Intraday overreaction of stock prices

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Preliminary Version

Abstract

We propose a concept of intraday overreaction characterized by intraday price movements which are corrected within the same trading day. It is a concept of relative overreaction in the sense that the price range within a trading day is large in comparison with the open-close return volatility. As a one-sided concept it allows to distinguish between upward and downward overreaction. A test for overreaction is proposed and applied to daily high, low, and open-close returns of the components of the S&P500 and to the German XETRA-DAX stock shares, providing strong support for intraday overreactions to bad news.

JEL-classifications: C22, C52, G10

Keywords: Intraday volatility, High-Low-Prices, Time-changed Brownian motion, Overreaction

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1 Introduction

Since the pioneering papers by Shiller (1984) and De Bondt and Thaler (1985) a large volume of theoretical and empirical research work has analyzed price overreaction in financial markets reflecting market inefficiency. Typically, the literature closely links price overreaction to forecastability of stock prices and the prospect for investors to earn above-average returns.

Given the rapidly growing empirical evidence of forecastable components of equity prices, however, financial economists have emphasized that in the context of intertemporal models providing for variation of required returns over time, forecastability is not necessarily inconsistent with the concept of market efficiency, see, among others, Fama and French (1988) and Balvers et al. (1990). Thus, it has been argued that predictable variations in long-term stock returns over a horizon of several years, resulting in profitable contrarian investment strategies as in the analysis of De Bondt and Thaler (1985), need not to be attributed to overreaction to extreme situations and a tendency to overweigh current information, or to other deviations from rationality, as for example considered by Barberis et al. (1998), but can be renconciled with market efficiency.

In order to distinguish stock price overreaction and market inefficiency from predictable changes in expected returns, Lehman (1990) suggested to examine returns over short time intervals.[1] In fact, the focus on long-term dynamics in stock returns in the papers by Shiller (1984) and De Bondt and Thaler (1985) was realigned to short-run return behavior, ranging over time periods from a few days up to a month, in the major part of the subsequent literature, including, for example, Zarowin (1989), Atkins and Dyl (1990), Cox and Peterson (1994), Park (1995), Bowman and Iverson (1998), and Nam et al. (2001). Factors such as seasonal (e.g., January), firm size, and bid-ask bounce effects have been a major concern of the research on short-term overreaction, both with respect to sample selection bias in identifying large price change events, and with respect to the construction of "winner" and "loser" portfolios for evaluating the relative profitability of a contrarian strategy which builds on a reverting behavior of stock prices in the short run.

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[1]Lehman argues that systematic reversals in fundamental valuation over intervals like a week should not occur in efficient markets, instead "asset prices should follow a martingale process over very short time intervals even if there are predictable variations in expected security returns over longer horizons", see Lehman (1990, p. 2).

In this paper we consider the measurement of *intraday* overreaction of stock prices. Besides our focus on the very short-run behavior of stock prices the proposed methodology differs from the previous research in several aspects. Firstly, by using a concept of relative overreaction, which compares the extremal intraday price fluctuation with the open-close price change, we do not concentrate on the price movements following large price change events, but rather take a full sample of subsequent trading days. Secondly, in developing our concept, we do not consider the profitability of contrarian intraday trading strategies, but prefer a more direct approach for analyzing the intraday price movements. More generally, we do not rely on any connection between price overreaction and forecastability, or any identification of non-forecastability with market efficiency.[2]

The crucial point for our attempt to identify intraday price overreaction directly from the analysis of the price process is to find an adequate measure of intraday excess volatility and to legitimate some benchmark behavior for this measure. We propose to use as separate measures of intraday upside and downside volatility the respective test statistics for Brownian motion, introduced in Becker et al. (2007), which follow an $F-$distribution under the assumption of a Brownian process for the log-prices. These measures capture the deviation of daily high and low prices from the starting and end point of the intraday price movement, normalized by the open-close return volatility. As Brandt and Diebold (2006) have emphasized, using daily open, close, high, and low prices, instead of ultra-high frequency returns, has the advantage that these data not only are widely available, in many cases over long historical spans, but also yield results being fairly robust against micromarket structure noise arising from bid-ask spread and asynchronous trading. Furthermore, the proposed one-sided concept, which allows to distinguish between upward and downward overreaction, has the advantage to potentially detect asymmetric return behaviour.

While the Brownian motion assumption may be considered as too restrictive

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[2]Shiller (1984) represents a notable exemption in the literature when he rejects the argument for the efficient market hypothesis, that because real returns are nearly unforecastable, the real price of stocks is close to the intrinsic value, as one of the most remarkable errors in the history of economic thought. Posing the question, why speculative asset prices fluctuate as much as they do, he insists on the important role of "fads", misperceptions of information, and psychologically driven price movements, irrespective of whether these price components are predictable or not.

at first view, we show that the implied behavior of our measures holds under much more general conditions. In particular, the distribution of the proposed measures of upside and downside volatility remains unaffected, when we allow for any intraday seasonal volatility pattern, such as the frequently observed U-shape of intraday volatility. Furthermore, the behavior of the proposed measures is shown to be conservative with respect to the benchmark behavior under a Brownian motion for a wide range of intraday price processes, including discrete random walks with leptokurtic increments, Merton jump diffusions (Merton (1976)), or variance gamma processes. With respect to another class of processes, including variation of the volatility parameter according to an interday GARCH model, we obtain results on the robustness of the distribution of the proposed statistics. On the other hand, for price processes featuring non-persistent price changes and mean reversion, such as an Ornstein Uhlenbeck log-price process, our test identifies overreaction. Comparing these results with our empirical findings, we claim to provide strong evidence for intraday overreaction to bad news.

The paper is organized as follows. Section 2 presents formal definitions for the suggested measures of upside and downside volatility, together with some basic properties under the assumption of a Brownian motion for the log-price process. In section 3 we provide some theoretical results on the distribution of the proposed measures for more general classes of stochastic price processes.

Our empirical findings in section 4 are based on the analysis of daily open, high, low, and close price data for the components of the S&P500, including the constituents of the Dow Jones Industrial Average, and on the 30 shares included in the German XETRA DAX. Generally, for the majority of individual shares we find highly significant increases of normalized downside volatility as compared with the benchmark. This is considered as strong evidence for intraday overreaction with respect to bad news. To further illustrate the discrimination obtained by the test results, we compare the in-sample performance of shares, for which our test indicates overreaction on bad news, with the performance of the other shares, under a "buy on bad news" intraday trading strategy. We conclude with a summary in section 5.

