3 Mechanism Design and Auction Theory in Computer Networks

In this chapter, we introduce mechanism design and required properties of a mechanism. In particular, we first define a mechanism and mechanism design. Then, we discuss one important principle, the revelation principle, when designing a mechanism. After that, we define and discuss required properties of the mechanism, including incentive compatibility, individual rationality, economic efficiency, and budget balance. Finally, we discuss optimal mechanisms.

3.1 Mechanism Design

3.1.1 Mechanism

We consider a trading market as follows:

- The market consists of one seller that has one indivisible item for trading to buyers. For example, in wireless resource trading markets, the seller can be a service provider, and the item is a resource unit, such as a bandwidth unit or a sensing data unit.
- There is a set of N buyers, such as mobile users, that is denoted by $\mathcal{N} = \{1, 2, \ldots, N\}$.
- The buyers have private values of the item. Let v_i denote the value of the item to buyer i. $v_i \in \mathcal{V}_i$, where $\mathcal{V}_i$ is the set of types of buyer i.
- v_i is also known as a *type* of the buyer since it refers to the true value or the preference of the item.
- Let $\mathbf{v} = (v_1, \ldots, v_N)$ denote the set of types reported by the buyers.
- We have $\mathcal{V} = \prod_{i=1}^{N} \mathcal{V}_i$, which is the product of the sets of the buyers' types.

To trade the item to the buyers, the seller can use different schemes/methods. For example, the seller can use auction schemes for trading the item. Accordingly, the buyers, called bidders, are asked to submit their bids to the seller. The bids are the prices that the buyers are willing to pay the seller for the item. The bids of the buyers are known as *messages* of the buyers. Upon receiving the bids, the seller needs to select the best buyer, the winner, for receiving the item. Also, the seller determines the price that the winner needs to pay for winning the item. For example, when the first-price sealed-bid auction is used, the seller selects the best buyer with the highest bid as the winner, and the highest bid is the price that the winner needs to pay. When the second-price sealed-bid auction is used, the seller selects the best buyer with the highest bid as the winner, and the second highest bid is the price that the winner needs to pay. Apart from the two auction schemes, the seller can use other schemes/methods for trading the item. For example, the seller can adopt a scheme in which the seller (i) posts a fixed price for the item, (ii) selects the best buyer that accepts the price first, and (iii) determines the fixed price as the price that the winner pays.

Any scheme or method that the seller adopts for trading the item is called the *mechanism*. For example, the first-price sealed-bid auction and the second-price sealed-bid auction, as just discussed, are mechanisms. Each mechanism contains *rules* selected or set by the seller. The rules of the mechanism consist of (i) the resource allocation or *allocation rule* and (ii) the payment determination or *payment rule*. The allocation rule determines the allocation of the item to the buyers, and the payment rule determines the prices that each buyer needs to pay if it receives the item. The rules are designed by the seller to achieve its objectives or required properties. Designing the allocation and payment rules of the mechanism to meet the desired objectives is called *mechanism design*. Mechanism design is discussed in the next section.

3.1.2 Mechanism Design

To design the mechanism, an input is required. Typically, the input includes messages submitted or reported by the buyers. The messages may be types, meaning the true values or preferences, of the buyers or the bids of the bidders when the auctions are used. A general mechanism is modeled by a tuple $(\mathcal{B}, f^1, f^2)$ where [183]

- $\mathcal{B}$ can be considered to be the input of the mechanism. $\mathcal{B}$ is a product of sets of the buyers' messages that is defined as $\mathcal{B} = \prod_{i=1}^{N} \mathcal{B}_i$, where $\mathcal{B}_i$ is the set of possible messages of buyer i. Again, the messages may be the types or bids of the buyers. Let b_i denote the message submitted by buyer i, and $\mathbf{b} = \{b_1, \ldots, b_N\}$ denote the vector of messages submitted by N buyers. When the message of buyer i is its type, $b_i = v_i$, we say that the buyer reports or submits its type truthfully. Otherwise, when $b_i \neq v_i$, we say that the buyer reports its type untruthfully.
- f^1 is the allocation rule that maps the sets of the buyers' messages to the winning probabilities of the buyers. f^1 is defined as $f^1 : \mathcal{B} \to \mathcal{G}$, where $\mathcal{G}$ is the set of the winning probabilities of the buyers. Each element $g_i \in \mathcal{G}$ refers to the probability that buyer i wins the item.
- f^2 is the payment rule that maps the sets of the buyers' messages to the prices that the buyers need to pay the seller. Thus, f^2 is expressed by $f^2 : \mathcal{B} \to \mathcal{P}$. $p_i \in \mathcal{P}$ is the price that buyer i is expected to pay the seller if it wins the item. p_i is sometimes called the *expected payment* that buyer i makes.
- f^1 and f^2 are the *social choice functions* of the mechanism. $\mathcal{G}$ and $\mathcal{P}$ are the *outcomes* of the mechanism. For convenience, we can use a common social choice function f to represent the two social choice functions f^1 and f^2.

To further understand the outcomes of the mechanism, we consider two well-known auctions, the first-price sealed-bid auction (described in detail in Chapter 5) and the second-price sealed-bid auction (described in detail in Chapter 6). Assume that the seller receives a vector of messages (i.e., bids) $\mathbf{b} = \{b_1, \ldots, b_N\}$ from the buyers (i.e., the bidders).

With the first-price sealed-bid auction, the seller implements a social choice function $f(\mathbf{b})$ to map the message vector $\mathbf{b}$ to the outcomes as follows:

- The allocation rule for each buyer i is

$$g_i = \begin{cases} 1, & \text{if } b_i > \max\limits_{j \neq i} b_j \\ 0, & \text{if } b_i < \max\limits_{j \neq i} b_j \end{cases} \tag{3.1}$$

- The payment rule for each buyer i is defined as

$$p_i = \begin{cases} b_i, & \text{if } b_i > \max\limits_{j \neq i} b_j \\ 0, & \text{if } b_i < \max\limits_{j \neq i} b_j \end{cases} \tag{3.2}$$

In summary, the first-price sealed-bid auction selects the best buyer with the highest bid as the winner, and the highest bid is the price that the winner needs to pay.

