SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

May 2020, Vol. 63 150203:1–150203:10
https://doi.org/10.1007/s11432-019-2692-9

Special Focus on Advanced Techniques for Event-Triggered Control and Estimation

Rotating consensus control of double-integrator multi-agent systems with event-based communication

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Received 19 May 2019/Revised 6 August 2019/Accepted 23 September 2019/Published online 25 March 2020

Abstract This paper solves the rotating consensus problem for a group of double-integrator agents with event-based communication only. We propose a distributed event-based rotating consensus protocol, which guarantees that a consensus regarding both position and velocity is achieved when all agents exhibit circular motion around the same center. It is observed that overall less communication is required as the communication between agents is only needed at event times. Moreover, with the proposed event-based protocol, it is proved that Zeno behavior can be strictly avoided for each agent. Numerical simulations show that this event-based control law can efficiently solve the rotating consensus problem.

Keywords rotating consensus, event-based control, double-integrator dynamics, multi-agent systems, complex systems

Citation Ding R, Hu W F, Yang Y H. Rotating consensus control of double-integrator multi-agent systems with event-based communication. Sci China Inf Sci, 2020, 63(5): 150203, https://doi.org/10.1007/s11432-019-2692-9

1 Introduction

With the recent development of communication and computing technology, researchers are more interested in the cooperative control of groups of autonomous agents. Presently, a lot of studies focus on the consensus [1, 2], formation control [3–5], and flocking [6] problems.

Because the consensus problem is a typical cooperative control problem, it has paved the way for research of other cooperative control problems, which requires the states of all agents converge to a common value of interest. Pioneering researchers focused on the consensus problem of multi-agent systems composed of single-integrator [1] or double-integrator [2, 7] agents. As an important extension of the consensus problem, the objective of the formation control problem is to guarantee that all agents form and maintain an arbitrary but specific formation through distributed control while in motion. Specifically, Chen et al. [8] studied a class of collective circular motion problem for a group of nonholonomic vehicles provided that the switching topologies among all the agents are jointly connected. It was shown that all vehicles moved in a circle around the common center and were distributed within the circle in a specific pattern. The authors in [9] studied the formation problem for a group of mobile robots modeled by single-integrator agents with a moving target. Herein, all robots asymptotically reach a regular polygon formation while surrounding the moving target as its centroid. Furthermore, Lin et al. [10] proposed a distributed control law for a second-order multi-agent system modeled by complex systems in such a way that all agents reach a consensus while rotating around a common center. This problem defined as the rotating consensus control problem differs from the classical consensus problem in the sense that apart

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from achieving the consensus with respect to both position and velocity, the agents are also required to move in a circular around a common center. Because the rotating consensus problem is a special case of the collective circular formation problem, its study serves as the basis for solutions of cooperative control of unmanned air vehicles (UAVs) flying in formation or problems pertaining to flight of satellites.

In engineering applications, owing to the limitation of agents' power, the conventional high-frequency sampling methods consume too much on-board energy. In [11, 12], the authors further discussed on the event-based strategy in comparison with the periodic sampling strategy and established that event-triggered sampling has a higher energy efficiency. Therefore, event-based strategies are adopted to reduce the communication in multi-agent systems. Following the idea proposed in [11], the authors in [13, 14] studied the event-based consensus problem for a group of single-integrator and double-integrator agents, respectively. For a group of agents with general linear dynamics, the leaderless consensus and the leader-follower consensus problems were solved by the proposed event-based control protocols in [15, 16], respectively. It is proved that a controlled system can reach the consensus asymptotically when the communication load is reduced significantly [13, 15]. Furthermore, to avoid that the distributed control law depends on the global information of the graph, novel fully distributed event-triggered protocols were proposed for linear multi-agent networks in [17]. As an important extension of the static event-triggered control, a new dynamic event-triggered control approach was proposed to address the consensus problem in [18]. Other related results on these topics can be found in [19–23].

In this paper, the rotating consensus problem is addressed by incorporating an event-based control strategy. Each agent is in the complex plane and follows double-integrator dynamics. Three main difficulties arise with this approach. First, the rotating consensus problem is more complicated than the classical consensus problem because it requires all agents not only to achieve the consensus with respect to both position and velocity but also move in a circle around a common center. Second, communication between all agents is restricted to event times of itself or its neighbors only and the Zeno-free property of the event-triggering mechanism should be guaranteed. Furthermore, the event-based rotating consensus problem is considered in the complex plane making the design of control law and stability analysis more challenging for complex systems. To overcome these difficulties, the theoretical analysis of the problem includes three steps: (i) the original rotating consensus problem is converted to the stabilization problem of the disagreement system and the resulting system matrix is shown to be Hurwitz; (ii) for the event-triggering mechanism, it is shown that the disagreement vector converges to zero exponential through the proof by contradiction, and the event-based control law can then be designed correspondingly; (iii) the feasibility of the proposed event-based function is guaranteed by proving that Zeno behavior is strictly avoided.

