DSA5203 Assignment1 Technical Report

ZHANG Youyang, 10 Mar 2023

Introduction

The goal of this assignment is to apply 2D Haar-based DWT and its inverse DWT for 2D grayscale image in Python without using PyWavelet packages. The result of DWT is presented by a visualization of decomposition in a certain level, and the result of inverse DWT is presented by a reconstructed image.

$$H = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}; G_1 = \frac{1}{2}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}; G_2 = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}; G_3 = \frac{1}{2}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Figure 1 The low-pass filter and 3 high-pass filters of a Haar Wavelet Transform.

Methodology

The image is decomposed into four sub-bands which are LL, LH, HL, and HH. This is called the first level of wavelet decomposition. The low frequency sub-band could be continuously decomposed into four sub-bands, which is called the second level of decomposition. The inverse of DWT is applied by reconstructing low frequency sub-band from its decomposed four sub-bands using reversed filters.

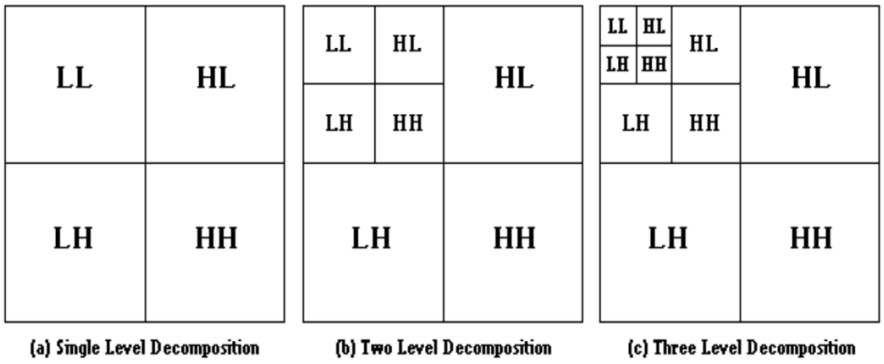

Figure 2 Level of decomposition.

Two important functions are expected in this assignment which are `haar2d` for DWT and `ihaar2d` for inverse DWT. Other functions developed are for visualization purposes and coding readability.

Function Name	Functionality	Inputs	Output
`normalize_image_array`	Normalizes the input array to a 0-255 range for plotting.	1. An array. 2. An integer representing the half size of the input array.	A normalized array that has the same size as the input array.
`down_sampling`	Performs one-level decomposition of the input array based on four filters of Haar Wavelet Transform.	1. An initialized array for storing result. 2. An array for decomposition. 3. Four arrays representing four filters.	A combined array that stores results from four filters and has the same size as the input array.
`up_sampling`	Performs one-level reconstruction of the input array based on four reversed filters.	1. An initialized array for storing temporarily calculated result. 2. An array for reconstruction. 3. Four arrays representing four filters.	A reconstructed array that has double size as the input array.

Function Name	Functionality	Inputs	Output
`haar2d`	Contains a for loop that decomposes the image array based on expected levels by calling `down_sampling` function. Each time after decomposition, it will call `normalize_image_array` function to normalize the array.	1. An image array. 2. An integer representing the level of decomposition.	An array representing the Haar wavelet coefficients.
`ihaar2d`	Contains a for loop that reconstruct the coefficient array back the original image array based on expected levels. Each time it will call `up_sampling` function to perform the reconstruction.	1. A coefficient array. 2. An integer representing the level of decomposition.	A reconstructed image array.

Results

All images used to run this program should be in size of $2^N * 2^N$, N is an integer. Three examples shown here are in size of 256*256, 512*512 and 1024*1024.

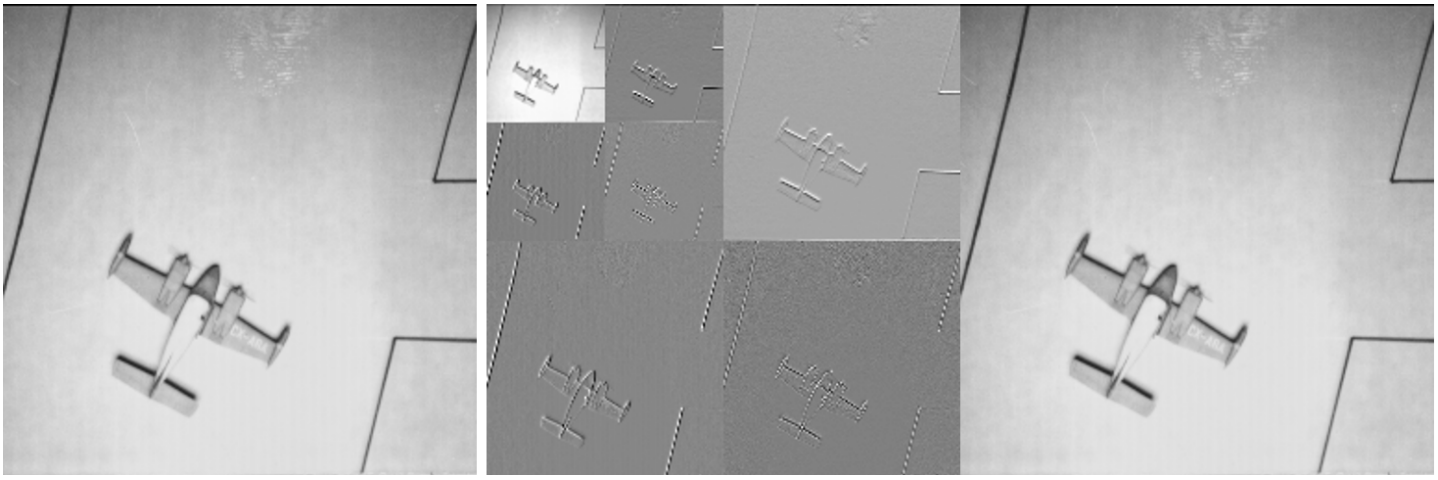

*Figure 3 Original, decomposed (2 levels), and reconstructed image of size 256*256.*