With the second-price sealed-bid auction, the seller implements a social choice function $f(\mathbf{b})$ to map the message vector $\mathbf{b}$ into the outcomes as follows:

- The allocation rule for each buyer i is

$$g_i = \begin{cases} 1, & \text{if } b_i > \max\limits_{j \neq i} b_j \\ 0, & \text{if } b_i < \max\limits_{j \neq i} b_j \end{cases} \tag{3.3}$$

- The payment rule for each buyer i is

what does this mean?

$$p_i = \begin{cases} \max\limits_{j \neq i} b_j, & \text{if } b_i > \max\limits_{j \neq i} b_j \\ 0, & \text{if } b_i < \max\limits_{j \neq i} b_j \end{cases} \tag{3.4}$$

In summary, the seller selects the best buyer with the highest bid as the winner, and sets the second highest bid as the price that the winner pays.

These two auctions are the mechanisms that are often used in single-item markets. They are used depending on the seller's objectives and required properties. For example, the seller adopts the first-price sealed-bid auction if it wants to gain revenue or profit. Conversely, the seller uses the second-price sealed-bid auction if it wants to achieve the incentive compatibility property. In fact, the seller can have multiple items for trading with other objectives and required properties. Thus, the seller needs to design an appropriate mechanism to achieve the objectives and required properties.

To design the mechanism, a mechanism designer (the seller) needs to determine or implement the social choice function f. For this, the mechanism designer may need to consider all possible mechanisms and choose the best one that has the objectives equal to or close to the desired objectives. This task is generally a complicated search problem, and the mechanism designer sometimes cannot solve it. Fortunately, the *revelation principle* can be used to reduce the search space and to facilitate the mechanism design task. Before presenting this principle in the next section, we briefly define the equilibrium of the mechanism.

Every mechanism can be defined as a game (e.g., a Bayesian game) of incomplete information among players, referring to the buyers or bidders. The information of each player refers to the type, meaning the true value or preference, of the player that is private. Again, each player i can report its message truthfully ($b_i = v_i$) or untruthfully ($b_i \neq v_i$). b_i is called the *strategy* of player i. The strategy is a complete decision defining an action that the player selects at a stage of the game. We have the following definition:

biding strategy?

DEFINITION 3.1 *[184] A vector of strategies* $\mathbf{b} = \{b_1, \ldots, b_N\}$ *of the players is called the equilibrium of the mechanism if given the strategies* $\mathbf{b}_{-i}$ *of other players, strategy* b_i *maximizes the utility or payoff of player* i*. Here,* $\mathbf{b}_{-i}$ *is the vector of strategies of the players excluding the strategy of player* i*.*

3.1.3 Revelation Principle

Here, we provide some important concepts related to the revelation principle (and the mechanism design).

- *Truth-revealing strategy:* In the mechanism, the truth-revealing strategy of a player is that the player reports the true information about its type. In particular, the truth-revealing strategy of player i is $b_i(v_i) = v_i$.
- *Direct-revelation mechanism:* A direct-revelation mechanism is the mechanism in which the players are asked/required to perform truth-revealing strategies by reporting directly their true types, consisting of their true values or preferences. This means that we have $\mathcal{B}_i = \mathcal{V}_i, \forall i \in \mathcal{N}$ and $\mathcal{B} = \mathcal{V}$. Thus, a direct-revelation mechanism is modeled as $(\mathcal{V}, f^1, f^2)$. For simplification, the direct-revelation mechanism can be modeled by a tuple with two functions f^1 and f^2 as (f^1, f^2) where $f^1 \colon \mathcal{V} \to \mathcal{G}$ and $f^2 \colon \mathcal{V} \to \mathcal{P}$.
- *Incentive-compatible direct-revelation mechanism:* If the truth-revealing strategies in the direct-revelation mechanism constitute an equilibrium, the mechanism is called an incentive-compatible direct-revelation mechanism or strategy-proof direct-revelation mechanism. The equilibrium of the incentive-compatible direct-revelation mechanism is $\mathbf{v} = (v_1, \ldots, v_N)$. For example, the second-price sealed-bid auction or the Vickrey auction is an incentive-compatible direct-revelation mechanism for the single-item allocation problem. The Vickrey–Clarke–Groves (VCG) auction is an incentive-compatible direct-revelation mechanism for the multi-item allocation problem. The first-price sealed-bid auction is a direct-revelation mechanism but not the incentive-compatible

multiple auctions

direct-revelation mechanism since each player bids half its true value at the equilibrium.

The revelation principle can be stated as follows:

THEOREM 3.2 *[183] If a social choice function f can be implemented by an arbitrary mechanism, namely the original mechanism, and if this mechanism has an equilibrium corresponding to implementing the social choice function, then*

- *It is the equilibrium for the players to submit their types truthfully.*
- *The social choice function f can be implemented by the incentive-compatible direct-revelation mechanism with the same outcome or the same objective as the original mechanism.*

The proof of Theorem 3.2 can be found in [183] and [185]. The revelation principle means that the social choice function or the outcome of any mechanism can be replicated/implemented by the incentive-compatible direct-revelation mechanism. The revelation principle thus simplifies the task of mechanism design. In particular, if the mechanism designer wants to design the mechanism to achieve a certain objective or a property, the mechanism designer can search for only those incentive-compatible direct-revelation mechanisms that have the same objective or the same property. The social choice function f of the incentive-compatible direct-revelation mechanisms can be used to design the original mechanism. If no such an incentive-compatible direct-revelation mechanism exists, there is no mechanism that can achieve the outcomes, objective, and property. For example, if the mechanism designer wants to design the mechanism to achieve social welfare maximization (the objective) and incentive compatibility (the required property), the mechanism designer can use the social choice function of the well-known VCG auction, which is an incentive-compatible direct-revelation mechanism. By narrowing the search area, the problem of designing as well as finding the mechanism becomes much easier for the mechanism designer to solve.

In the next sections, we discuss important properties of the mechanism, which are summarized here [185]:

- *Incentive compatibility or truthfulness:* This property guarantees that players report their types, or true values, truthfully. This property is important when designing the mechanism to overcome the self-interest or rationality of the players.
- *Individual rationality:* This property guarantees a positive utility/payoff for every player who is participating in the game.
- *Economic efficiency:* This property aims to maximize the total utility or payoff of players.
- *Budget balance:* This property guarantees balanced transfers across players; that is, there are no transfers out of the system or into the system.
- *Revenue maximization:* This objective aims to maximize the revenue of one of the players.
- *Fairness:* This property seeks to guarantee fairness, such an equal winning probability, among players.