2 Intraday upside and downside volatility

For any trading day $n = 0, 1, 2, \ldots$ we consider the movement of the log-price $P(t)$ of a security from the opening of the market at time t_n until market close at t_n^c. Taking the length of the daily trading time, $t_n^c - t_n$, as the time unit we have $t_n^c = t_n + 1$, where $t_{n+1} \geq t_n + 1$. Using data on the daily open, high, low, and close log-prices, $P_n^o = P(t_n)$, P_n^h, P_n^l, and $P_n^c = P(t_n + 1)$, respectively, we define measures $V_{n,max}$ $(V_{n,min})$ of intraday upside (downside) volatility as

$$V_{n,\max} = 2 \ (P_n^h - P_n^o)(P_n^h - P_n^c), \tag{1}$$
$$V_{n,\min} = 2 \ (P_n^o - P_n^l)(P_n^c - P_n^l). \tag{2}$$

Both $V_{n,\max}$ and $V_{n,\min}$ are nonnegative and can be considered as measuring the distance of the daily extremal prices from open and close price. If the intraday return process is denoted with

$$X_n(t) := P(t_n + t) - P(t_n), \quad 0 \leq t \leq 1, \tag{3}$$

with the intraday final returns X_n and intraday maximal (minimal) returns $Y_{n,\max}$ $(Y_{n,\min})$ given as

$$X_n := X_n(1) = P_n^c - P_n^o, \tag{4}$$
$$Y_{n,\max} := \sup_{0 \leq t \leq 1} X_n(t) = P_n^h - P_n^o, \tag{5}$$
$$Y_{n,\min} := \inf_{0 \leq t \leq 1} X_n(t) = P_n^l - P_n^o, \tag{6}$$

the definitions of intraday upside and downside volatility can be rewritten equivalently as

$$V_{n,\max} = 2 \ Y_{n,\max}(Y_{n,\max} - X_n), \tag{7}$$
$$V_{n,\min} = 2 \ Y_{n,\min}(Y_{n,\min} - X_n). \tag{8}$$

Under the benchmark assumption that the intraday log-price process follows a Brownian motion with drift rate μ_n and volatility σ_n, the suggested measures of intraday upside and downside volatility have some attractive properties, which are summarized in the following lemma.

Lemma 1: If the intraday log-price process at day n follows a Brownian motion with drift parameter μ_n and volatility σ_n, intraday upside (downside) volatility $V_{n,\max}$ $(V_{n,\min})$ satisfy the following properties:

(i) The distribution of $V_{n,\max}$ ($V_{n,\min}$) is drift independent with

$$E(V_{n,\max}) = E(V_{n,\min}) = \sigma_n^2.$$

(ii) The distribution of $\chi^2 = 2V_{n,\max}/\sigma_n^2$ ($\chi^2 = 2V_{n,\min}/\sigma_n^2$) is chi-square with two degrees of freedom.

(iii) $V_{n,\max}$ ($V_{n,\min}$) is stochastically independent of the contemporary intra-day final return X_n.[3]

(iv) The distribution of the ratios obtained by normalizing upside (down-side) volatility with the variance estimate of the intraday open-to-close return,

$$F_{n,\max} = \frac{V_{n,\max}}{(X_n - \mu_n)^2}, \quad \text{and} \quad F_{n,\min} = \frac{V_{n,\min}}{(X_n - \mu_n)^2}, \tag{9}$$

is an $F-$distribution with two degrees of freedom in the numerator and one degree of freedom in the denominator.

Proof. See Yor (1997) and Becker et al. (2007).

Notice that the proposed measures of upside (downside) volatility as well as the intraday final return volatility used for normalization in (9) refer to the daily trading period from market open to market close. Hence, price changes from the market close price to the next open price (opening jumps), which are important to account for in estimating the common close-to-close return volatility using intraday high and low prices, do not interfere with our analysis.

For a sample of daily open, high, low, and close prices over N days we define the following test statistics for intraday overreaction[4]

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[3]Notice that, although each of the random variables $V_{n,\max}$, $V_{n,\min}$ is independent of the final return X_n, the vector $(V_{n,\max}, V_{n,\min})$ is not jointly independent of X_n. Furthermore, the joint distribution of $(V_{n,\max}, V_{n,\min})$ and in particular the correlation between $V_{n,\max}$, $V_{n,\min}$ depends on the drift rate. This follows from the joint trivariate distribution of the final return, and the minimal and maximal return, see Billingsley (1968), p. 79.

[4]For the sake of an intuitive interpretation of the test statistics as measures of intraday overreaction we have taken the inverse of the $F-$statistics which were originally proposed in Becker et al. (2007). This should be kept in mind, in particular with respect to the results of small sample power studies, to which we refer below.

$$F_{N,\max} = \frac{\frac{1}{N}\sum\limits_{n=1}^{N} V_{n,\max}}{\frac{1}{N-1}\sum\limits_{n=1}^{N}(X_n - \bar{X})^2} \quad \text{and} \quad F_{N,\min} = \frac{\frac{1}{N}\sum\limits_{n=1}^{N} V_{n,\min}}{\frac{1}{N-1}\sum\limits_{n=1}^{N}(X_n - \bar{X})^2}, \qquad (10)$$

which are called "volatility ratios" in the following. Thus, the volatility ratios give the mean upside (downside) volatility, normalized by the ordinary estimator of the intraday return variance. According to lemma 1, under the null hypothesis that the intraday log-price processes at different days follow independently distributed Brownian motions with constant drift rate μ and volatility parameter σ^2 the distribution of the volatility ratios is an $F-$distribution with $2N$ degrees of freedom in the numerator and $N-1$ degrees of freedom in the denominator.

As Becker et al. (2007) mention, Brownian motion implies a continuous flow of news, which all have a persistent impact on the log-price. Accordingly, if log-prices follow a Brownian motion, there is no overreaction, as the influence of news does not die out. This is different for Ornstein-Uhlenbeck processes, which exhibit a mean-reverting, stationary behaviour and correspond to non-persistent news, whose impact is very likely to be corrected due to the mean-reversion. Therefore, OU processes can be considered as describing overreaction. For OU processes and for 'overreacting' prices in general we expect the volatility ratio to be significantly higher than in the benchmark case of Brownian motion, because given the same level of extremal values $Y_{n,i}$, the 'overreacting' process will tend back to the initial level with higher probability than Brownian motion does, thereby resulting in a lower daily variance than that of Brownian motion. Therefore it is possible to test for overreaction by testing whether the volatility ratios $F_{N,i}$ are significantly higher than in the benchmark case. For some simple Ornstein-Uhlenbeck processes this has been done by Becker et al. (2007), who found that this test for overreaction has good power against the alternative of OU processes.

