# Random Notes

Random Course

Random Person

1. 1
2. 2
3. 3
4. (a) a

   (b) b

   (c) We want to translate the following sentence:

   Eventually, all natural numbers are not prime gaps.

   First we want to quantify ”eventually, all natural numbers”. By definition of *eventually true*, we want to quantify $n_0 \in \mathbb{N}$ such that for any $n > n_0$, $n$ is not a prime gap. This is equivalent to

   $$\exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, n > n_0 \implies n \text{ is not a prime gap.} \tag{1}$$

   Now let us move onto translating “$n$ is not a prime gap.” We do this by negating $n$ is a prime gap.

   By definition of **prime gap**, we say $a$ is a prime gap if

   - there exists prime $p$: $\exists p \in \mathbb{N}, Prime(p)$
   - with such $p$, $p + a$ is also prime: $Prime(p + a)$
   - for all number $d$ between $p$ and $p + a$ exclusively, $d$ is not prime. This is equivalent to saying “If $d$ is between $p$ and $p + a$, then $d$ is not prime.” Hence we can translate this into

   $$\forall d \in \mathbb{N}, p < d < p + a \implies \neg Prime(d)$$

   Now we combine these and get

   $$\exists p \in \mathbb{N}, Prime(p) \wedge Prime(p + a) \wedge (\forall d \in \mathbb{N}, p < d < p + a \implies \neg Prime(d)) \tag{2}$$

   Back to predicate (1). We want to express that $n$ is not a prime gap, hence we negate predicate (2) and get

   $$\forall p \in \mathbb{N}, \neg Prime(p) \vee \neg Prime(p + n) \vee (\exists d \in \mathbb{N}, p < d < p + a \wedge Prime(d)). \tag{3}$$

   Putting (3) to (1) yields

   $$\exists n_0 \in \mathbb{N}, \forall n \in \mathbb{N}, n > n_0 \implies \Big(\forall p \in \mathbb{N}, \neg Prime(p) \vee \neg Prime(p + n) \vee \big(\exists d \in \mathbb{N}, p < d < p + a \wedge Prime(d)\big)\Big)$$