1.

(a)

(Ⅳ-39) (X)

(課本 204~205) use a 1 less bit → use a $\frac{1}{2}$ less bit

(b)

(Ⅳ-42) (X)

(課本 206 ~ 209) input signal unchanged

(c)

(Ⅴ-25) (X)

allpass system can't be a minimum-phase system

(d)

(Ⅵ-19) (O)

(e)

(Ⅵ-19) (X)

parallel : FIR + FIR ≠ FIR

(f)

(Ⅵ-31) (O)

(g)

(Ⅵ-42 ~Ⅵ-44) (O)

(h)

(課本 518) (X)

only the bilinear transform will result in an allpass filter

(i)

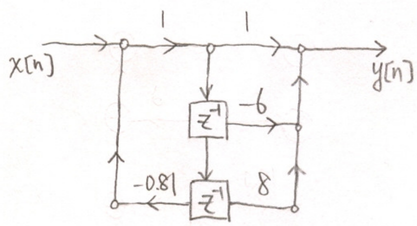
(課本 518) (O)

(j)

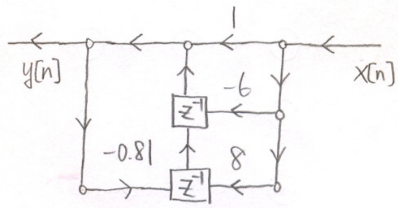
(O)

2. $$H(z) = \frac{(1-2z^{-1})(1-4z^{-1})}{1+0.81z^{-2}} = \frac{1-6z^{-1}+8z^{-2}}{1+0.81z^{-2}} = \frac{(1-2z^{-1})(1-4z^{-1})}{(1-0.9iz^{-1})(1+0.9iz^{-1})}$$

(a)

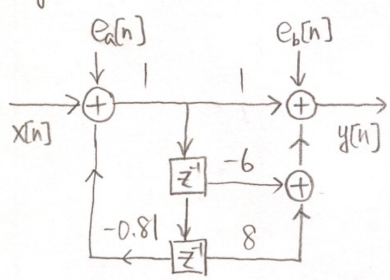


(b)



(c)
```
b=[1 -6 8];
a=[1 0 0.81];
freqz(b,a);
```

(d)



where $e_a[n]$, $e_b[n]$ are round-off noise sources.

(e) power of $e_a[n] = \sigma_B^2$

power of $e_b[n] = 2\sigma_B^2$

Output noise power $$\sigma_{eo}^2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sigma_B^2 \left|H(e^{j\omega})\right|^2 d\omega + 2\sigma_B^2.$$

(f) $$H(z) = \frac{(1-2z^{-1})(1-4z^{-1})}{(1-0.9iz^{-1})(1+0.9iz^{-1})} = \underbrace{\frac{8(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}{(1-0.9iz^{-1})(1+0.9iz^{-1})}}_{H_\ell(z)} \cdot \underbrace{\frac{(z^{-1}-\frac{1}{2})(z^{-1}-\frac{1}{4})}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}}_{H_{ap}(z)}$$

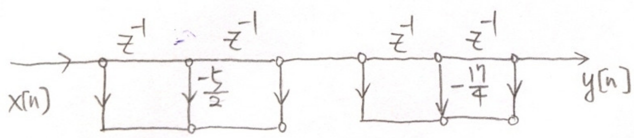
$$= H_\ell(z) \cdot H_{ap}(z).$$

(g) $$H(z) = \frac{(1-2z^{-1})(1-4z^{-1})}{(1-0.9iz^{-1})(1+0.9iz^{-1})}$$

$$= \underbrace{\frac{1}{(1-0.9iz^{-1})(1+0.9iz^{-1})} \cdot \frac{1}{(1-\frac{1}{2}z^{-1})(1-\frac{1}{4}z^{-1})}}_{H_2(z)} \cdot \underbrace{(1-\tfrac{1}{2}z^{-1})(1-\tfrac{1}{4}z^{-1})(1-2z^{-1})(1-4z^{-1})}_{H_{lin}(z)}$$

$$= H_2(z) \cdot H_{lin}(z).$$

(h)



Because the linearity of $H(z) = 1 + az^{-1} + z^{-2}$ only depends on the value of a, even the quantizer is coarse, we can still have linear-phase subsystems as well as $H_{lin}(z)$.