In the following section we consider the distribution of the proposed measures of intraday overreaction under more general, alternative assumptions for the intraday price processes.

3 Robustness of volatility ratios

First of all, we mention robustness results obtained by Becker et al. (2007), regarding the effects of interday variation of the drift and volatility parameters: variation of the volatility parameter leads asymptotically to only a small perturbation of the test statistic's distribution (see Becker et al. (2007), p. 10), whereas variation of the drift parameter shifts the test statistic to the left, resulting in an asymptotic probability of 0 for wrongly detecting overreaction (for details, esp. small sample properties, see Becker et al. 2007, p. 12-13).

With respect to intraday deviations from Brownian motion we start our analysis by considering the effect of well-known typical intraday volatility patterns such as deterministic U-shaped intraday seasonality. A deterministic intraday seasonality in volatility, described by a (U-shaped) function

$$\phi : [0,1] \to \mathbb{R}, t \mapsto \phi(t), \tag{11}$$

can be modelled by subjecting Brownian motion to the (deterministic) time change

$$T(t) := \int_0^t \phi(u)du, \tag{12}$$

i.e. by modelling intraday returns as time-changed Brownian motion,

$$X_n(t) = W_n(T(t)), \tag{13}$$

with W_n being a Brownian motion with drift μ and volatility σ^2 for every day n. In this case, high-frequency returns from t to $t+h$ will have volatility $\sigma^2 \int_t^{t+h} \phi(u)du$, which approximately equals $\sigma^2 \phi(t)h$ for small h, reflecting the intraday seasonality. For processes of this kind, the following lemma holds:

Lemma 2: If returns follow a time-changed Brownian motion with a deterministic continuous time change T, the distribution of the volatility ratios $F_{N,\max}$ and $F_{N,\min}$ is still $F_{2N,N-1}$.

Proof: Because of $T(0) = 0$ and the continuity of T, the paths of $X_n = (W_n(T(t)))_{t \in [0,1]}$ take the same values as those of $(W_n(\tau))_{\tau \in [0,T(1)]}$, which completes the proof due to the well-known scaling properties of Brownian motion.

In the following, we generalise (12) by allowing for non-continuous as well as

random time changes.

Example: For instance, consider the non-continuous, deterministic time change

$$T(t) := c\lfloor tL \rfloor \tag{14}$$

for $c > 0$, $L \in \mathbb{N}$, which is constant on $[\frac{l-1}{L}, \frac{l}{L}[$ and jumps by c on $\frac{l}{L}$, $l = 1, ..., L$. The corresponding time-changed Brownian motion obviously is just a subsampling of the Brownian motion at the points $0, c, 2c, ..., Lc$, in other words it is an L-step random walk with $N(\mu c, \sigma^2 c)$-distributed innovations. L-step random walks have been considered by Becker et al. (2007) (cf. their table 2, p. 17[5]), esp. with respect to small sample properties of the F-test and other innovations' distributions.

Example:
The following time-change is an example for a random, non-continuous time change:

$$T(t) := c_1 t + c_2 N(t), \tag{15}$$

where $c_1, c_2 > 0$ and N is a Poisson process with intensity λ. It is easily seen that for an independent Brownian motion W_n with drift μ and volatility σ^2, the time-changed Brownian motion

$$X(n, t) := W_n(T(t)) = W_n(c_1 t + c_2 N(t)) \tag{16}$$

will be a Merton jump-diffusion consisting of

- a Brownian motion with drift μc_1 and volatility $\sigma^2 c_1$,

- a jump-component with jump intensity λ and $N(\mu c_2, \sigma^2 c_2)$-distributed jumps.

Apart from theoretical asymptotical results, which will be given later (see Theorem 1 below), we conducted MC studies to investigate the small sample properties of the volatility ratios $F_{N,\min}, F_{N,\max}$. Because Merton jump-diffusions are Lévy processes (see, e.g., Cont and Tankov (2004)), $F_{N,\min}$ and $F_{N,\max}$ are identically distributed (see Klößner (2006)), so that we can limit our MC studies to $F_{N,\min}$. In our simulation studies, we chose different settings of the following parameters:

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[5]Notice that there significant downside deviations of F from $F_{2N, N-1}$ have been tested.

- the drift rate μ_{BM} and the diffusion volatility σ_{BM}^2 of the Brownian motion,

- the jump intensity λ,

- the mean μ_J and the variance σ_J^2 of the $N(\mu_J, \sigma_J^2)$-distributed jump sizes.

Thus, the return X_n over the unit period is distributed as

$$X_n \sim N(\mu, \sigma^2) \quad \text{with } \mu = \mu_{BM} + \lambda \mu_J, \quad \sigma^2 = \sigma_{BM}^2 + \lambda \sigma_J^2 + \lambda \mu_J^2. \quad (17)$$

For the Monte Carlo simulation of the Merton model the jump intensity λ is varied over $\lambda = 0.01, 0.1, 1, 5, 10$, corresponding to an average occurence of one jump in one hundred days, one jump in ten days, one jump per day, five jumps per day, and ten jumps per day, respectively. We denote the proportion of the contribution of the jumps to $\text{Var}(X_n)$ by ρ,

$$\rho = \frac{\lambda \sigma_J^2 + \lambda \mu_J^2}{\sigma_{BM}^2 + \lambda \sigma_J^2 + \lambda \mu_J^2}. \quad (18)$$

In our simulation study ρ takes the values $\rho = 0.1, 0.25, 0.5$. We assume $\mu = 0$, hence $\mu_J = -\mu_{BM}/\lambda$, first with $\mu_{BM} = 0$ and, as another case, with $\mu_{BM} = 0.03$. With normalization by $\sigma^2 = 1$, for a fixed variance proportion ρ, the variance of the jump size varies with the jump intensity,

$$\sigma_J^2 = \frac{\rho}{\lambda} - \mu_J^2 = \frac{\rho}{\lambda} - \frac{\mu_{BM}^2}{\lambda^2}. \quad (19)$$

