

Research Experience 1

# Topography in 2-D turbulence

# Abstract

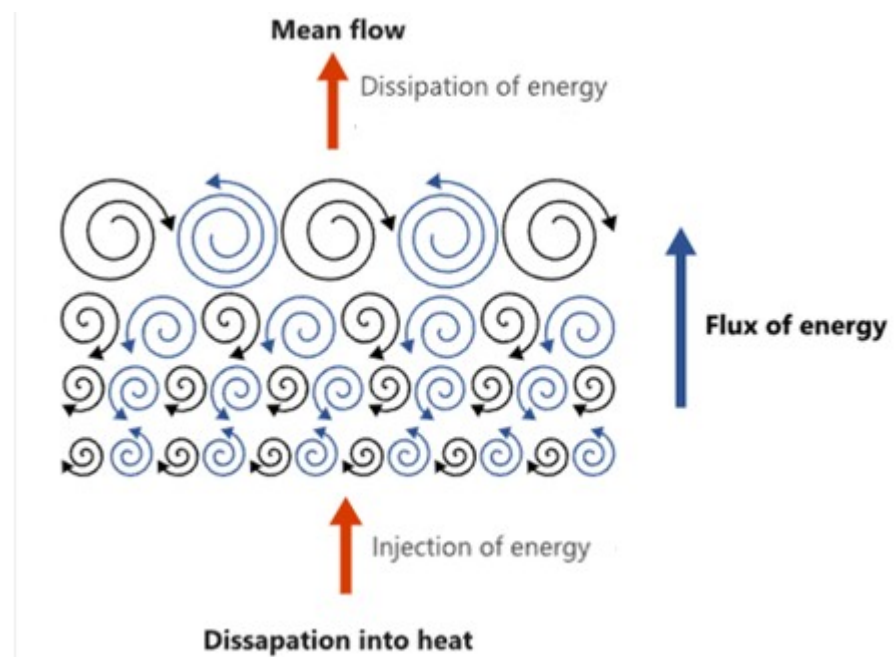
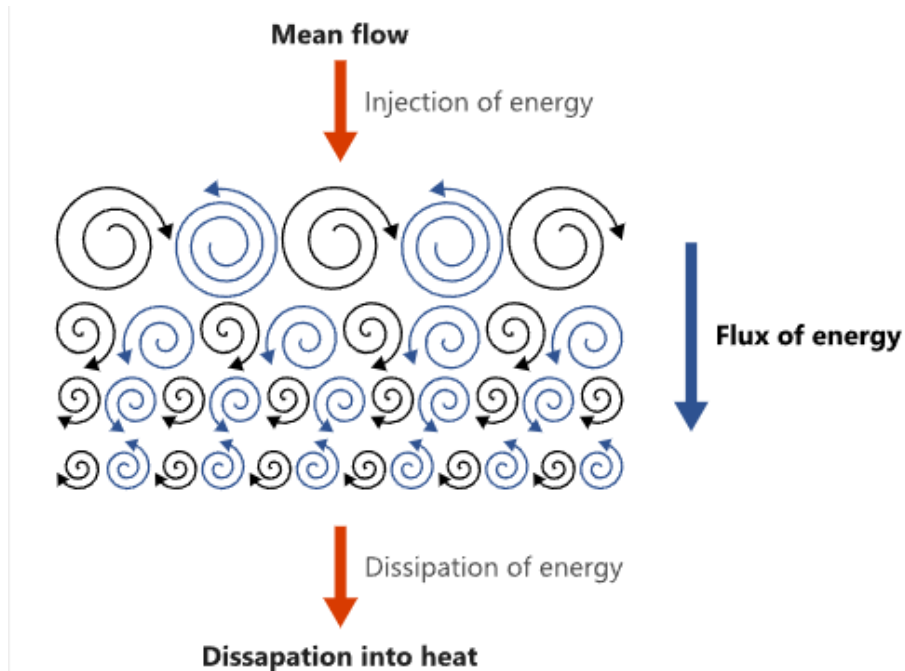
Seafloor topography is essential for oceanic fluid dynamics from many perspectives, and it is believed to enhance energy dissipation in oceanic flows. This study numerically examines the impact of small-scale topography on the dynamics of quasi-geostrophic barotropic flows. We found that the direction of energy transfer varies depending on the range of topographic amplitude.

