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# Liquidity Risk and Expected Stock Returns

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This study investigates whether marketwide liquidity is a state variable important for asset pricing. We find that expected stock returns are related cross-sectionally to the sensitivities of returns to fluctuations in aggregate liquidity. Our monthly liquidity measure, an average of individual-stock measures estimated with daily data, relies on the principle that order flow induces greater return reversals when liquidity is lower. From 1966 through 1999, the average return on stocks with high sensitivities to liquidity exceeds that for stocks with low sensitivities by 7.5 percent annually, adjusted for exposures to the market return as well as size, value, and momentum factors. Furthermore, a liquidity risk factor accounts for half of the profits to a momentum strategy over the same 34-year period.

## I. Introduction

In standard asset pricing theory, expected stock returns are related cross-sectionally to returns' sensitivities to state variables with pervasive effects on investors' overall welfare. A security whose lowest returns tend to accompany unfavorable shifts in that welfare must offer additional compensation to investors for holding the security. Liquidity appears to be a good candidate for a priced state variable. It is often viewed as an important feature of the investment environment and macroeconomy, and recent studies find that fluctuations in various measures of liquidity are correlated across assets.[1] This empirical study investigates whether marketwide liquidity is indeed priced. That is, we ask whether cross-sectional differences in expected stock returns are related to the sensitivities of returns to fluctuations in aggregate liquidity.

It seems reasonable that many investors might require higher expected returns on assets whose returns have higher sensitivities to aggregate liquidity. Consider, for example, any investor who employs some form of leverage and faces a margin or solvency constraint, in that if his overall wealth drops sufficiently, he must liquidate some assets to raise cash. If he holds assets with higher sensitivities to liquidity, then such liquidations are more likely to occur when liquidity is low, since drops in his overall wealth are then more likely to accompany drops in liquidity. Liquidation is costlier when liquidity is lower, and those greater costs are especially unwelcome to an investor whose wealth has already dropped and who thus has higher marginal utility of wealth. Unless the investor expects higher returns from holding these assets, he would prefer assets less likely to require liquidation when liquidity is low, even if these assets are just as likely to require liquidation on average.[2]

The well-known 1998 episode involving Long-Term Capital Management (LTCM) seems an acute example of the liquidation scenario above.

[1] Chordia, Roll, and Subrahmanyam (2000), Lo and Wang (2000), Hasbrouck and Seppi (2001), and Huberman and Halka (2002) empirically analyze the systematic nature of stock market liquidity. Chordia, Sarkar, and Subrahmanyam (2002) find that improvements in stock market liquidity are associated with monetary expansions and that fluctuations in liquidity are correlated across stocks and bond markets. Eisfeldt (2002) develops a model in which endogenous fluctuations in liquidity are correlated with real fundamentals such as productivity and investment.

[2] This economic story has yet to be formally modeled, but recent literature presents related models that lead to the same basic result. Lustig (2001) develops a model in which solvency constraints give rise to a liquidity risk factor, in addition to aggregate consumption risk, and equity's sensitivity to the liquidity factor raises its equilibrium expected return. Holmström and Tirole (2001) also develop a model in which a security's expected return is related to its covariance with aggregate liquidity. Unlike more standard models, their model assumes risk-neutral consumers and is driven by liquidity demands at the corporate level. Acharya and Pedersen (2002) develop a model in which each asset's return is net of a stochastic liquidity cost, and expected returns are related to return covariances with the aggregate liquidity cost (as well as to three other covariances).

The hedge fund was highly levered and by design had positive sensitivity to marketwide liquidity, in that many of the fund's spread positions, established across a variety of countries and markets, went long less liquid instruments and short more liquid instruments. When the Russian debt crisis precipitated a widespread deterioration in liquidity, LTCM's liquidity-sensitive portfolio dropped sharply in value, triggering a need to liquidate in order to meet margin calls. The anticipation of costly liquidation in a low-liquidity environment then further eroded LTCM's position. (The liquidation was eventually overseen by a consortium of 14 institutions organized by the New York Federal Reserve.) Even though exposure to liquidity risk ultimately spelled LTCM's doom, the fund performed quite well in the previous four years, and presumably its managers perceived high expected returns on its liquidity-sensitive positions.[3]

Liquidity is a broad and elusive concept that generally denotes the ability to trade large quantities quickly, at low cost, and without moving the price. We focus on an aspect of liquidity associated with temporary price fluctuations induced by order flow. Our monthly aggregate liquidity measure is a cross-sectional average of individual-stock liquidity measures. Each stock's liquidity in a given month, estimated using that stock's within-month daily returns and volume, represents the average effect that a given volume on day $d$ has on the return for day $d + 1$, when the volume is given the same sign as the return on day $d$. The basic idea is that, if signed volume is viewed roughly as "order flow," then lower liquidity is reflected in a greater tendency for order flow in a given direction on day $d$ to be followed by a price change in the opposite direction on day $d + 1$. Essentially, lower liquidity corresponds to stronger volume-related return reversals, and in this respect our liquidity measure follows the same line of reasoning as the model and empirical evidence presented by Campbell, Grossman, and Wang (1993). They find that returns accompanied by high volume tend to be reversed more strongly, and they explain how this result is consistent with a model in which some investors are compensated for accommodating the liquidity demands of others.

We find that stocks' "liquidity betas," their sensitivities to innovations in aggregate liquidity, play a significant role in asset pricing. Stocks with higher liquidity betas exhibit higher expected returns. In particular, between January 1966 and December 1999, a spread between the top and bottom deciles of predicted liquidity betas produces an abnormal return ("alpha") of 7.5 percent per year with respect to a model that accounts for sensitivities to four other factors: the market, size, and value factors of Fama and French (1993) and a momentum factor. The alpha

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[3] See, e.g., Jorion (2000) and Lowenstein (2000) for accounts of the LTCM experience.

with respect to just the three Fama-French factors is over 9 percent per year. The results are both statistically and economically significant, and similar results occur in both halves of the overall 34-year period.

This study investigates whether expected returns are related to systematic liquidity risk in returns, as opposed to the level of liquidity per se. The latter's relation to expected stock returns has been investigated by numerous empirical studies, including Amihud and Mendelson (1986), Brennan and Subrahmanyam (1996), Brennan, Chordia, and Subrahmanyam (1998), Datar, Naik, and Radcliffe (1998), and Fiori (2000).[4] Using a variety of liquidity measures, these studies generally find that less liquid stocks have higher average returns. Amihud (2002) and Jones (2002) document the presence of a time-series relation between their measures of market liquidity and expected market returns. Instead of investigating the level of liquidity as a characteristic that is relevant for pricing, this study entertains marketwide liquidity as a state variable that affects expected stock returns because its innovations have effects that are pervasive across common stocks. The potential usefulness of such a perspective is recognized by Chordia, Roll, and Subrahmanyam (2000, 2001).

Chordia, Subrahmanyam, and Anshuman (2001) find a significant cross-sectional relation between stock returns and the variability of liquidity, where liquidity is proxied by measures of trading activity such as volume and turnover. The authors report that stocks with more volatile liquidity have lower expected returns, an unexpected result. Liquidity risk in that study is measured as firm-specific variability in liquidity. Our paper focuses on systematic liquidity risk in returns and finds that stocks whose returns are more exposed to marketwide liquidity fluctuations command higher expected returns.

Section II explains the construction of the liquidity measure and briefly describes some of its empirical features. The sharpest troughs in marketwide liquidity occur in months easily identified with significant financial and economic events, such as the 1987 crash, the beginning of the 1973 oil embargo, the 1997 Asian financial crisis, and the 1998 collapse of LTCM. Moreover, in months of large liquidity drops, stock returns are negatively correlated with fixed-income returns, in contrast to other months. This observation seems consistent with "flight-to-quality" effects. We also find significant commonality across stocks in our monthly liquidity measure. That result, in accord with the high-frequency evidence of previous studies, enhances the prospect that marketwide liquidity could be a priced state variable.

[4] Theoretical studies that investigate the relation between liquidity and asset prices include Amihud and Mendelson (1986), Constantinides (1986), Heaton and Lucas (1996), Vayanos (1998), Lo, Mamaysky, and Wang (2001), and Huang (in press), among others.

Section III presents the asset pricing investigation. We find that stocks' liquidity betas can be predicted not only by their simple historical estimates but by other variables as well. Each year, we sort stocks by their predicted liquidity betas and form 10 portfolios. This procedure yields a substantial spread in the estimated postformation liquidity betas as well as the large spread in abnormal returns reported above. Sorting stocks on their historical liquidity betas alone produces results that are slightly less strong but still significant. A sort on firm size reveals that stocks of the smallest firms tend to have high liquidity betas as well as significantly positive alphas with respect to the four-factor model.

Section IV provides an investment perspective on liquidity risk by examining the degree to which spreads between stocks with high and low liquidity risk expand the mean-variance opportunity set. In an investment universe that also includes the market portfolio and spreads based on size, value, and momentum, we find that liquidity risk spreads receive substantial weight in the portfolio with the highest ex post Sharpe ratio. The importance of the momentum spread in that portfolio is especially reduced as compared to a universe without a liquidity risk spread. Moreover, an equally weighted liquidity risk spread reduces momentum's alpha by half in the overall 34-year period and eliminates it completely (driving it to a small negative value) in the more recent 17-year subperiod 1983–99. Section V briefly reviews our conclusions and suggests directions for future research.

## II. Marketwide Liquidity

### A. Constructing a Measure

Liquidity has many dimensions. This study focuses on a dimension associated with temporary price changes accompanying order flow. We construct a measure of market liquidity in a given month as the equally weighted average of the liquidity measures of individual stocks on the New York Stock Exchange (NYSE) and American Stock Exchange (AMEX), using daily data within the month.[5] Specifically, the liquidity

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[5] All the individual-stock return and volume data used in the study are obtained from the Center for Research in Security Prices (CRSP) at the University of Chicago. Daily returns and volume are taken from the CRSP daily stock file; all month-end (or year-end) codes and values are taken from the CRSP monthly stock file. We exclude NASDAQ in constructing the aggregate liquidity measure because NASDAQ returns and volume data are available from CRSP for only part of this period (beginning in 1982). Also, reported volumes on NASDAQ include interdealer trades, unlike the volumes reported on the NYSE and the AMEX. To exclude NASDAQ, we omit stocks with exchange codes of 3 or 33 as of the end of the previous year. We use only stocks classified as ordinary common shares (CRSP share codes 10 and 11), excluding American depository receipts, shares of beneficial interest, certificates, units, real estate investment trusts, closed-end funds, companies incorporated outside the United States, and Americus trust components.

measure for stock $i$ in month $t$ is the ordinary least squares estimate of $\gamma_{i,t}$ in the regression

$$r_{i,d+1,t}^e = \theta_{i,t} + \phi_{i,t} r_{i,d,t} + \gamma_{i,t} \text{sign}(r_{i,d,t}^e) \cdot v_{i,d,t} + \epsilon_{i,d+1,t}, \quad d = 1, \ldots, D, \quad (1)$$

where quantities are defined as follows: $r_{i,d,t}$ is the return on stock $i$ on day $d$ in month $t$; $r_{i,d,t}^e = r_{i,d,t} - r_{m,d,t}$, where $r_{m,d,t}$ is the return on the CRSP value-weighted market return on day $d$ in month $t$; and $v_{i,d,t}$ is the dollar volume for stock $i$ on day $d$ in month $t$. A stock's liquidity is computed in a given month only if there are more than 15 observations with which to estimate the regression (1) ($D > 15$), and we exclude a stock for the first and last partial month that it appears on the CRSP tape. The daily observations are not required to be consecutive (except that each observation requires data for two successive days). Stocks with share prices less than \$5 and greater than \$1,000 at the end of the previous month are excluded, and volume is measured in millions of dollars.

The basic idea is that "order flow," constructed here simply as volume signed by the contemporaneous return on the stock in excess of the market, should be accompanied by a return that one expects to be partially reversed in the future if the stock is not perfectly liquid. We assume that the greater the expected reversal for a given dollar volume, the lower the stock's liquidity. That is, one would expect $\gamma_{i,t}$ to be negative in general and larger in absolute magnitude when liquidity is lower.[6] Viewing volume-related return reversals as arising from liquidity effects is motivated by Campbell et al. (1993). Those authors present a model in which risk-averse "market makers," defined in the general sense of Grossman and Miller (1988), accommodate order flow from liquidity-motivated traders and are compensated with a higher expected return (by buying at a low price or selling at a high one). The greater the order flow, the greater the compensation, so this liquidity-induced effect on expected future return is larger when current volume is high. Campbell et al. present empirical evidence consistent with this argument.

As illustrated below, the estimates of the liquidity measure $\gamma_{i,t}$ are typically negative, although there are months in which the average estimate is positive. The preponderance of negative values is consistent with the basic intuition underlying our liquidity measure, but it must be recognized that the measure abstracts from other potential roles that volume can play in the relation between current and lagged return. For example, Llorente et al. (2001) explain that asymmetric information (not considered by Campbell et al. [1993]) can weaken the volume-

[6] An alternative class of liquidity measures is based on a positive *contemporaneous* relation between returns and order flow. Typically, these measures are estimated with intraday transactions data, and the volume for a transaction is signed by comparing the transaction price to the bid-ask midpoint (see, e.g., Hasbrouck 1991; Foster and Viswanathan 1993; Brennan and Subrahmanyam 1996).

related reversal effect and even produce volume-related continuations in returns on stocks for which information-motivated trading is sufficiently important. Using daily data, the authors report empirical evidence consistent with that prediction. Other related evidence is reported by Lee and Swaminathan (2000), who conclude that momentum effects in monthly returns are stronger for stocks with high recent volume.

The specification of the regression in (1) is somewhat arbitrary, as is any liquidity measure. We use $r_{i,d,t}^e$, the return in excess of the market, as the dependent variable as well as to sign volume, in order to remove marketwide shocks and better isolate the individual-stock effect of volume-related return reversals. Moreover, daily returns of zero are not uncommon with lower-priced stocks for which a one-tick move represents a greater relative price change. Signing volume on the basis of total return is problematic in those zero-return cases, whereas returns in excess of the market are unlikely to be zero. On a day in which a stock's price does not change but the market goes up, it seems reasonable to identify the stock's order flow on that day as more likely initiated by sellers than by buyers. We also include the lagged stock return as a second independent variable with the intention of capturing lagged-return effects that are not volume-related, such as reversals due to a minimum tick size. Since we use $r_{i,d,t}^e$ to sign volume, we use the total return $r_{i,d,t}$ as this second variable to have it be less correlated with the variable whose coefficient we take as the liquidity measure. (A higher correlation between the independent variables generally reduces the precision with which one can measure the individual slope on either one.) The precise specification of the variables in (1), as compared to seemingly close alternative specifications, is addressed below in subsection C.