Table 1:
Overreaction detection for Merton jump-diffusion (right-sided test, $\alpha = 5\%$)

ρ	$\mu_J = \mu_{BM} = 0$ $(N = 250)$				
	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = 1$	$\lambda = 5$	$\lambda = 10$
0.1	2.35	2.17	3.23	3.91	4.25
0.25	1.24	0.36	0.80	2.31	2.87
0.5	0.92	0.04	0.06	0.23	0.68

ρ	$\mu_J = -0.03/\lambda$, $\mu_{BM} = 0.03$, $F_{\min} - \text{test}$ $(N = 250)$				
	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = 1$	$\lambda = 5$	$\lambda = 10$
0.1	1.89	2.03	3.63	4.20	4.35
0.25	1.42	0.33	0.85	1.98	2.59
0.5	1.09	0.00	0.04	0.33	0.60

Rejection frequencies $(F_{N,\min} > F_{2N,N-1,0.95})$ in 10,000 replications in percent.

Generally, according to our simulation results and in complete accordance with the asymptotic results of Theorem 1, imposing price jumps on the Brownian motion leads to a decrease of the volatility ratios. From Table 1 it can be seen that Merton jump-diffusion as data generating process implies frequencies of detecting overreaction of less than 5%, if the test is calibrated to the 5% level. It even is possible, by checking for significantly small F-values, to discriminate between Merton jump-diffusions and Brownian motion, as Table 2 reveals.

Table 2:
Merton jump-diffusion vs. Brownian motion (left-sided test, $\alpha = 5\%$)

ρ	$\mu_J = \mu_{BM} = 0$ $(N = 250)$				
	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = 1$	$\lambda = 5$	$\lambda = 10$
0.10	21.79	13.07	8.01	6.23	5.85
0.25	49.32	48.28	20.90	11.69	9.37
0.50	71.06	94.80	75.72	37.64	25.36

ρ	$\mu_J = -0.03/\lambda$, $\mu_{BM} = 0.03$ $(N = 250)$				
	$\lambda = 0.01$	$\lambda = 0.1$	$\lambda = 1$	$\lambda = 5$	$\lambda = 10$
0.10	20.47	13.04	7.83	6.34	5.81
0.25	50.17	47.81	21.63	11.11	9.87
0.50	71.83	94.75	76.24	36.70	25.28

Rejection frequencies $(F_{N,\min} < F_{2N,N-1,0.05})$, in 10,000 replications in percent.

In the following, we investigate theoretically the impact of time-changing a Brownian motion by any independent time change. Recall for this purpose the definition of a time change as an increasing càdlàg process T with $T(0) = 0$ (e.g. Cherny and Shiryaev (2002)). As the next theorem shows, there are two important (non-distinct) cases, where the volatility ratios are shifted to the left, resulting in an asymptotic probability of 0 for the F-test to indicate overreaction.

Theorem 1: Let W_n be independent Brownian motions with drift μ and volatility σ^2 for each day n and T_n be iid time changes with existing second moment that are independent of the Brownian motions W_n. If

- $\mu \neq 0$ and T_n is not deterministic or

- T_n is non-continuous with positive probability,

11

then for the time-changed returns

$$X_n(t) := W_n(T_n(t)), \tag{20}$$

the volatility ratio

$$F_{N,i} = \frac{\frac{1}{N}\sum\limits_{n=1}^{N} V_{n,i}}{\frac{1}{N-1}\sum\limits_{n=1}^{N}(X_n - \overline{X})^2} \tag{21}$$

converges for $N \to \infty$ a.s. towards some constant $C_i < 1$ for $i = \max, \min$.

Proof: Provided that $\mathrm{E}\,(V_i)$ and $\mathrm{Var}\,(X)$ are finite[6], the strong law of large numbers tells us that $F_{N,i}$ converges a.s. to $\dfrac{\mathrm{E}\,(V_i)}{\mathrm{Var}\,(X)}$. Thus, the proof will be complete if we have shown

$$\mathrm{E}\,(V_i) < \mathrm{Var}\,(X) < \infty \text{ for } i = \max, \min. \tag{22}$$

Using the notations $\mu_T := \mathrm{E}\,(T(1))$, $\sigma_T^2 := \mathrm{Var}\,(T(1))$, we have:

- $\mathrm{E}\,(X) = \mathrm{E}\,(W(T(1))) = \mathrm{E}\,(\mathrm{E}\,(W(T(1))|T(1))) = \mathrm{E}\,(\mu T(1)) = \mu\,\mu_T$,

- $\mathrm{E}\,(X^2) = \mathrm{E}\,(\mathrm{E}\,(W(T(1))^2|T(1))) = \mathrm{E}\,((\mu T(1))^2 + \sigma^2 T(1))$
  $= \mu^2(\sigma_T^2 + \mu_T^2) + \sigma^2\mu_T$,

- $\mathrm{Var}\,(X) = \mathrm{E}\,(X^2) - (\mathrm{E}\,(X))^2 = \mu^2\sigma_T^2 + \sigma^2\mu_T < \infty$.

For a given realisation of the time change we consider now

- $Y_{\max} = \sup\limits_{0 \le t \le 1} X(t) = \sup\limits_{0 \le t \le 1} W(T(t)) = \sup\limits_{\tau \in T([0,1])} W(\tau)$ and

- $\widetilde{Y}_{\max} := \sup\limits_{\tau \in [0,T(1)]} W(\tau)$.