# 2-D turbulence

Unlike three-dimensional turbulence, two-dimensional turbulence lacks vortex stretching, resulting in many unusual behaviors.

3-D: Kolmogorov's  $4/5$ th inertial range law  
energy cascade

2-D: inverse energy cascade



# Governing equations

Including the forcing and dissipation effects, the dimensionless quasi-geostrophic equation becomes:

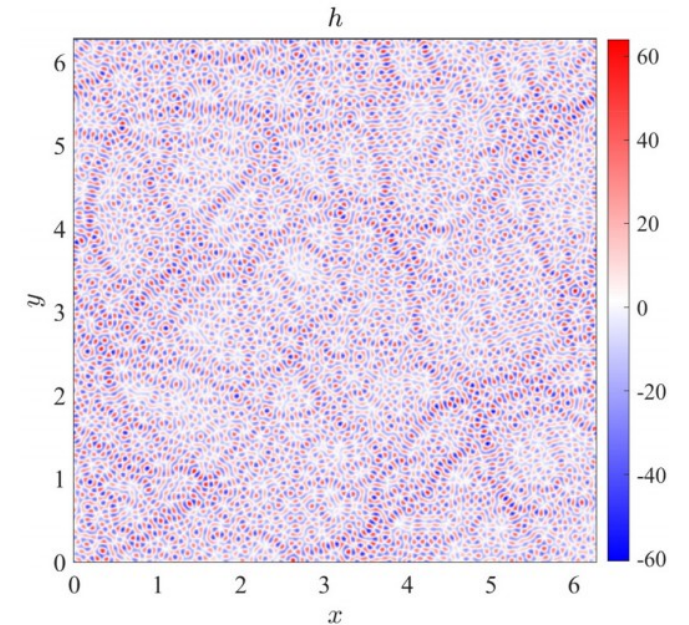
$$\frac{\partial Q}{\partial t} + J(\psi, Q) + J(\psi, h) = -\alpha Q + \nu \nabla^6 Q + M_F k_f^{1/2} F$$

where  $Q$  is the vorticity with  $\psi$  being the stream function, and topography  $h$  is the local variation of layer thickness normalized by the Rossby number.

# Topography

Topography influences the ocean circulation on various temporal and spatial scales. It affects the stability of large-scale flows and changes their associated mixing characteristics.

Topography at different scales has varying effects on the system.