In order to investigate the ability of the regression slope $\gamma_{i,t}$ in (1) to capture a liquidity effect, we examine a simple model in which the return on a given day has an order flow component that is partially reversed on the subsequent day. Specifically, the return on stock $i$ on day $d$ is given by

$$r_{i,d} = f_d + u_{i,d} + \phi_i(q_{i,d-1} - q_{i,d}) + \eta_{i,d} - \eta_{i,d-1}. \qquad (2)$$

The first two terms on the right-hand side represent permanent changes in the price, where $f_d$ is a marketwide factor and $u_{i,d}$ is a stock-specific effect. The term $\phi_i(q_{i,d-1} - q_{i,d})$ is intended to capture the liquidity-related effect arising from order flow $q_{i,d}$, in the sense that both current and lagged order flow enter the return, but in the opposite directions. The coefficient $\phi_i$ is negative and represents the stock's liquidity. We assume that $q_{i,d} = q_{i,d}^* + q_d$, where $q_{i,d}^*$ is independent across stocks and $q_d$ is a marketwide component whose standard deviation is one-third as large as that of $q_{i,d}^*$, so the marketwide component then explains 10 percent

of the total variance of order flow. (Hasbrouck and Seppi [2001] report that the first principal component explains 7.8 percent of total order flow variance.)

We use (2) to simulate returns on 10,000 stocks. The quantities $f_d$, $u_{i,d}$, $q_{i,d}^*$, and $q_d$ are all mean zero draws from normal distributions. The values of $f_d$ are drawn independently across $d$ with standard deviation $\sigma = 0.20/250$; $u_{i,d}$ and $q_{i,d}^*$ are drawn independently across $d$ and $i$ with standard deviations equal to $\sigma$; and $q_d$ is drawn independently across $d$ with standard deviation equal to $\frac{1}{3}\sigma$. The liquidity coefficient $\phi_i$ is drawn independently across $i$ from a uniform $[-1, 0]$ distribution. The term $\eta_{i,d} - \eta_{i,d-1}$ represents an additional reversal effect that is independent of the order flow effect, and this component of the return is best viewed as bid-ask bounce or a tick size effect. On a given day, $\eta_{i,d}$ takes the value $-s_i$, zero, or $s_i$ with probabilities one-fourth, one-half, and one-fourth, and the realizations are independent across days and stocks. The value of $s_i$ for a given stock is drawn as $0.01(U_{[0,1]} - \phi_i)$, where $U_{[0,1]}$ is a uniform $[0, 1]$ variate, so the mean value of $s_i$ across stocks is 0.01, and there is some association between the typical magnitude of $\eta_{i,d}$ and the stock's liquidity (less liquid stocks tend to have larger $s_i$'s). In this simulation setting, the average standard deviation of a daily stock return is 0.023, the average standard deviation of each of the first three right-hand-side terms in (2) is 0.013, and the average standard deviation of $\eta_{i,d} - \eta_{i,d-1}$ is 0.010. The regression in (1) requires returns in excess of the market, so we also construct a market return as

$$r_{m,d} = \frac{1}{n}\sum_{i=1}^{n} r_{i,d},$$

for $n = 10,000$. (The average $R^2$ in a regression of $r_{i,d}$ on $r_{m,d}$ is .33.) We also specify a stock's "volume" on day $d$ as $v_{i,d} = |q_{i,d}|$.[7] For each stock, we then compute the population value of the coefficient $\gamma_i$ in (1) by estimating that regression across 50,000 simulated daily values. We find that the cross-sectional correlation between $\phi_i$ and $\gamma_i$ is .98, which suggests that the regression in (1) is a reasonable specification for estimating the hypothesized liquidity effect.

The use of signed volume as a predictor of future return can also be motivated using the equilibrium model of Campbell et al. (1993). In their model, the stock's excess return $Q_t$ and order flow $\Delta_t$ are jointly normal, along with $Q_{t+1}$, and the regression relating expected future

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[7] Because of the common factor in order flow, the market return is correlated with lagged order flow. Moreover, if we compute a lagged aggregate "volume" measure as $V_d = \Sigma_i |q_{i,d}|$, then the correlation between $r_{m,d}$ and $r_{m,d-1}V_d$ is $-.03$. This feature of our simulation is consistent with the negative relation between the market return and the lagged product of return and volume reported by Campbell et al. (1993).

return to current return and volume $V_t$ ($= |\Delta_t|$) is given by a relation of the form

$$E(Q_{t+1}|Q_t,\ V_t) = \phi_1 Q_t - \phi_2 \tanh(\phi_3 V_t Q_t)V_t, \tag{3}$$

where $\phi_2 < 0$ and $\phi_3 < 0$. As the correlation between $Q_t$ and $\Delta_t$ increases, (3) becomes well approximated by

$$E(Q_{t+1}|Q_t,\ V_t) = \phi_1 Q_t + \phi_2 \text{sign}(Q_t)V_t, \tag{4}$$

which is roughly analogous to (1).[8] To the extent that order flow plays an important role in determining high-frequency return variation, a conjecture that seems plausible, we see that the model of Campbell et al. gives some justification for the use of signed volume. Of course, their model of a single-stock economy with continuous price variables (no minimum tick) is only suggestive when applied to our empirical setting, but the intuition underlying their model corresponds to our interpretation of $\gamma_{i,t}$ as a liquidity measure.

Although the ordinary least squares slope coefficient $\hat{\gamma}_{i,t}$ is an imprecise estimate of a given stock's $\gamma_{i,t}$, the marketwide average liquidity in month $t$ is estimated more precisely. The disturbances in (1) are less than perfectly correlated across stocks (recall that the dependent variable is the return in excess of the market). Thus, as the number of stocks, $N$, grows large, the true unobserved average $\gamma_t = (1/N)\sum_{i=1}^{N}\gamma_{i,t}$ becomes more precisely estimated by

$$\hat{\gamma}_t = \frac{1}{N}\sum_{i=1}^{N}\hat{\gamma}_{i,t}. \tag{5}$$

We construct the marketwide measure above for each month from August 1962 through December 1999. The number of stocks in the index ($N$) ranges from 951 to 2,188.

Given the regression specification in (1), the value of $\gamma_{i,t}$ can be viewed as the liquidity "cost," in terms of return reversal, of "trading" $1 million of stock $i$, so the average in (5) can be viewed as the cost of a $1 million trade distributed equally across stocks. Obviously, a dollar trade size of

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[8] Equation (3) relies on a result given in Wang (1994). It is straightforward to show that Wang's eq. B.6 allows (3) to be restated as

$$E(Q_{+1}|Q_t,\ V_t) = \phi_1 Q_t + \phi_2 \tanh\left[\left(\frac{\rho}{1-\rho^2}\right)\left(\frac{Q_t}{\sigma_Q}\right)\left(\frac{V_t}{\sigma_\Delta}\right)\right]V_t,$$

where $\rho$ is the correlation between $Q_t$ and $\Delta_t$, and $\sigma_Q$ and $\sigma_\Delta$ are the standard deviations of those variables. Note that as $\rho \to 1$,

$$\tanh\left[\left(\frac{\rho}{1-\rho^2}\right)\left(\frac{Q_t}{\sigma_Q}\right)\left(\frac{V_t}{\sigma_\Delta}\right)\right]$$

converges in distribution to $\text{sign}(Q_t)$ since $V_t \geq 0$ and $\tanh(x) \to 1$ $(-1)$ as $x \to \infty$ $(-\infty)$.
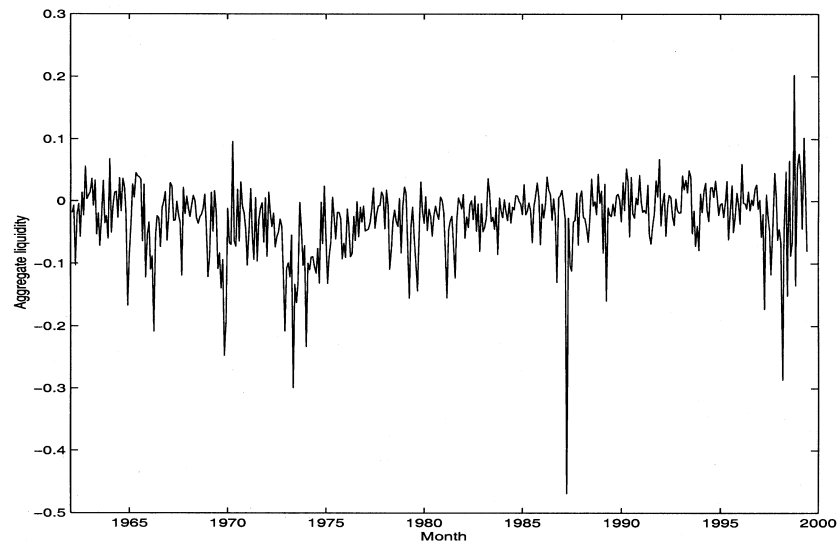
FIG. 1.—Aggregate liquidity. Each month's observation is constructed by averaging individual-stock measures for the month and then multiplying by $m_t/m_1$, where $m_t$ is the total dollar value at the end of month $t-1$ of the stocks included in the average in month $t$, and month 1 corresponds to August 1962. An individual stock's measure for a given month is a regression slope coefficient estimated using daily returns and volume data within that month. Tick marks correspond to July of the given year.

$1 million was more substantial in relative terms in the 1960s than in the 1990s, so, not surprisingly, the raw values of $\hat{\gamma}_t$ tend to be smaller in magnitude later in the period. It seems reasonable to construct a liquidity measure that reflects the cost of a trade whose size is commensurate with the overall size of the stock market, so we construct the scaled series $(m_t/m_1)\hat{\gamma}_t$, where $m_t$ is the total dollar value at the end of month $t-1$ of the stocks included in the average in month $t$, and month 1 corresponds to August 1962. This scaled series is plotted in figure 1. It can essentially be viewed as an estimate of the liquidity cost, averaged across stocks at a given point in time, of trading $1 million in 1962 "stock market" dollars (about $34 million at the end of 1999). The average value of this liquidity measure over time is −0.03 (the median is −0.02), indicating about a 2–3 percent cost for such a trade. Chordia, Roll, and Subrahmanyam (2001) report that the average daily dollar volume per stock over the 1988–98 period is about $7 million. For the same period, the average value of $1 million in 1962 stock market dollars is $14 million. On the basis of this comparison, one can also view our measure as the cost, for the average stock, of trading twice the daily volume.

   In the next section we explore the importance of liquidity risk, mea-

sured as comovement between returns and unanticipated innovations in liquidity. The liquidity series plotted in figure 1 has a first-order serial correlation of .22. In constructing innovations, we do not work directly with that series, since to do so could introduce a return component through fluctuations in the scaling factor $(m_t/m_1)$. Although any such return effects would be lagged, since $m_t$ uses values at the end of month $t - 1$, we nevertheless wish to minimize the possibility that any estimated relation between returns and liquidity innovations could arise in that fashion. At the same time, the innovation series should also appropriately reflect the growth in size of the stock market. Therefore, rather than difference the scaled series, we first difference and then scale. Specifically, to construct innovations in liquidity, we first scale the monthly difference in liquidity measures, averaged across the $N_t$ stocks with available data in both the current and previous month,

$$\Delta \hat{\gamma}_t = \left(\frac{m_t}{m_1}\right) \frac{1}{N_t} \sum_{i=1}^{N_t} (\hat{\gamma}_{i,t} - \hat{\gamma}_{i,t-1}). \tag{6}$$

We then regress $\Delta \hat{\gamma}_t$ on its lag as well as the lagged value of the scaled level series:

$$\Delta \hat{\gamma}_t = a + b \Delta \hat{\gamma}_{t-1} + c \left(\frac{m_{t-1}}{m_1}\right) \hat{\gamma}_{t-1} + u_t. \tag{7}$$

This regression allows the predicted change to depend on the most recent change as well as on the deviation of the most recent level from its long-run mean (impounded in $a$). Aside from the scaling issues, the regression is analogous to a second-order autoregression in the level series, and it produces residuals that appear serially uncorrelated.[9] The innovation in liquidity, $L_t$, is taken as the fitted residual divided by 100:

$$L_t = \frac{1}{100} \hat{u}_t. \tag{8}$$

The arbitrary scaling by 100 simply produces more convenient magnitudes of the liquidity betas reported in the next section. If expected changes in liquidity are correlated with time variation in expected stock returns, then failure to use liquidity innovations can contaminate risk measures. We find that expected liquidity changes can indeed predict

[9] Note that the equation

$$y_t - y_{t-1} = a + b(y_{t-1} - y_{t-2}) + cy_{t-1} + u_t$$

is equivalent to

$$y_t = a + b'y_{t-1} + c'y_{t-2} + u_t,$$

with $b' = 1 + b + c$ and $c' = -b$.

future stock returns one month ahead, thereby confirming the desirability of forming innovations.[10]

## B.    *Empirical Features of the Liquidity Measure*

Perhaps the most salient features of the liquidity series plotted in figure 1 are its occasional downward spikes, indicating months with especially low estimated liquidity. Many of these spikes occur during market downturns, consistent with the evidence in the studies by Chordia, Roll, and Subrahmanyam (2001) and Jones (2002), who use different liquidity measures. Chordia, Roll, and Subrahmanyam observe that their liquidity measures plummet in down markets, and Jones finds that his average spread measure exhibits frequent sharp spikes that often coincide with market downturns.

The largest downward spike in our measure of aggregate liquidity occurs in October 1987, the month of the stock market crash. Grossman and Miller (1988) argue that both spot and futures stock markets were "highly illiquid" on October 19, the day of the crash, and Amihud, Mendelson, and Wood (1990) contend that the crash occurred in part because of a rise in market illiquidity during and before October 19. The second largest spike is in November 1973, the first full month of the Mideast oil embargo. Estimated liquidity is generally low in the early 1970s, again consistent with the evidence in Jones (2002). The third largest negative value is in September 1998, when liquidity is widely perceived to have dried up because of the LTCM collapse and the recent Russian debt crisis.[11] The next largest spike occurs in May 1970, a month of significant domestic political unrest.[12] The third biggest spike in the second half of the sample is observed in October 1997 at the height of the Asian financial crisis. There is obviously a risk in pushing such anecdotal analysis very far, but a drop in stock market liquidity during these months seems at least plausible.

