DSP 2009 Spring Final Solution

1.

(a) Circular Shift Properties

$$x[(n-n_0)_N] \xleftrightarrow{DFT} e^{-j\frac{2\pi k}{N}n_0} \cdot X[k]$$

$$\because h_1[n] = h_2[(n-4)_8]$$

$$\leftrightarrow H_1[k] = e^{-j\frac{2\pi k}{8}\cdot 4} \cdot H_2[k]$$

$$\therefore |H_1[k]| = |H_2[k]|$$

(b) $$H_3[k] = W_8^{3k} \cdot H_1[k] = e^{j\frac{2\pi}{8}3k} \cdot H_1[k]$$

$$\therefore h_3[n] = h_1[(n-3)_8] = [f, g, h, a, b, c, d, e]$$

(c) $$H_4[k] = [A, E]$$

$$h_4[n] = \frac{1}{2}[A e^{j\frac{2\pi n}{2}0} + E e^{j\frac{2\pi n}{2}1}]$$

$$= \frac{1}{2}[A + E\cdot(-1)^n]$$

$$= \begin{cases} \frac{1}{2}[A+E] & \text{for } n=0 \\ \frac{1}{2}[A-E] & \text{for } n=1 \end{cases}$$

and $$A = \sum_{n=0}^{7} h_1[n] e^{-j\frac{2\pi}{8}n\cdot 0} = a+b+c+d+e+f+g+h$$

$$E = \sum_{n=0}^{7} h_1[n] e^{-j\frac{2\pi}{8}n\cdot 4} = a-b+c-d+e-f+g-h$$

$$= \begin{cases} [a+c+e+g] & \text{for } n=0 \\ [b+d+f+h] & \text{for } n=1 \end{cases}$$

另：圖解　8-DFT → 2-IDFT may aliasing

```
... a b c d e f g h ...
      ... a b c d e f g h ...
            ... a b c d e f g h ...
                  ... a b c d e f g f ...
```

$h_4[0]$ $h_4[1]$

(d)

$$H_5[k] = \sum_{n=0}^{7} h_1[n]\, z^{-n} \Big|_{z = e^{-j\frac{2\pi}{16}k}}$$

$$= a\cdot e^{-j\frac{2\pi}{16}k\cdot 0} + b\cdot e^{-j\frac{2\pi}{16}k\cdot 1} + c\cdot e^{-j\frac{2\pi}{16}k\cdot 2} + d\cdot e^{-j\frac{2\pi}{16}k\cdot 3}$$

$$+ e\cdot e^{-j\frac{2\pi}{16}k\cdot 4} + f\cdot e^{-j\frac{2\pi}{16}k\cdot 5} + g\cdot e^{-j\frac{2\pi}{16}k\cdot 6} + h\cdot e^{-j\frac{2\pi}{16}k\cdot 7}$$

$$= \sum_{n=0}^{15} h_5[n]\, e^{-j\frac{2\pi}{16}nk}$$

$$\therefore h_5[n] = [a, b, c, d, e, f, g, h, 0, 0, 0, 0, 0, 0, 0, 0]$$

(e)

$$H_6[k] = \begin{cases} 2H_1[\frac{k}{2}] & , k \text{ is even} \\ 0 & , k \text{ is odd} \end{cases}$$

$$h_6[n] = \frac{1}{16}\sum_{k=0}^{15} H_6[k]\, e^{j\frac{2\pi}{16}kn}$$

$$= \frac{1}{\cancel{16}}\sum_{k=0,2,4\cdots}^{15} \cancel{2}H_1[\tfrac{k}{2}]\, e^{j\frac{2\pi}{16}kn}$$

$\ell = \frac{k}{2}$, $k = 0, 2, 4 \cdots$
$\ell = 0, 1, 2 \cdots$

$$= \frac{1}{8}\sum_{\ell=0,1,}^{15} H_1[\ell]\cdot e^{j\frac{2\pi}{\cancel{16}}\cancel{2}\ell n}$$

$$= \frac{1}{8}\sum_{\ell=0}^{7} H_1[\ell]\, e^{j\frac{2\pi}{8}\ell n}$$

$= h_1[n]$ 16-point sequence

$$= [a, b, c, d, e, f, g, h, a, b, c, d, e, f, g, h]$$

2. (10%)

(a)