3.1.4 Incentive Compatibility

Incentive compatibility is an important property of the mechanism. The reason is that players are rational, meaning that they have an incentive to not report their types truthfully. This behavior reduces the efficiency of the resource allocation. The mechanism holding the incentive compatibility overcomes the rationality or the self-interest of the players, so that the players have an incentive to report their types truthfully. We have the following concepts related to the incentive compatibility:

- *Utility function:* The utility function refers to the welfare or satisfaction of a player when the player receives an item. Let u_i be a utility function of player i. u_i is typically a function of parameters including (i) its type, v_i; (ii) its reported type, b_i; and (iii) the types of other players, $\mathbf{v}_{-i}$, the reported types of other players, $\mathbf{b}_{-i}$, and the social choice function f of the mechanism. Thus, we have $u_i(v_i, b_i, \mathbf{b}_{-i}, f)$. Note that one or some of the parameters in the utility function can be removed for simplification.
- *Dominant strategy:* Strategy b_i of player i is dominant if this strategy maximizes the player's utility given all possible strategies of other players. In other words, the dominant strategy of a player maximizes the player's utility regardless of the strategies of other players. Formally, we have

$$u_i(b_i, \mathbf{b}_{-i}, v_i) \geq u_i(b_i', \mathbf{b}_{-i}, v_i), \forall i, \text{and } \forall b_i' \neq b_i \tag{3.5}$$

  For example, in the second-price sealed-bid auction, the dominant strategy of each player is the truth-revelation strategy, which entails reporting its type truthfully, $b_i = v_i$. The reason is that the bid of the player is the price that the player accepts, but it is not the actual price that the player pays. The price that the player pays is completely independent of its bid; it is the second highest bid. This means that the auction guarantees a gain utility for the player when it becomes the winner. This is further explained and discussed in Chapter 6.
- *Nash equilibrium:* In a Nash equilibrium, every player selects a utility-maximizing strategy given the strategies of other players.

  DEFINITION 3.3 *A strategy profile* $\mathbf{b} = (b_1, \ldots, b_N)$ *is the Nash equilibrium if*

$$u_i(b_i, \mathbf{b}_{-i}, v_i) \geq u_i(b_i', \mathbf{b}_{-i}, v_i), \forall i \textit{ and } \forall b_i' \neq b_i \tag{3.6}$$

- *Bayesian Nash equilibrium:* In a Bayesian game, each player has incomplete information about the types of other players, but the player is assumed to know a common prior about the distribution of other players' types. The Bayesian Nash equilibrium is defined as follows:

  DEFINITION 3.4 *A strategy profile* $\mathbf{b} = (b_1, \ldots, b_N)$ *is the Bayesian Nash equilibrium if*

$$E_{\mathbf{v}_{-i}}[u_i(b_i, \mathbf{b}_{-i}, v_i)] \geq E_{\mathbf{v}_{-i}}[u_i(b_i', \mathbf{b}_{-i}, v_i)], \forall i \textit{ and } \forall b_i' \neq b_i \tag{3.7}$$

Definition 3.4 means that each player selects a strategy to maximize the expected utility in conjunction with the expected-utility maximizing strategies of other players.

Now we can define the incentive compatibility property as follows. This mechanism is called incentive compatible or truthful or strategy-proof if every player can achieve the highest utility by reporting their types truthfully. There are two common types of incentive compatibility: *Bayesian–Nash incentive-compatibility* and *dominant-strategy incentive-compatibility*.

DEFINITION 3.5 *The mechanism is said to hold the Bayesian–Nash incentive-compatibility if*

$$E_{\mathbf{v}_{-i}}[u_i(v_i, \mathbf{v}_{-i}, v_i)] \geq E_{\mathbf{v}_{-i}}[u_i(v_i, (\mathbf{v}_{-i}, b_i)], \forall i, \forall v_i, and\ \forall b_i \tag{3.8}$$

Definition 3.5 means that each player that reports its type truthfully achieves the highest expected utility given that other players report their types truthfully.

DEFINITION 3.6 *The mechanism is said to hold the dominant-strategy incentive-compatibility if*

$$u_i(v_i, \mathbf{b}_{-i}, v_i) \geq u_i(v_i, (\mathbf{b}_{-i}, b_i), \forall i, \forall v_i, \forall b_i, and\ \forall \mathbf{b}_{-i}. \tag{3.9}$$

Definition 3.6 means that the player achieves the maximum utility by reporting its type truthfully no matter what other players submit.

It can be seen from Definition 3.5 and Definition 3.6 that every dominant-strategy incentive-compatibility mechanism is the Bayesian–Nash incentive-compatibility mechanism, but the Bayesian–Nash incentive-compatibility mechanism may exist even if no dominant-strategy incentive-compatibility mechanism exists. Therefore, the Bayesian–Nash incentive-compatibility is said to be "weaker" and the dominant-strategy incentive-compatibility is said to be "stronger."

3.1.5 Individual Rationality

Individual rationality is known as a "voluntary participation" constraint. The reason is that the mechanism, such as a game or an auction, may not attract a player to participate in it if the player's expected utility is negative. For example, a bidder may not have an incentive to participate in an auction if its expected utility is negative due to a high price that the bidder needs to pay. Let $u_i(f(\mathbf{v}))$ denote the expected utility of player i at the equilibrium of the outcome when the player participates in the mechanism. Also, let $u_i^0(v_i)$ be the expected utility achieved by the player for non-participation – that is, the player is outside of the mechanism. We have the following definition.

DEFINITION 3.7 *[186] A mechanism is said to hold individual rationality (i.e., the mechanism is individually rational) if the mechanism implements a social choice function $f(\mathbf{v})$ such that*

$$u_i(f(\mathbf{v})) \geq u_i^0(v_i), \forall i\, and\ \forall v_i \tag{3.10}$$

In practice, we often assume that the expected utility of the player when it does not participate in the mechanism is zero, $u_i^0(v_i) = 0$. Thus, we can define that the mechanism holds individual rationality if for every player i, $u_i(f(\mathbf{v})) \geq 0$, or if every player achieves a non-negative utility when participating in the mechanism.

3.1.6 Economic Efficiency and Budget Balance

Economic Efficiency

Economic efficiency refers to an economic state at which items are optimally allocated to buyers in the best way while minimizing waste of resources and inefficiency. Thus, the mechanism that holds the economic efficiency ensures that the items are allocated to those buyers that value them the most. We also say that the mechanism is allocatively efficient.

