Rotating Consensus of Second-Order Multi-Agent Systems with Signed Directed Graphs ⋆

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Abstract: This paper investigates the rotating motion control for a class of second-order multi-agent systems with both cooperative and antagonistic interactions. Compared with some existing results, the multi-agent systems are assumed to have a signed directed graph rather than an undirected graph. By using the local relative information, we design a control protocol and give a sufficient condition for rotating consensus problem with antagonistic networks. Furthermore, we derive the lower bound of parameters in the control protocol. Finally, the correctness of our results is confirmed by the simulation results.

Keywords: Multi-agent systems, rotating consensus, antagonistic network, signed directed graph.

1. INTRODUCTION

In the last few decades, the study on the multi-agent dynamical systems has received a major attention within the control field. This is partly due to their broadly application value in sensor networks, robots cooperation, cooperative control of unmanned air vehicles, and so on Fax and Murray (2004); Oh et al. (2015); Ge et al. (2018); Zuo et al. (2018). As the fundamental problem of coordinated control, consensus problems have been studied widely since it was firstly proposed in Reynolds (1987). For example, Olfati-Saber *et al.* addressed the consensus problem of first-order multi-agent systems with undirected graphs in both discrete-time and continuous-time domains Olfati-Saber and Murray (2004). In Ren (2007), Ren introduced some consensus protocols for the double-integrator multi-agent systems with more general directed graphs. Furthermore, with taken time-varying communication into consideration, it is shown in Zhu and Cheng (2010) that consensus can be achieved asymptotically if the union of the directed graphs has a spanning tree.

Apart from the consensus problems, more and more researchers have paid much attention to the rotating consensus problem, which is used to describe the circular motions for a class of moving robots. To mention a few, in Lin and Jia (2010), all agents finally move together along a circle around a common point with the proposed distributed rotating consensus protocol. The authors in Li et al. (2018) further considered the rotating consensus problem under undirected graphs by taking time delays into account. In Li et al. (2015), the rotating consensus problems with and without mixed uncertainties and communication delay are solved by utilizing the Lyapunov method and linear matrix inequality (LMI) techniques. We recommend the readers to refer to Zhang and Duan (2018); Mo et al. (2019) for more related results.

The above-mentioned results are based on the cooperative interactions in multi-agent systems. However, in some real scenarios, communication topologies are often subjected to antagonistic networks, in which the cooperative and competitive interaction exist simultaneously see Wasserman and Faust (1994); Easley and Kleinberg (2012). With such antagonistic networks, it is important to recognize its impact on the behaviour of multi-agent systems. Thus, much attention has been paid to the consensus problem of multi-agent systems with antagonistic interactions Altafini (2012); Valcher and Misra (2014); Meng et al. (2016); Meng (2017); Shi et al. (2018). It is worth noting that, under the assumption of the so-called structural balance, proper bipartite consensus protocols were first proposed in Altafini (2012), where antagonistic interactions were considered in the first-order multi-agent systems. The control performance of multi-agent systems with antagonistic interactions was further considered in Shi et al. (2018), where a fixed-time bipartite consensus protocol is proposed.

Note that most studies on rotating consensus problem usually focus on cooperative interactions, while few results are reported on the rotating consensus problem with antagonistic interactions. Moreover, the multi-agent systems involved are usually assumed to have an undirected graph, which means that those results are not applicable for multi-agent systems with directed graphs. Nevertheless, a large number of applications of multi-agent systems require that the communication graphs should be directed. Therefore, it is practical and significant to address the rotating consensus problem with directed graphs, which motivates this study.

In this paper, we focus on the rotating consensus control of multi-agent systems with antagonistic directed networks.

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Firstly, we give the rotating consensus definition under the so-called structural balance assumption. Secondly, we convert the rotating consensus problems with signed networks into the case with unsigned networks by a transformation matrix. Then, the original problem is equivalently addressed by using the spectral property of the system matrix, where the lower bound of the design parameters can be derived.

The main contributions of the paper are listed as follows:

- We extend the results in Lin and Jia (2010) to general cases, where the multi-agent systems are allowed to have a directed graph rather than an undirected graph. Thus, the obtained results in this paper include that of Lin and Jia (2010) as a special case. Moreover, compared with Lin and Jia (2010), a new rotating protocol is devised, and a sufficient condition is proposed to design suitable parameters of the protocol; and
- We further consider the antagonistic interactions in the rotating consensus problem in this paper. By analyzing the spectral property of the system matrix, a more general result is obtained, which is applicable to both cooperative and antagonistic interactions.

