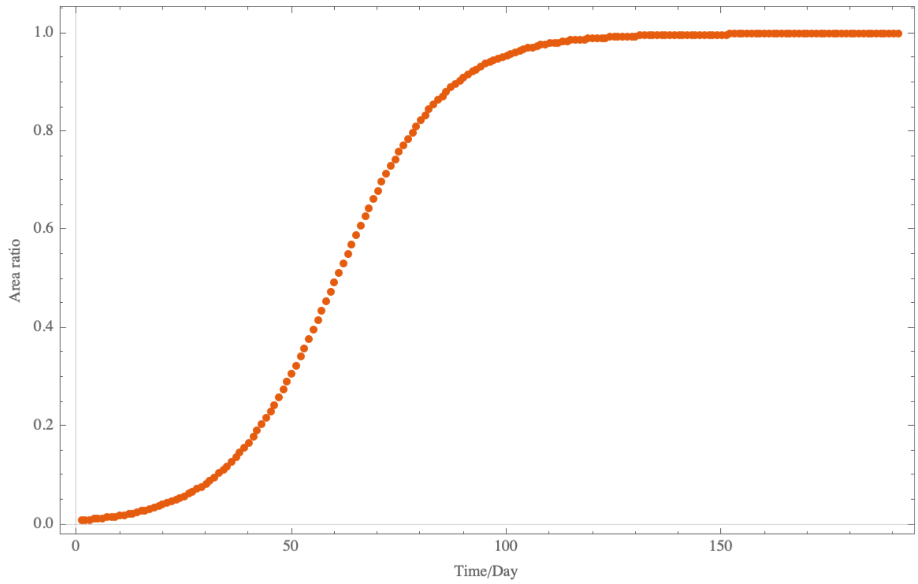

Figure 2: The relation between area proportion and time

Result analysis: we assumed that the number of fungi in the initial colony was very small and the colony area was negligible, and the white rot fungi grew and multiplied rap-idly under the condition of sufficient nutrients for their growth. In this process, mycelia continue to grow, and after reaching a certain density, due to the limitation of the autogenic conditions of organisms, they cannot grow indefinitely and the reduction of the consump-tion of nutrients at the location of the fungus, some of these fungi continue to spread and reproduce towards the edge of the colony, and some die near the center of the colony.

The graph shows the relationship between the proportion of colony area and time, which is the process of colony area expanding continuously. This process is essentially the growth process of white rot fungi. The slope of the curve can reflect the growth rate of the white rot fungus, which increases first and then decreases with time, as described above.

3.3 Mixing model of litter decomposition

On the basis of the above model, the surface density $\mu\left(t\right)$ of litter is introduced

$$m_0 = \iint\limits_D \mu\left(0\right) dS$$

Where D is the integral interval on the litter area. When the colony area is $S\left(t\right)$, the remaining mass in the colony is expressed by $S\left(t\right) = \pi R^2$ and:

$$m_t = \iint\limits_S \mu\left(t\right) dS$$

Where, is the mass within the colony, and the colony area S is the integral interval.

The simultaneous equations, we can get the following curve:

Result analysis: On the whole, the mass ratio of litter to initial mass decreased with the decomposition of lignin by fungi. The slope of the curve can reflect the magnitude of the decomposition rate, but it is not the same. We can see that in the initial stage of decom-position,