DEFINITION 3.8 *The mechanism is said to hold the economic efficiency property if it can implement a social choice function f that maximizes the total value over all buyers.*

For example, the double auction, as presented in Chapter 8, is a mechanism that holds the economic efficiency property since it guarantees that the items of sellers are allocated to buyers that value them the most.

It is worth mentioning the concept of "social welfare." In general, social welfare is defined as the sum of utilities of all players. Since the utility of each player is proportional to the value of the item to the player, social welfare is closely related to economic efficiency. In particular, the mechanism that maximizes the total utility over players also maximizes the total value of all the players. Thus, we can define the economic efficiency as follows: the mechanism is efficient if it maximizes social welfare.

Budget Balance

Consider a mechanism with N players in which there are $N - K$ buyers and K sellers. Also, there is a broker that conducts the trading. Let p_i denote the price that buyer i pays for receiving items, and let p'_j denote the price that seller j receives for selling its items. In general, the budget balance introduces constraints over the total monetary transfer made from the players to the broker. Depending on the total monetary transfer, there are two different degrees of the budget balance.

- The mechanism is said to hold the strong budget balance if

$$\sum_{i=1}^{N-K} p_i + \sum_{j=1}^{K} p'_j = 0 \qquad (3.11)$$

an intermediate person

  Equation (3.11) means that there are no monetary transfers to the broker; that is, the monetary transfers are done only between the buyers and the sellers.
- The mechanism is said to hold the weak budget balance if

$$\sum_{i=1}^{N-K} p_i + \sum_{j=1}^{K} p'_j \geq 0. \qquad (3.12)$$

  Equation (3.12) means that there can be some monetary transfers to the broker.

Double auction with the average payment rule [187] is a mechanism that assures a strong budget balance since all monetary transfers are among buyers and sellers, and the auctioneer (i.e., the broker), does not gain money.

3.2 Optimal Mechanisms

In this section, we introduce two most fundamental objectives of an optimal mechanism design: social surplus or total welfare maximization and profit or revenue maximization. We first define the social surplus and profit of the mechanism. Then, we discuss the problem formulations corresponding to the two objectives. Further details of the optimal mechanism design can be found in [188].

3.2.1 Social Surplus and Profit

indivisible item: a product can not be divided into small parts

We consider again the market model in Section 3.1.1. In particular, there is one seller that has one indivisible item for trading to buyers. There are N buyers that are willing to buy the item. The value of the item to buyer i is v_i, and let $\mathbf{v} = (v_1, \ldots, v_N)$ be the value profile of the buyers. We assume that the seller designs a mechanism that has an allocation $\mathbf{g} = (g_1, \ldots, g_N)$, where g_i indicates whether player i receives the item, and the payment $\mathbf{p} = (p_1, \ldots, p_N)$, where p_i is the payment made by buyer i given the allocation g_i. Note that given the allocation $\mathbf{g}$, the seller may need to pay the cost $c(\mathbf{g})$. In particular, for computer network environments, the cost can be the resource maintenance cost. For example, when delivering streaming live videos to viewers (i.e., buyers), a content provider needs to pay the cost for leasing network links. Also, to provide cloud network resources to cloud tenants, a cloud provider needs to pay the bandwidth cost to network providers. The cost $c(\mathbf{g})$ is sometimes called service cost. We have the following definitions [188].

- *Buyer surplus:* The surplus of buyer i, denoted by S_i^b, is defined as the difference between the value of the item to the buyer and the price that the buyer pays. Thus, S_i^b is defined as $S_i^b = g_i v_i - p_i$.
- *Seller surplus:* The surplus of the seller, denoted by S^s, is defined as the difference between the price that the seller receives from selling the item and the cost of the item. Thus, S^s is defined as $S^s = \sum_i^N p_i - c(\mathbf{g})$.
- *Social surplus:* The social surplus or social welfare of the mechanism, denoted by S, is the sum of the buyer surplus and the seller surplus:

$$\begin{aligned} S &= \sum_i^N S_i^b + S^s \\ &= \sum_i^N g_i v_i - c(\mathbf{g}) \end{aligned} \tag{3.13}$$

From (3.13), the social surplus is actually the difference between the total value of buyers and the service cost. The service cost $c(\mathbf{x})$ is typically fixed. Thus, the mechanism that maximizes the social surplus also maximizes the total value of the buyers. According to Definition 3.8, such a mechanism is allocatively efficient.

- *Profit:* The profit of the mechanism is defined as the seller surplus, the difference between the total payment made by the buyers and the service cost:

$$\pi = \sum_{i}^{N} p_i - c(\mathbf{x}) \tag{3.14}$$

Note that $\sum_i^N p_i$ is defined as the revenue of the seller. Since the service cost $c(\mathbf{x})$ is typically fixed, the mechanism that maximizes the profit also maximizes the revenue of the seller.

profit maximization mechanism doesn't exist

In general, designing the optimal mechanism in terms of social surplus maximization seems to be simpler than designing the optimal mechanism in terms of profit maximization. For example, to design the optimal mechanism in terms of social surplus maximization, we can simply adopt the VCG auction, a generalization of the second-price sealed-bid auction described in Section 6.2. For the optimal mechanism in terms of profit or revenue maximization, such an optimal mechanism does not exist. The mechanism designer needs to know the distribution of the players' types to derive the optimal mechanism.

3.2.2 Social Surplus Maximization Problem

In this section, we derive the optimal mechanism for the social surplus. The optimization problem of maximizing social surplus is to find an allocation rule $\mathbf{g}$ to maximize the surplus $S(\mathbf{v}, \mathbf{g})$:

$$\arg\max_{\mathbf{g}} S(\mathbf{v}, \mathbf{g}) \tag{3.15}$$

The allocation rule is implemented as follows:

- Assume that the optimal social surplus obtained by solving Equation (3.15) is $S^*(\mathbf{v})$. This means that

$$S^*(\mathbf{v}) = \max_{\mathbf{g}} S(\mathbf{v}, \mathbf{g}) \tag{3.16}$$

- Consider a particular buyer i. There are two possible cases for the buyer [188]:
  - Case I: The item is assigned to buyer i, $g_i = 1$, so $S^*(\mathbf{v})$ can be expressed as follows:

$$S^*(\mathbf{v}) = v_i + \max_{\mathbf{g}_{-i}} S\big((\mathbf{v}_{-i}, 0), (\mathbf{g}_{-i}, 1)\big) \tag{3.17}$$

  We define $S^*_{-i}(\mathbf{v}) = \max_{\mathbf{g}_{-i}} S((\mathbf{v}_{-i}, 0)(\mathbf{g}_{-i}, 1))$, and we have $S^*(\mathbf{v}) = v_i + S^*_{-i}(\mathbf{v})$.