The rest of the paper is organized as follows: problem formulation and some preliminaries are introduced in section 2; main results and detailed proofs are included in section 3; some examples are shown in section 4 to confirm our results; finally, we give a brief conclusion in section 5.

Through this article, we use $\mathbf{R}^n$ and $\mathbf{C}^n$ to denote the set of n dimensional column vectors in real and complex, respectively; $\mathbf{R}^+$ represents the positive real set; $\mathbf{C}$ denotes the complex set; 0 denotes zero vectors or zero matrixes with appropriate dimensions; $\mathbf{1}_n$ denotes the n dimensional column vector of all ones; I_m refers to the m dimensional identity matrix; x^T and x^* denote the transposition and conjugate transposition of vectors, respectively; j denotes the imaginary unit; $\otimes$ denotes the kronecker product; $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ represent the real part and the imaginary part of a complex number, respectively; $\det(\cdot)$ denotes the determinant of a matrix. $|\cdot|$ gets the absolute value of a number. $\mathrm{sign}(\cdot)$ denotes a sign function. $\mathrm{diag}(d_1, d_2, \cdots, d_n)$ is a diagonal matrix whose diagonal entries are $d_1, d_2, ...,$ and d_n.

2. PRELIMINARIES AND PROBLEM FORMULATION

2.1 Some Preliminaries

A signed graph is denoted by $\mathcal{G}(\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{G}$ denotes a graph, $\mathcal{V} = \{v_1, \cdots, v_n\}$ is the nodes set, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edges set and $\mathcal{A} \in R^{n\times n}$ is the weights matrix. The element $a_{ik} \neq 0$ $(i \neq k)$ if and only if the edge $(v_k, v_i) \in \mathcal{E}$ and $a_{ik} = 0$, otherwise. For a directed signed graph, we will always suppose that the edge pairs of any two nodes always have the same sign. The Laplacian matrix L_c is defined as $L_c = [l_{ik}]$, where $l_{ii} = \sum_{k=1}^{n} |a_{ik}|$ and $l_{ik} = -a_{ik}$, $i \neq k$. A (directed) path from v_1 to v_l is a sequence of edges of the form $(v_1, v_2), (v_2, v_3), ..., (v_{l-1}, v_l)$ with distinct nodes in a (directed) graph. A directed spanning tree is a path which consists of all the nodes and some edges in $\mathcal{G}$. If there exists a directed path between any two nodes, the graph is strongly connected. The set of neighbors of node v_i is denoted by $N_i = \{v_k \in \mathcal{V} : (v_k, v_i) \in \mathcal{E}\}$.

As for the signed graphs, we have the following preliminary results.

Lemma 2.1. (Altafini (2012)). A connected signed graph $\mathcal{G}$ is structurally balanced if and only if any of the following conditions hold:

1. All nodes can be partitioned into two sets (one possibly empty) in such a way that edges joining two nodes in the same set are positive while edges joining two nodes in different sets are negative, i.e., it admits a bipartition of the nodes $\mathcal{V}_1, \mathcal{V}_2$, $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$, $\mathcal{V}_1 \cap \mathcal{V}_2 = \varnothing$.

2. There is a Gauge transformation matrix $D \in \mathcal{D}$ satisfying DAD has all nonnegative entries where $\mathcal{D} = \{D = \mathrm{diag}(\sigma_1, \sigma_2, \cdots, \sigma_n),\ \sigma_i \in \{+1, -1\}\}$.

Lemma 2.2. (Altafini (2012)). If graph $\mathcal{G}$ is structurally balanced, L_c and $L_D = DL_cD$ are isospectral, i.e., $\mathrm{sp}(L_c) = \mathrm{sp}(L_D)$

Lemma 2.3. (Ren and Beard (2005)). The Laplacian matrix L has a simple eigenvalue 0 and all the other eigenvalues have positive real parts if and only if the directed network has a directed spanning tree.

2.2 Problem Formulation

Consider a group of n agents, and the ith agent has the following dynamics:

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= u_i(t), \end{aligned} \tag{1}$$

where $x_i(t)$, $v_i(t)$, $u_i(t) \in \mathbf{C}$ represent the position, velocity and the control input of the ith agent at time t, respectively, with the initial conditions $x_i(0)$, $v_i(0)$. Consider the control protocol as

$$u_i(t) = f_i(x_i(t), x_k(t), v_i(t), v_k(t), k \in N_i). \tag{2}$$

The following definition is given to illustrate the concept of rotating consensus with antagonistic networks.