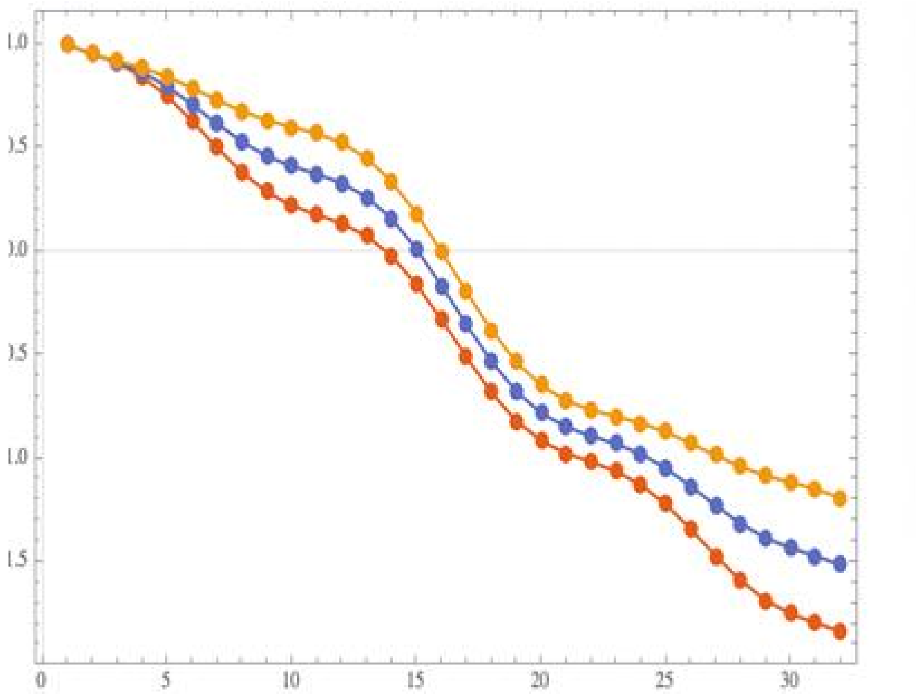

Figure 3: The relation between area proportion and time

because the colony is not large enough, and the number of fungi is not large enough, the decomposition rate is relatively slow and gradually increases. Such growth is not sustainable. With the expansion of the colony area, the decomposition rate slows down due to the competition between the nutrients of litters and the fungi inside the colony, and the macroscopic performance is that the mass decay of litters slows down. The fungus eventually decomposes 98

4 Colony interaction

4.1 Short-term Effects

When studying the interaction of multiple colonies, there is ample space in the medium and sufficient food in the early stage. We assume that multiple colonies grow independently on the medium, which all conform to the model, a species decomposition model, and their decomposition is simply regarded as a multiple effect of fungi on litter decomposition.

Different types of fungal colonies live in the same environment, and they may promote or compete with each other. From the biological point of view, some of the organic matter produced by fungi in the process of growth and reproduction will benefit other organisms and promote their growth. And some organic matter produced may have a toxic effect on them, thus inhibiting their growth.

However, in a short period of time, the number of fungi in the colony has not reached a certain amount, showing that the colony area is still very small. The colonies are so isolated from each other that they have little impact. From this we assume that multiple colonies grow independently on it

4.2 Long-term effect

In order to better study colony interaction, we assume that there are two colonies A and B. When they live alone in the environment, the evolution rule of colony area follows the rule of colony area model. Remember $3S_1(t)\, S_2(t)$ are the areas of two colonies respectively, are their growth rates, and Sm is the area of litters.

Therefore, for colony A, there are:

$$S_1(t) = r_1 S_1 \left(1 - \frac{S_1}{S_m}\right)$$

Wherein, the above formula reflects the blocking effect of colony A on its own growth due to its consumption of limited resources. When two colonies survive in the same environment, the influence of colony B's con-sumption of the same limited resource on the growth of colony A can be reasonably sub-tracted from Equation (7), which is in direct proportion to the ratio of the area S2 of colony B to the total area.

Therefore, the equation of colony A area growth is modified as follows:

$$S_1(t) = r_1 S_1 \left(1 - \frac{S_1}{N_1} - \sigma_1 \frac{x_2}{N_2}\right)$$

Where σ_1 means that the amount of food that colony B consumes to support colony A per unit area is σ_1 times that of colony A to support colony B. Similarly, the existence of colony A also affects the growth of colony B. The equation for the growth of colony B can be as follows:

$$S_2(t) = r_2 S_2 \left(1 - \frac{S_2}{S_m} - \sigma_2 \frac{S_1}{N_1}\right)$$

In the competition between the two populations, $\sigma_1\sigma_2$ are two key indicators. From the explanation given to them above, $\sigma_1 > 1$means that the consumption of colony B is more than that of colony A in the re-source consumption of supporting colony A, so the corresponding understanding of $\sigma_1 > 1$ can be made. Generally speaking, there is no definite relationship between $\sigma_1\ and\ \sigma_2$ and here we only discuss the case where $\sigma_1\ and\ \sigma_2$are independent of each other. Simultaneous Equations (8) and (9)

$$\begin{cases} f(S_1, S_2) = r_1 S_1 \left(1 - \frac{S_1}{N_1} - \sigma_1 \frac{x_2}{N_2}\right) \\ g(S_1, S_2) = r_2 S_2 \left(1 - \sigma_2 \frac{S_1}{N_1} - \frac{S_2}{S_m}\right) \end{cases}$$

Let's assume that $\sigma_1 > \sigma_2, intheecologicalsense, meansthatBisweakerthanAinthecompetitionforresour$

Using the data of 34 species of fungi [2], lignin fiber was decomposed on the same litter. Using the established colony interaction model, the following curve of colony area ratio changes over time was obtained.