Obviously, $\widetilde{Y}_{\max} \ge Y_{\max}$, with the inequality being strict with positive probability if T is non-continuous. Together with

$$Y_{\max}(Y_{\max} - X) = \widetilde{Y}_{\max}(\widetilde{Y}_{\max} - X) - (\widetilde{Y}_{\max} - Y_{\max})(\widetilde{Y}_{\max} + Y_{\max} - X),$$

this implies for $V_{\max} = 2\,Y_{\max}(Y_{\max} - X)$ the inequality

$$V_{\max} \le 2\,\widetilde{Y}_{\max}(\widetilde{Y}_{\max} - X) =: \widetilde{V}_{\max},$$

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[6]For notational simplicity, we omit the index n.

with the inequality again being strict with positive probability if T is non-continuous. Due to the scaling properties of Brownian motion and the invariance of the distribution of $V_{\max}$ with respect to the Brownian motion's drift rate, $\widetilde{V}_{\max}$ is distributed as $\sigma^2 T(1)\chi_2^2/2$ conditional on T, which implies $\mathrm{E}\left(\widetilde{V}_{\max}\right) = \sigma^2 \mu_T$. Summing up, we have

$$\mathrm{E}\left(V_{\max}\right) \leq \sigma^2 \mu_T \leq \sigma^2 \mu_T + \mu^2 \sigma_T^2 = \mathrm{Var}\left(X\right) < \infty,$$

where

- the first inequality is strict if the time-change T is non-continuous with positive probability and

- the second inequality is strict if $\mu \neq 0$ and the time-change is non-deterministic,

which completes the proof for $F_{N,\max}$. The proof for $F_{N,\min}$ follows in a completely analogous manner. $\diamond$

Under the assumptions of Theorem 1, the volatility ratios converge so some constants smaller than 1, whereas $F_{2N,N-1}$ converges to 1. Thus, the probability that the F-test (wrongly) indicates overreaction will vanish asymptotically.

Time changes have been introduced to the finance literature by Clark (1973). They have recently been reconsidered by many authors, see e.g. Ané and Geman (2000) and Geman (2005) for an overview. Typically, $\tau := T(t)$ is interpreted as 'business time', as opposed to t as 'physical time'. Many processes can be written as time-changed Brownian motions with an independent time-change, including VG (variance gamma) processes, NIG (normal inverse Gaussian) processes and many others (see, e.g., Geman et al. (2001)).

The next theorem states the asymptotic properties of the overreaction test for the remaining case of a non-deterministic, a.s. continuous time change for a Brownian motion with zero drift. Inspection of the proof of the previous theorem reveals that in this case the volatility ratios will converge to 1, which does not allow to draw conclusions about the asymptotic significance level of the overreaction test. That's why we study the behaviour of

$$Z_{N,i} := \frac{3N}{\sqrt{3N-2}} \frac{\frac{1}{N}\sum\limits_{n=1}^{N} V_{n,i} - \frac{1}{N-1}\sum\limits_{n=1}^{N}(X_n - \overline{X})^2}{2\frac{1}{N}\sum\limits_{n=1}^{N} V_{n,i} + \frac{1}{N-1}\sum\limits_{n=1}^{N}(X_n - \overline{X})^2} \tag{23}$$

for $i = \max, \min$. It is easily seen that Z is a monotone transformation of F:

$$Z_{N,i} = \frac{3N}{\sqrt{3N-2}} \frac{F_{N,i} - 1}{2F_{N,i} + 1} = \frac{3N}{\sqrt{3N-2}} \left(\frac{1}{2} - \frac{3/2}{2F_{N,i} + 1} \right).$$

From Becker et al. (2007), we know that under the benchmark assumption of Brownian motion the exact distribution of $Z_{N,i}$ is a Beta distribution, which converges to the standard normal distribution as $N \to \infty$. Therefore, testing for overrreaction by comparing $F_{N,i}$ to a quantile of the $F_{2N,N-1}$-distribution is asymptotically equivalent to comparing $Z_{N,i}$ to the corresponding normal quantile.

Theorem 2: Let W_n be independent Brownian motions with zero drift and volatility σ^2 for each day n and T_n be iid non-deterministic, a.s. continuous time changes with existing second moment that are independent of the Brownian motions W_n. Then for the time-changed returns

$$X_n(t) := W_n(T_n(t)), \tag{24}$$

the test statistic

$$Z_{N,i} = \frac{3N}{\sqrt{3N-2}} \frac{\frac{1}{N} \sum\limits_{n=1}^{N} V_{n,i} - \frac{1}{N-1} \sum\limits_{n=1}^{N} (X_n - \overline{X})^2}{2 \frac{1}{N} \sum\limits_{n=1}^{N} V_{n,i} + \frac{1}{N-1} \sum\limits_{n=1}^{N} (X_n - \overline{X})^2} \tag{25}$$

converges towards $N(0, 1 + \gamma)$ as $N \to \infty$ for $i = \max, \min$, where

$$\gamma = \frac{\mathrm{Var}\,(T_n(1))}{(\mathrm{E}\,(T_n(1)))^2}.$$

Proof: From the proof of the previous theorem we know for $i = \max, \min$:

- $\mathrm{E}\,(X) = 0$, $\mathrm{Var}\,(X) = \mathrm{E}\,(X^2) = \sigma^2 \mu_T$ as well as

- $\mathrm{E}\,(V_i) = \sigma^2 \mu_T$.

Thus, the second denominator in (25) converges a.s. to $3\,\sigma^2 \mu_T$. The second nominator can be decomposed into[7]

$$\frac{1}{N} \sum\limits_{n=1}^{N} V_n - \frac{1}{N-1} \sum\limits_{n=1}^{N} (X_n - \overline{X})^2 = \overline{V - X^2} - \frac{\overline{X^2}}{N-1} + \frac{N}{N-1} \overline{X}^2.$$

Because $\overline{X^2}$ converges a.s. to $\mathrm{Var}\,(X) < \infty$ and $(\sqrt{N}\overline{X})^2$ converges in distribution to $\mathrm{Var}\,(X)\,\chi_1^2$, both the terms

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[7]We omit the index $i = \max, \min$.

- $\dfrac{3N}{\sqrt{3N-2}}\dfrac{\overline{X^2}}{N-1}$ and

- $\dfrac{3N}{\sqrt{3N-2}}\dfrac{N}{N-1}\overline{X}^2$

converge to 0 in distribution. That's why it suffices to investigate the limit distribution of

$$\frac{3N}{\sqrt{3N-2}}\frac{\overline{V-X^2}}{3\,\sigma^2\mu_T},$$

which will be normal due to the central limit theorem. The theorem now follows from

$$\mathrm{Var}\left(V-X^2\right) = 3\,\sigma^4(\sigma_T^2 + \mu_T^2),$$

which can be seen after some simple, but tedious calculations using the following facts from lemma 1:

- the conditional distribution of V given $T(1)$ is $\sigma^2 T(1)\chi_2^2/2$,

- the conditional distribution of X given $T(1)$ is $N(0, \sigma^2 T(1))$,

- V and X are independent conditional on $T(1)$.

