# Linear regression

# Supervised learning example



$a(x)$

13

# Supervised learning
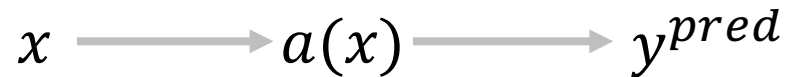
$x_i$ — example

$y_i$ — target value

$x_i = (x_{i1}, \ldots, x_{id})$ — features

$X = \big((x_1, y_1), (x_2, y_2), \ldots, (x_\ell, y_\ell)\big)$ — training set

$a(x)$ — model, hypothesis

$$x \longrightarrow a(x) \longrightarrow y^{pred}$$

# Regression and classification

$y_i \in \mathbb{R}$ — regression task

- Salary prediction

- Movie rating prediction
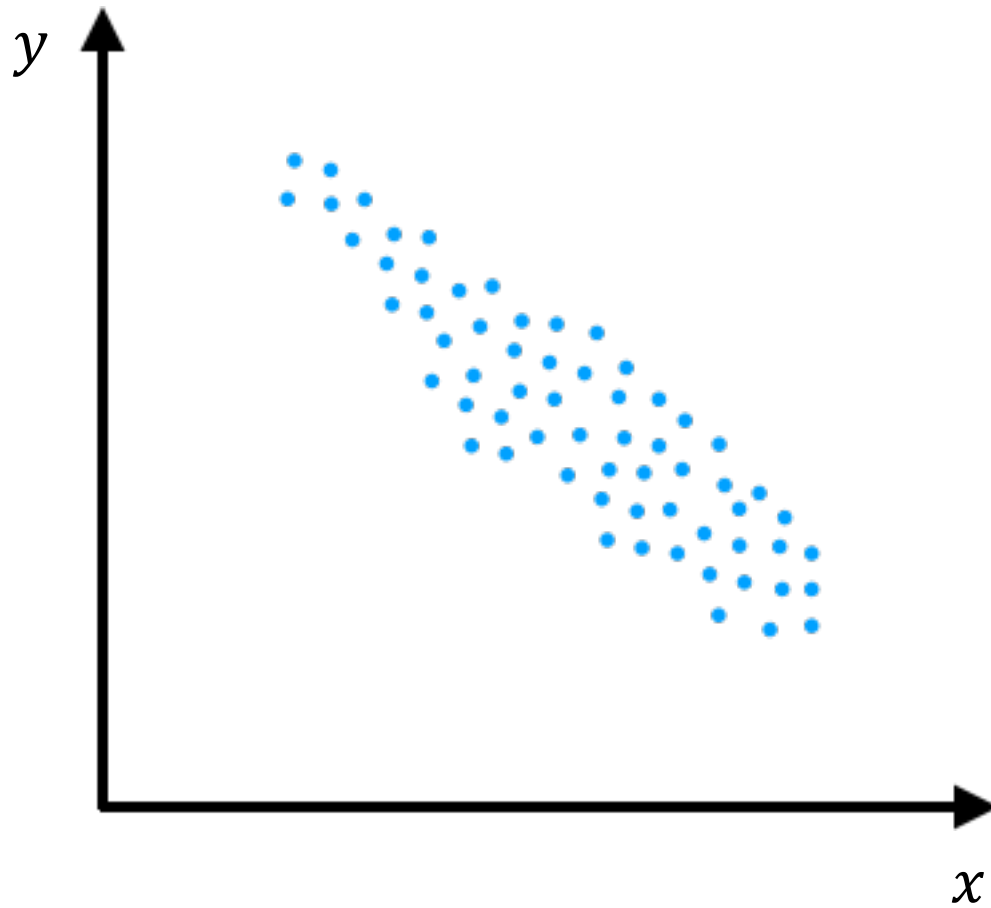
# Regression and classification