5 Impact factor analysis

The growth of fungal mycelium is affected by environmental conditions, especially air temperature and humidity. We transformed the influence of air temperature and relative humidity changes on mycelium growth into an analysis and study of the influence on colony area.

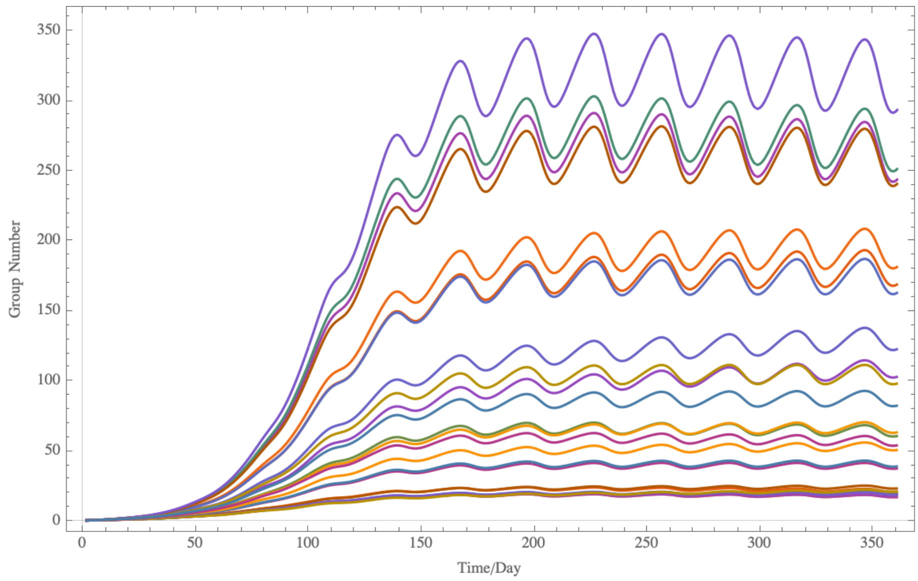

Figure 4: Simulation diagram of interspecific interaction relationship

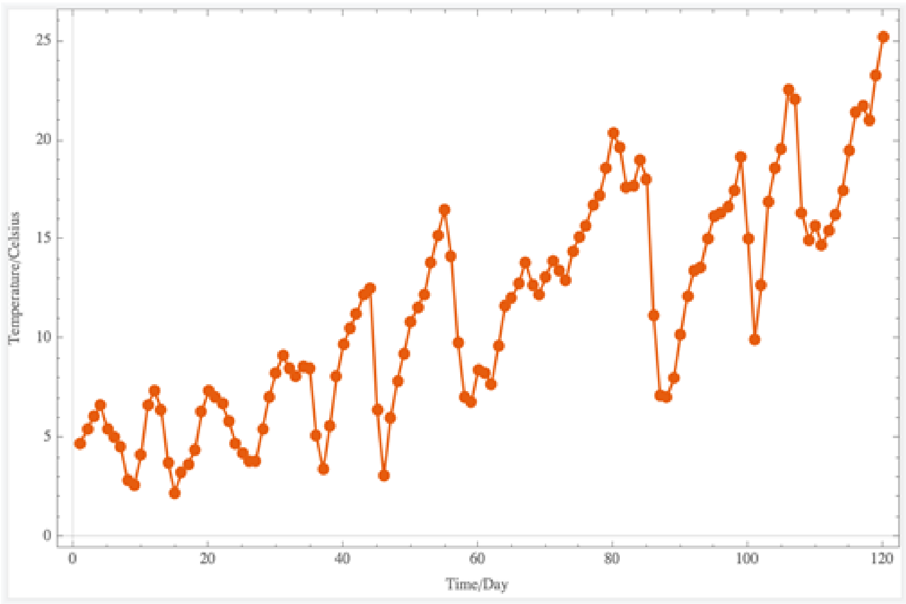

Figure 5: (December March) 120-day mean temperature data

We collected the 120-day average temperature data of a region from December to the following March and from March to July respectively, and made a line chart as shown in the figure below. We will use the above data to analyze the influence of different air temperatures and relative humidity on mycelium growth.

