

Least square

$$Ax=b \quad x \in \mathbb{R}^n, b \in \mathbb{R}^m \quad A \in \mathbb{R}^{m \times n}$$

$m \geq n$ (more equations than variables)

$$m \begin{bmatrix} A \end{bmatrix}^n = \begin{bmatrix} b \end{bmatrix}^m \quad \text{Solve } x \quad x_{ls} = \underset{x}{\operatorname{argmin}} \|Ax - b\|^2$$

least square.

set derivative $\|Ax - b\|^2$ should be zero **solved! but...**

$$\text{Solve } \Rightarrow A^T A x_{ls} = A^T b$$

$$x_{ls} = (A^T A)^{-1} A^T b$$

$$\begin{bmatrix} A^T \end{bmatrix} \begin{bmatrix} A \end{bmatrix} = \begin{bmatrix} A^T A \end{bmatrix}$$

Linear algebra — useful tool to solve multiple variables with multiple equations

For EIT

~~$A^T A$ is horribly large since~~

Inverting $A^T A$ is problematic since the condition number of $A^T A$ is big (ill-posed) (many solutions to a same measurement)

That is where regularization can help: (filter out abnormal solutions)

$$x_{tik} = \underset{x}{\operatorname{argmin}} \|Ax - b\|^2 + \alpha^2 \|Lx\|^2$$

(Tikhonov regularization). penalize size large x

This is the same as assuming no single pixel of conductivity pixel will be significantly bigger than the neighboring pixels.

So L takes difference of neighbors.

$$\text{In the end } x_{tik} = (A^T A + \alpha^2 L^T L)^{-1} A^T b$$

makes it better conditioned. **4**