(i) $$H_3(z) = \frac{(1-2z^{-1})(1-4z^{-1})}{(z^{-2}+0.81)}, \quad \text{R.O.C}: \ |z| < \frac{1}{0.9}.$$

$$\frac{(1-2z^{-1})(1-4z^{-1})}{(z^{-1}-0.9i)(z^{-1}+0.9i)} \qquad \frac{(z^{-1}-0.9i)(z^{-1}+0.9i)}{(1-0.9iz^{-1})(1+0.9iz^{-1})} \qquad z^{-2}+0.81$$

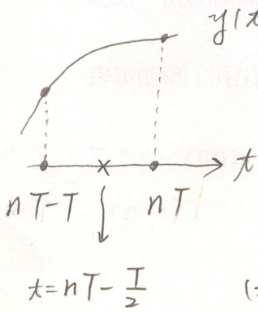
$z^{-2}+0.81$

3. (a)

consider a 1-st order analog filter $H_c(s) = \frac{b}{s-a}$,

the differential equation is $y'(t) = a\,y(t) + b\,x(t)$

suppose now two sampled data $y(nT)$ and $y(nT-T)$ are given.



Consider the halfway time instant $t = nT - \frac{T}{2}$, we have

$$(*)\begin{cases} y'(t) \approx \dfrac{y(nT) - y(nT-T)}{T} \\[2ex] y(t) \approx \dfrac{y(nT) + y(nT-T)}{2} \\[2ex] x(t) \approx \dfrac{x(nT) + x(nT-T)}{2} \end{cases}$$

In discrete time, $y'(t) = a \cdot y(t) + b \cdot x(t)$ with $(*)$ becomes:

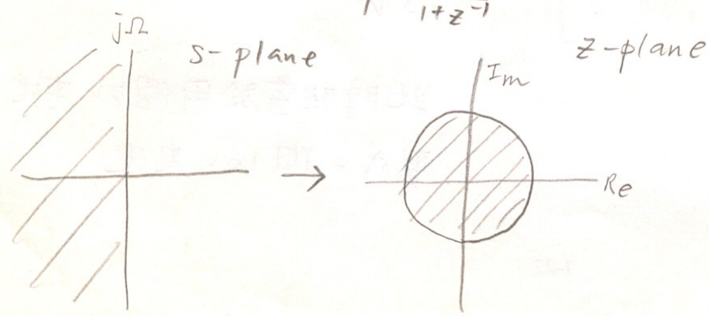
$$\frac{y[n] - y[n-1]}{T} = a \cdot \frac{y[n] + y[n-1]}{2} + b\,\frac{x[n] + x[n-1]}{2}$$

Transform to z-domain:

$$H(z) = \frac{Y}{X} = \frac{\frac{b(1+z^{-1})}{2}}{\frac{1-z^{-1}}{T} - \frac{a(1+z^{-1})}{2}} = \frac{b}{\frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}} - a}$$

comparing with $H_c(s) = \frac{b}{s-a}$

$$\therefore H(z) = H_c(s)\Big|_{s = \frac{2}{T} \cdot \frac{1-z^{-1}}{1+z^{-1}}}$$



(b) Given the digital filter spec's

$H(\omega)$

$\downarrow$

$\boxed{\text{Prewarping}}$ $\quad \Omega_p = \frac{2}{T}\tan\frac{\omega_p}{2}$

$\Omega_s = \frac{2}{T}\tan\frac{\omega_s}{2}$

$\downarrow$

analog spec's

$\downarrow$

$\boxed{\text{Analog filter design}}$ $\quad$ find N, Ω_c

$\downarrow$

$H_c(s)$

$\downarrow$

$\boxed{\text{Bilinear Transformation}}$ $\quad s = \frac{2}{T}\cdot\frac{1-z^{-1}}{1+z^{-1}}$

$\downarrow$

$H(z)$

done!