Firstly, considering the effect of temperature on the decomposition rate, a mathemati-cal model of the effect of air temperature and humidity on the growth rate was established based on the growth model of colony area. It is assumed that the relationship between air temperature and colony area proportion is a single peak curve. The mathematical model of the influence of temperature on mycelium growth is as follows:

$$JS = a_1 \cdot T^2 + b_1 \cdot T + c_1$$

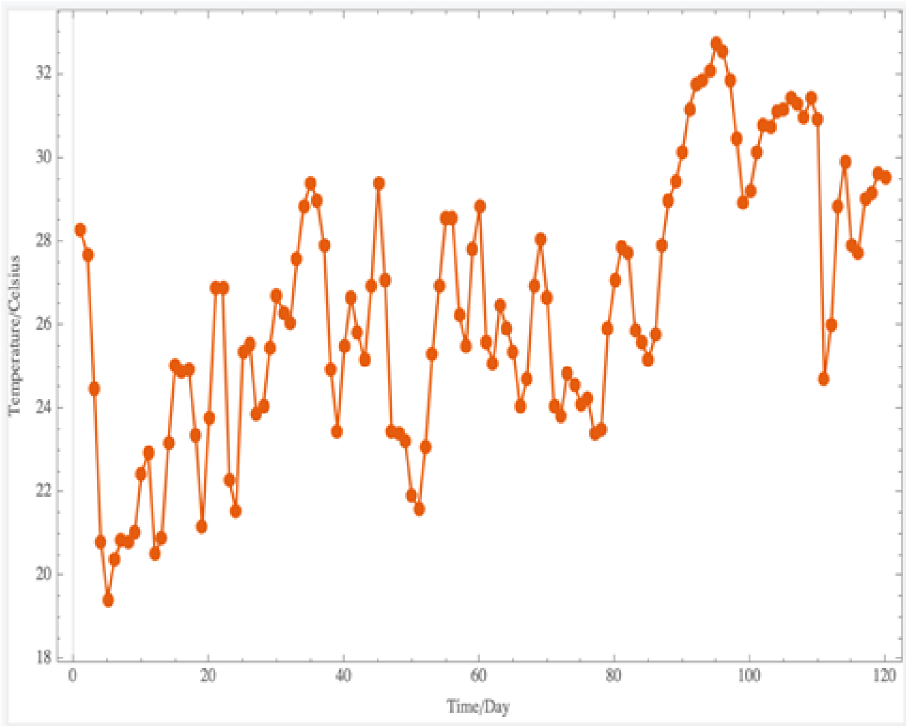

Figure 6: (March   July) 120-day mean temperature data

Whereinto, JS is the proportion of colony area, T is air temperature, and $a_1b_1c_1$ are parameters.

Although in practice, different air humidity has little effect on the ratio of mycelium to substrate, air humidity is almost certainly an important environmental factor affecting fun-gal production. Therefore, the mathematical relationship between air humidity and the pro-portion of colony area is also simulated. The mathematical relationship between the propor-tion of fungal colony area and air humidity is as follows:

$$JS = a_2 \cdot H^2 + b_2 \cdot H + c_2$$

Where H is relative air humidity and $a_2b_2c_2$ are both parameters

There is an optimum range of air temperature and humidity for mycelium growth. When the air temperature is lower than optimum value, the growth of mycelia will be re-stricted, but when the air temperature is higher than optimum value, the growth of mycelia will also be inhibited.

The search data was put into the model, and the relationship between temperature and mycelial growth rate was obtained as shown in the figure: When the air temperature is lower than the optimum value, the growth of mycelia will be restricted, but when the air temperature is higher than the optimum value, the growth of mycelia will also be inhibited.