Definition 2.1. Consider multi-agent system (1) with the signed directed graph $\mathcal{G}$ which is structurally balanced. Develop a distributed control scheme such that, for any finite $x_i(0)$, $v_i(0)$, $i = 1, \cdots, n$, if the following conditions are satisfied: all agents can be partitioned into two sets with $\mathcal{V}_q (q \in \{1, 2\})$ such that for any agents $v_i, v_k \in \mathcal{V}_q (q \in \{1, 2\})$

$$\begin{aligned} &\lim_{t\to+\infty} [v_i(t) - v_k(t)] = 0, \\ &\lim_{t\to+\infty} [(x_i(t) + j\varpi^{-1} v_i(t)) - (x_k(t) + j\varpi^{-1} v_k(t))] = 0, \\ &\lim_{t\to+\infty} [\dot{v}_i(t) - j\varpi v_i(t)] = 0, \end{aligned} \tag{3}$$

and for any agents $v_i \in \mathcal{V}_q, v_k \in \mathcal{V}_r, q \neq r, (q, r \in \{1, 2\})$

$$\begin{aligned} &\lim_{t\to+\infty} [v_i(t) + v_k(t)] = 0, \\ &\lim_{t\to+\infty} [(x_i(t) + j\varpi^{-1} v_i(t)) + (x_k(t) + j\varpi^{-1} v_k(t))] = 0, \\ &\lim_{t\to+\infty} [\dot{v}_i(t) - j\varpi v_i(t)] = 0, \end{aligned} \tag{4}$$

where ϖ is the angular velocity, it is said that the rotating consensus problem with antagonistic networks is solved.

Remark 2.1. The definition of the rotating consensus problem was firstly given in Lin and Jia (2010). In Definition 2.1, the first two conditions of (3) imply that all agents reach the consensus with respect to both the velocity and the center of the circle, where $x_i(t) + j\varpi^{-1}v_i(t)$ tends to the center of the circle in the complex plane. The last condition means that the acceleration of each agent tends to be perpendicular to its velocity, such that each agent finally moves in a circle with the angular velocity ϖ. To simplify the analysis below, we assume $\varpi = 1$ without loss of generality. Different from Lin and Jia (2010), where only cooperation interactions were considered, in this definition, all agents are divided into two groups, where each group of agents reaches the rotating consensus while the centers of the two groups are symmetric with respect to the origin in the complex plane.

Here, we give some assumptions which will be used later.

Assumption 2.1. The signed directed graph has a directed spanning tree.

Assumption 2.2. The signed directed graph is structurally balanced.

Remark 2.2. It is noted that both Assumptions 2.1 and 2.2 are quite standard. Assumption 2.1 is a general assumption commonly used in many papers Ren (2007); Lin and Jia (2009), where only unsigned graphs are considered. In addition, Assumption 2.2 is required due to the consideration of signed graphs, which is also used in the existing papers Altafini (2012); Meng (2017); Shi et al. (2018).

3. MAIN RESULTS

The control protocol is designed as follows

$$\begin{aligned} u_i(t) = jv_i(t) &- \sum_{k\in N_i} |a_{ik}|(x_i(t) - \mathrm{sign}(a_{ik})x_k(t)) \\ &-\eta \sum_{k\in N_i} |a_{ik}|(v_i(t) - \mathrm{sign}(a_{ik})v_k(t)), \end{aligned} \tag{5}$$

where $i \in \{1, 2, \cdots, n\}$ and $\eta \in \mathbf{R}^+$ is the design parameter.

Let $\xi(t) = [x^T(t), v^T(t)]^T$. Equivalently, by applying the control protocol (5), system (1) can be rewritten in a compact form as

$$\dot{\xi}(t) = \Gamma_c \xi(t), \tag{6}$$

where $\Gamma_c = \begin{bmatrix} 0 & I_n \\ -L_c & jI_n - \eta L_c \end{bmatrix}$.