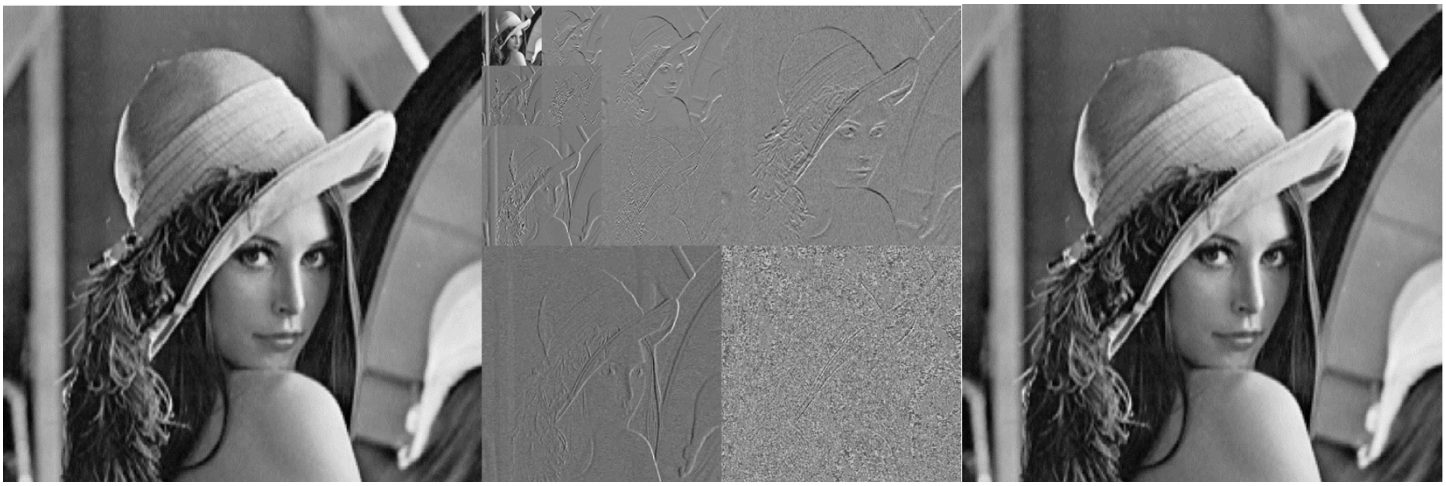

*Figure 4 Original, decomposed (3 levels), and reconstructed image of size 512*512.*

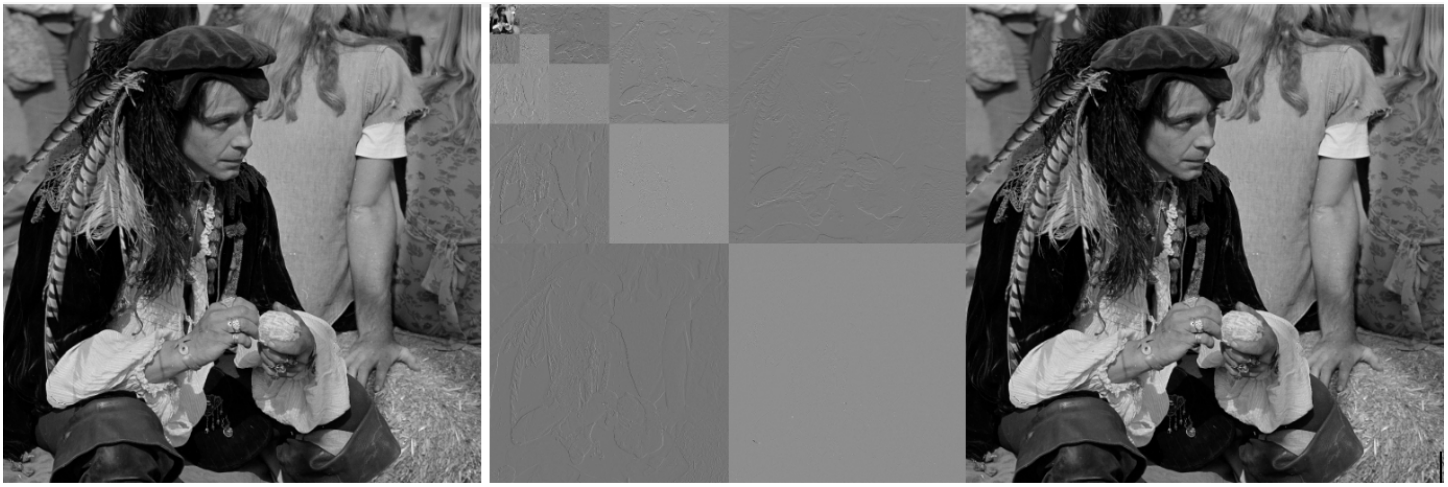

*Figure 5 Original, decomposed (4 levels), and reconstructed image of size 1024*1024.*

Bibliography

Sowmyashree, M., Saritha, I., & Naveen, I. (2015, December). Implementation of DWT using Haar Transform with Felics Algorithm. *IARJSET, 2*(12), 40-44.