- Case II: The item is not allocated to buyer i, $g_i = 0$, so $S^*(\mathbf{v})$ can be expressed as follows:

$$S^*(\mathbf{v}) = \max_{\mathbf{g}_{-i}} S\big((\mathbf{v}_{-i}, 0), (\mathbf{g}_{-i}, 0)\big) \tag{3.18}$$

We define $S^*(\mathbf{v}_{-i}) = \max_{\mathbf{g}_{-i}} S((\mathbf{v}_{-i}, 0), (\mathbf{g}_{-i}, 0))$, and we have $S^*(\mathbf{v}) = S^*(\mathbf{v}_{-i})$.

- To maximize the social surplus, the item is allocated to buyer i whenever the surplus in case I is greater than or equal to the social surplus in case II [188]:

$$S^*(\mathbf{v}) = v_i + S^*_{-i}(\mathbf{v}) \geq S^*(\mathbf{v}) = S^*(\mathbf{v}_{-i}) \tag{3.19}$$

- Let $S_i^0 = S^*(\mathbf{v}_{-i}) - S^*_{-i}(\mathbf{v})$. Then, we can say that the item is allocated to buyer i whenever its value is greater than or equal to S_i^0. Note that S_i^0 does not depend on v_i of buyer i, and thus S^0 is considered to be a *critical value* [188].

The critical value $S^0 = S^*(\mathbf{v}_{-i}) - S^*_{-i}(\mathbf{v})$ is known as the externality that buyer i imposes on the other buyers due to receiving the item [188]. In other words, since buyer i receives the item, the social surplus of the other buyers is $S^*_{-i}(\mathbf{v})$ instead of $S^*(\mathbf{v}_{-i})$. Buyer i needs to pay a price that is equal to the externality that it imposes on the other buyers.

One well-known optimal mechanism in term of social surplus maximization is the VCG auction described in Section 6.2. The allocation and payment rules of the VCG auction aim to maximize the social welfare while guaranteeing the incentive compatibility. Thus, the VCG auction is applicable in several scenarios of computer networks that aim to satisfy buyers' QoS or the fairness among the buyers. For example, the VCG auction is adopted for allocating data rates to M2M applications as proposed in [189], for spectrum allocation in the 4G LTE network as presented in [190], and for bandwidth reservation in cloud networking as proposed in [191]. In particular, for the bandwidth reservation, the considered model consists of a cloud provider, or seller, which owns a number of distributed data centers, and cloud tenants, or buyers, which act as application and service providers. Cloud tenants rent bandwidth from the cloud provider to serve their subscribers. To avoid the high bandwidth reservation payment, the cloud tenants can lie about their revenues obtained by serving subscribers. The VCG auction is adopted for the bandwidth reservation to achieve both optimal social welfare and incentive compatibility such that the cloud tenants have no incentive to lie about their revenue information. Specifically, the cloud tenants are required to submit their bids to compete for bandwidth to the cloud provider. Each bid consists of bandwidth demands and the price per unit of bandwidth for which the cloud tenant is willing to pay. To achieve the highest social welfare for the allocation, the winners are determined through a linear programming model that can be solved in polynomial time. The VCG mechanism is then applied to calculate the charge for each winner. The charge is the difference between the social welfare when the winner does not participate and that when the winner participates in the auction. Further details are presented in [191]. Since the proposed approach has an optimal allocation and calculates the charge based on the VCG auction, it is concluded to be the optimal auction mechanism in terms of social welfare maximization for the bandwidth reservation.

3.2.3 Profit Maximization Problem

Profit =
Revenue - Cost

Profit here refers to the profit of the seller as defined in (3.14). Since the service cost is typically fixed, the profit maximization problem is equivalent to the revenue maximization problem. In general, designing the optimal mechanism in terms of profit maximization is more difficult than designing the optimal mechanism in terms of social surplus maximization. The main reason is that improving the profit of the seller also means reducing the utility/payoff/benefit of the buyers. Thus, maximizing the profit of the seller may make the utility of the buyers negative, and the mechanism may not attract the desired buyers. Therefore, the mechanism designer needs to optimize the trade-off between the profit of the seller and the utility of the buyers. In other words, the mechanism designer needs to solve an optimization problem that maximizes the expected profit or expected revenue of the seller while guaranteeing the rational individuality property. In addition, incentive compatibility is an important property that needs to be introduced in the optimization problem. For this, the mechanism designer needs to know the distribution of the values of the items to the buyers. Based on the values drawn from the distribution, the mechanism designer determines the allocation and payment rules to maximize the expected profit while guaranteeing the required properties – that is, the rational individuality and incentive compatibility. The mechanism with the allocation and payment rules is called *Bayesian optimal mechanism*. Here, "Bayesian" means that the probability distribution of a particular buyer's value is known to other buyers and even the mechanism designer. A simple distribution function is $F(v) = v$, as the buyer's value follows the uniform distribution $U[0, 1]$.

The optimization problem for the optimal mechanism in terms of profit maximization is defined as follows [192]:

$$\max_{\mathbf{g},\mathbf{p}} \Big[\sum_i p_i(\mathbf{v}) \Big] \tag{3.20}$$
$$\text{s.t. [IC], [IR]}$$

where $\mathbf{g}$ is the allocation rule, $\mathbf{p}$ is the payment rule, and [IC] and [IR] are the incentive compatibility and individual rationality constraints, respectively.

The problem in (3.20) can be found in different scenarios of computer network environments. For example, it can be found in fog resource trading markets [193], where

- The market consists of one service provider, the seller, that has M computing resource units for trading.
- There are N users, and each user i has a value, a type, v_i of the computing resource unit. The valuation profile of the users is $\mathbf{v} = (v_1, \ldots, v_N)$. v_i is drawn independently from distribution F_i over a possible valuation profile $\mathbf{V}_i$.