$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{kn}, \quad k = 0, 1, 2, \cdots, N-1$$

$$= \sum_{n \bmod 3 = 0} x[n]\, W_N^{kn} + \sum_{n \bmod 3 = 1} x[n]\, W_N^{kn} + \sum_{n \bmod 3 = 2} x[n]\, W_N^{kn}$$

$$= \sum_{r=0}^{\frac{N}{3}-1} x[3r]\, W_N^{k\cdot 3r} + \sum_{r=0}^{\frac{N}{3}-1} x[3r+1]\, W_N^{k(3r+1)} + \sum_{r=0}^{\frac{N}{3}-1} x[3r+2]\, W_N^{k(3r+2)}$$

$$= \sum_{r=0}^{\frac{N}{3}-1} x[3r]\, W_{N/3}^{kr} + W_N^{k} \sum_{r=0}^{\frac{N}{3}-1} x[3r+1]\, W_{N/3}^{kr} + W_N^{2k} \sum_{r=0}^{\frac{N}{3}-1} x[3r+2]\, W_{N/3}^{kr}$$

$$= G[k] + W_N^{k} H[k] + W_N^{2k} I[k], \quad k = 0, 1, 2, \cdots, N-1,$$

where $G[k]$ is an $\frac{N}{3}$-point DFT of $x[n]$, $n = 0, 3, 6, \cdots$;

$H[k]$ is an $\frac{N}{3}$-point DFT of $x[n]$, $n = 1, 4, 7, \cdots$;

$I[k]$ is an $\frac{N}{3}$-point DFT of $x[n]$, $n = 2, 5, 8, \cdots$.

(5%)

(b) $N = 3$, $\quad X[k] = \sum_{n=0}^{2} x[n]\, W_3^{kn}, \quad k = 0, 1, 2.$

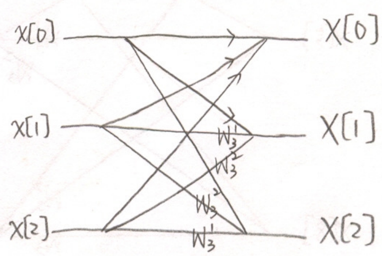
$$= x[0] + W_3^{k} x[1] + W_3^{2k} x[2]$$