$\diamond$

The result of Theorem 2 resembles very much the result for interday varying volatility by Becker et al. (2007). It holds exactly for the so-called Ocone martingales, as can be seen from Proposition 3.6 by Cherny and Shiryaev (2002), who also state that stochastic volatility processes of the form $\int \sigma_u dW_u$ with a Brownian motion W and an independent stochastic volatility process σ belong to this class (Cherny and Shiryaev (2002), Exercise 2.2).

4 Empirical findings

Our analysis uses daily price data (open, high, low, close) for the components of the S&P500, with a closer look on the 30 components which are also included in the Dow Jones Industrial Average (DJIA), furthermore daily quotes for the 30 securities included in the German XETRA DAX. The presented results refer to a sample of $N = 1000$ daily price vectors up to November

30, 2005, starting in December 2001.[8] From using other time periods and sample sizes we find that our conclusions do not substantially depend on the selected time period.

Generally for every security in our analysis we compute the test statistics $F_{\min}, F_{\max}$, as defined in (10), giving the normalized downside volatility and upside volatility, respectively. Our focus is on testing for overreaction through a right-sided $F-$test. From our analysis in the previous section we know that common features of intraday price movements such as discrete information arrival and jumps pull the volatility ratios downside. Despite this fact, we find considerable evidence for short run overreaction in the case of bad news.
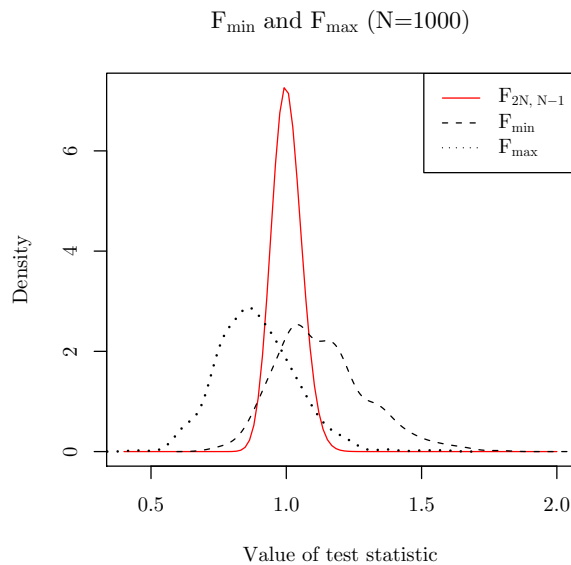
$F_{\min}$ and $F_{\max}$ (N=1000)

Figure 1: Distribution of volatility ratios $F_{\min}, F_{\max}$ for S&P500 shares

Regarding the S&P500 components, for 258 (52.4%) of the companies the normalized downside volatility $F_{\min}$ is greater than the 5%−critical value $F_{2N,N-1,0.95} = 1.095$, and there are still 220 (44.7%) significant at the 1% level. On the other hand, the normalized upside volatility $F_{\max}$ does not indicate comparable non-persistent upside price movements, with only 39 (7.9%) of the test statistics exceeding the 5%−critical value, of which 23

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[8]We have excluded 8 out of the set of 500 shares of the S&P500 portfolio, because the available sample size is less than $N = 1000$; the excluded shares are AMP, CIT, FSL-B, GNW, HSP, MHS, PRU, SHLD.

(4.7%) remain to be significant at the 1% level. The distribution of the test statistics $F_{\min}$ and $F_{\max}$ is shown in figure 1.
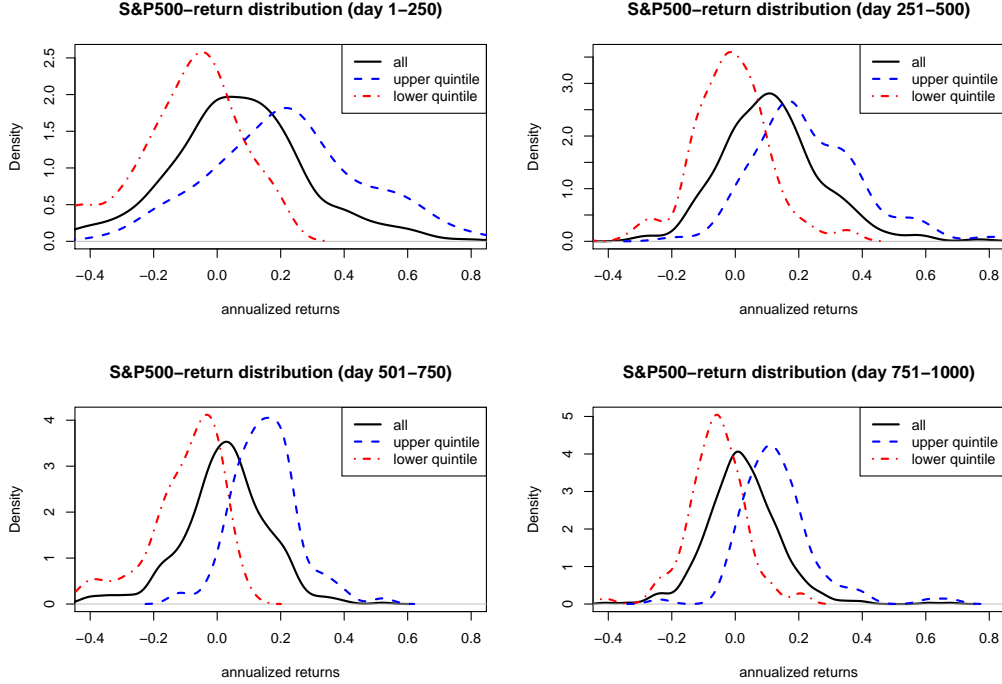

Figure 2: Distribution of annualized returns under a "buy on bad news" intraday trading strategy

To further illustrate the discrimination obtained by the test statistic $F_{\min}$, we compare the in-sample performance of shares, for which our test statistic indicates overreaction to bad news, with the performance of the other shares, under a "buy on bad news" intraday trading strategy. More specifically, the "buy on bad news" strategy, applied to an individual share with estimated volatility parameter $\hat{\sigma}$, means to buy at a price decline by $\hat{\sigma}$ and to sell at the close price of the same day. Thus the return of this trading strategy is $X_n + \hat{\sigma}$ on days where such a price decline occurs, i.e., if $Y_{n,\min} \leq -\hat{\sigma}$, and zero on days where $Y_{n,\min} > -\hat{\sigma}$ and no buy-signal is triggered. The performance of the "buy on bad news" trading strategy is evaluated by the mean annualized return of a share under this strategy. In order to get an impression of the robustness of the results, we split the sample into four subsamples, each with a sample size of 250 days. For each of the four subsamples the shares are ordered according to the value of the $F_{\min}$ statistic. Figure 2 compares the

distribution of mean annualized returns under the "buy on bad news" intra-day trading strategy for the upper quintile of shares, which are all significant at the 1% level even for the reduced sample size, with the respective return distribution of the lower quintile. For comparison, figure 2 also displays the return distribution for all shares.