At the same time, considering the influence of humidity on mycelial growth rate, the following results were obtained: When the air humidity is lower than the appropriate value, mycelia will dry and wither; when the air humidity is higher than the appropriate value, mycelia will grow rapidly and have difficulty breathing. Therefore, the precise environment is very important for the growth of mycelium.

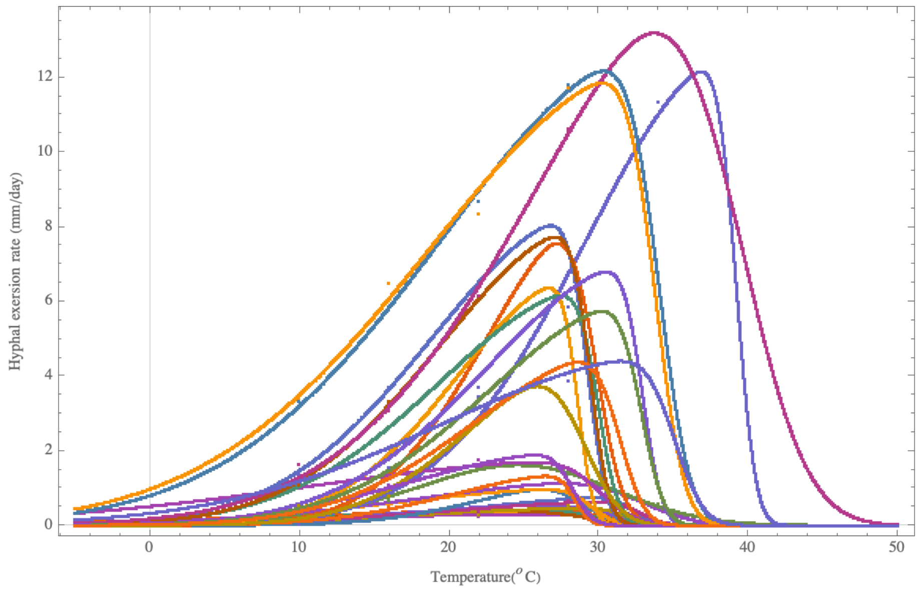

Figure 7: Effect of temperature on Hyphal exersion rate

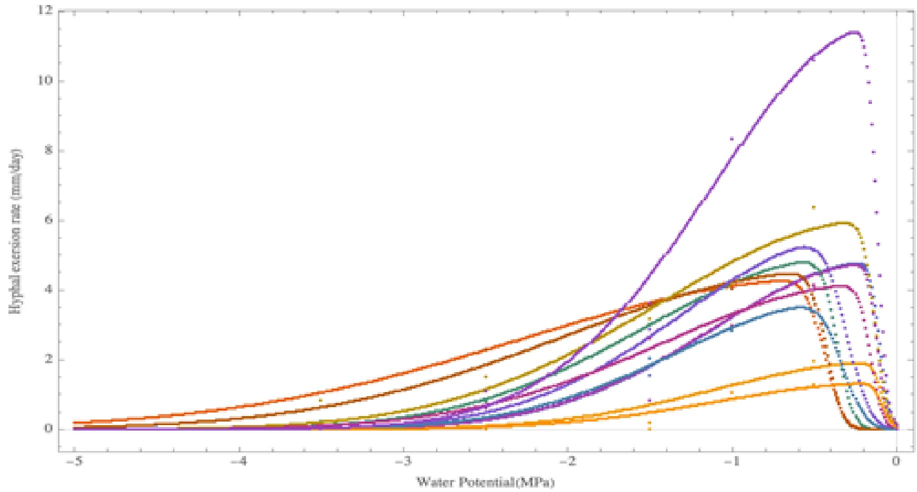

Figure 8: Effect of Water Potential on Hyphal exersion rate

6 Studies on the diversity of fungal communities

In order to study the diversity of the fungal community, we took the number of fungal species as the independent variable to design a model to analyze the influence of the number of fungal species on the decomposition efficiency. We set the initial condition to the number of fungal species.

With other conditions being the same, we changed the initial value of fungus species and used the species interaction model to study the proportions of various colonies in the presence of multiple fungi, so as to study the macroscopic perfor-mance of diversity.

The experimental results showed that the decomposition rate of litter was dif-ferent from that of single species with the increase of species diversity. When the number of fungal populations increases (the richness is high), the organisms are in a highly competitive environment, because of the fierce competition for shared re-sources, and the efficiency of individual populations is low.