$y_i \in \mathbb{R}$ — regression task
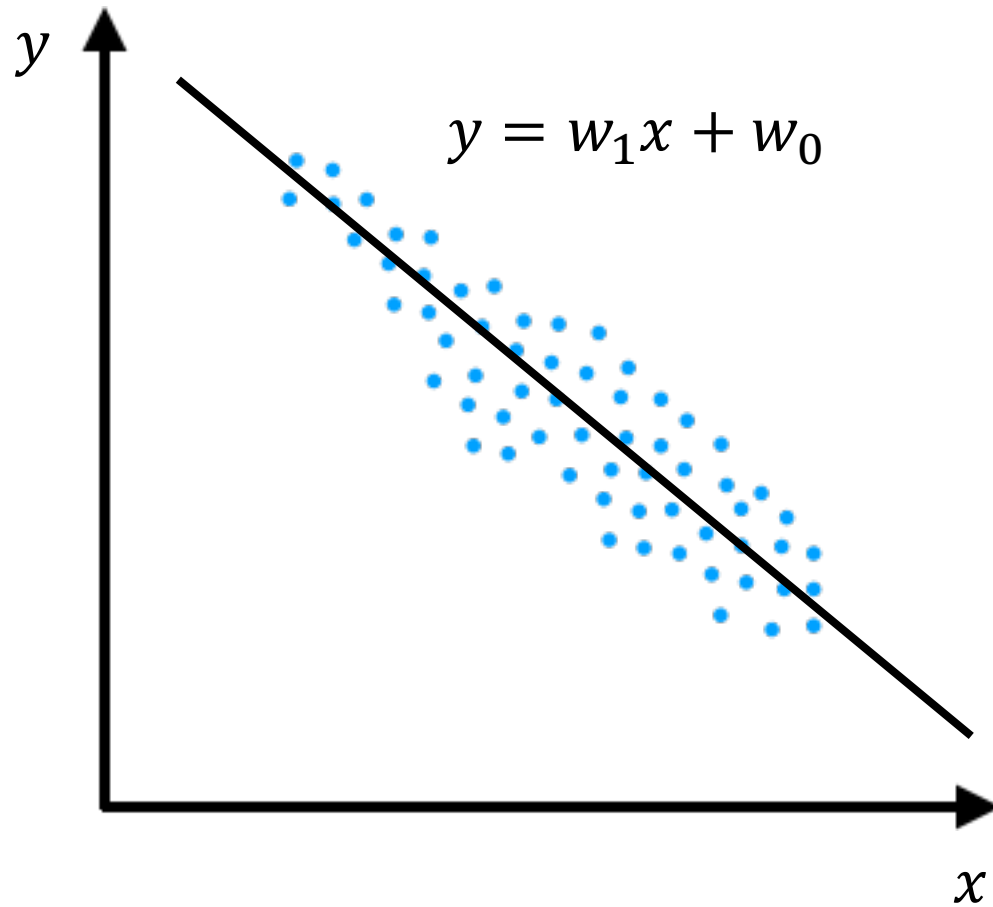
- Salary prediction

- Movie rating prediction

$y_i$ belongs to a finite set — classification task

- Object recognition

- Topic classification

# Linear model for regression example

# Linear model for regression example



$$y = w_1 x + w_0$$

# Linear model for regression

$$a(x) = b + w_1 x_1 + w_2 x_2 + \cdots + w_d x_d$$

- $w_1, \ldots, w_d$ — coefficients (weights)

- $b$ — bias

- $d + 1$ parameters

- To make it simple: there's always a constant feature

# Linear model for regression

Vector notation:

$$a(x) = w^T x$$

For a sample $X$:

$$a(X) = Xw$$

$$X = \begin{pmatrix} x_{11} & \cdots & x_{1d} \\ \vdots & \ddots & \vdots \\ x_{\ell 1} & \cdots & x_{\ell d} \end{pmatrix}$$

# Loss function

How to measure model quality?

Mean squared error:

$$L(w) = \frac{1}{\ell} \sum_{i=1}^{\ell} (w^T x_i - y_i)^2$$

$$= \frac{1}{\ell} \|Xw - y\|^2$$

# Training a model

Fitting a model to training data:

$$L(w) = \frac{1}{\ell} \|Xw - y\|^2 \to \min_w$$

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Exact solution:

$$w = (X^T X)^{-1} X^T y$$

But <u>inverting a matrix is hard</u> for high-dimensional data!

# Summary

- Linear models are very simple

- MSE can be used as a loss function

- There is an analytical solution, but we need more generic and scalable learning method