For those of the S&P500 components which constitute the DJIA, the test results for F_{min} and F_{max} are given in table 3. The normalized downside volatility F_{min} leads to significant overreaction at the 5% level for 22 (73.3%) of the shares, while F_{max} indicates significant upside overreaction only for 3 (10%) of the Dow Jones shares.

As an example for a European stock exchange, we consider the 30 components of the German XETRA DAX. The test results are presented in table 4. Here we find an extremely pronounced asymmetry in downside and upside volatility. The evidence for downside overreaction is even stronger than for the DJIA or S&P500 components. For 29 companies (96.7%) F_{min} indicates significant overreaction at the 5% level, and still 26 (86.7%) are significant at the 1% level. Only for Deutsche Telekom the normalized downside volatility is quite low. On the other hand, for only three companies (Deutsche Lufthansa, Deutsche Post and TUI) we find significant upside overreaction. For 27 (90.0%) of the shares, the normalized upside volatility F_{max} is quite low and there is no significant upside overreaction.

Table 3: $F-$test for overreaction (DJIA shares)

Symbol	Name	$F_{\min}$	$p.F_{\min}$	$F_{\max}$	$p.F_{\max}$
AA	Alcoa	**1.2139**	0.0002	0.9750	0.6802
AIG	American Intern. Group	**1.1499**	0.0058	0.8630	0.9967
AXP	American Express	**1.1155**	0.0240	1.0444	0.2161
BA	Boeing	**1.2164**	0.0002	1.0215	0.3517
C	Citigroup	**1.4841**	0.0000	0.9896	0.5782
CAT	Caterpillar	1.0909	0.0577	0.8300	0.9997
DD	Du Pont	**1.1183**	0.0216	1.0038	0.4749
DIS	Walt Disney	**1.4749**	0.0000	1.0151	0.3948
GE	General Electric	**1.2911**	0.0000	0.9605	0.7714
GM	General Motors	1.0205	0.3582	0.7851	1.0000
HD	Home Depot	**1.2473**	0.0000	1.0592	0.1491
HON	Honeywell Intern.	**1.2187**	0.0002	1.0188	0.3694
HPQ	Hewlett$-$Packard	**1.1553**	0.0046	**1.1330**	0.0120
IBM	IBM	0.9331	0.8988	0.9857	0.6059
INTC	Intel	0.8905	0.9837	0.8932	0.9813
JNJ	Johnson & Johnson	**1.2054**	0.0004	**1.2513**	0.0000
JPM	JPMorgan Chase and Co.	**1.1412**	0.0085	0.9571	0.7908
KO	Coca$-$Cola	**1.1636**	0.0031	1.0322	0.2841
MCD	McDonald's	**1.1384**	0.0096	1.0625	0.1365
MMM	3M Company	1.0297	0.2993	0.8676	0.9956
MO	Altria Group	1.0241	0.3344	0.8720	0.9942
MRK	Merck & Co.	1.0434	0.2214	0.9610	0.7683
MSFT	Microsoft Corp.	0.9048	0.9672	0.9638	0.7521
PFE	Pfizer	**1.5042**	0.0000	0.9684	0.7236
PG	Procter & Gamble	**1.1335**	0.0118	0.9202	0.9370
T	AT & T	**1.2228**	0.0001	**1.1258**	0.0161
UTX	United Technologies	**1.1192**	0.0209	0.8547	0.9981
VZ	Verizon Communications	**1.1878**	0.0010	0.9777	0.6620
WMT	Wal$-$Mart Stores	**1.1446**	0.0074	1.0412	0.2331
XOM	Exxon Mobil Group	**1.1758**	0.0017	1.0201	0.3610

Sample size $N = 1000$ until 2005/11/30, $F_{\min}, F_{\max}$: values of the volatility ratio (test statistic), bold: signif. at 5% level (right-sided test), $F_{2N,N-1,0.95} = 1.095$, $p.F_i = P(F_{2N,N-1} > F_i)$, $i = \min, \max$.

Table 4: F−test for overreaction (DAX shares)

Symbol	Name	$F_{\min}$	$p.F_{\min}$	$F_{\max}$	$p.F_{\max}$
ADS.DE	Adidas Salomon	**1.3063**	0.0000	0.9945	0.5427
ALT.DE	Altana	**1.5093**	0.0000	1.0666	0.1217
ALV.DE	Allianz	**1.1654**	0.0029	0.8028	1.0000
BAS.DE	BASF	**1.5189**	0.0000	1.0640	0.1308
BAY.DE	Bayer	**1.2409**	0.0001	0.7490	1.0000
BMW.DE	BMW	**1.4284**	0.0000	0.9569	0.7913
CBK.DE	Commerzbank	**1.3628**	0.0000	1.0303	0.2956
CON.DE	Continental	**1.5592**	0.0000	1.0193	0.3664
DB1.DE	Deutsche Boerse	**1.4972**	0.0000	1.0393	0.2434
DBK.DE	Deutsche Bank	**1.0984**	0.0448	0.8907	0.9834
DCX.DE	DaimlerChrysler	**1.3978**	0.0000	1.0023	0.4860
DPW.DE	Deutsche Post	**1.7050**	0.0000	**1.1755**	0.0018
DTE.DE	Deutsche Telekom	0.9084	0.9614	0.8827	0.9892
EOA.DE	E.ON	**1.3228**	0.0000	1.0136	0.4052
FME.DE	Fresenius	**1.1206**	0.0198	0.8484	0.9988
HEN3.DE	Henkel	**1.3823**	0.0000	0.7466	1.0000
HVM.DE	HVB	**1.3111**	0.0000	0.9479	0.8377
IFX.DE	Infineon	**1.3185**	0.0000	0.9458	0.8476
LHA.DE	Lufthansa	**1.6595**	0.0000	**1.1563**	0.0044
LIN.DE	Linde	**1.2444**	0.0000	1.0296	0.2999
MAN.DE	MAN	**1.2774**	0.0000	0.8762	0.9926
MEO.DE	Metro	**1.4772**	0.0000	0.9679	0.7266
MUV2.DE	Muenchner Rueck	**1.1894**	0.0009	0.8860	0.9871
RWE.DE	RWE	**1.4174**	0.0000	1.0436	0.2203
SAP.DE	SAP	**1.1294**	0.0139	0.8685	0.9953
SCH.DE	Schering	**1.4348**	0.0000	1.0310	0.2914
SIE.DE	Siemens	**1.2883**	0.0000	0.9348	0.8927
TKA.DE	ThyssenKrupp	**1.6461**	0.0000	1.0762	0.0919
TUI1.DE	TUI	**1.4440**	0.0000	**1.1505**	0.0057
VOW.DE	Volkswagen	**1.3030**	0.0000	0.9024	0.9706