7 Predictions for different environments

In reality, fungi exist in a myriad of environments. By consulting data, we analyzed the climatic characteristics of five environments, namely arid, semi-arid, temperate, arboreal and tropical rain forest, mainly including temperature and humidity.

As a leading factor affecting life activities, temperature has an important effect on the number of microorganisms and enzyme activity, and then plays a leading role in the de-composition of litter.The research results show that due to the effect of temperature, the decomposition rate of litter of zonal climate, each climatic zone of litter decomposition rate from big to small in the order: tropical > > temperate zone, in the case of high temperature, also affected by the humidity of the air, due to the tropical rain forest of rainwater, rainfall in arid regions, therefore,

In conclusion, the influence of each climate on litter decomposition rate from large to small is as follows: tropical > arid BBB>mi-arid BBB2 t>rate BBB3 tree> By changing the parameters of temperature and humidity on the interaction model of fungi, the influences of the five environments of drought, semi-drought, temperate zone, trees and tropical rain for-est on the decomposition of litter by fungi (DR is the decomposition rate of litter) were pre-dicted: $Dr_{tropical} > Dr_{arid} > Dr_{semi-arid} > Dr_{temperate} > Dr_{arboreal}$

8 Sensitivity Analysis

Impact of Litterfall area on Decomposition Rate.In our model, we took a leaf leaf as the study object and abstracted it into a flat sheet of a certain shape (as a culture medium for the fungus to provide nutrients). The area of leaf litter is fixed. So we keep the temperature and air humidity constant and change the area of litters. According to Fig. 11, we can see that

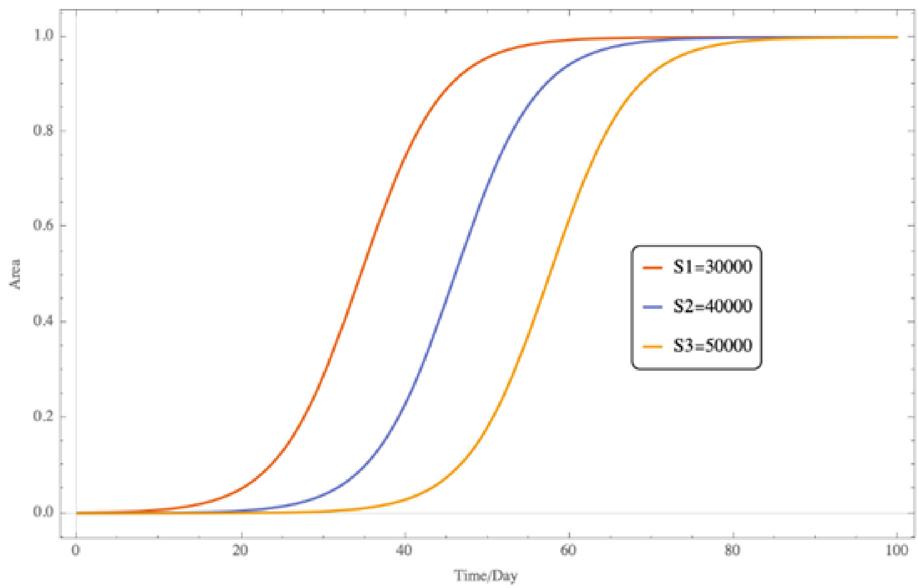

Figure 9: Sensitivity Analusis of area

the area of litters has a certain influence on the decomposition of litters. With the increase of the limit value of leaf litter area, the decom-position rate of bacterial colonies to litters is also faster. This is because the increase of leaf litter area will lead to the increase of colony species. According to our study on the diversity of fungal community, when the population number increases, the decomposition rate of leaf litter will be affected, which is in line with the natural law of biodiversity and also illus-trates the rationality of our model.

9 Strengths and weaknesses

9.1 Strengths

- We reflected the expansion of mycelium from the growth of area, effectively avoiding the calculation of the micro number which is difficult to measure, and reasonably es-tablished the relationship between the growth of area and the expansion rate of myce-lium.
- In the study of colony interaction, it is divided into two main processes to avoid discussing the monotony of the interaction, and the result is more in line with the natural law.
- In this paper, the sensitivity analysis of the model shows that the model is reasonable and reliable.