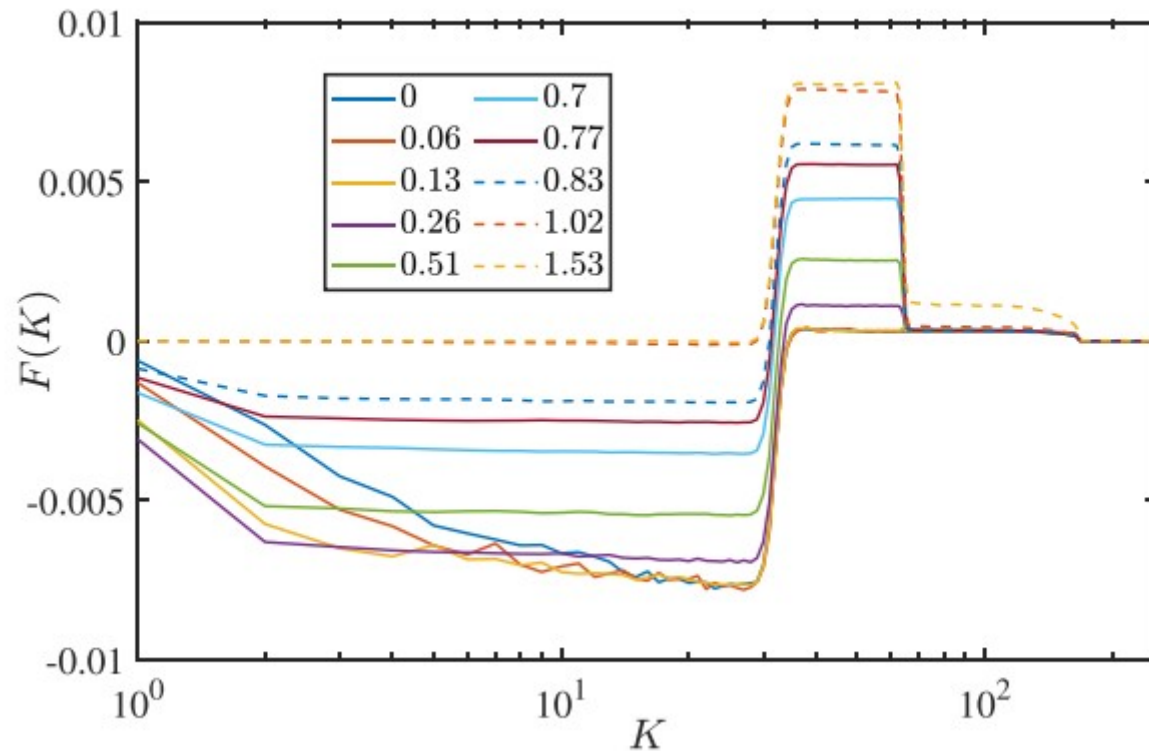


An illustration of small-scale topography,

# Numerical simulation

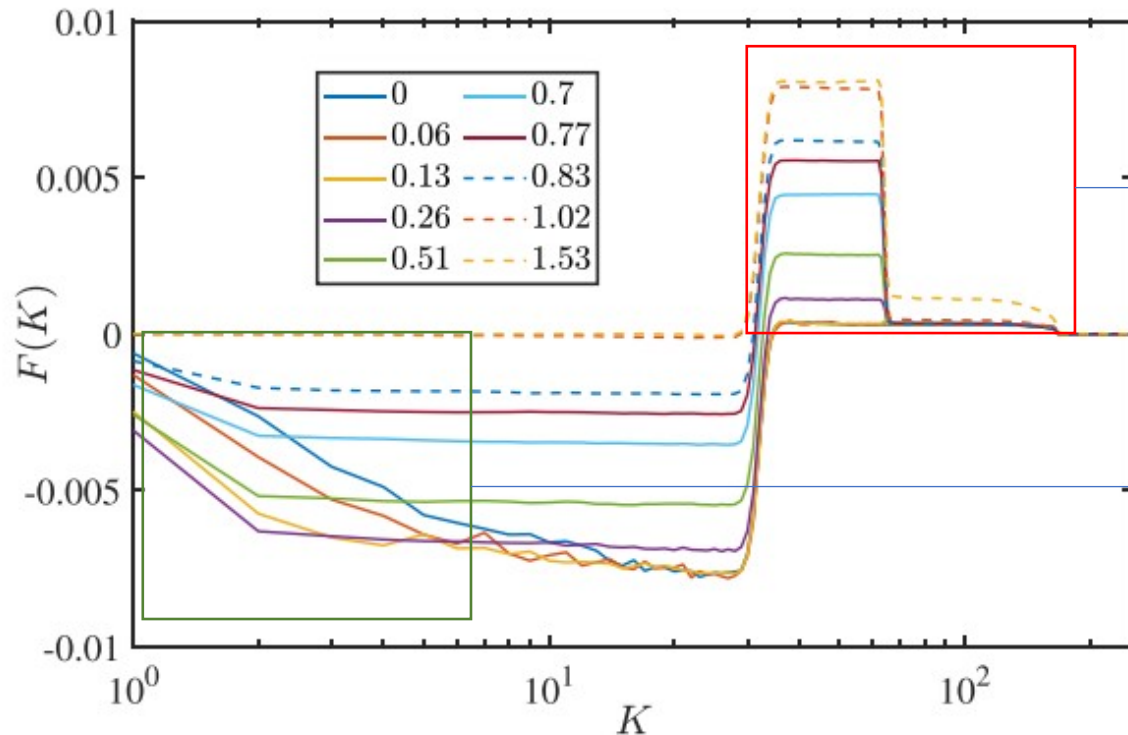
We applied a Fourier transform to the governing equations for numerical simulation and calculated the residual

$$F(K) = - \sum_{|k|=0}^K \frac{1}{2} \widehat{\psi}^* J(\widehat{\psi}, q) + c.c.$$



Energy fluxes with , and varying

# Numerical simulation



Topography enhances energy transfer to smaller scales.

When  $h$  exceeds a certain threshold, on scales much larger than the topography, the topography can facilitate energy transfer to even larger scales before dissipation.