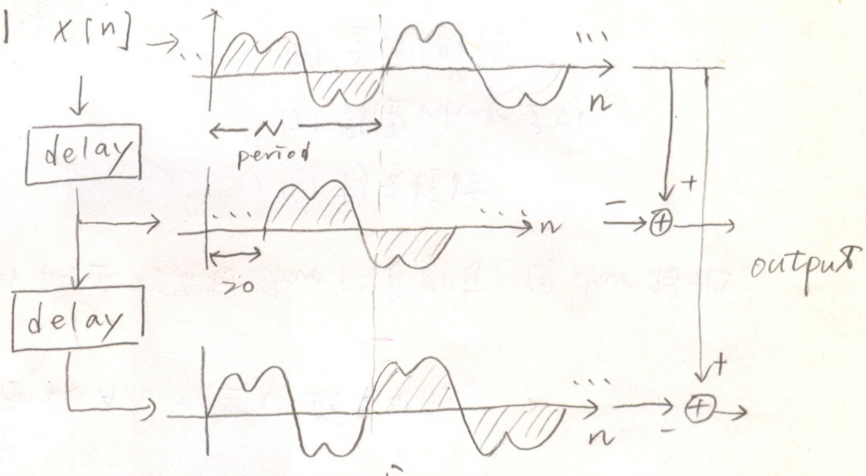
(c)

Bilinear transformation is 1-to-1 mapping from s-plane to z-plane, where impulse invariance isn't. Digital IIR filter design using impulse invariance needs to recheck because aliasing is possible.

4.

(a) from time domain perspectives,

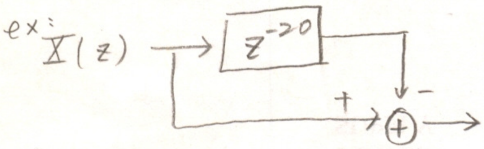
a periodic signal $x[n]$



我們 shift 原訊號，再和原訊號相減，當 shift 到正確 period 時，得到的相減 output 就會很小，即可 detect 原訊號的週期。

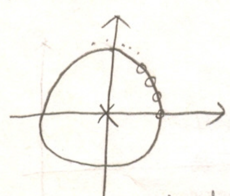
(b) from frequency domain perspectives.

ex:



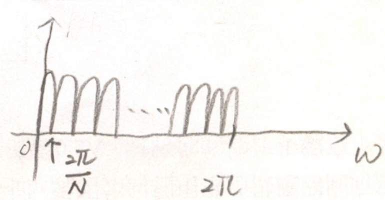
$Y(z) = (1 - z^{-20})\, X(z)$

pole-zero plot:



unit circle 上
均分 20 個點

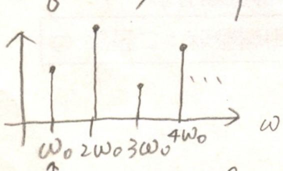
frequency response:



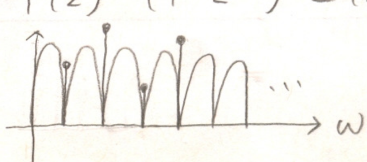
$0 \sim 2\pi$ 上有 20 個
掉到 zero 的點

∴ if 原訊號 $X(\omega)$ 是週期訊號，

frequency response can be:



$Y(z) = (1 - z^{-20}) \cdot X(z)$



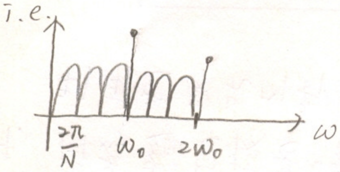
所以，我們調整 filter 為均分 $0 \sim 2\pi$ 20個 zero, 21個 zero...

當 zero 點剛好都把原訊號週期濾掉時，output 就會很小，即可 detect 原訊號週期。

$\frac{2\pi}{N} = \omega_0$, N is the Number of delay.

* 有一點需要注意，如果 ω_0 是 $\frac{2\pi}{N}$ 的整數倍，output 也都會很小．所以要取 N 最小的那一個為週期。

i.e.



(c)

sample at 10 kHz.

$\frac{10k}{N} = 50$, $\Rightarrow N = 200$

$\frac{10k}{N} = 500$, $\Rightarrow N = 20$

(or one 20-time delay plus 180 unit-time delays)

So, in the possible range, we need 200 unit-time delays.

Because N is integer, the fundamental frequency can only be : $\frac{10k}{20}$, $\frac{10k}{21}$, $\frac{10k}{22}$, ... , $\frac{10k}{200}$

So, it is not uniformly distributed in $50 \sim 500$ Hz.