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[10] Regressions of the value-weighted and equally weighted liquidity risk spreads $LIQ^V$ and $LIQ^E$ (defined in Sec. III) on the lagged fitted values in (7) produce *t*-statistics of $-3.33$ and $-2.30$. The correlation between the innovations and the level series in fig. 1 is .88. We repeated the historical beta analysis reported in table 8 below using the level series in place of the innovations and obtained weaker results that go in the same direction as those reported.

[11] According to the *Economist* ("When the Sea Dries Up," 1999), "In August 1998, after the Russian government had defaulted on its debts, liquidity suddenly evaporated from many financial markets, causing asset prices to plunge" (p. 93). The article also asserts that "the possibility that liquidity might disappear from a market … is a big source of risk to an investor."

[12] On April 30, President Richard Nixon announced the invasion of Cambodia and the need to draft 150,000 more soldiers. The Kent State and Jackson State shootings occurred on May 4 and May 14, and nearly 500 colleges and universities closed that month because of antiwar protests.

The monthly innovation in liquidity, $L_t$, has a correlation of .36 with the returns on both the value-weighted and equally weighted NYSE-AMEX indexes, constructed by CRSP. This result goes in the same direction as that reported by Chordia, Roll, and Subrahmanyam (2001), who find a positive association at a daily frequency between stock returns and changes in other marketwide liquidity measures. As mentioned earlier, the downward spikes in our liquidity series often coincide with market downturns, and this observation is confirmed by comparing correlations between $L_t$ and the value-weighted market return for months in which that return is negative versus positive. The correlation is .52 in negative-return months but only .03 in positive-return months, and the difference between the liquidity-return relation in these two subsamples is statistically significant.[13] The simple correlation between $L_t$ and stock market returns is larger than those between $L_t$ and other factors typically included in empirical asset pricing studies. In particular, $L_t$'s correlations with SMB and HML, the size and value factors constructed by Fama and French (1993), are .23 and $-.12$.[14] Recall that SMB is the difference in returns between small and large firms, whereas HML is the return difference between stocks with high and low book-to-market ratios (i.e., value minus growth). The correlation between $L_t$ and a momentum factor is only .01. The inclusion of momentum as an asset pricing factor, here and in other studies, is motivated by the evidence in Jegadeesh and Titman (1993) that ranking stocks by performance over the past year produces abnormal returns.[15]

Our measure of aggregate liquidity also tends to be low when market volatility is high. Specifically, the within-month daily standard deviation of the value-weighted market return has a correlation of $-.57$ with the liquidity series in figure 1. This association between volatility and our liquidity measure seems reasonable, in that the compensation required by providers of liquidity for a given level of order flow could well be greater when volatility is higher.

To describe further the nature of months with exceptionally low liquidity, we note that a kind of "flight-to-quality" effect appears in such

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[13] We run the regression

$$L_t = a + bR_{S,t} + cD_tR_{S,t} + e_t,$$

where $R_{S,t}$ is the market return, and $D_t = 1$ if $R_{S,t} > 0$ and zero otherwise. The estimate of $b$ is 1.01 with a $t$-statistic of 9.7, and the estimate of $c$ is $-0.99$ with a $t$-statistic of $-6.2$.

[14] We are grateful to Ken French for supplying the Fama-French factors.

[15] To construct the momentum factor in month $t$, which we denote as MOM, all stocks in the CRSP file with return histories back to at least month $t-12$ are ranked at the end of month $t-1$ by their cumulative returns over months $t-12$ through $t-2$, and MOM is the payoff on a spread consisting of a $1 long position in an equally weighted portfolio of the top decile of the stocks in that ranking and a corresponding $1 short position in the bottom decile. This particular specification is the same as the "12–2" portfolio in Fama and French (1996).

TABLE 1
Correlations of Monthly Stock Market Returns with Other Variables in
Months with Large Liquidity Drops

| | Correlation of $R_{S,t}$ with | | | | Number of Observations |
|---|---|---|---|---|---|
| | $-\Delta R_{f,t}$ | $R_{GB,t}$ | $R_{CB,t}$ | $Vol_t$ | |
| | A. January 1962–December 1999 | | | | |
| All months | .047 | .323 | .372 | .491 | 449 |
| Low-liquidity months* | −.387 | −.197 | −.278 | −.360 | 14 |
| Other months | .092 | .362 | .406 | .522 | 435 |
| $p$-value | .087 | .045 | .018 | .002 | |
| | B. August 1962–March 1981 | | | | |
| All months | .077 | .285 | .376 | .567 | 224 |
| Low-liquidity months* | −.194 | .247 | −.370 | −.362 | 7 |
| Other months | .079 | .285 | .378 | .572 | 217 |
| $p$-value | .279 | .426 | .070 | .016 | |
| | C. April 1981–December 1999 | | | | |
| All months | .007 | .353 | .365 | .394 | 225 |
| Low-liquidity months* | −.573 | −.401 | −.307 | −.306 | 8 |
| Other months | .105 | .433 | .434 | .459 | 217 |
| $p$-value | .048 | .033 | .040 | .038 | |

Note.—The table reports the correlation between the monthly return on the CRSP value-weighted NYSE-AMEX index, $R_{S,t}$, and (i) minus the change in the rate on one-month Treasury bills, $-\Delta R_{f,t}$; (ii) the return on long-term government bonds, $R_{GB,t}$; (iii) the return on long-term corporate bonds, $R_{CB,t}$; and (iv) the equally weighted average percentage change in monthly dollar volume for NYSE-AMEX stocks, $Vol_t$. The $p$-values for the hypothesis that the correlations during these months are equal to those in other months are computed by a bootstrap approach.

* "Low-liquidity" months are those in which the innovation in the liquidity series is at least two standard deviations below zero.

months. That is, months in which liquidity drops severely tend to be months in which stocks and fixed-income assets move in opposite directions. Table 1 reports correlations between the monthly return on the CRSP value-weighted NYSE-AMEX index ($R_{S,t}$) and three fixed-income variables: minus the change in the rate on one-month Treasury bills ($-\Delta R_{f,t}$), the return on long-term government bonds ($R_{GB,t}$), and the return on long-term corporate bonds ($R_{CB,t}$).[16] The first row reports the correlations across all months, and the next two rows report correlations in subsamples split according to the values of $L_t$. The second row of table 1 shows the correlation between $R_{S,t}$ and the other variables during the 14 months in which $L_t$ is at least two standard deviations below its mean. The correlations between stock returns and the three fixed-income series during those months are negative, in contrast to the correlations during the remaining months, and the bootstrap $p$-values indicate that those differences are significant at levels of either 5 percent (for the bond returns) or 10 percent (for the Treasury bill rate

[16] The fixed-income data are obtained from Ibbotson Associates.

change).[17] The results across both subperiods generally support the inference drawn for the overall period, in that five of the six correlations between $R_{S,t}$ and the fixed-income series are negative in the months of large liquidity drops.

Also shown in table 1 is the correlation between the stock return $R_{S,t}$ and the change in volume, $Vol_t$, defined as the equally weighted average percentage change in monthly dollar volume for NYSE-AMEX stocks. Stock returns are positively correlated with volume changes in all months, but the correlation is negative in months with large liquidity drops, and the bootstrap $p$-value for the overall period is .002. The subperiod results again support the inference that the correlation is lower in the months of severe liquidity drops. There is no obvious story here, other than perhaps that, in such months, higher volume accompanying a larger liquidity drop is another manifestation of a flight to quality. We also find that, in low-liquidity months, the correlation between volume changes and $L_t$ is equal to $-.27$, whereas it equals .18 in other months (and in all months). But, again, we do not wish to push the descriptive analysis of the marketwide liquidity series too far. The primary goal of the paper is to investigate whether liquidity is a source of priced systematic risk in stock returns, and we use the series constructed here for that purpose.

An important motive for entertaining a marketwide liquidity measure as a priced state variable is evidence that fluctuations in liquidity exhibit commonality across stocks. Chordia, Roll, and Subrahmanyam (2000) and Huberman and Halka (2001) find significant commonality in various liquidity measures at a daily frequency, whereas Hasbrouck and Seppi (2001) find only weak commonality in intraday (15-minute) fluctuations in liquidity. Our stock-by-stock measure $\hat{\gamma}_{it}$ affords an additional perspective on commonality, since it measures liquidity differently, it is constructed at a monthly frequency, and our sample period is substantially longer. We conduct a simple exploration of commonality in $\hat{\gamma}_{it}$ across stocks by first sorting all stocks at the end of each year by market value and then assigning them to decile portfolios on the basis of NYSE break points (i.e., each decile has an equal number of NYSE stocks). Each decile portfolio's change in liquidity for a given month is then computed as the cross-sectional average change in the individual-stock measures, and this procedure yields a 1963–99 monthly series of liquidity changes for each decile. The sample correlation of these series between any two deciles is positive. If the decile series are averaged separately across the odd-numbered and even-numbered deciles, the sample correlation between the two resulting series is .56, and the $t$-statistic for a

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[17] The $p$-values are computed by resampling the original series and then randomly assigning observations to subsamples of the same size as in the reported results.

test of zero correlation is 14.20. This commonality in our liquidity measure across stocks enhances the prospect that marketwide liquidity represents a priced source of risk.

### C. Specification Issues

Our liquidity measure relies on a large cross section of stocks and yields a monthly series spanning more than 37 years. As such, the series seems well suited for this study's focus on liquidity risk and asset pricing. Aggregate stock market liquidity is measured in a variety of alternative ways by recent studies that explore other interesting issues. Those studies include Chordia, Roll, and Subrahmanyam (2000, 2001, 2002), Lo and Wang (2000), Amihud (2002), and Jones (2002). Chordia, Roll, and Subrahmanyam form daily time series of various measures of liquidity (such as depth and bid-ask spread) and trading activity (such as dollar volume), averaged across NYSE stocks over the period 1988–98. Lo and Wang form a weekly series of average turnover across NYSE and AMEX stocks from July 1962 to December 1996. Amihud constructs an annual aggregate liquidity series for the period 1963–97 by averaging across NYSE stocks the ratios of average absolute price change to trading volume. Jones collects an annual time series of average quoted bid-ask spreads on the stocks in the Dow Jones index, covering the period 1898–1998.

While measures of trading activity such as volume and turnover seem useful in explaining cross-sectional differences in liquidity, they do not appear to capture time variation in liquidity. Although liquid markets are typically associated with high levels of trading activity, it is often the case that volume is high when liquidity is low. One example is October 19, 1987, when the market was highly illiquid in many respects but trading volume on the NYSE set its historical record. More generally, the previous subsection shows that the positive time-series correlation between our liquidity measure and dollar volume turns negative when calculated only across low-liquidity months. For this reason, we do not proxy for time variation in liquidity using measures of trading activity. Bid-ask spreads and depth are not used either because suitable data are not available for a long enough sample period. The data of Chordia, Roll, and Subrahmanyam span 11 years, which is too short for an asset pricing study. Notably, their liquidity measures (quoted share and dollar depth, quoted absolute and proportional spreads, and effective absolute and proportional spreads) covary with ours in the expected direction (depth positively and spreads negatively). These measures are also jointly significant in explaining the time variation in our measure, as one might

expect from measures that capture different dimensions of market liquidity.[18]

As explained earlier, our liquidity measure reflects reversals in returns in excess of the market. Another potential source of negative serial correlation in excess returns is nonsynchronous trading. (When returns are measured with reported closing prices, an infrequently traded security is more likely to outperform the market on a day following one on which it underperforms.) With nonsynchronous trading, however, a reversal on day $d + 1$ is more likely when volume on day $d$ is low, as opposed to high as under the liquidity interpretation of $\gamma_{i,t}$ in (1). Moreover, nonsynchronous trading is likely to be more important when trading activity is low, but we find that average turnover is in fact slightly higher in the months identified as having the lowest liquidity by our measure. Nevertheless, it remains possible that nonsynchronous trading makes some contribution to a negative value of $\gamma_{i,t}$. If the negative serial correlation in excess returns, arising from either liquidity-related reversals or nonsynchronous trading, is relatively more stable through time than volatility, then fluctuations in volatility are likely to be reflected in the value of $\gamma_{i,t}$. Recall from the earlier discussion that our aggregate liquidity series exhibits a negative association with market volatility.

The liquidity measure used in this paper has substantial ex ante appeal and a number of empirical liquidity-like features, as argued earlier. One class of alternative measures involves merely changing the precise specification of regression (1). In fact, one can consider 24 different specifications (including ours). The variable on the left-hand side of (1) can be either the excess or total stock return. On the right-hand side, the first regressor can be either total return or excess return, or it can be absent. Next, one can use not only excess return but also total return to sign volume for the purpose of obtaining a proxy for order flow. Finally, the return sign can be replaced by the return itself, for both excess and total return, motivated by the empirical implementation of Campbell et al. (1993).

The various specifications described above produce aggregate series that are all substantially different from ours. The correlations between the innovations in the aggregate series produced by our specification and those for the remaining 23 choices are low, ranging from −.47 to .80 and averaging .21. The highest correlation is achieved by the specification that differs from ours only in replacing $\text{sign}(r^e_{i,d,t})$ by $r^e_{i,d,t}$ itself. The plot of the resulting series, shown in figure 2a, departs noticeably

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[18] Changes in our measure are regressed on changes in theirs in the overlapping period of January 1988 through December 1998, excluding the change between June and July 1997, when the quoted depth dropped sharply because of a reduction in the tick size on the NYSE. The regression $R^2$ is .115, and the $F$-test rejects the hypothesis that all slopes are jointly equal to zero with a $p$-value of .03.

from the plot of our series in figure 1. For example, the well-known low-liquidity episodes of October 1987 and September 1998 are much less prominent, and there are a number of downward spikes (e.g., in the late 1990s) in months that are not commonly identified with low-liquidity events. Moreover, this alternative series does not exhibit the flight-to-quality effects documented for our measure in table 1: the stock-bond correlations in low-liquidity months are actually positive. Our liquidity measure therefore seems more appealing than its most highly correlated alternative.

Figure 2 also plots the aggregate series for two other specifications of regression (1) obtained by making only one change to ours. In figure 2b, the lagged total return $r_{i,d,t}$ is replaced by its excess counterpart $r_{i,d,t}^{e}$. In figure 2c, the excess return is replaced by total return throughout, on the left-hand side as well as within the sign operator on the right-hand side. Both alternative series have a correlation of only .41 with ours, and the flight-to-quality effects are again absent from both measures. In addition, both series exhibit a negative correlation with the market in negative-return months, in contrast to the significantly positive correlation obtained for our measure (.52) as well as for liquidity measures such as bid-ask spreads and depth considered in other studies. Finally, the first series does not pick up the best-known low-liquidity periods at all, and its time-series average is in fact positive, not negative. All these facts make the alternative specifications less appealing than ours.