When the scale of the topography is relatively small, it can, in certain cases, unexpectedly influence larger scales.

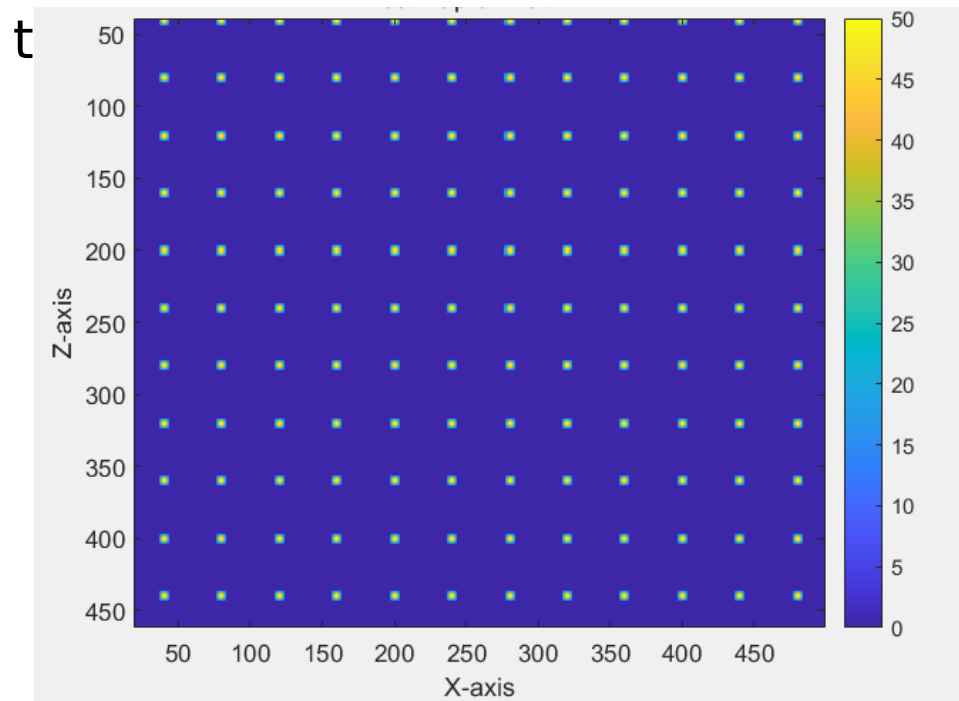
# Summary and discussion

In summary, we discover a novel phenomenon where small-scale topography enhances energy condensation in quasi-geostrophic turbulence. Current ocean parameterizations are inadequate for capturing the topography-enhanced energy flux presented in this paper. Therefore, our findings bring about challenges in ocean modeling.



# Other work

The previous discussion explored the impact of small-scale topography on two-dimensional turbulence. Other work includes examining the effects of large-scale topography with embedded small-scale structures. When  $\epsilon$  is large, it is expected that large-scale structures containing similar small-scale features to the



In this case, the topography height is  $H$ , the spacing between topographic features is  $L$  (representing large-scale information), and the topography width is  $l$  (representing small-scale information). Previous experience suggests that the resulting vorticity structure is related to the ratio of  $H/l$  and  $L$ .

# Personal gains

- (1) I have mastered the basic theory of turbulence and gained a deeper understanding of classical theories such as those by Kolmogorov.
- (2) I have reviewed the methods of using MATLAB to solve computational fluid dynamics problems.
- (3) have improved my skills in literature research, as well as in communication and discussions with my professors.

# Reference

- [1] Erik Lindborg, Can the atmospheric kinetic energy spectrum be explained by two-dimensional turbulence, J. Fluid Mech. (1999), vol. 388, pp. 259–288.
- [2] Jin-Han Xie and Oliver Bühler, Exact third-order structure functions for two-dimensional turbulence, J. Fluid Mech. (2018), vol. 851, pp. 672–686.
- [3] Lin-Fan Zhang and Jin-Han Xie, Spectral condensation and bidirectional energy transfer in quasi-geostrophic turbulence above small-scale topography, Phys. Fluids 36, 086601 (2024)
- [4] Lennard Miller, Bruno Deremble, and Antoine Venaille, Gyre Turbulence: Anomalous Dissipation in a Two-Dimensional Ocean Model