- The service provider may not know type v_i of each user i, but the service provider can know the distribution functions, for example through the observation.
- The users are required to submit the prices that they are willing to pay to the service provider. Let $b_i \in \mathbf{V}_i$ denote the price submitted by user i, and $\mathbf{b} = (b_1, \ldots, b_N)$ denote the price profile of the users.

Upon receiving the price profile **b** from the users, the service provider determines an allocation rule and a pricing rule. The allocation rule includes the winning probabilities $g_i, i = 1, \ldots, N$, of the users, and the pricing rule includes the conditioned prices $p_i, i = 1, \ldots, N$, for the users. The service provider needs to determine the allocation and pricing rules to maximize its revenue. Moreover, to provide an incentive to the users to participate in the market, the utility of the users must be non-negative. For this, the service provider can formulate its problem as shown in (3.20). The problem in (3.20) is the constrained optimization. In general, solving such a constrained optimization problem to derive the optimal mechanism is difficult [192]. The Myerson's optimal mechanism [194] can be adopted by using the concept of *virtual values* and *monotone transform functions*. However, the Myerson's optimal mechanism is limited to a single item.

In recent years, machine learning technique that has the ability to automatically identify relevant features of data has gained considerable attention. Recent theoretical results in [195] show that machine learning using stochastic gradient descent can successfully find globally optimal solutions for complex problems. Thus, machine learning can be used to solve the constrained optimization problem in (3.20). The use of machine learning for the optimal mechanism design is proposed in [192] and described in detail in Sections 10.3 and 10.4.

There are various properties for an auction. First, allocative efficiency means that in all such auctions the highest bidder always wins (i.e., there are no reserve prices). Second, it is desirable for an auction to be computationally efficient. Finally, to study the revenue (expected selling price) of different auctions, we have one of the major findings of auction theory: the celebrated revenue equivalence theorem. The revenue equivalence theorem is used to predict the strategy of each bidder in the auctions, and determine the equilibrium in the auctions. The revenue equivalence theorem is presented in Section 4.3.2.

3.3 Auction Theory in Computer Networks

Auctions are known as mechanisms that are widely used in computer networks. The history of auction theory in computer networks dates from the use of spectrum license distribution in wireless systems. Prior to the application of pricing and auction theory, static resource management approaches were used. In these approaches, the spectrum licenses are assigned to users in a static manner. One example of the static resource management approaches is the first-come-first-serve approach [196]. However, such a static approach is inefficient since the demand and supply of the resources do not always match. In particular, the resources may not be assigned to the users that value the resources most. To enhance the efficiency of the resource allocation, pricing and auction theory can be adopted. In the auction, the users submit their bids for the spectrum licenses. The bids are the prices that the users are willing to pay for the spectrum licenses. These prices reflect the demands or the values of the spectrum licenses to the users. The users with the highest bids are the winners of the spectrum licenses. As such, the adoption of the auction methodology increases the market competition and enhances the efficiency of the resource allocation. This further improves the revenue of the seller. For example, by applying the auction approach for spectrum license distribution, the Federal Communications Commission (FCC), an independent agency of the US government, gained $40 billion from 1994 to 2001.

In this section, we first present the basics of auction. Then, we discuss the motivations for and significance of applying the auctions to computer networks. Finally, we define basic terminologies in auction theory.

3.3.1 Auction Basics

As mentioned earlier, auctions are regarded as market mechanisms in which the item allocation and pricing and payment determination are performed by a *bidding process* [197]. Various auctions are designed with different objectives and economic properties. In general, auctions can take many forms, but they share two major characteristics. First, auctions are universal since they can be used in anywhere to sell any item. Second, auctions are anonymous since the outcome of the auction does not depend on the identity of participants, referring to the buyers or the bidders. Moreover, in most auctions, participants are required to submit their bids. Here, the bids refer to the prices, or amounts of money, that the participants are willing to pay for the items. The winner of the auction is the participant with the highest bid. The auction can be defined as follows.

DEFINITION 3.9 *[197] An auction is a market mechanism that includes an explicit set of rules for determining item allocation and the corresponding prices on the basis of bids from the market participants, the bidders. The traditional mechanisms include English and Dutch auctions, and the first-price and second-price sealed-bid auctions.*

From the perspective of game theory, an auction can be considered to be an incomplete information game. The game is represented by a set of players, a set of strategies available to each player, and a payoff vector that corresponds to each combination of the strategies, the strategy profile, of the players. Here, the players can be the buyers (i.e., the bidders) or the sellers. In the case that the players are the buyers, the strategy of the player is a bid function that maps its value of the item to a bid, a bidding price. The payoff obtained by the player corresponding to each strategy profile is the expected utility or profit of the player given the strategy profile. The strategy profile constitutes the equilibrium of the auction if each strategy b_i in the profile maximizes the payoff of the corresponding player, player i, given the strategies of the other players.

There are two game-theoretic models of auctions: *common value auctions* and *private value auctions*.

- *Common value auctions:* In the common value auctions, bidders have equal values of the item; that is, the value of the item is identical among the bidders. However, the bidders do not have perfectly accurate information about this value, so they do not know an exact value of the item. Each bidder can assume that any other bidder obtains a random signal, which is used to estimate the true value, from a probability distribution common to all bidders. Examples of common value auctions are Treasury Bill auctions, auctions of timber, spectrum auctions, and auctions of oil and gas leases. In each case, the value of the item is the same to all the bidders, but different bidders have different information about what that value actually is.
- *Private value auctions:* In contrast to common value auctions, in private value auctions, each bidder knows the value, or private value, of the item, but the bidder does not know the values of the other bidders. We say that the values are *independent* across bidders since the value that a particular bidder assigns to the item is independent of the values assigned by the other bidders.

Various types of auctions exist, and there are also many ways to categorize auctions. Some simple examples of categorization follow.

- *Forward and reverse auctions:* The forward auction typically consists of multiple bidders (i.e., buyers) and one seller (see Figure 3.1(a)). The bidders bid for the items by offering increasingly higher prices. The reverse auction has multiple sellers and one buyer (see Figure 3.1(b)). The sellers compete for the buyer's attraction by submitting their asks to the buyer or the auctioneer. Here, each ask refers to the price that the seller is willing to receive for trading the items.