Another test of the usefulness of the various alternative specifications of (1) is to what extent they capture the liquidity effect modeled in the simulation exercise described in Section II$A$. To explore this issue, we repeated the simulation described there for each of the other 23 specifications. The version with the same independent variables as ours but total returns on the left achieves the same correlation (.98) with the true liquidity value $\phi_i$ as our measure does. This is not surprising since the additional noise in the dependent variable under this alternative matters little in the population (large-sample) value for the slope. More interesting is that all 22 remaining specifications produce smaller correlations with true liquidity, which lends additional support to our measure as being a sensible specification relative to seemingly close alternatives.

Figure 2$d$ plots the aggregate series obtained by value-weighting our individual stock measures across stocks. This series differs substantially from our equal-weighted measure, since the correlation between the innovations in the two series is only .77. One less attractive feature of the value-weighted series is that certain months in which liquidity was notoriously low are relatively unimportant, largely because of the high volatility of the series in the first half of the sample. When all months

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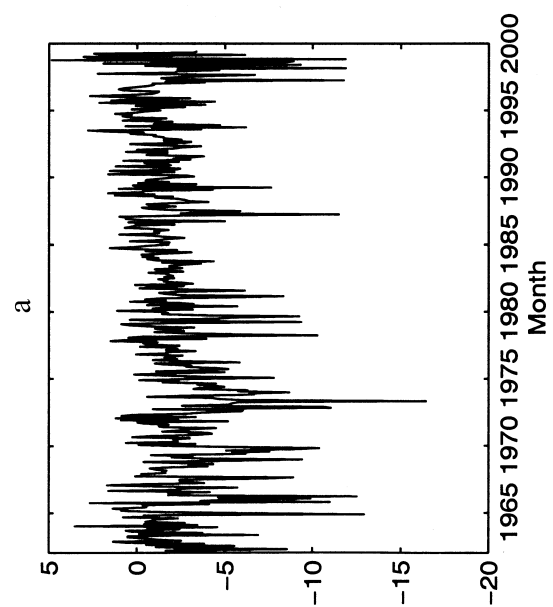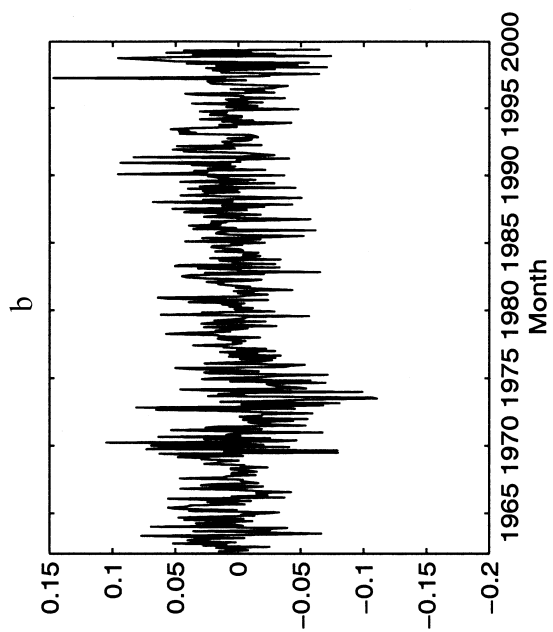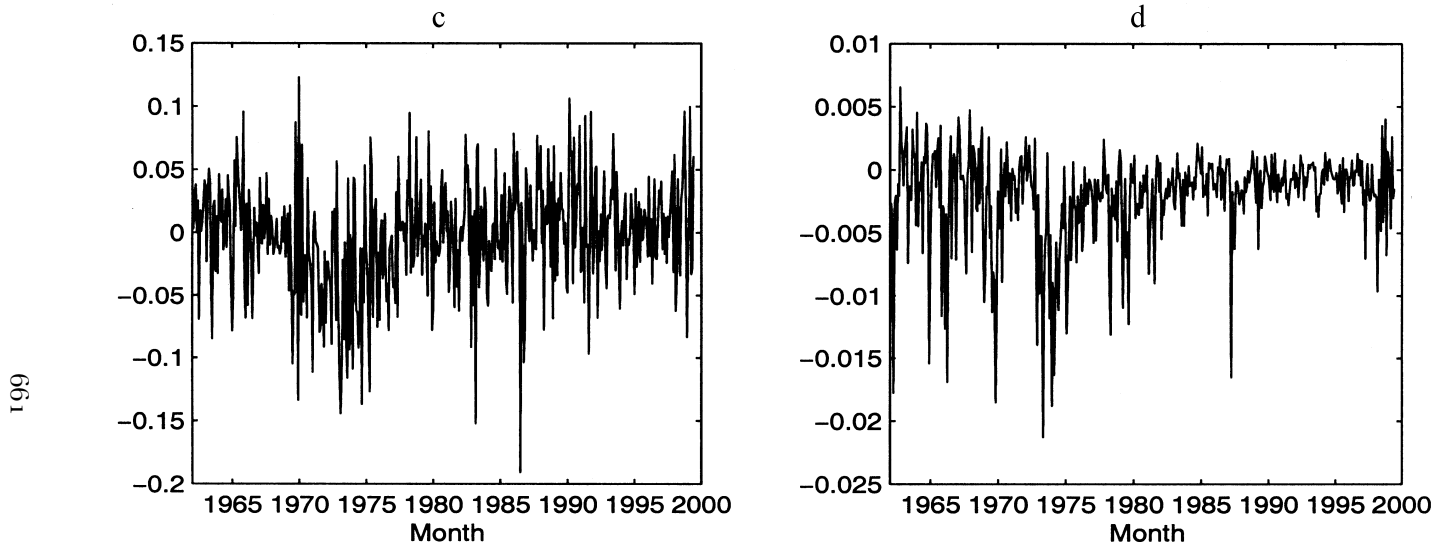
FIG. 2.—Alternative aggregate series. Each panel plots an alternative to our aggregate liquidity measure. The first three panels are based on a simple modification of the liquidity-defining regression (1). $a$, sign$(r^e_{i,d,t})$ is replaced by $r^e_{i,d,t}$ on the right-hand side of the regression. $b$, $r_{i,d,t}$ is replaced by $r^e_{i,d,t}$ on the right-hand side. $c$, $r^e_{i,d+1,t}$ is replaced by $r_{i,d+1,t}$ on the left-hand side and sign$(r^e_{i,d,t})$ is replaced by sign$(r_{i,d,t})$ on the right-hand side. These three panels plot equal-weighted averages of the slope coefficients on order flow in regression (1), multiplied by $m_t/m_1$, where $m_t$ is the total dollar value at the end of month $t - 1$ of the stocks included in the average in month $t$, and month 1 corresponds to August 1962. $d$, The value-weighted average of the individual stock measures from the unaltered version of regression (1). Tick marks correspond to July of the given year.

are sorted by their value-weighted liquidity measures, the October 1987 liquidity crunch appears sixth and September 1998 appears only twenty-fifth in the order of importance. Moreover, the value-weighted series fails to exhibit any flight-to-quality effects: the correlations between stocks and bonds in low-liquidity months are in fact positive. These unappealing features of the value-weighted measure are likely due to its domination by large-cap stocks, whose liquidity often remains high even when smaller-cap stocks experience a liquidity crunch. Our interest centers on a broad liquidity measure, as opposed to a large-stock liquidity measure, so we attempt to measure changes in aggregate liquidity using an equally weighted average of the liquidity measures for individual stocks.[19]

One might prefer to replace dollar volume on the right-hand side of regression (1) by turnover, defined as dollar volume divided by market capitalization. Note that, with such a change, the resulting gamma coefficients are very close to our coefficients multiplied by the stock's market cap at the beginning of the month, since the effects on the independent variable of within-month variation in market cap are likely to be small. Equal-weighting such modified gamma coefficients across stocks hence produces the same series as value-weighting our original coefficients and scaling them by the average market cap of all stocks used to compute the average. The resulting series therefore looks very similar to the series discussed in the previous paragraph, and it inherits all the unappealing features of that series.

In summary, the various series produced by alternative specifications and weightings of our regression-based liquidity measure are significantly different from our measure and exhibit various features that render them less appealing as measures of aggregate liquidity.

## III.    Is Liquidity Risk Priced?

This section investigates whether a stock's expected return is related to the sensitivity of its return to the innovation in aggregate liquidity, $L_t$. That sensitivity, denoted for stock $i$ by its liquidity beta $\beta_i^L$, is the slope coefficient on $L_t$ in a multiple regression in which the other independent variables are additional factors considered important for asset pricing. To investigate whether the stock's expected return is related to $\beta_i^L$, we follow a straightforward portfolio-based approach to create a universe of assets whose liquidity betas are sufficiently disperse. At the end of each year, starting with 1965, we sort stocks on the basis of their predicted values of $\beta_i^L$ and form 10 portfolios. The postformation returns

[19] We did repeat the historical beta analysis reported in table 8 below using the value-weighted series; the results are weaker but go in the same direction as those reported.

on these portfolios during the next 12 months are linked across years to form a single return series for each decile portfolio. The excess returns on those portfolios are then regressed on return-based factors that are commonly used in empirical asset pricing studies. To the extent that the regression intercepts, or alphas, differ from zero, $\beta_i^L$ explains a component of expected returns not captured by exposures to the other factors.

For the purpose of portfolio formation, we define $\beta_i^L$ as the coefficient on $L_t$ in a regression that also includes the three factors of Fama and French (1993):

$$r_{i,t} = \beta_i^0 + \beta_i^L L_t + \beta_i^M \text{MKT}_t + \beta_i^S \text{SMB}_t + \beta_i^H \text{HML}_t + \epsilon_{i,t}, \qquad (9)$$

where $r_{i,t}$ denotes asset $i$'s excess return, MKT denotes the excess return on a broad market index, and the other two factors, SMB and HML, are payoffs on long-short spreads constructed by sorting stocks according to market capitalization and book-to-market ratio. This definition of $\beta_i^L$ captures the asset's comovement with aggregate liquidity that is distinct from its comovement with other commonly used factors. We allow $\beta_i^L$ for any given stock to vary through time, and the predicted values of $\beta_i^L$ used to sort stocks are obtained using two methods. The first allows the predicted $\beta_i^L$ to depend on the stock's historical least-squares estimate as well as a number of additional stock characteristics observable at the time of the sort. The results using that method, reported in subsection A, reveal large differences in expected returns on $\beta_i^L$-sorted portfolios that are unexplained by the other factors. The second method uses only historical betas and is presented to confirm that the first set of results is not driven solely by sorting stocks on the other characteristics that help predict liquidity betas. The results from that method, reported in subsection B, also reveal large and significant differences in alphas on the $\beta_i^L$-sorted portfolios. Subsection C reports results obtained for portfolios formed by sorting stocks on market capitalization.

Our analysis covers all stocks traded on the NYSE, AMEX, and NASDAQ that are ordinary common shares (CRSP share codes 10 and 11). Stocks with prices below $5 or above $1,000 are also excluded from the portfolio sorts. The portfolio formation procedure uses data available only as of the formation date, and this requirement applies to the liquidity series as well. Thus the formation procedure each year begins with a reestimation of (7) using only the raw liquidity series ($\hat{\gamma}_t$) available up to that point in time. The historical values of $L_t$ used in that formation year are then recomputed using (8), where $\hat{u}_t$ is the fitted residual from that reestimated regression.

TABLE 2
DETERMINANTS OF PREDICTED LIQUIDITY BETAS

| | AUGUST 1962 THROUGH | | |
| --- | --- | --- | --- |
| | December 1998 | December 1983 | December 1968 |
| Intercept | −1.79 | −4.39 | −2.75 |
| | (−6.75) | (−12.94) | (−2.95) |
| Historical beta | 2.30 | 3.75 | 9.18 |
| | (9.97) | (10.87) | (9.99) |
| Average liquidity | −.87 | −.02 | −.48 |
| | (−4.12) | (−.08) | (−.61) |
| Average volume | 1.54 | −3.37 | .07 |
| | (3.29) | (−5.03) | (.05) |
| Cumulative return | −.04 | 1.00 | .93 |
| | (−.14) | (2.86) | (.86) |
| Return volatility | −.24 | −1.13 | −2.61 |
| | (−1.60) | (−3.39) | (−2.25) |
| Price | .59 | 7.51 | 4.32 |
| | (1.85) | (15.00) | (3.38) |
| Shares outstanding | −1.43 | .67 | −.69 |
| | (−3.37) | (1.26) | (−.54) |

NOTE.—Each column reports the results of estimating a linear relation between a stock's liquidity beta and the seven characteristics listed (in addition to the intercept, shown first). At each year end shown, the estimation uses all stocks defined as ordinary common shares traded on the NYSE, AMEX, or NASDAQ with at least three years of monthly returns continuing through the given year end. The estimation uses a two-stage pooled time-series and cross-sectional approach. Each value reported is equal to the coefficient estimate multiplied by the time-series average of the annual cross-sectional standard deviations of the characteristic. The $t$-statistics are in parentheses.

## A.    Sorting by Predicted Liquidity Betas

## 1.    Predicting Liquidity Betas

We model each stock's liquidity beta as a linear function of observable variables

$$\beta_{i,t-1}^{L} = \psi_{1,i} + \psi_{2,i}' \mathbf{Z}_{i,t-1}. \tag{10}$$

The vector $\mathbf{Z}_{i,t-1}$ contains seven characteristics: (i) the historical liquidity beta estimated using all data available from months $t-60$ through $t-1$ (if at least 36 months are available), (ii) the average value of $\hat{\gamma}_{i,t}$ from months $t-6$ through $t-1$, (iii) the natural log of the stock's average dollar volume from months $t-6$ through $t-1$, (iv) the cumulative return on the stock from months $t-6$ through $t-1$, (v) the standard deviation of the stock's monthly return from months $t-6$ through $t-1$, (vi) the natural log of the price per share from month $t-1$, and (vii) the natural log of the number of shares outstanding from month $t-1$. (These seven characteristics are listed in table 2.) The list of characteristics is necessarily arbitrary, although they do possess some appeal ex ante. Historical liquidity beta should be useful if the true beta is fairly stable over time. The average of the stock's $\hat{\gamma}_{i,t}$ and volume can matter if liquidity risk is related to liquidity per se. Stocks with different market capitalization could have different liquidity

betas, so we include shares outstanding and stock price, whose product is equal to the stock's market capitalization. The level and variability of recent returns simply allow some role for short-run return dynamics. Each characteristic is "demeaned" by subtracting the time-series average (through month $t-1$) of the characteristic's cross-sectional average in each previous month.

