Lecture One

$-\nabla u = E$ (E, electric field vector)

$J = \sigma E$ (σ, conductivity)

(J, current density)

$\nabla \cdot J = 0$ (Kirchhoff's law)

$\Rightarrow \nabla \cdot (\sigma \nabla u) = 0$

note that $\nabla \cdot A = \frac{\delta A_x}{\delta x} + \frac{\delta A_y}{\delta y} + \frac{\delta A_z}{\delta z}$

$=$

rate of change of A in x-direction

$+$ rate of change of A in y-direction

$+$ rate of change of A in z-direction

$$\Rightarrow \frac{\delta\left(\sigma \frac{\delta u}{\delta x_1}\right)}{\delta x_1} + \frac{\delta\left(\sigma \frac{\delta u}{\delta x_2}\right)}{\delta x_2} + \frac{\delta\left(\sigma \frac{\delta u}{\delta x_3}\right)}{\delta x_3} = 0$$

which is a 2nd order partial differential equation.

A second-order partial differential equation eg. $AU_{xx} + BU_{xy} + CU_{yy} + DU_x + EU_y + F = 0$ is called elliptic if the matrix is positive definite.

$$Z \equiv \begin{bmatrix} A & B \\ B & C \end{bmatrix}$$

So in a forward problem we know σ and try to solve u.

To get a unique solution of this, we need to apply some boundary conditions:

① Dirichlet boundary Condition : $u|_{\delta\Omega} = f$ (given)

(these are voltage measurements)