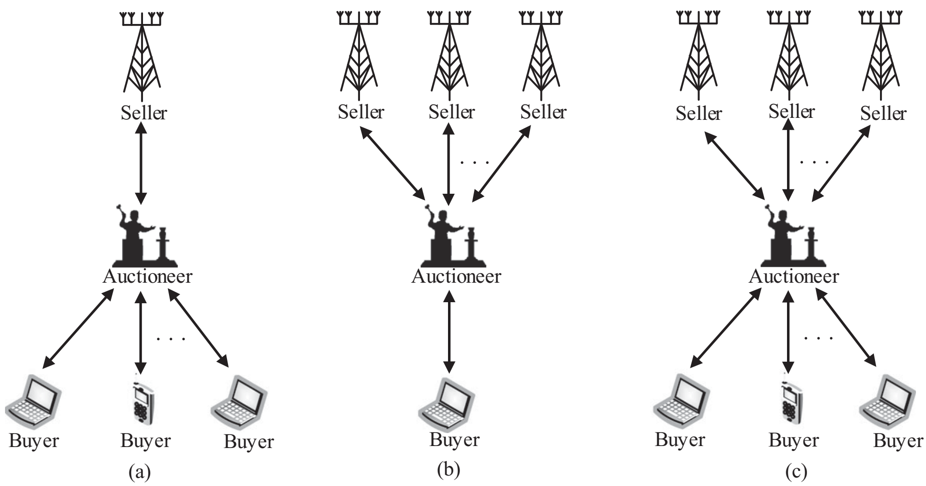

Figure 3.1 Different types of auctions: (a) forward auction, (b) reverse auction, and (c) double auction. The arrows indicate transactions of items and money among auction players [86].

The price in the reverse auction typically decreases to the lowest price that the buyer can accept.

- *Single-sided and double-sided auctions:* In the single-sided auction, there are only buyers or sellers that submit bids or asks. In the double-sided auction or double auction (see Figure 3.1(c)), both buyers and sellers submit their bids and asks, respectively.
- *Open-cry and sealed-bid auctions:* In the open-cry auction, bids of bidders are made out in the open market. Since the bids of the bidders are disclosed to the other bidders during the auction, the open-cry auction is regarded as a public auction that gives the bidders a chance to compete for the item with the best price. In the sealed-bid auction, all bidders simultaneously submit their sealed bids to the seller (or the auctioneer). Thus, no bidder knows how much the other bidders bid.
- *Single-item and multi-item auctions:* In the single-item auction, there is a single item for sale. In the multi-item auction, there are multiple items for sale.

Apart from these categorizations, another typical approach is based on the auction rules. This approach facilitates the selection of proper auctions for the mechanism designers or the sellers. There are four traditional auctions:

- *English auction:* The English auction or open ascending-bid auction is the oldest and perhaps the most popular auction. Here, *open* refers to the fact that the bids of all bidders are disclosed to the other bidders during the auction. The general idea of the English auction is that the auctioneer initially sets a low price for the item and raises the price gradually until only one bidder expresses its willingness to buy the item at the price. The last bidder that is willing to buy the item is the winner of the auction. The winner receives the item and pays the price at which the last-second bidder dropped out.
- *Dutch auction:* In contrast to the English auction, the Dutch auction is a descending-bid auction in which the auctioneer initially sets a high asking price, a ceiling price, for the item and then decreases the price until one of the bidders accepts the price. The winning bidder pays the final price and receives the item.
- *First-price sealed-bid auction:* In the first-price sealed-bid auction, the bidders submit their bids in sealed envelopes to the auctioneer. Upon receiving the bids, the auctioneer selects the bidder with the highest bid as the winner. The winning bidder receives the item and pays the highest bid.
- *Second-price sealed-bid auction:* This auction is similar to the first-price sealed-bid auction. However, the winning bidder pays the second-highest bid.

Apart from these auctions, there are other auctions such as the VCG auction, double auction, and combinatorial auction. They have different objectives, such as social welfare maximization and revenue maximization, and advance various desired properties, such as truthfulness, economic efficiency, and individual rationality. These auctions are discussed in the next chapters of this book.

3.3.2 Auction Theory for Computer Networks

This section explains the motivations and significance of applying the auctions for the resource management in computer networks.

To fully support various emerging multimedia applications, modern computer networks such as IoT, 5G wireless networks, cognitive radio, and cloud networking have been advancing rapidly in recent years. However, the adoption of emerging technologies introduces new challenges in the design and optimization of the network resource management.

- The computer network may consist of billion of devices, such as IoT devices. These devices are required to make optimal decisions without or with minimal human intervention given their constrained resources and the dynamic nature of the network environment. This leads to many challenges in efficiently controlling and managing the devices. Thus, new approaches with higher efficiency and more flexibility to adapt to dynamic networks need to be developed.
- The computer network has become more decentralized and ad hoc in nature. The traditional network resource management methods, such as system optimization, face many challenges or even may not work since they usually require a centralized entity. Thus, it is crucial to develop and adopt new resource allocation and control schemes that are suitable for distributed autonomous decision making.
- The decentralization of the computer network further increases the need for online resource sharing and resource reallocation among the network entities. To cope with the dynamic and unpredicable resource demand as well as to match resource supply and demand profiles in time and space in the network, dynamic, flexible, and scalable resource management schemes need to be considered.
- The computer network is a large-scale entity with a high density of network devices. Perfect global network state information may be too costly, impractical, and impossible to obtain. Thus, control decisions on resource management have to be made with partial or no knowledge of the parameters of the optimization problem.
- The computer network may include a number of rational and selfish entities. These entities may seek to maximize their own utilities by misreporting local parameters that may reduce the socially optimal and global resource allocation within the network. For example, they can misreport information related to channel demands and channel values that increases their own utilities. This can reduce the spectrum utilization as well as the revenue of the seller. Such behaviors need to be understood through game theoretic models and prevented through mechanisms that promote truthfulness and cooperation.
- The entities and stakeholders in the computer network are diverse and heterogeneous. They have different objectives, such as high data rates, low latency, utility maximization, cost minimization, and profit maximization, which may conflict with each other. The traditional methods merely focus on the system performance metrics given system parameters and constraints rather than economic factors, including profit, cost, and revenue. Thus, resource management methods that incorporate economic implications into the solution need to be adopted.

These complexity characteristics make modern computer networks analogous to real markets [86]. In particular, both have various participants in the system, and those participants perform transacting items or commodities, including sharing information and network resources, under certain regulations. Therefore, economics and business management approaches [198] can be employed to dynamically and efficiently manage the resources of the computer networks. Auction [199] is one kind of interdisciplinary method used to solve the resource management issues. A major advantage of using the auction mechanisms is the ability to guarantee the efficiency of the resource management by allocating the resources to those buyers that value the resources most. Moreover, the auction mechanisms can address the challenges of resource management in the computer networks in the following ways:.