Substituting the right-hand side of (10) for $\beta_i^L$ in (9), we obtain

$$r_{i,t} = \beta_i^0 + \beta_i^M \text{MKT}_t + \beta_i^S \text{SMB}_t + \beta_i^H \text{HML}_t$$
$$+ (\psi_{1,i} + \psi'_{2,i} \mathbf{Z}_{i,t-1}) L_t + \epsilon_{i,t}. \qquad (11)$$

This regression for stock $i$ contains 11 independent variables, seven of which are cross products of the elements of $\mathbf{Z}_{i,t-1}$ with $L_t$. (This approach to incorporating time variation in betas follows Shanken [1990].) To increase precision in the face of the substantial variance in individual-stock returns, we restrict the coefficients $\psi_{1,i}$ and $\psi_{2,i}$ in equation (10) to be the same across all stocks and estimate them using the whole panel of stock returns. Specifically, at the end of each year between 1965 and 1998, we first construct for each stock the historical series of

$$e_{i,t} = r_{i,t} - \hat{\beta}_i^M \text{MKT}_t - \hat{\beta}_i^S \text{SMB}_t - \hat{\beta}_i^H \text{HML}_t, \qquad (12)$$

where the $\hat{\beta}$'s are estimated from the regression of the stock's excess returns on the Fama-French factors and $L_t$, using all data available up to the current year end. Then we run a pooled time-series, cross-sectional regression of $e_{i,t}$ on the characteristics,

$$e_{i,t} = \psi_0 + \psi_1 L_t + \psi'_2 \mathbf{Z}_{i,t-1} L_t + \nu_{i,t}, \qquad (13)$$

again using all data available up to the current year end. The first year end considered here is that of 1965, since the data on $L_t$ begin in August 1962, and it seems reasonable to use at least three years of data to conduct the estimation. A stock is excluded for any month in which it has any missing characteristics.

Table 2 reports the estimated coefficients $\hat{\psi}_1$ and $\hat{\psi}_2$ from the pooled regression, together with their $t$-statistics.[20] Results are reported for several periods, each beginning in August 1962 but ending in December of a different year; the estimated coefficients are those used in the ranking at that year end. Each coefficient is multiplied by the time-series average of the cross-sectional standard deviation of the corresponding demeaned characteristic. This scaling helps clarify the relative contributions of the individual characteristics to the predicted betas. Historical

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[20] The $t$-statistics are computed assuming independence of the regression residuals, which are purged of common variation in returns attributable to the three Fama-French factors together with $L_t$.

liquidity beta is the most important determinant of the predicted beta in the longest sample period, used for the most recent ranking in December 1998. The coefficient of 2.30 ($t = 9.97$) indicates that if a stock's historical liquidity beta is one cross-sectional standard deviation above the cross-sectional mean of the historical betas, then the stock's predicted liquidity beta is higher by 2.30, when we hold the other characteristics constant and average the effect over time. Historical beta is also the most robust determinant of the predicted beta across the different periods. The coefficient on stock price is significantly positive early in the sample, but its effect weakens in the longer period. Volatility enters negatively, again more strongly in the earlier periods. The coefficients on the stock's past return, shares outstanding, and average volume are less stable over time.[21] The coefficient on the stock's recent average $\hat{\gamma}_{it}$ is significantly negative in the longest period (and insignificantly negative in the subperiods), suggesting that stocks with lower liquidity (as measured by $\hat{\gamma}_{it}$) tend to be more exposed to aggregate liquidity fluctuations.

### 2.  Postranking Portfolio Betas

At the end of each year, stocks are sorted by their predicted liquidity betas and assigned to 10 portfolios. The predicted beta for each stock is calculated from equation (10), using the year-end values of the stock's characteristics along with the values of $\hat{\psi}_1$ and $\hat{\psi}_2$ estimated using data through the current year end. Portfolio returns are computed over the following 12 months, after which the estimation/formation procedure is repeated. The postranking returns are linked across years, generating a single return series for each decile covering the period from January 1966 through December 1999. On average, there are 187 stocks in each portfolio, and no portfolio ever contains fewer than 103 stocks.

  Panel A of table 3 reports the postranking liquidity betas of the decile portfolios when the stocks within each portfolio are value-weighted. (The results for equally weighted portfolios, not shown, are nearly identical.) The liquidity betas are estimated by running the regression in

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[21] As mentioned earlier, the trading volume of the NASDAQ stocks is overstated relative to the NYSE/AMEX volume. When the NASDAQ stocks are excluded from the pooled regression, the coefficient on volume remains negative in the first two subperiods and turns insignificantly negative in the overall period. In addition, the results presented in this section lead to similar conclusions about the relation between liquidity risk and expected stock returns. We retain the NASDAQ stocks in the analysis because their inclusion increases the dispersion of the postranking liquidity betas of the portfolios sorted on predicted betas, in line with the purpose of the sorting procedure. Stocks with prices outside the $5–$1,000 range are also included in the pooled regression for the same reason: their inclusion increases the spread in the postranking betas, even if these stocks are subsequently excluded from the portfolio sorts.

TABLE 3
Properties of Portfolios Sorted on Predicted Liquidity Betas

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 10–1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Decile Portfolio | | | | | | |
| | | | | | A. Postranking Liquidity Betas | | | | | | |
| Jan. 1966–Dec. 1999 | −5.75 | −6.54 | −4.66 | −3.16 | .90 | −.63 | −.86 | .68 | 2.44 | 2.48 | 8.23 |
| | (−2.22) | (−2.98) | (−2.59) | (−2.18) | (.69) | (−.54) | (−.68) | (.52) | (1.77) | (1.35) | (2.37) |
| Jan. 1966–Dec. 1982 | −7.28 | −8.29 | −3.47 | −3.15 | 2.58 | −.34 | −.47 | .73 | −2.51 | 4.19 | 11.47 |
| | (−1.84) | (−2.54) | (−1.19) | (−1.36) | (1.23) | (−.17) | (−.22) | (.33) | (−1.10) | (1.38) | (2.06) |
| Jan. 1983–Dec. 1999 | −3.00 | −4.27 | −5.09 | −2.36 | −1.10 | −.84 | −1.60 | 1.94 | 5.67 | .85 | 3.85 |
| | (−.85) | (−1.37) | (−2.12) | (−1.22) | (−.63) | (−.57) | (−1.06) | (1.22) | (3.23) | (.36) | (.84) |
| | | | | | B. Additional Properties, January 1966–December 1999 | | | | | | |
| Market cap | 2.83 | 5.90 | 8.30 | 7.65 | 10.67 | 16.61 | 15.99 | 16.02 | 16.05 | 14.28 | |
| Liquidity | −.46 | −.16 | −.10 | −.15 | −.08 | −.07 | −.03 | −.03 | −.04 | −.10 | |
| MKT beta | 1.24 | 1.21 | 1.09 | 1.05 | 1.04 | 1.03 | 1.00 | 1.01 | .98 | .94 | −.30 |
| | (37.70) | (44.61) | (48.31) | (56.83) | (62.83) | (68.89) | (62.56) | (60.75) | (55.76) | (40.75) | (−6.85) |
| SMB beta | .70 | .31 | .05 | .01 | −.09 | −.12 | −.12 | −.09 | −.12 | .05 | −.65 |
| | (14.47) | (7.64) | (1.61) | (.26) | (−3.51) | (−5.63) | (−5.04) | (−3.82) | (−4.76) | (1.36) | (−10.14) |
| HML beta | .07 | .19 | .23 | .20 | .11 | .14 | .08 | −.00 | −.01 | −.34 | −.40 |
| | (1.31) | (4.36) | (6.45) | (6.69) | (4.02) | (5.68) | (3.07) | (−.06) | (−.37) | (−9.04) | (−5.74) |
| MOM beta | −.06 | −.10 | −.07 | −.03 | −.03 | −.01 | .01 | −.01 | .03 | .05 | .11 |
| | (−2.43) | (−5.35) | (−4.29) | (−2.19) | (−2.51) | (−.72) | (.53) | (−.72) | (2.72) | (3.02) | (3.41) |

Note.—At each year end between 1965 and 1998, eligible stocks are sorted into 10 portfolios according to predicted liquidity betas. The betas are constructed as linear functions of seven stock characteristics at the current year end, using coefficients estimated from a pooled time-series, cross-sectional regression approach. The estimation and sorting procedure at each year end uses only data available at that time. Eligible stocks are defined as ordinary common shares traded on the NYSE, AMEX, or NASDAQ with at least three years of monthly returns continuing through the current year end and with stock prices between $5 and $1,000. The portfolio returns for the 12 postranking months are linked across years to form one series of postranking returns for each decile. Panel A reports the decile portfolios' postranking liquidity betas, estimated by regressing the value-weighted portfolio excess returns on the aggregate liquidity innovation and the Fama-French factors. Panel B reports the time-series averages of the decile portfolios' market capitalization and liquidity, obtained as value-weighted averages of the corresponding measures across the stocks within each decile. Market capitalization is reported in billions of dollars. A stock's liquidity in any given month is the slope coefficient $\gamma_{i,t}$ from eq. (1), multiplied by 100. Also reported are postranking betas with respect to the three Fama-French factors and a momentum factor, estimated by regressing value-weighted portfolio excess returns on the four factors. The t-statistics are in parentheses.

(9) over the whole sample period, January 1966 through December 1999, as well as over two subperiods. The postranking liquidity betas increase across deciles, consistent with the objective of the sorting procedure. The "10–1" spread, which goes long decile 10 (stocks with high liquidity betas) and short decile 1 (stocks with low liquidity betas), has an overall-period liquidity beta of 8.23, with a $t$-statistic of 2.37.

Panel B of table 3 reports some additional properties of portfolios sorted by predicted liquidity betas. The low-beta portfolios contain stocks of somewhat smaller firms: the value-weighted average size in portfolio 1 is \$2.83 billion, as compared to \$14.28 billion in portfolio 10 (averaged over time). Stocks in the low-beta portfolios also tend to be less liquid, as measured by the average value of $\hat{\gamma}_{i,}$ although this pattern is not monotonic. Panel B also reports the decile portfolios' betas with respect to the Fama-French factors, MKT, SMB, and HML, and the previously described momentum factor, MOM. The Fama-French and momentum betas are estimated by regressing the decile excess returns on the returns of the four-factor portfolios. All three Fama-French betas of the 10–1 spread are significantly negative: $-0.30$ for MKT, $-0.65$ for SMB, and $-0.40$ for HML. The SMB betas confirm the pattern in average capitalizations, and the HML betas indicate that the 10–1 spread has a tilt toward growth stocks. The 10–1 spread's momentum beta is significantly positive (0.11), suggesting some tilt toward past winners.

## 3.    Alphas

If our liquidity risk factor is priced, we should see systematic differences in the average returns of our beta-sorted portfolios. The evidence in table 4 indeed favors the pricing of liquidity risk. The table reports the value-weighted portfolios' postranking alphas estimated under three different factor specifications. The capital asset pricing model (CAPM) alpha is computed with respect to MKT, the Fama-French alpha with respect to the Fama-French factors, and the four-factor alpha with respect to the Fama-French factors and MOM. All three alphas of the 10–1 spread are significantly positive: the CAPM alpha is 6.40 percent per year ($t = 2.54$), the Fama-French alpha is 9.23 percent per year ($t = 4.29$), and the four-factor alpha is 7.48 percent per year ($t = 3.42$). (Annual alphas are computed as 12 times the monthly estimates.) The alphas are also robust across the subperiods. For example, the subperiod Fama-French alphas of the 10–1 spread are 8.50 percent ($t = 2.77$) and 10.74 percent ($t = 3.53$), and the subperiod four-factor alphas are 6.21 percent ($t = 1.95$) and 9.49 percent ($t = 3.12$). Table 5 reports alphas when the decile portfolios are equally weighted rather than value-weighted. These results are even slightly stronger. For example, the full-period CAPM, Fama-French, and four-factor alphas of the equally

TABLE 4
ALPHAS OF VALUE-WEIGHTED PORTFOLIOS SORTED ON PREDICTED LIQUIDITY BETAS

| | DECILE PORTFOLIO | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 10–1 |
| A. January 1966–December 1999 | | | | | | | | | | | |
| CAPM alpha | −5.16 | −1.88 | −.66 | −.07 | −1.48 | 1.48 | 1.22 | 1.38 | 1.68 | 1.24 | 6.40 |
| | (−2.57) | (−1.24) | (−.56) | (−.08) | (−1.80) | (1.93) | (1.52) | (1.72) | (1.93) | (1.01) | (2.54) |
| Fama-French alpha | −6.05 | −3.36 | −2.15 | −1.23 | −2.10 | .78 | .86 | 1.41 | 1.90 | 3.18 | 9.23 |
| | (−3.77) | (−2.47) | (−1.93) | (−1.37) | (−2.61) | (1.08) | (1.11) | (1.76) | (2.22) | (2.82) | (4.29) |
| Four-factor alpha | −5.11 | −1.66 | −1.02 | −.76 | −1.61 | .91 | .76 | 1.55 | 1.34 | 2.36 | 7.48 |
| | (−3.12) | (−1.23) | (−.91) | (−.83) | (−1.96) | (1.22) | (.96) | (1.88) | (1.54) | (2.06) | (3.42) |
| B. January 1966–December 1982 | | | | | | | | | | | |
| CAPM alpha | −2.26 | 1.63 | .54 | .67 | −3.09 | 1.44 | .61 | 1.78 | 1.43 | −.93 | 1.34 |
| | (−.81) | (.76) | (.31) | (.50) | (−2.69) | (1.29) | (.54) | (1.46) | (1.14) | (−.52) | (.36) |
| Fama-French alpha | −7.32 | −2.22 | −1.80 | −.75 | −3.29 | 1.03 | .20 | 1.91 | 2.32 | 1.18 | 8.50 |
| | (−3.36) | (−1.23) | (−1.13) | (−.59) | (−2.85) | (.95) | (.17) | (1.56) | (1.86) | (.71) | (2.77) |
| Four-factor alpha | −6.43 | −.25 | −.22 | −.03 | −2.46 | 1.09 | .31 | 2.89 | 1.67 | −.22 | 6.21 |
| | (−2.82) | (−.13) | (−.13) | (−.02) | (−2.05) | (.95) | (.25) | (2.28) | (1.28) | (−.13) | (1.95) |
| C. January 1983–December 1999 | | | | | | | | | | | |
| CAPM alpha | −8.01 | −5.33 | −1.76 | −1.01 | .20 | 1.55 | 1.74 | .70 | 1.81 | 3.38 | 11.39 |
| | (−2.76) | (−2.49) | (−1.08) | (−.77) | (.17) | (1.46) | (1.54) | (.67) | (1.47) | (1.98) | (3.36) |
| Fama-French alpha | −5.23 | −5.08 | −2.69 | −1.80 | −.82 | .37 | .89 | .76 | 1.25 | 5.51 | 10.74 |
| | (−2.23) | (−2.46) | (−1.67) | (−1.41) | (−.72) | (.38) | (.89) | (.72) | (1.05) | (3.51) | (3.53) |
| Four-factor alpha | −4.43 | −3.72 | −1.94 | −1.52 | −.63 | .53 | .70 | .47 | .84 | 5.06 | 9.49 |
| | (−1.88) | (−1.85) | (−1.21) | (−1.17) | (−.54) | (.54) | (.69) | (.44) | (.70) | (3.20) | (3.12) |

NOTE.—See the note to table 3. The table reports the decile portfolios' postranking alphas, in percentages per year. The alphas are estimated as intercepts from the regressions of excess portfolio postranking returns on excess market returns (CAPM alpha), on the Fama-French factor returns (Fama-French alpha), and on the Fama-French and momentum factor returns (four-factor alphas). The t-statistics are in parentheses.

