CSC240 Notes

Week 11

0.1 Languages

Definition. A *language* over Σ is a subset of Σ^*.

i.e. $L \subseteq \Sigma^*$ is a language of Σ.

Definition. (Kleene Star *)

$$L^* = \bigcup_{k \in \mathbb{N}} L^k$$

- $L^+ = \bigcup_{k \in \mathbb{Z}^+} L^k$
  - Notice that $L^* = L^+ \cup \{\lambda\}$, so $L^+ = L^*$ iff $\lambda \in L$.
- Let $L_0, L_1 \subseteq \Sigma^*$ be languages. Then $L_0 \cup L_1, L_0 \cap L_1$ are languages.
- The *complement* of a language $L \subseteq \Sigma^*$, $\bar{L} = \Sigma^* - L$, is also a language

0.2 Regular Expressions

Regular Expressions are a concise way to describe some language.

Definition. Given a finite alphabet Σ, R_Σ denotes the set of regular expressions over Σ. R_Σ is the smallest set of strings such that

- Base case: $\emptyset \in R_\Sigma, \lambda \in R_\Sigma$, for every $a \in \Sigma, a \in R_\Sigma$.
- Constructor cases: For every $r, r' \in R_\Sigma$,

$$(r + r') \in R_\Sigma, (r \cdot r') \in R_\Sigma, r^* \in R_\Sigma.$$

Given a regular expression