Sample size $N = 1000$ until 2005/11/30, $F_{\min}, F_{\max}$: values of the volatility ratio (test statistic), bold: signif. at 5% level (right-sided test), $F_{2N,N-1,0.95} = 1.095$, $p.F_i = P(F_{2N,N-1} > F_i)$, $i = \min, \max$.

5　Summary

In this paper we provide empirical evidence for short run, intraday overreaction to bad news for the majority of the constituent shares of the S&P500, and, even more pronounced, for most of the blue chips which constitute the DJIA and the German XETRA DAX. Our analysis of overreaction is based on the measurement of the deviation of daily high and low prices from the respective open and close prices. Regarding the empirical evidence provided by the S&P500 shares, the ordering of shares according to the suggested test statistic for downside overreaction is fairly compatible with their performance under an intraday "buy on bad news" strategy.

According to Becker et al. (2007) the proposed upside and downside volatility ratio follows an F-distribution under the benchmark assumption of a Brownian motion for the intraday log-price process. Using the concept of time-changed Brownian motion, however, we have proved that the suggested $F-$test for intraday overreaction holds exactly or is even conservative under much more general conditions, including deterministic intraday volatility patterns, jump-diffusions and discrete information arrival. Further the distribution of the volatility ratios is shown to be only slightly affected through interday variation of the volatility parameter. On the other hand short term mean reversion, which is captured by the model of a (stationary) Ornstein Uhlenbeck process, is identified to imply overreaction by the proposed test.

References

Ané, T. and H. Geman (2000) Order Flow, Transaction Clock, and Normality of Asset Returns, *The Journal of Finance*, 55(5), pp. 2259–2284.

Atkins, A. B. and E. A. Dyl (1990) Price Reversals, Bid-Ask Spreads, and Market Efficiency, *The Journal of Financial and Quantitative Analysis*, 25(4), pp. 535–547.

Balvers, R. J., T. F. Cosimano, and B. McDonald (1990) Predicting Stock Returns in an Efficient Market, *The Journal of Finance*, 45(4), pp. 1109–1128.

Barberis, N., A. Shleifer, and R. W. Vishny (1998) A model of investor sentiment, *Journal of Financial Economics*, 49, pp. 307–343.

Becker, M., R. Friedmann, S. Klößner, and W. Sanddorf-Köhle (2007) A Hausman test for Brownian motion, *Allgemeines Statistisches Archiv - Advances in Statistical Analysis*, 91(1), pp. 3–21.

Billingsley, P. (1968) *Convergence of Probability Measures*, Wiley Series in Probability and Mathematical Statistics, Wiley, New York et al.

Bowman, R. G. and D. Iverson (1998) Short-run overreaction in the New Zealand stock market, *Pacific-Basin Finance Journal*, 6(5), pp. 475–491.

Brandt, M. W. and F. X. Diebold (2006) A No-Arbitrage Approach to Range-Based Estimation of Return Covariances and Correlations, *Journal of Business*, 79(1), pp. 61–74.

Cherny, A. and A. Shiryaev (2002) Change of Time and Measure for Lévy Processes, MaPhySto Lecture Notes 2002-13.

Clark, P. K. (1973) A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices, *Econometrica*, 41(1), pp. 135–155.

Cont, R. and P. Tankov (2004) *Financial Modelling with Jump Processes*, CRC Financial Mathematics Series, Chapman & Hall, Boca Raton et al.

Cox, D. R. and D. R. Peterson (1994) Stock Returns Following Large One-Day Declines: Evidence on Short-Term Reversals and Longer-Term Performance, *The Journal of Finance*, 49(1), pp. 255–267.

De Bondt, W. F. M. and R. Thaler (1985) Does the Stock Market Overreact?, *The Journal of Finance*, 40(3), pp. 793–805.

Fama, E. F. and K. R. French (1988) Permanent and Temporary Components of Stock Prices, *The Journal of Political Economy*, 96(2), pp. 246–273.

Geman, H. (2005) From measure changes to time changes in asset pricing, *Journal of Banking & Finance*, 29(5), pp. 2701–2722.

Geman, H., D. B. Madan, and M. Yor (2001) Time changes for Lévy processes, *Mathematical Finance*, 11(1), pp. 79–96.

Klößner, S. (2006) Empirical Evidence: Intraday Returns are neither Symmetric nor Lévy Processes, Paper presented at Statistische Woche, Dresden, 2006, September 18-21.

Lehman, B. N. (1990) Fads, martingales, and market efficiency, *The Quarterly Journal of Economics*, 105(1), pp. 1–28.

Merton, R. C. (1976) Option pricing when underlying stock returns are discontinuous, *Journal of Financial Economics*, 3(1–2), pp. 125–144.

Nam, K., C. S. Pyun, and S. L. Avard (2001) Asymmetric reverting behavior of short-horizon stock returns: An evidence of stock market overreaction, *Journal of Banking & Finance*, 25(4), pp. 807–824.

Park, J. (1995) A Market Microstructure Explanation for Predictable Variations in Stock Returns following Large Price Changes, *The Journal of Financial and Quantitative Analysis*, 30(2), pp. 241–256.

Shiller, R. J. (1984) Stock Prices and Social Dynamics, *Brookings Papers on Economic Activity*, XII(2), pp. 457–498.

Yor, M. (1997) Some remarks about the joint law of Brownian motion and its supremum, *Séminaire de probabilités*, 31, pp. 306–314.

Zarowin, P. (1989) Short-run market overreaction: Size and seasonality effects, *The Journal of Portfolio Management*, 15, pp. 26–29.