TABLE 5
Alphas of Equally Weighted Portfolios Sorted on Predicted Liquidity Betas

| | Decile Portfolio | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 10–1 |
| | A. January 1966–December 1999 | | | | | | | | | | |
| CAPM alpha | −5.46 | −1.47 | −.73 | .34 | .02 | .94 | 1.97 | 2.78 | 2.43 | 2.77 | 8.23 |
| | (−2.27) | (−.75) | (−.46) | (.24) | (.02) | (.84) | (1.91) | (2.71) | (2.37) | (2.06) | (4.12) |
| Fama-French alpha | −7.53 | −3.47 | −3.04 | −1.58 | −1.67 | −.76 | .46 | 1.49 | 1.46 | 2.96 | 10.49 |
| | (−6.35) | (−3.39) | (−3.63) | (−2.13) | (−2.47) | (−1.21) | (.78) | (2.50) | (2.18) | (3.14) | (6.50) |
| Four-factor alpha | −5.80 | −1.64 | −1.68 | −.68 | −1.02 | −.17 | .16 | 1.32 | .95 | 1.86 | 7.66 |
| | (−4.98) | (−1.68) | (−2.07) | (−.92) | (−1.50) | (−.26) | (.26) | (2.16) | (1.40) | (1.98) | (4.95) |
| | B. January 1966–December 1982 | | | | | | | | | | |
| CAPM alpha | 1.74 | 5.52 | 5.22 | 4.49 | 2.99 | 4.15 | 4.76 | 6.00 | 4.11 | 4.68 | 2.95 |
| | (.49) | (1.90) | (2.22) | (2.25) | (1.70) | (2.49) | (3.15) | (4.10) | (2.77) | (2.54) | (.98) |
| Fama-French alpha | −6.50 | −1.12 | −.80 | −.51 | −1.21 | .02 | 1.12 | 2.70 | 1.23 | 2.76 | 9.25 |
| | (−4.02) | (−.76) | (−.69) | (−.52) | (−1.25) | (.02) | (1.32) | (3.17) | (1.22) | (1.95) | (4.19) |
| Four-factor alpha | −5.32 | 1.00 | 1.28 | .80 | .20 | .93 | .85 | 2.79 | .84 | 1.18 | 6.49 |
| | (−3.16) | (.67) | (1.14) | (.81) | (.20) | (1.00) | (.96) | (3.12) | (.79) | (.81) | (2.91) |
| | C. January 1983–December 1999 | | | | | | | | | | |
| CAPM alpha | −11.47 | −7.36 | −6.09 | −3.06 | −2.21 | −1.58 | .06 | .29 | 1.77 | 1.78 | 13.25 |
| | (−3.70) | (−2.94) | (−2.92) | (−1.63) | (−1.41) | (−1.10) | (.04) | (.21) | (1.34) | (.92) | (5.13) |
| Fama-French alpha | −8.90 | −5.83 | −5.58 | −2.58 | −2.08 | −1.56 | .13 | .54 | 2.37 | 4.12 | 13.02 |
| | (−5.02) | (−4.07) | (−4.62) | (−2.26) | (−2.16) | (−1.70) | (.15) | (.64) | (2.72) | (3.33) | (5.50) |
| Four-factor alpha | −7.10 | −4.34 | −4.73 | −2.00 | −1.92 | −1.19 | −.13 | .29 | 1.87 | 3.42 | 10.51 |
| | (−4.40) | (−3.35) | (−4.05) | (−1.77) | (−1.97) | (−1.30) | (−.15) | (.34) | (2.18) | (2.80) | (4.94) |

Note.—See the note to table 3. The table reports the decile portfolios' postranking alphas, in percentages per year. The alphas are estimated as intercepts from the regressions of excess portfolio postranking returns on excess market returns (CAPM alpha), on the Fama-French factor returns (Fama-French alpha), and on the Fama-French and momentum factor returns (four-factor alphas). The $t$-statistics are in parentheses.

weighted 10–1 spread are 8.23 percent, 10.49 percent, and 7.66 percent, respectively. The subperiod results are comparably strong, too.

We also test the hypothesis that all 10 alphas are jointly equal to zero, using the test of Gibbons, Ross, and Shanken (1989). For both equally weighted and value-weighted portfolios and for all three models, the hypothesis is rejected at a 1 percent significance level in the overall period. The hypothesis is also rejected at the 5 percent level in both subperiods, for both equally weighted and value-weighted portfolios and for all three models. The only exception occurs with the four-factor alphas for the value-weighted portfolios in the second subperiod, in which case the hypothesis is rejected at the 10 percent level.

Overall, the evidence strongly supports the hypothesis that our liquidity risk factor is priced. The premium for this risk is positive, in that stocks with higher sensitivity to aggregate liquidity shocks offer higher expected returns. The latter result is consistent with the notion that a pervasive drop in liquidity is seen as undesirable by the representative investor, so that the investor requires compensation for holding stocks with greater exposure to this risk.

## 4. Estimating the Premium Using All 10 Portfolios

The discussion above relies on the 10–1 spread to infer that the expected-return premium associated with liquidity risk is positive. We also estimate the liquidity risk premium using all 10 decile portfolios. Define the multivariate regression

$$\mathbf{r}_t = \boldsymbol{\beta}_0 + \mathbf{B}\mathbf{F}_t + \boldsymbol{\beta}^L L_t + \mathbf{e}_t, \tag{14}$$

where $\mathbf{r}_t$ is a $10 \times 1$ vector containing the excess returns on the decile portfolios; $\mathbf{F}_t$ is a $4 \times 1$ vector containing the realizations of the "traded" factors MKT, SMB, HML, and MOM; $\mathbf{B}$ is a $10 \times 4$ matrix; and $\boldsymbol{\beta}_0$ and $\boldsymbol{\beta}^L$ are $10 \times 1$ vectors. We also consider a specification with only three traded factors, excluding MOM. Assume that the decile portfolios are priced by the returns' sensitivities to the traded factors and the non-traded liquidity factor:

$$E(\mathbf{r}_t) = \mathbf{B}\boldsymbol{\lambda}_F + \boldsymbol{\beta}^L \lambda_L, \tag{15}$$

where $E(\cdot)$ denotes the unconditional expectation. Taking expectations of both sides of equation (14) and substituting from equation (15) gives

$$\boldsymbol{\beta}_0 = \boldsymbol{\beta}^L [\lambda_L - E(L_t)], \tag{16}$$

since the vector of premia on the traded factors, $\boldsymbol{\lambda}_F$, is equal to $E(\mathbf{F}_t)$. The liquidity factor $L_t$ is not the payoff on a traded position, so in general the liquidity risk premium $\lambda_L$ is not equal to $E(L_t)$. We estimate $\lambda_L$ using the generalized method of moments (GMM) of Hansen (1982). Let $\boldsymbol{\theta}$

TABLE 6
LIQUIDITY RISK PREMIUM AND ITS CONTRIBUTION TO EXPECTED RETURN

| | January 1966–December 1999 | January 1966–December 1982 | January 1983–December 1999 |
|---|---|---|---|
| | A. Value-Weighted Portfolios Sorted on Predicted Betas | | |
| Three traded factors: | | | |
| $\lambda_L$ | .91 | .81 | 1.13 |
| | (2.92) | (2.05) | (2.73) |
| $(\beta_{10}^L - \beta_1^L)\lambda_L$ | 9.63 | 8.37 | 10.59 |
| | (4.57) | (2.91) | (3.22) |
| Four traded factors: | | | |
| $\lambda_L$ | .78 | .23 | .82 |
| | (2.43) | (1.36) | (2.93) |
| $(\beta_{10}^L - \beta_1^L)\lambda_L$ | 7.56 | 2.61 | 9.27 |
| | (3.42) | (1.32) | (2.78) |
| | B. Equally Weighted Portfolios Sorted on Predicted Betas | | |
| Three traded factors: | | | |
| $\lambda_L$ | 1.65 | 1.28 | 1.10 |
| | (2.74) | (1.82) | (3.38) |
| $(\beta_{10}^L - \beta_1^L)\lambda_L$ | 11.06 | 9.90 | 10.77 |
| | (7.19) | (4.26) | (4.05) |
| Four traded factors: | | | |
| $\lambda_L$ | 1.72 | 3.01 | 1.02 |
| | (2.33) | (.74) | (3.49) |
| $(\beta_{10}^L - \beta_1^L)\lambda_L$ | 8.56 | 8.20 | 10.14 |
| | (5.53) | (3.03) | (4.07) |

NOTE.—The table reports the estimates of the risk premium associated with the liquidity factor, as well as the contribution of liquidity risk to the expected return on the "10–1" spread. Stocks are sorted into 10 portfolios by their predicted liquidity betas at each year end. The premium $\lambda_L$ is estimated using postranking returns on all 10 portfolios. The decile portfolios are value-weighted in panel A and equally weighted in panel B. The premium is reported as a monthly value multiplied by 1,200, so that the product of the liquidity beta and the reported premium can be interpreted as the annual percentage return. The 10–1 spread goes long decile 10, with high liquidity beta $\beta_{10}^L$, and short decile 1, with low liquidity beta $\beta_1^L$. The contribution of liquidity risk to the portfolio's expected return, $(\beta_{10}^L - \beta_1^L)\lambda_L$, is also expressed in percentage per year. The asymptotic $t$-statistics are in parentheses.

denote the set of unknown parameters: $\lambda_L$, $\beta^L$, $\mathbf{B}$, and $E(L_t)$. The GMM estimator of $\theta$ minimizes $\mathbf{g}(\theta)'\mathbf{W}\mathbf{g}(\theta)$, where $\mathbf{g}(\theta) = (1/T)\sum_{t=1}^{T}\mathbf{f}_t(\theta)$,

$$\mathbf{f}_t(\theta) = \begin{pmatrix} \mathbf{h}_t \otimes \mathbf{e}_t \\ L_t - E(L_t) \end{pmatrix}, \tag{17}$$

$$\mathbf{h}_t' = (1 \quad \mathbf{F}_t' \quad L_t),$$

$$\mathbf{e}_t = \mathbf{r}_t - \beta^L[\lambda_L - E(L_t)] - \mathbf{B}\mathbf{F}_t - \beta^L L_t, \tag{18}$$

and $\mathbf{W}$ is a consistent estimator of the optimal weighting matrix.[22]

Estimates of the liquidity risk premium $\lambda_L$ are reported in table 6, along with asymptotic $t$-statistics. Results are reported for both value-

[22] Following Hansen (1982), we estimate $\mathbf{W}$ as the inverse of $(1/T)\sum_{t=1}^{T}\hat{\mathbf{f}}_t\hat{\mathbf{f}}_t'$, where $\hat{\mathbf{f}}_t = \mathbf{f}_t(\theta)$ evaluated at a consistent estimator of $\theta$, obtained by minimizing $\mathbf{g}(\theta)'\mathbf{g}(\theta)$.

weighted and equally weighted portfolios. The full-period estimate of $\lambda_L$ is significantly positive for both sets of portfolios under both specifications (three traded factors or four). The subperiod estimates are all positive, and the majority are statistically significant. Overall, estimating the liquidity risk premium using all 10 portfolios confirms the previous inferences based on the extreme deciles.[23] Again, liquidity risk appears to be an economically important determinant of expected stock returns.

The magnitude of the liquidity risk premium $\lambda_L$ depends on the arbitrary scaling of $L_t$ described earlier, but that scaling does not affect the $t$-statistic or the product $\beta_i^L \lambda_L$, the contribution of liquidity risk to asset $i$'s expected return. Table 6 also reports the GMM estimates of $(\beta_{10}^L - \beta_1^L)\lambda_L$, the difference between expected returns on the extreme decile portfolios implied by their liquidity betas. In the overall period, the annualized estimate of $(\beta_{10}^L - \beta_1^L)\lambda_L$ is 9.63 percent with three traded factors and 7.56 percent with four (the corresponding values for equally weighted portfolios are 11.06 percent and 8.56 percent). These values are close to the 10–1 spread alphas in table 4 of 9.23 percent and 7.48 percent (the corresponding values in table 5 for equally weighted portfolios are 10.49 percent and 7.66 percent). Thus, even when the liquidity premium is estimated using all 10 portfolios, the contribution of liquidity risk to the 10–1 expected-return difference remains virtually unchanged. The contributions of the traded factors to the expected return of the 10–1 spread are much smaller, all below 2 percent per year in absolute value for the overall period.

### B.    Sorting by Historical Liquidity Betas

As discussed earlier, a stock's historical liquidity beta is the most important predictor of its future liquidity beta (table 2). If liquidity betas are sufficiently stable over time, sorting on the historical liquidity betas alone could produce dispersion in the postranking betas. This subsection shows that this is indeed the case, although the dispersion in the betas is not as large as when liquidity betas are predicted using additional variables. Although our study focuses primarily on the results produced by sorts on betas predicted with the larger set of variables, we present here some results based on historical beta sorts in order to show that the results do not hinge on the inclusion of the additional variables.