Take a block-diagonal matrix as $\bar{D} = \mathrm{diag}\{D, D\}$ and let $z(t) = [z_1^T(t), z_2^T(t)]^T = \mathrm{diag}\{D, D\}\xi(t)$. Then, the closed-loop network dynamics (6) can be rewritten as

$$\begin{aligned} \dot{z}(t) = \bar{D}\dot{\xi}(t) = \bar{D} \begin{bmatrix} 0 & I_n \\ -L_c & jI_n - \eta L_c \end{bmatrix} \bar{D}\bar{D}\xi(t) \\ = \Gamma_D z(t), \end{aligned} \tag{7}$$

where $\Gamma_D = \begin{bmatrix} 0 & I_n \\ -L_D & jI_n - \eta L_D \end{bmatrix}$ and $L_D = DL_cD$ is a new Laplacian matrix of an unsigned graph.

It follows from (7) that

$$z(t) = e^{\Gamma_D t} z(0). \tag{8}$$

Thus, the spectral property of matrix Γ_D plays an important role for the convergence of analysis.

Here, we give the main theorem of this paper.

Theorem 3.1. Consider the multi-agent system (1). Under Assumptions 2.1 and 2.2, the control protocol (5) solves the rotating consensus problem with antagonistic networks if

$$\eta > \max_{i\in\{2,\cdots,n\}} \{\frac{a_ib_i + \sqrt{(a_ib_i)^2 + 4b_i^2(a_i^3 + a_ib_i^2)}}{2(a_i^3 + a_ib_i^2)}\}, \tag{9}$$

where $a_i = \mathrm{Re}(\lambda_i)$, $b_i = \mathrm{Im}(\lambda_i)$, with $\lambda_i, i = 2, \cdots, n$ being eigenvalues of matrix L_D. Specifically, $\sigma_i x_i(t) \to \sum_{i=1}^n \sigma_i\omega^i x_i(0) + j\sum_{i=1}^n \sigma_i\omega^i v_i(0)(1 - e^{jt})$, $\sigma_i v_i(t) \to \sum_{i=1}^n \sigma_i\omega^i v_i(0)e^{jt}$ as $t \to \infty$, where $\omega^T L_D = 0$, $\omega = [\omega^1, \cdots, \omega^n]^T \in \mathbf{R}^n$ with $\omega^T \mathbf{1}_n = 1$ and σ_i is given in Lemma 2.1.

Before presenting the proof of Theorem 3.1, we further need the following lemma.

Lemma 3.1. Let $\rho_\pm = \frac{j-\eta_1\mu\pm\sqrt{(j-\eta_1\mu)^2-4\mu}}{2}$, where $\mu = a + bj$, $a \in R^+, b \in \mathbf{R}$, $\eta_1 \in \mathbf{R}^+$. If

$$\eta_1 > \frac{ab + \sqrt{(ab)^2 + 4b^2(a^3 + ab^2)}}{2(a^3 + ab^2)}, \tag{10}$$

then $\mathrm{Re}(\rho_\pm) < 0$.

Proof. The proof is omitted due to the limit of the space.

Remark 3.1. From Lemma 3.1, it is easy to conclude that if μ is a positive real number, namely, $b = 0$, then $\mathrm{Re}(\rho_\pm) < 0$ if and only if $\eta_1 > 0$.

Now we are ready to give the proof of Theorem 3.1.

Proof. Firstly, we study the spectral property of matrix Γ_D. The solutions of $\det\begin{pmatrix} sI_n & -I_n \\ L_D & sI_n - jI_n + \eta L_D \end{pmatrix} = 0$ are the eigenvalues of matrix $\begin{bmatrix} 0 & I_n \\ -L_D & jI_n - \eta L_D \end{bmatrix}$. Denote the eigenvalues of matrix L_D as $\lambda_1, \cdots, \lambda_n$. Under Assumptions 2.1 and 2.2, it follows from Lemma 2.3 that $0 = \lambda_1 < \mathrm{Re}(\lambda_2) \le \cdots \le \mathrm{Re}(\lambda_n)$. The eigenvalues of matrix Γ_D are equal to solutions of polynomials

$$s^2 - js + \eta\lambda_i s + \lambda_i = 0, i \in \{1, 2, ..., n\}. \tag{11}$$

Thus, the matrix Γ_D has two eigenvalues at $s_{11} = 0$ and $s_{12} = j$ corresponding to $\lambda_1 = 0$.