$\Rightarrow X[0] = x[0] + x[1] + x[2]$

$X[1] = x[0] + W_3^1 x[1] + W_3^2 x[2]$

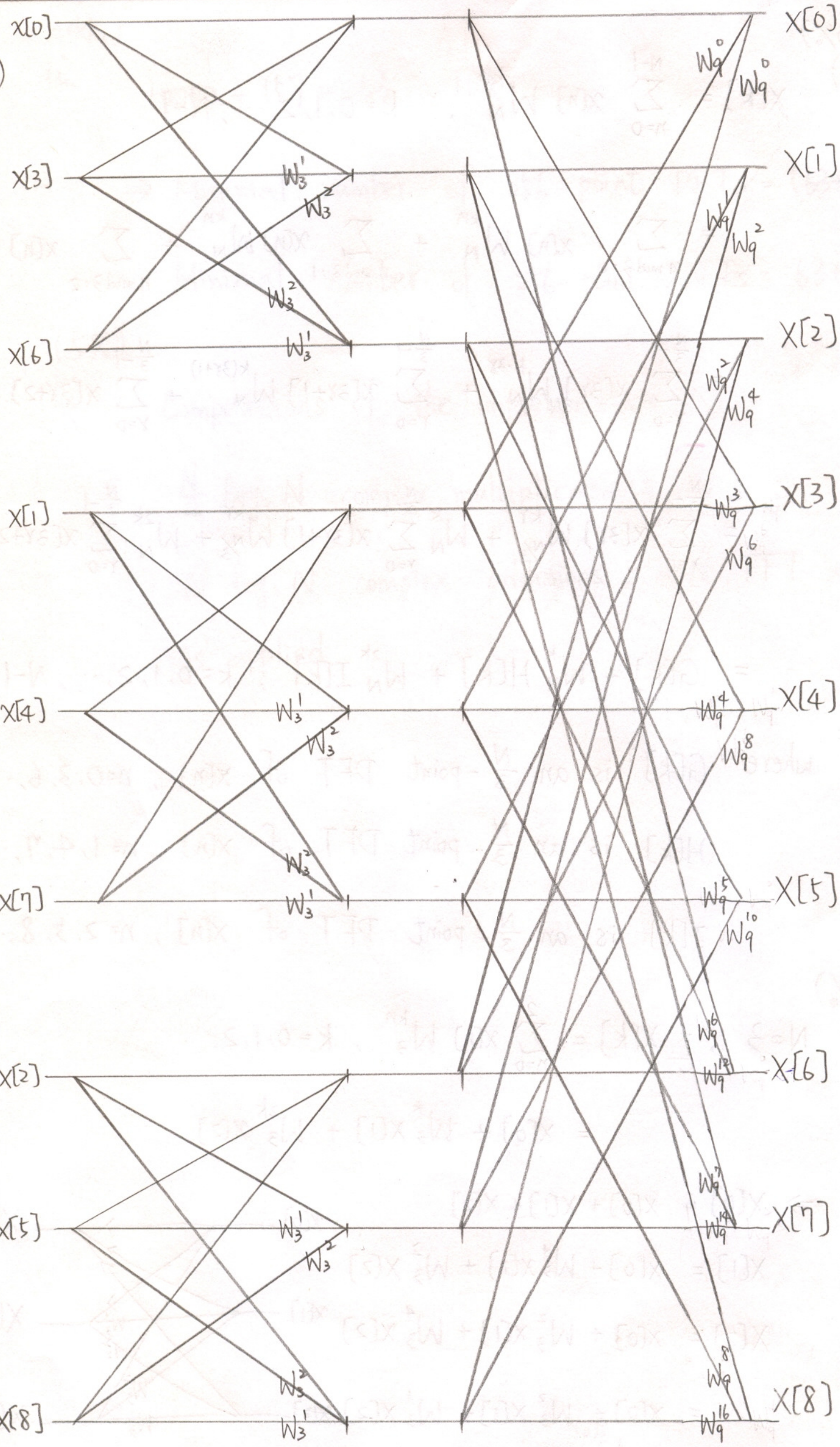
$X[2] = x[0] + W_3^2 x[1] + W_3^4 x[2]$

$= x[0] + W_3^2 x[1] + W_3^1 x[2]$



(5%)

(c)



x[0]
x[3]
x[6]
x[1]
x[4]
x[7]
x[2]
x[5]
x[8]
W_3^1
W_3^2
W_9^0
W_9^1
W_9^2
W_9^3
W_9^4
W_9^5
W_9^6
W_9^7
W_9^8
W_9^{10}
W_9^{12}
W_9^{14}
W_9^{16}
X[0]
X[1]
X[2]
X[3]
X[4]
X[5]
X[6]
X[7]
X[8]

3\. (5%)

(a)

```
fft_xN = fft(x,N);
fft_hN = fft(h,N);
CirConv = ifft(fft_xN .* fft_hN, N);
```

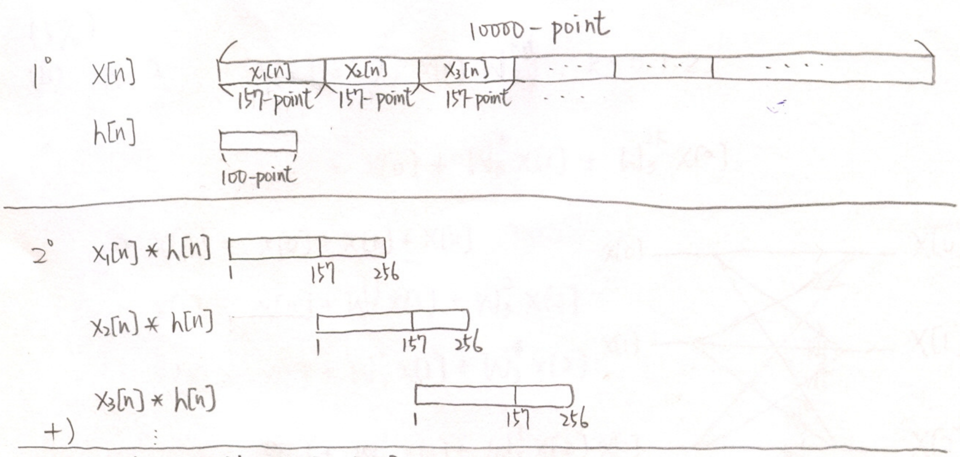
(5%)

(b)

```
fft_xlin = fft(x, 2N-1);
fft_hlin = fft(h, 2N-1);
LinConv = ifft(fft_xlin.* fft_hlin, 2N-1);
```

(c) (6%)

i. Since only 256-point DFT/IDFT is performed, we have to break x[n] into several subsequences. And the maximial size of every subsequence $x_i[n]$ is restricted to be $256+1-100 = 157$.