At the end of each year between 1967 and 1998, we identify stocks with at least five years of monthly returns continuing through the current year end. For each stock, we estimate its historical liquidity beta by running the regression in (9) using the most recent five years of monthly

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[23] In no case does the asymptotic $\chi^2$ test reject the restriction in (15) at standard significance levels.

data. We impose a five-year minimum here in estimating the historical beta, as compared to the minimum of three years required to compute historical betas in the previous analysis. With no other information about liquidity beta brought to bear, it seems reasonable to require a somewhat more precise historical estimate. The series of innovations ($L_t$'s) is again recomputed at the end of each year. Stocks are then sorted by these historical betas into 10 value-weighted portfolios. Analogous to our sort on the predicted betas, we obtain a January 1968 through December 1999 series of monthly returns on each decile portfolio by linking across years the postranking returns during the next 12 months. On average, there are 217 stocks in each decile portfolio, and no portfolio ever contains fewer than 108 stocks.

Table 7 reports, in the same format as table 3, the postranking liquidity betas as well as the average market capitalization, liquidity, and Fama-French and momentum betas of the decile portfolios. Note that, although the pattern in the postranking liquidity betas is not monotonic, sorting on historical betas achieves some success in spreading the postranking betas. The liquidity beta of the 10–1 spread is positive at 5.99 ($t = 1.88$), not as large as the corresponding value of 8.23 ($t = 2.37$) obtained by sorting on the predicted betas. The SMB beta of the 10–1 spread is significantly negative, as in table 3, but the low-beta portfolio no longer has the lowest market capitalization. Rather, smaller firms now occupy both extremes of the historical beta sort. The latter result is consistent with smaller (and more volatile) stocks producing noisier historical liquidity betas. Also, average liquidity is now lower at both extremes, contrary to the pattern in table 3. Finally, the tilt toward growth stocks and past winners observed in table 3 disappears when sorting on historical liquidity betas.

Table 8 reports the value-weighted decile portfolios' postranking alphas. The dispersion in the alphas is now smaller than in the previous results, which is consistent with the smaller dispersion in the postranking liquidity betas. Nevertheless, all three alphas of the 10–1 spread are still significantly positive in the overall period: the CAPM alpha is 4.66 percent per year ($t = 2.36$), the Fama-French alpha is 4.15 percent per year ($t = 2.08$), and the four-factor alpha is 4.87 percent per year ($t = 2.38$).[24] Moreover, the liquidity risk premium estimated from the universe of all 10 portfolios, obtained by the same GMM procedure used to produce the values in table 6, is positive and significant at the 10 percent level. With three traded factors the estimated premium is 0.80 with a $t$-statistic of 1.77, and with four traded factors it is 1.04 with a $t$-

[24] When the decile portfolios are equally weighted, the postranking betas are less disperse than when the portfolios are value-weighted, and the alphas lose significance but are still positive. This is consistent with greater estimation error in historical liquidity betas for smaller stocks, which are typically more volatile.

TABLE 7
PROPERTIES OF PORTFOLIOS SORTED ON HISTORICAL LIQUIDITY BETAS

| | | | | | DECILE PORTFOLIO | | | | | | |
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 10–1 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | A. Liquidity Betas | | | | | | |
| Jan. 1968–Dec. 1999 | −6.02 | −.65 | −.62 | −.54 | 1.12 | −1.58 | 1.37 | 2.00 | 3.04 | −.04 | 5.99 |
| | (−2.57) | (−.37) | (−.48) | (−.41) | (.96) | (−1.24) | (1.00) | (1.49) | (1.99) | (−.02) | (1.88) |
| Jan. 1968–Dec. 1983 | −7.59 | −1.17 | 3.87 | −1.54 | −.48 | 1.65 | −1.18 | .02 | 1.26 | .41 | 7.99 |
| | (−1.84) | (−.44) | (1.86) | (−.68) | (−.25) | (.71) | (−.55) | (.01) | (.54) | (.14) | (1.60) |
| Jan. 1984–Dec. 1999 | −4.17 | −1.49 | −4.10 | −.30 | 2.55 | −2.75 | 2.80 | 3.79 | 4.38 | 1.18 | 5.35 |
| | (−1.52) | (−.63) | (−2.46) | (−.18) | (1.72) | (−2.00) | (1.56) | (2.08) | (2.07) | (.39) | (1.26) |
| | | | | B. Additional Properties, January 1968–December 1999 | | | | | | | |
| Market cap | 7.11 | 7.69 | 10.44 | 17.65 | 16.76 | 22.18 | 16.26 | 11.64 | 9.89 | 6.97 | |
| Liquidity | −.52 | −.19 | −.06 | −.04 | −.02 | −.05 | −.05 | −.05 | −.05 | −.12 | |
| MKT beta | 1.12 | 1.09 | 1.02 | .96 | .98 | .99 | 1.02 | 1.01 | 1.02 | 1.09 | −.03 |
| | (37.25) | (48.37) | (61.23) | (56.63) | (65.92) | (59.99) | (58.01) | (58.52) | (51.53) | (40.84) | (−.74) |
| SMB beta | .37 | −.00 | −.13 | −.16 | −.09 | −.15 | −.11 | −.00 | .04 | .16 | −.20 |
| | (8.02) | (−.02) | (−5.11) | (−6.03) | (−4.21) | (−6.10) | (−4.19) | (−.02) | (1.20) | (4.06) | (−3.25) |
| HML beta | −.20 | −.05 | .02 | −.02 | .10 | .12 | .07 | .09 | −.01 | −.15 | .05 |
| | (−4.04) | (−1.31) | (.87) | (−.80) | (4.22) | (4.40) | (2.60) | (3.27) | (−.38) | (−3.39) | (.76) |
| MOM beta | .04 | −.00 | .02 | .01 | −.02 | −.00 | −.01 | .01 | −.02 | −.01 | −.05 |
| | (1.64) | (−.18) | (1.25) | (1.13) | (−1.91) | (−.17) | (−.76) | (.65) | (−1.11) | (−.46) | (−1.51) |

NOTE.—At each year end between 1967 and 1998, eligible stocks are sorted into 10 portfolios according to historical liquidity betas. The betas are estimated as the slope coefficients on the aggregate liquidity innovation in regressions of excess stock returns on that innovation and the three Fama-French factors. The regressions are estimated using the most recent five years of data, and eligible stocks are defined as ordinary common shares traded on the NYSE, AMEX, or NASDAQ with five years of monthly returns continuing through the current year end and with stock prices between $5 and $1,000. The portfolio returns for the 12 postranking months are linked across years to form one series of postranking returns for each decile. Panel A reports the decile portfolios' postranking liquidity betas, estimated by regressing value-weighted portfolio excess returns on the liquidity innovation and the Fama-French factors. Panel B reports the time-series averages of each decile's market capitalization and liquidity, obtained as value-weighted averages of the corresponding measures across the stocks within each decile. Market capitalization is reported in billions of dollars. A stock's liquidity in any given month is the slope coefficient $\gamma_{i,t}$ from eq. (1), multiplied by 100. Also reported are postranking betas with respect to the Fama-French and momentum factors, estimated by regressing value-weighted portfolio excess returns on the four factors. The $t$-statistics are in parentheses.

TABLE 8
Alphas of Value-Weighted Portfolios Sorted on Historical Liquidity Betas

| | Decile Portfolio | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 10–1 |
| A. January 1968–December 1999 | | | | | | | | | | | |
| CAPM alpha | −2.06 | −.36 | .63 | .49 | .07 | .49 | 1.42 | 1.36 | −.02 | 2.60 | 4.66 |
| | (−1.30) | (−.34) | (.76) | (.57) | (.10) | (.58) | (1.64) | (1.63) | (−.02) | (1.96) | (2.36) |
| Fama-French alpha | −.62 | −.09 | .46 | .57 | −.62 | −.28 | .90 | .84 | .03 | 3.53 | 4.15 |
| | (−.42) | (−.08) | (.57) | (.68) | (−.86) | (−.35) | (1.06) | (1.00) | (.03) | (2.71) | (2.08) |
| Four-factor alpha | −1.20 | −.04 | .22 | .34 | −.29 | −.25 | 1.05 | .71 | .29 | 3.67 | 4.87 |
| | (−.79) | (−.04) | (.26) | (.40) | (−.40) | (−.31) | (1.20) | (.82) | (.29) | (2.74) | (2.38) |
| B. January 1968–December 1983 | | | | | | | | | | | |
| CAPM alpha | −1.10 | 1.04 | .94 | .35 | −.28 | .46 | .09 | .83 | .33 | 2.51 | 3.62 |
| | (−.46) | (.70) | (.79) | (.27) | (−.26) | (.34) | (.08) | (.72) | (.25) | (1.51) | (1.32) |
| Fama-French alpha | −1.24 | 2.32 | 1.66 | 1.53 | −1.05 | −.49 | −.06 | −.07 | .17 | 1.61 | 2.85 |
| | (−.53) | (1.56) | (1.41) | (1.21) | (−.98) | (−.38) | (−.05) | (−.06) | (.13) | (1.01) | (1.01) |
| Four-factor alpha | −3.74 | 1.50 | .87 | .86 | −.20 | .21 | .59 | −.18 | .59 | 1.64 | 5.38 |
| | (−1.58) | (.96) | (.71) | (.66) | (−.18) | (.16) | (.47) | (−.15) | (.43) | (.98) | (1.86) |
| C. January 1984–December 1999 | | | | | | | | | | | |
| CAPM alpha | −2.79 | −1.63 | .21 | .40 | .37 | .23 | 3.12 | 1.70 | −.11 | 2.70 | 5.49 |
| | (−1.31) | (−1.04) | (.18) | (.36) | (.36) | (.23) | (2.51) | (1.40) | (−.08) | (1.28) | (1.90) |
| Fama-French alpha | .03 | −2.04 | −.60 | −.33 | −.40 | −.55 | 2.21 | 1.50 | −.11 | 4.41 | 4.38 |
| | (.02) | (−1.29) | (−.53) | (−.30) | (−.40) | (−.59) | (1.83) | (1.22) | (−.07) | (2.20) | (1.54) |
| Four-factor alpha | .57 | −1.50 | −.50 | −.28 | −.39 | −.87 | 2.06 | 1.35 | .02 | 4.55 | 3.98 |
| | (.30) | (−.94) | (−.44) | (−.25) | (−.38) | (−.93) | (1.68) | (1.08) | (.01) | (2.23) | (1.38) |

Note.—See the note to table 7. The table reports the decile portfolios' postranking alphas, in percentage per year. The alphas are estimated as intercepts from the regressions of excess portfolio postranking returns on excess market returns (CAPM alpha), on the Fama-French factor returns (Fama-French alpha), and on the Fama-French and momentum factor returns (four-factor alphas). The $t$-statistics are in parentheses.

statistic of 1.76. Note that the magnitude of the premium in either case is fairly close to its counterpart in table 6. In summary, the analysis based solely on historical betas supports the conclusion that stocks with greater sensitivity to innovations in aggregate liquidity offer higher expected returns.

### C. Sorting by Size

Total market capitalization, or "size," is a common criterion for sorting stocks in empirical investment studies, and size sorts often produce dispersion in a number of other characteristics. Table 9 reports various properties of decile portfolios formed by sorting on size at the end of each year, where the break points are based on all eligible NYSE, AMEX, and NASDAQ stocks. Not surprisingly, smaller stocks are less liquid, in that the average value of $\hat{\gamma}_{it}$ increases nearly monotonically across deciles. The liquidity betas of the two or three portfolios containing the smallest stocks are large and significantly positive, whereas the betas for the other deciles exhibit no discernible pattern and are not significantly different from zero. When the size sort is instead based on break points for NYSE stocks only, so that each decile contains the same number of NYSE stocks but more AMEX and NASDAQ stocks are assigned to the lower deciles, then the pattern in liquidity betas is fairly flat across all 10 deciles, except for the large liquidity beta for the first decile. In other words, in a sort on size, the very smallest firms tend to be those with high liquidity betas.

It seems plausible that small and illiquid stocks might be those whose values are most affected by drops in marketwide liquidity, particularly if those drops prompt some investors concerned with the overall liquidity of their portfolios to "flee" such stocks and move to assets with greater liquidity. At the same time, though, size and liquidity are not the sole determinants of liquidity betas. Recall from table 3 that, when we sort by predicted liquidity betas, the high-beta portfolios actually have somewhat higher liquidity and average market capitalizations than the low-beta portfolios. It is easy to see why stocks with high liquidity betas need not be illiquid. When market liquidity declines, many investors sell stocks and buy bonds (see table 1 for indirect evidence), and those investors might prefer to sell liquid stocks in order to save on transaction costs. As a result, the price reaction to aggregate liquidity changes could actually be stronger for stocks that are more liquid. Also, prices of liquid stocks could have greater sensitivity to aggregate liquidity shocks if such stocks are held in larger proportion by the more liquidity-conscious investors. In general, liquidity betas need not bear a simple relation to size and liquidity.