9.2 Weaknesses

- In the litter decomposition model established in this paper, the decomposition process of litter under the action of a fungus was studied. Ignoring other creatures that break down litter. Although the model is simplified, the reality of the model is reduced.
- When studying the decomposition rate of litters, we only focus on the two factors of temperature and humidity. In fact, there are many other factors that can affect the decomposition rate of litter, so our research results may be biased from the facts.

10 Enter the world of fungi

Fungi is a kind of microbial classification, and its parallel classification method also includes bacteria and viruses, but fungi and them have a big difference, all kinds of mushrooms, mushrooms are the most typical fungi, living in a parasitic, saprophytic way, mainly rely on the host nutrition, such as mushrooms with the help of the nutrition of the old wood reproduction. Unlike bacteria, fungi can make their own nutrients; It's not like a virus, which has its own whole orga-nelle. There are a vast number of species of fungi, most of which are harmless or even beneficial to humans, and only a few that cause disease.

Figure 10: https://image.baidu.com

Fungi are an important part of terrestrial ecosystems in terms of biomass and diversity, and they influence almost every aspect of terrestrial ecosystem function. For example, a key com-ponent of the carbon cycle is the breakdown of plant materials and wood fibers. Fungi are a major factor in the breakdown of organic matter, and the carbon cycle allows carbon to be re-newed and used in other forms, so fungi have a direct impact on global carbon and nutrient dynamics. Microbes that break down cellulose include bacteria and fungi. Both bacteria and fungi may produce cellulases that break down fibrin. Cellulase is widely found in natural organ-isms. Bacteria, fungi, etc. can produce cellulase. There are fewer bacteria that can produce cellu-lase, and the amount of cellulase produced is not large, so it is generally not used. Commonly used in the production of cellulase from fungi, more common are Trichoderma, Aspergillus and Penicillium fungi.

Fungi play an important role in the ecosystem of the Earth's biosphere. For the convenience of illustration, taking forest ecosystem as an example, soil and above-ground vegetation constitute a complex functional system. Soil provides the growth and material basis for plants and soil organisms. Plants are the important source and material basis of soil organic matter and various nutrient elements, while various microorganisms in the soil are the main force of de-composing plant and animal residues and organic matter into the soil. For example, 85% of forest orange litter is decomposed by microorganisms, among which fungi play an obvious role.

In a paper on fungal decomposition of wood, Lustenhouwer et al., characteristics of fungal decomposition rates were identified. The researchers examined a large number of traits associated with different fungi and their role in ground litter (dead plant material) and lignin fiber decomposition. Based on the research of Lustenhouwer et al., some scholars established various models to describe the decomposition of litters under the interaction of multiple fungi, studied the importance of fungi in biodiversity, and made the process of fungi decomposition of litters clearer. This is of great significance for our further study on fungal biodiversity.

References

[1] Juan Y , Yin Z J , Hailong Y , et al. Mathematical study of the effects of temperature and humidity on the mycelium growth of Pleurotus eryngii[C]// First International Conference on Agro-geoinformatics. IEEE, 2012.

[2] Song Xinzhang, Jiang Hong, Zhang Huiling, et al. Effects of global environmental change on forest litter decomposition [J]. Acta Ecologica Sinica, 2008(9)

[3] Lin Chengfang, Gao Ren, Chen Guangshui, et al. Research progress of litter decomposition model [J]. Fujian Forestry Science and Technology, 2007(03):227-233.

[4] Rui-qing zhang, zhen-jun sun, Wang Chong, etc. Xishuangbanna tropical rain forest litter decomposition process of ecological . Dynamics of leaf litter decomposition [J]. Chin J Plan Ecolo, 2006, 32(003):622-631.

[5] Effects of fungal community on litter decomposition in Pinus tabulaeformis plantation

[6] Hattenschwiler S, Tiunov A V, Scheu S. Biodiversity and litter decomposition in terrestrial ecosystems[J]. Annual Review of Ecology, Evolution, and Systematics, 2005: 191-218.

[7] Fioretto A, Papa S, Curcio E, et al. Enzyme dynamics on decomposing leaf litter of Cistus incanus and Myrtus communis in a Mediterranean ecosystem[J]. Soil Biology and Biochemistry, 2000, 32(13): 1847-1855.