- Auctions as game models can model and analyze complex interactions among the network entities and stakeholders [74]. Through these interactions, each entity can observe, learn, and predict the status/actions of other entities, and then make the best decisions based on the equilibrium analysis. Therefore, auctions are inherently suitable for distributed autonomous decision making and can cope with the diverse and conflicting interests of autonomous network entities in the computer networks.
- Auctions can support different objectives, ranging from revenue maximization for the auctioneer to social welfare maximization for the entities, and own various desired properties, including truthfulness, economic efficiency, and individual rationality. Thus auctions can be used to design incentive mechanisms that cope with the rationality, selfishness, and even maliciousness of the network entities.
- Auctions offer the flexibility of setting prices for items or commodities dynamically and efficiently based on current supply and demand in a market. They meet the requirement for matching the dynamic spatiotemporal patterns of demand and supply in the computer networks.
- By adopting auctions, desired resource allocation schemes can be achieved without knowledge of the utility functions of the network entities [200]. Thus auctions are able to provide control decisions for resource management under conditions of limited or no network state and node utility information.

3.3.3 Basic Terminology in Auction Theory

This section provides the basic terminology that is used through this book. These terms are essential to understand the auction approaches discussed in subsequent chapters.

- *Seller:* A seller offers its items for sale. The items are things that the seller sells in the auction. In computer networks, the items can be network resource, (e.g., bandwidth, spectrum, energy, cloud, and storage) or network services (e.g., relay, caching, and offloading services). Sellers can be wireless service providers, cloud providers, or even users.
- *Bidder:* A bidder is a buyer that wants to buy the items from the seller. In computer networks, bidders can be end-users, mobile users, mobile devices, or even

service providers. Bidders want to buy network resources and have to compete with each other for the resources.

- *Auctioneer:* An auctioneer is as an intermediate agent that conducts the auction. In particular, the auctioneer initializes the auction, determines the winners, and identifies the prices that the bidders need to pay or that the sellers receive. In many cases, the auctioneer is the seller itself. In computer networks, the auctioneer can be a base station or an access point that can conduct resource auctions using its auction controller.
- *Player:* The auction can be considered to be a game model in which the players are the sellers or the bidders.
- *Bidding price* and *asking price:* The bidding price is the price that the bidder is willing to pay for a requested item. The asking price is the price of the item that the seller is willing to sell/offer, and the price that the seller accepts. In computer networks, the asking price can be the cost for maintaining the network resources or network services.
- *Bid:* The bid is typically the bidding price, the price that the bidder is willing to pay for the item. However, in computer networks, the bid can refer to the resource demand of the bidder. For example, a bid can be a power demand, the number of power units, as presented in [145] and [201].
- *Ask:* The ask is typically the asking price, the price of the item that the seller accepts for trading. The ask is determined by the seller.
- *Price:* The price in the auction can refer to the bid or the ask. Also, it can be the price that winning bidders need to pay for winning the item or the price that the winning sellers receive for trading the items.
- *Strategy:* In the auction, the bidders and the sellers are players that have their strategies. The strategy of the bidder is to determine its bid, and that of the seller is to determine its ask. For example, the strategy of a service provider, the seller, is to determine the resource price, and the strategy of a user, the bidder, is to determine the resource price that it is willing to buy. The objective is to achieve a desired outcome or payoff. The payoff of a player depends on not only the player's own strategy, but also on the strategies of others. The payoff can be the revenue, profit, or utility. In particular, the utility is related to the values of the items to the buyers and the sellers. In particular, the value of the item to a particular bidder can be of the following types:
  - *Private value:* The value that a particular bidder assigns to the item is independent of the values of the other bidders. The private value of the bidder is typically unknown to other bidders.
  - *Interdependent value:* The value that a particular bidder assigns to the item depends on or is a function of the other bidders' values.
  - *Common value:* The bidders assign the same interdependent value to the item.

Other terms used in auction theory are as follows:

- *Efficiency:* An auction is considered to be efficient if the item is sold to the bidder that has the highest value.
- *Signal:* Each bidder has only an estimate/private information of the value, and the estimate or the private information is called a *signal* of the bidder. For example, in fog computing, users in different locations may have different estimates or signals of the same computing unit [193]. This is because the users have different latency, which results in different experiences.
- *Winner's curse:* A bidding strategy of each bidder entails increasing in its signal, and the bidder with the highest signal wins the item in the auction. If the bidder discovers that the value of the resource is less than its bid, then this case is considered to be *winner's curse*. To avoid the winner's curse issue, each bidder has to shade its bid. Sealed-bid auctions such as first-price sealed-bid and second-price sealed-bid auctions can guarantee this requirement.

We often use the following assumptions when analyzing an auction [202].

- *A1:* Bidders in the auction are risk neutral; that is, the bidders seek to maximize their expected utilities. Here, the utility of the bidder is the difference between its value and the price that the bidder needs to pay. For example, in the first-price sealed-bid auction, risk-averse bidders are willing to bid more to increase their chances of winning, which increases their expected utility. This allows the first-price sealed-bid auction to generate higher expected revenue than the English auction.
- *A2:* Bidders in the auction have independent private values; that is the values of the items to different bidders are independently distributed.
- *A3:* Bidders in the auction are symmetric; that is the values of the bidders are distributed according to same distribution function F. We also say that the bidders possess symmetric information.
- *A4:* Payment is a function of bids alone. In particular, let $\mathbf{b} = (b_1, \ldots, b_N)$ denote the bid profile of the bidders. Then, the payment p_i made by bidder i is a function of the bid profile, $p_i(\mathbf{b})$.

3.4 Summary

In this chapter, we introduce mechanism design and auction theory. In particular, we first define the mechanism and the allocation rule and the payment rule of the mechanism design. The mechanism design task is generally a complicated search problem. Thus, we introduce the revelation principle that can be used for facilitating the mechanism design task. We also present the required properties of the mechanism, which include incentive compatibility, individual rationality, economic efficiency, and budget balance. We further define and discuss optimal mechanisms in terms of social surplus maximization and profit maximization. After that, we introduce basics of auction theory and present the motivations as well as the significance of applying auctions to computer networks.