3° overlap-Add ⇒ $x[n] * h[n]$

(4%)

ii. $10000 \div 157 = 63 \ldots 109$

$\Rightarrow$ Minimal number of 256-point DFTs = $(63+1)+1 = 65$

Minimal number of 256-point IDFTs = $63+1 = 64$.

(5%)

iii. Computations of the implementation:

$\frac{N}{2}\log_2 N$ complex multiplications plus

$N\log_2 N$ complex additions, since FFT algorithm is applied.

4.

(a)

$$Y[n,\lambda) = \sum_{m=-\infty}^{\infty} Y[n+m]\, w[m]\, e^{-j\lambda m}$$

$$= \sum_{m=-\infty}^{\infty} \sum_{k=0}^{M} h[k]\, x[n+m-k]\, w[m]\, e^{-j\lambda m}$$

$$= \sum_{k=0}^{M} h[k] \sum_{m=-\infty}^{\infty} x[n+m-k]\, w[m]\, e^{-j\lambda m}$$

$$= \sum_{k=0}^{M} h[k]\, X[n-k,\lambda) = h[n] * X[n,\lambda)$$

(b)

$$X[n,\lambda) = \sum_{m=-\infty}^{\infty} x[n+m]\, w[m]\, e^{-j\lambda m}$$

$$= \sum_{m'=-\infty}^{\infty} x[m']\, w[-(n-m')]\, e^{-j\lambda(m'-n)}$$

$$= x[n] * w[-n]\, e^{j\lambda n} = x[n] * h_\lambda[n]$$

$$\therefore\ h_\lambda[n] = w[-n]\, e^{j\lambda n} \quad \text{and} \quad H_\lambda(e^{jw}) = W(e^{j(\lambda-w)})$$

(c)

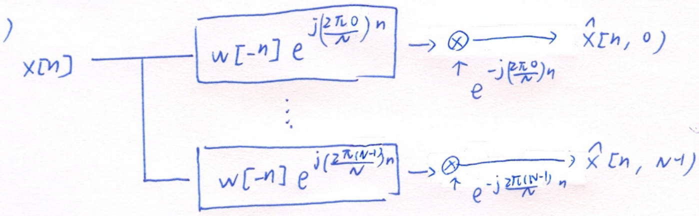
$$\hat{X}[n,\lambda) = \sum_{m'=-\infty}^{\infty} x[m']\, w[m'-n]\, e^{-j\lambda m'} \qquad (\text{let } m' = n+m)$$

$$= \sum_{m'=-\infty}^{\infty} x[n+m]\, w[m]\, e^{-j\lambda n+m} = e^{-j\lambda n} \cdot X[n,\lambda)$$

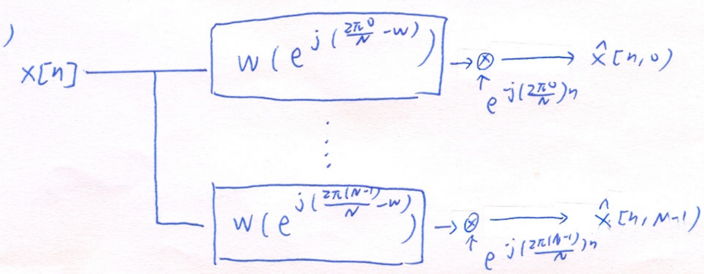
(d)

兩種畫法

(I)



(II)



(e) $f_s = 16\,kHz$

$$\Delta\lambda = \lambda_k - \lambda_{k-1} = \frac{1}{NT} = \frac{16K}{1024} \doteqdot 15 \text{ or } 16\ (Hz)$$
$$= 15.625\ (Hz)$$

5.

(a)
window length ↑ ⇒ the transition band becomes sharper,
because the width of lobe of $W(e^{jw})$ is reduced,
but long processing time

(b)
① window duration ↑ ⇒ frequency resolution ↑
time resolution ↓

② (a)