For $\lambda_i \neq 0, i = 2, \cdots, n$, the solutions of (11) are

$$\begin{aligned} s_{i1} &= \frac{j - \eta\lambda_i + \sqrt{(j - \eta\lambda_i)^2 - 4\lambda_i}}{2}, \\ s_{i2} &= \frac{j - \eta\lambda_i - \sqrt{(j - \eta\lambda_i)^2 - 4\lambda_i}}{2}. \end{aligned} \tag{12}$$

Define $\lambda_i = a_i + b_ij$, where $a_i > 0$ and $b_i \in \mathbf{R}$. We could get that $\mathrm{Re}(j - \eta\lambda_i) = -\eta a_i < 0$. By using Lemma 3.1, we could find that the eigenvalues $s_{i1}, s_{i2}, i = 2, \cdots, n$ have negative real part if $\eta > \max_{i\in\{2,\cdots,n\}} \{\frac{a_ib_i+\sqrt{(a_ib_i)^2+4b_i^2(a_i^3+a_ib_i^2)}}{2(a_i^3+a_ib_i^2)}\}$.

Then, we could conclude that matrix Γ_D exactly has two eigenvalues 0, j and all other eigenvalues have negative real part if $\eta > \max_{i\in\{2,\cdots,n\}} \{\frac{a_ib_i+\sqrt{(a_ib_i)^2+4b_i^2(a_i^3+a_ib_i^2)}}{2(a_i^3+a_ib_i^2)}\}$.

Since L_D has eigenvalue 0, it can be concluded that there exists $\omega = [\omega^1, \cdots, \omega^n]^T \in \mathbf{R}^n$, such that $\omega^T L_D =$

0, namely, ω is the left eigenvector corresponding to eigenvalue 0. Let $\omega^T \mathbf{1}_n = 1$.

The matrix Γ_D exactly has two eigenvalues 0 and j, and we calculate the corresponding right eigenvectors $w_1, w_2 \in \mathbf{C}^{2n}$ corresponding to eigenvalues 0 and j, respectively, which means that $\Gamma_D w_1 = 0$ and $\Gamma_D w_2 = j w_2$. With detailed calculation, we could get that

$$\begin{aligned} w_1 &= [\mathbf{1}_n^T, 0_n^T]^*, \\ w_2 &= [\mathbf{1}_n^T, -j\mathbf{1}_n^T]^*. \end{aligned} \tag{13}$$

Similarity, we define $v_1, v_2 \in \mathbf{C}^{2n}$ are left eigenvectors corresponding to eigenvalues 0 and j, respectively. After a series calculations, one obtains

$$\begin{aligned} v_1 &= [\omega^T, j\omega^T]^*, \\ v_2 &= [0_n^T, -j\omega^T]^*. \end{aligned} \tag{14}$$

Meanwhile, $v_1^* w_1 = 1$, $v_2^* w_2 = 1$.

It is noted that there exists a nonsingular matrix $P \in \mathbf{C}^{2n \times 2n}$ such that $P^{-1}\Gamma_D P = \Lambda$, where $\Lambda = \mathrm{diag}\{0, j, J'\}$ in which J' is the upper diagonal Jordan block matrix corresponding to eigenvalues $s_{i1}, s_{i2}, i = 2, \cdots, n$.

Without loss of generality, we choose $P = [w_1, w_2, \cdots, w_{2n}]$, $P^{-1} = [v_1, v_2, \cdots, v_{2n}]^*$, where w_i, v_i $(i = 1, 2, \cdots, 2n)$ are the right and left eigenvectors or generalized eigenvectors of matrix Γ_D.

Then,

$$\begin{aligned} e^{\Gamma_D t} &= P e^{\Lambda t} P^{-1} = [w_1, \cdots, w_{2n}] \\ &\times \begin{bmatrix} 1 & 0 & 0_{1\times(2n-2)} \\ 0 & e^{jt} & 0_{1\times(2n-2)} \\ 0_{(2n-2)\times 1} & 0_{(2n-2)\times 1} & e^{J' t} \end{bmatrix} \begin{bmatrix} v_1^* \\ \vdots \\ v_{2n}^* \end{bmatrix}. \end{aligned} \tag{15}$$