Table 9 also reports the size-sorted portfolios' alphas computed with

TABLE 9
Portfolios Sorted on Market Capitalization

| | Decile Portfolio | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 1–10 |
| | A. General Properties | | | | | | | | | | |
| Market cap | 13.00 | 23.85 | 38.13 | 57.34 | 84.98 | 129.90 | 206.61 | 373.89 | 837.34 | 17,068.24 | |
| Liquidity | −3.35 | −3.16 | −1.47 | −1.87 | −1.48 | −1.14 | −.92 | −.46 | −.19 | −.01 | |
| | B. Return-Based Measures for Value-Weighted Portfolios | | | | | | | | | | |
| Liquidity beta | 5.26 | 3.84 | 1.95 | −.42 | .34 | −1.13 | −.48 | −1.02 | −1.60 | .17 | 5.09 |
| | (2.57) | (2.46) | (1.52) | (−.43) | (.37) | (−1.25) | (−.54) | (−1.04) | (−1.66) | (.67) | (2.51) |
| Four-factor alpha | 3.01 | 1.09 | .57 | −.67 | −.75 | −.91 | −.33 | −1.05 | −.81 | .50 | 2.51 |
| | (2.34) | (1.12) | (.71) | (−1.07) | (−1.30) | (−1.64) | (−.61) | (−1.73) | (−1.34) | (3.14) | (1.96) |
| | C. Return-Based Measures for Equal-Weighted Portfolios | | | | | | | | | | |
| Liquidity beta | 4.73 | 4.22 | 2.61 | .43 | .82 | .06 | −.25 | −1.29 | −.63 | .59 | 4.14 |
| | (2.18) | (2.64) | (1.89) | (.40) | (.79) | (.07) | (−.28) | (−1.23) | (−.63) | (.80) | (1.75) |
| Four-factor alpha | 3.15 | .47 | −.45 | −1.48 | −1.42 | −1.83 | −.70 | −1.40 | −1.04 | −.68 | 3.83 |
| | (2.34) | (.48) | (−.55) | (−2.34) | (−2.27) | (−3.10) | (−1.24) | (−2.12) | (−1.65) | (−1.48) | (2.59) |

Note.—At each year end between 1962 and 1998, eligible stocks are sorted into 10 portfolios according to market capitalization. Eligible stocks are defined as ordinary common shares traded on the NYSE, AMEX, or NASDAQ with stock prices between $5 and $1,000. The break points for the sort are based on all eligible stocks, so that all decile portfolios contain approximately the same number of stocks in each month. The portfolio returns for the 12 postranking months are linked across years to form one series of postranking returns for each decile. Panel A reports the time-series averages of the deciles portfolios' market capitalization and liquidity, obtained as value-weighted averages of the corresponding measures across the stocks within each decile. Market capitalization is reported in millions of dollars. A stock's liquidity in any given month is the slope coefficient $\gamma_{i,t}$ from eq. (1), multiplied by 100. Panels B and C report the decile portfolios' postranking liquidity betas, estimated by regressing excess portfolio returns on the aggregate liquidity innovation and the Fama-French factors. Also reported are the portfolios' alphas, estimated as intercepts from the regressions of excess portfolio postranking returns on the Fama-French and momentum factor returns. The $t$-statistics are in parentheses. All statistics are calculated over the period January 1963 through December 1999.

respect to the four factors used previously (the excess market return and size, value, and momentum spreads). Note that, for both the value- and equal-weighted portfolios, the estimated alpha for the decile of smallest firms is over 3 percent annually, with a *t*-statistic of 2.3. This 3 percent positive abnormal return can be compared to the portion of expected return attributable to liquidity risk, computed as the product of the portfolio's liquidity beta and the estimate of the liquidity risk premium $\lambda_L$ reported earlier. If we take the premium estimated using the value-weighted beta-sorted portfolios, reported in panel A in table 6 (and the lower of the overall-period estimates), that product is 3.7 percent ( $= 4.73 \times 0.78$ ) for the equally weighted lowest-size decile and 4.1 percent ( $= 5.26 \times 0.78$ ) for the value-weighted version. In other words, the liquidity risk of the small-firm portfolio appears to be more than sufficient to explain its abnormal return with respect to the other four factors.

### D.   *Individual Stock Liquidity*

This paper investigates whether the cross section of returns is related to stocks' liquidity betas. A natural separate question is whether stocks whose liquidity is high according to our measure earn high average returns, in the spirit of Amihud and Mendelson (1986). This question cannot be conclusively answered here. While our estimated liquidity measure seems appealing at the aggregate level, it is too noisy to be useful at the individual stock level. In particular, when stocks are sorted into 10 portfolios on the basis of their $\hat{\gamma}_{it}$ coefficients averaged over the past one or three years, the pattern in the postranking portfolio $\hat{\gamma}$'s is rather flat across the deciles. The sorting procedure fails because of the large sampling error in the individual stock $\hat{\gamma}_{it}$'s. For this reason, we do not work with individual stock liquidity. It seems that our most successful sort on liquidity is the simple size sort described in the previous subsection.

  Our result that stocks with high liquidity betas tend to have high average returns does not appear to be explained by liquidity effects à la Amihud and Mendelson. Although the least liquid stocks in table 9 tend to have the highest liquidity betas, recall that when stocks are sorted on their predicted liquidity betas, as in table 3, stocks with the highest liquidity betas actually have somewhat higher average postranking liquidity measures than stocks with the smallest betas. The pricing results in tables 4 and 5 therefore seem distinct from any pure liquidity effects.

  Given the mounting evidence on commonality in liquidity, it seems natural to ask whether the sensitivity of stock liquidity to market liquidity is related to the cross section of returns. This question is different from the one addressed in this paper, since stocks whose liquidities are the

most sensitive to market liquidity are not necessarily those whose prices are the most sensitive to market liquidity; the issue is of independent interest nonetheless. One might conjecture, for example, that stocks whose liquidity dries up the most during marketwide liquidity crises need to compensate investors by higher average returns (e.g., Acharya and Pedersen 2002). Our lack of reliable time series of liquidity for individual stocks prevents us from investigating this hypothesis, since sorts on betas (or correlations) of individual liquidity with respect to aggregate liquidity are unable to achieve any significant postranking spread in those quantities. This intriguing topic presents an obvious direction for future research.

## IV.    An Investment Perspective

The evidence presented in the previous section reveals that liquidity risk is related to expected-return differences that are not explained by stocks' sensitivities to MKT, SMB, HML, and MOM. An equivalent characterization of this evidence is that no combination of these four factors (and riskless cash) is mean-variance efficient with respect to the universe of common stocks.[25] In particular, the large and significant alphas for the 10–1 spreads reported in tables 4 and 5 imply that adding such positions to an opportunity set consisting of the other four factors increases the maximum Sharpe ratio.

   In a linear pricing model in which expected returns are explained by betas with respect to nontraded factors, expected returns are also explained by betas with respect to portfolios whose returns are maximally correlated with those factors.[26] Constructing a maximum correlation portfolio for $L_t$ from the universe of common stocks is a challenging problem that lies beyond the scope of this study. It is the case that, if the ex post maximum correlation portfolio is constructed from the six-asset universe consisting of the first and last decile portfolios of the liquidity beta sort as well as the four factors MKT, SMB, HML, and MOM, then the weight on the high–liquidity beta portfolio is positive and the weight on the low–liquidity beta portfolio is negative (for both the value-weighted and equally weighted versions of those portfolios). In this sense, adding the 10–1 spread to an investment universe con-

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[25] The equivalence between multibeta asset pricing and mean-variance efficiency of some combination of benchmark portfolios is well known. For an early recognition of this point, see Merton (1973); for later discussions, see Jobson and Korkie (1982, 1985), Grinblatt and Titman (1987), and Huberman, Kandel, and Stambaugh (1987).

[26] Huberman et al. (1987) characterize the "mimicking" portfolios that can be used in place of nontraded factors when betas with respect to these factors explain expected returns.

TABLE 10
LIQUIDITY RISK SPREADS AND INVESTMENT OPPORTUNITIES: WEIGHTS IN THE EX POST
TANGENCY PORTFOLIO, JANUARY 1966–DECEMBER 1999

| MKT | SMB | HML | MOM | LIQ$^V$ | LIQ$^E$ | Sharpe Ratio |
|---|---|---|---|---|---|---|
| 100.00 | … | … | … | … | … | .12 |
| 35.08 | 5.83 | 59.10 | … | … | … | .22 |
| 20.05 | 16.07 | 43.03 | 20.85 | … | … | .33 |
| 22.34 | 18.77 | 36.41 | … | 22.49 | … | .31 |
| 17.32 | 22.33 | 29.10 | … | … | 31.25 | .40 |
| 17.70 | 20.62 | 34.23 | 11.86 | 15.59 | … | .37 |
| 15.88 | 22.51 | 29.56 | 6.47 | … | 25.58 | .42 |

NOTE.—Each row reports the ex post tangency portfolio weights (percent) as well as the ex post monthly Sharpe ratio of the tangency portfolio in the given asset universe. The assets available for investment are various subsets of six traded factors. This set comprises the Fama-French factors MKT, SMB, and HML; a momentum factor MOM; and two liquidity risk spreads, both of which go long decile 10, containing the stocks with the highest predicted liquidity betas, and short decile 1, containing the stocks with the lowest betas. Each leg of the spread is value-weighted in LIQ$^V$ and equally weighted in LIQ$^E$.

sisting of the original four factors is motivated by a model in which expected returns are related to liquidity risk.

Let LIQ$^V$ denote the payoff on the 10–1 spread constructed using value-weighted decile portfolios sorted on predicted liquidity betas, and let LIQ$^E$ denote the payoff on the equally weighted version. To provide an additional perspective on the importance of liquidity risk, we examine here the degree to which the mean-variance opportunity set is enhanced by adding LIQ$^V$ or LIQ$^E$ to MKT, SMB, HML, and MOM. Of course, a mean-variance-efficient portfolio is not necessarily the optimal choice of an investor in a world that gives rise to multibeta pricing, but we believe that a mean-variance setting is of interest to many investors nevertheless. Table 10 reports, for the overall 1966–99 period, the maximum ex post Sharpe ratio and the weights in the corresponding tangency portfolio for various subsets of the six factors. For ease of discussion, let $S^*$ denote the maximum Sharpe ratio for a given set of assets. The original four factors have an $S^*$ of 0.33 (on a monthly basis). When LIQ$^V$ is added, $S^*$ increases to 0.37, and LIQ$^V$ receives a greater weight in the ex post tangency portfolio than MOM (15.6 percent vs. 11.9 percent). When LIQ$^E$ is added to the original four, $S^*$ increases to 0.42, and the weight in MOM drops by more than two-thirds, from 20.9 percent to 6.5 percent. In contrast, the weight on LIQ$^E$ in that case is 25.6 percent, which is higher than the weights on all but HML (29.6 percent). Moreover, we see that when we add a fourth factor to the three Fama-French factors, which by themselves have an $S^*$ of 0.22, LIQ$^E$ is more valuable than MOM by the mean-variance comparison: LIQ$^E$ raises $S^*$ to 0.40 whereas MOM raises it to 0.33.

Since LIQ$^V$ and LIQ$^E$ figure prominently in the ex post tangency portfolio, at the expense of MOM especially, we are led to investigate a bit further the extent to which the momentum factor's importance is

TABLE 11
LIQUIDITY RISK SPREADS AND INVESTMENT OPPORTUNITIES: ALPHAS FROM THE
REGRESSION OF MOMENTUM ON PORTFOLIOS LISTED

| | January 1966–December 1999 | January 1966–December 1982 | January 1983–December 1999 |
|---|---|---|---|
| MKT, SMB, HML | 16.30 | 21.65 | 11.10 |
| | (4.85) | (4.53) | (2.29) |
| MKT, SMB, HML, $LIQ^V$ | 13.89 | 19.46 | 8.03 |
| | (4.09) | (4.04) | (1.63) |
| MKT, SMB, HML, $LIQ^E$ | 8.41 | 16.11 | −1.29 |
| | (2.55) | (3.35) | (−.28) |

NOTE.—The table reports the alphas (percent per year) of the momentum portfolio MOM with respect to the factors listed in each row. These factors include the Fama-French factors MKT, SMB, and HML and two liquidity risk spreads, both of which go long decile 10, containing the stocks with the highest predicted liquidity betas, and short decile 1, containing the stocks with the lowest betas. Each leg of the spread is value-weighted in $LIQ^V$ and equally weighted in $LIQ^E$. The $t$-statistics are in parentheses.

reduced by our liquidity-risk spreads. Table 11 reports the alpha for MOM when regressed on the three Fama-French factors plus either $LIQ^V$ or $LIQ^E$. In the overall period, momentum's annualized alpha with respect to just the three Fama-French factors is 16.3 percent with a $t$-statistic of 4.85, confirming a well-known result. Adding $LIQ^V$ reduces MOM's alpha somewhat, to 13.9 percent with a $t$-statistic of 4.09. The momentum factor MOM is a spread between equally weighted portfolios, and perhaps for that reason the effect on its alpha of adding $LIQ^E$ to the Fama-French factors is more dramatic. That equally weighted liquidity risk spread cuts momentum's full-period alpha nearly in half, to 8.4 percent with a $t$-statistic of 2.55. In the more recent 17-year subperiod from 1983 through 1999, MOM's estimated alpha in the presence of $LIQ^E$ is actually negative, at −1.29.

Although such evidence is tantalizing, it is difficult to conclude that liquidity risk provides a partial explanation for momentum. On one hand, MOM's alpha is substantially reduced by the addition of liquidity risk spreads, and MOM's loadings on those spreads are highly significant in the overall period as well as in both subperiods (in the full period, MOM's beta on $LIQ^V$ is 0.26 with $t = 3.41$, and MOM's beta on $LIQ^E$ is 0.75 with $t = 7.77$, in a multiple regression that includes the three Fama-French factors). On the other hand, the liquidity beta of MOM, estimated as the multiple-regression coefficient on the nontraded factor $L_t$, is positive but not statistically significant at conventional levels in the overall period (6.9 with a $t$-statistic of 1.3). Moreover, in the later subperiod, when $LIQ^E$ eliminates MOM's alpha, the estimated liquidity beta of MOM is negative (−1.65 with a $t$-statistic of −0.23). At the same time, though, we must remember that $L_t$ is at best an imperfect proxy for whatever correct measure of liquidity could be relevant for asset pricing. It remains possible that the 10–1 spread constructed by ranking on betas with respect to $L_t$ comes closer to the correct mimicking portfolio than

$L_t$ does to the correct liquidity measure. At this point, however, we can simply observe that momentum's importance in an investment context is affected significantly by the addition of spreads based on liquidity risk.

## V.    Conclusions

Marketwide liquidity appears to be a state variable that is important for pricing common stocks. We find that expected stock returns are related cross-sectionally to the sensitivities of stock returns to innovations in aggregate liquidity. Stocks that are more sensitive to aggregate liquidity have substantially higher expected returns, even after we account for exposures to the market return as well as size, value, and momentum factors.

Our liquidity measure captures a dimension of liquidity associated with the strength of volume-related return reversals. Over the last four decades, this measure of marketwide liquidity exhibits a number of sharp declines, many of which coincide with market downturns and apparent flights to quality. Our liquidity measure is also characterized by significant commonality across stocks, supporting the notion of aggregate liquidity as a priced state variable. Smaller stocks are less liquid, according to our measure, and the smallest stocks have high sensitivities to aggregate liquidity.

One direction for future research is to explore whether liquidity risk plays a role in various pricing anomalies in financial markets. This study takes a step on this path by showing that the momentum strategy of buying recent winning stocks and selling recent losing stocks becomes less attractive from an investment perspective when portfolio spreads based on liquidity risk are also available for investment. Future research could investigate whether expected returns are related to stocks' sensitivities to fluctuations in other aspects of aggregate liquidity. It would also be useful to explore whether some form of systematic liquidity risk is priced in other financial markets, such as fixed income markets or international equity markets.

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