Under the conditions that the matrix Γ_D exactly has two eigenvalues 0 and j, and all other eigenvalues have negative real parts, then $\lim_{t\to+\infty} e^{J' t} = 0_{(2n-2)\times(2n-2)}$.
Thus,

$$\begin{aligned} \lim_{t\to+\infty} \begin{bmatrix} z_1(t) \\ z_2(t) \end{bmatrix} &= \lim_{t\to+\infty} e^{\Gamma_D t} z(0) \\ &= \lim_{t\to+\infty} \begin{bmatrix} \mathbf{1}_n\omega^T & j\mathbf{1}_n\omega^T(1-e^{jt}) \\ 0_{n\times n} & \mathbf{1}_n\omega^T e^{jt} \end{bmatrix} \begin{bmatrix} Dx(0) \\ Dv(0) \end{bmatrix}, \end{aligned} \tag{16}$$

Thus, the position of all agents is $\lim_{t\to+\infty} \sigma_i x_i(t) = \sum_{i=1}^n \sigma_i \omega^i x_i(0) + j\sum_{i=1}^n \sigma_i \omega^i v_i(0)(1-e^{jt})$ and the velocity of all agents is $\lim_{t\to+\infty} \sigma_i v_i(t) = \sum_{i=1}^n \sigma_i \omega^i v_i(0) e^{jt}$. Meanwhile, $\sigma_i \dot{v}_i(t) - j\sigma_i v_i(t) = 0$.

Based on Lemma 2.1, all agents are divided into two sets where $\mathcal{V}_1 = \{v_i : \sigma_i = 1\}$, $\mathcal{V}_2 = \{v_i : \sigma_i = -1\}$. Thus, the conditions defined in Definition 2.1 are satisfied, namely, the rotating consensus with antagonistic networks is achieved.

In particular, if all eigenvalues of Laplacian matrix L_D are real, it follows from Theorem 3.1 that $\eta > 0$. Thus, we could get the following corollary.

Corollary 3.1. Consider multi-agent system (1) with control protocol (5). Under Assumptions 2.1 and 2.2, if all eigenvalues of matrix L_D are real, the rotating consensus with antagonistic networks is achieved if $\eta > 0$.

Remark 3.2. Consider the special case where the graph is undirected and unsigned, the corresponding Laplacian matrix L_D is symmetric. In this case, all eigenvalues of L_D are positive real numbers, thus the rotating consensus is achieved according to Corollary 3.1, which is in accordance with the results in paper Lin and Jia (2010).

4. SIMULATION

To validate the theoretical results, we will apply the control protocol to a numerical example.

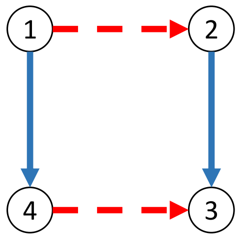

Fig. 1. Directed graph $\mathcal{G}$ with 4 agents.

As shown in Fig. 1, the signed directed graph consists of 4 nodes, where the solid lines represent cooperative relationship with weights +1, while the dotted lines represent antagonistic relationship with weights −1. It can be observed that the network is structurally balanced with $\mathcal{V}_1 = \{v_2, v_3\}$ and $\mathcal{V}_2 = \{v_1, v_4\}$. The eigenvalues of corresponding Laplacian matrix L_c are 0.0, 1.0, 1.0, and 2.0. The exact bound of the design parameter is $\eta > 0$ according to Corollary 3.1. Thus, we choose $\eta = 2$ in our simulation. Under the control law (5), the simulation results are shown in Fig. 2 and Fig. 3, it is clear that the rotating consensus can be reached as we expected.

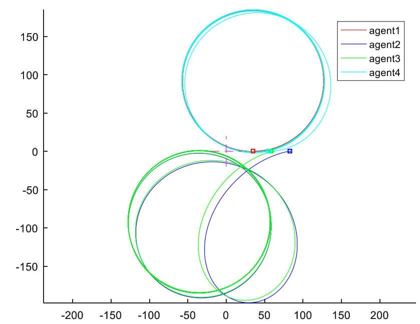

Fig. 2. Position trajectory of all agents.

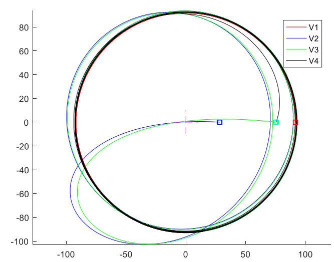

Fig. 3. Velocity trajectory of all agents.

5. CONCLUSIONS

This paper has dealt with the rotating consensus problem of second-order multi-agent systems with both cooperative and antagonistic interactions. Under the structurally balance condition, all agents could reach the rotating consensus with the proposed control protocol, and the lower bound of the control parameter is figured out. In future, we will further consider the problem with switching antagonistic networks.

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