The rest of this paper is organized as follows: Section 2 briefly reviews some preliminaries of graph theory; Section 3 defines the system models and problem studied; Section 4 presents the main event-based consensus protocol; Section 5 provides a simulation example to compare event-based sampling with periodic sampling; Section 6 draws conclusion and discusses future work.

2 Some preliminaries of graph theory

For convenience, we use the following notations:

R^n: the set of n dimensional real column vectors.

C^n: the set of n dimensional complex column vectors.

ℓ: index set $\{1,\ldots,n\}$.

N: number of agents.

N_i: neighbor set of agent i.

$\mathbf{1}$: $[1,\ldots,1]^{\mathrm{T}}$

j: the imaginary unit.

x^{T}: transpose of x.

$\otimes$: Kronecker product.

$\|\cdot\|$: Euclidean norm.

$|N_i|$: cardinality of a set N_i.

Re$(\cdot)$: real part of a complex number.

An undirected graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ consists of a node set $\mathcal{V}=\{v_1,\ldots,v_n\}$ and the edge set $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$. The weighted adjacency matrix of $\mathcal{G}$ is defined by $\mathcal{A}=[a_{ij}]_{N\times N}$, where $a_{ij}=a_{ji}=1, i,j\in\ell$ if and only if $(i,j)\in\mathcal{E}$, otherwise $a_{ij}=0$. N_i is defined as $\{v_j\in\mathcal{V}:(v_i,v_j)\in\mathcal{E}\}$. The degree matrix of $\mathcal{G}$ is $\mathcal{D}=\operatorname{diag}\{d_1,d_2,\ldots,d_n\}$ where each diagonal element satisfies $d_i=|N_i|$. Then, the Laplacian matrix is defined as $L=\mathcal{D}-\mathcal{A}$. A sequence of ordered edges from node v_i to v_j is called a path. If there exists a path between any two nodes, the undirected $\mathcal{G}$ is called connected.

Lemma 1 ([24])**.** If the undirected graph $\mathcal{G}$ is connected, the following two properties hold:

(1) L always has a simple zero eigenvalue and its corresponding eigenvector is $\mathbf{1}$;

(2) The rest $N-1$ eigenvalues are real and positive.

3 Problem formulation

Consider a group of n agents moving in a complex plane with the position and velocity denoted by $r_i, v_i\in C$, respectively. Each agent can be regarded as a node and the information flow can be regarded as an edge of an undirected graph $\mathcal{G}$. Moreover, each agent is assumed to have the following dynamics:

$$\dot{r}_i=v_i,\quad \dot{v}_i=u_i, \tag{1}$$

where $u_i(t)\in C$ is the control input.

Remark 1. Complex systems are chosen to describe the agents' dynamics owing to their obvious physical meaning. For example, in our system, the position and velocity of each agent are both complex numbers, which can be geometrically represented in the complex plane. In this case, all agents are moving in a 2D plane, which coincides with the complex plane. Besides, in the complex plane, when a complex number is multiplied by j, it is rotated by $\pi/2$ radians counterclockwise. Consequently, the circle center of each agent can be expressed as $r_i(t)+w^{-1}\mathrm{j}v_i(t)$, where w is a constant to be specified later. Note that the stability analysis becomes more challenging when complex theory is involved.

This paper mainly focuses on the rotating consensus problem, which has been previously studied in [10]. Particularly, the rotating consensus problem is defined as follows.

Definition 1 ([10])**.** The multi-agent system (1) reaches a rotating consensus if

$$\lim_{t\to+\infty}[v_i(t)-v_m(t)]=0, \tag{2}$$

$$\lim_{t\to+\infty}\left[\left(r_i(t)+w^{-1}\mathrm{j}v_i(t)\right)-\left(r_m(t)+w^{-1}\mathrm{j}v_m(t)\right)\right]=0, \tag{3}$$

$$\lim_{t\to+\infty}[\dot{v}_i(t)-\mathrm{j}wv_i(t)]=0, \tag{4}$$

where $i,m\in\ell$. w is the constant angular velocity satisfying $0<w<+\infty$. Without any loss of generality, we can assume w to be 1.

Different from the control law in [10], where each agent needs continuous communication with other agents, we incorporate an event-based control strategy to the control law, such that the communication is only needed at the event times of the agent itself or its neighbors. Specifically, for agent i, the event-based controller is given by

$$u_i(t)=u_{i1}(t)+u_{i2}(t), \tag{5}$$

where

$$u_{i1}(t)=\mathrm{j}v_i(t),$$

and

$$u_{i2}(t)=-\sum_{m\in N_i}a_{im}\left[v_i\left(t_k^i\right)-v_m\left(t_{k'(t)}^m\right)\right]-\sum_{m\in N_i}a_{im}\left[r_i\left(t_k^i\right)+\mathrm{j}v_i\left(t_k^i\right)-\left(r_m\left(t_{k'(t)}^m\right)+\mathrm{j}v_m\left(t_{k'(t)}^m\right)\right)\right],$$

where $i \in l$ and $k'(t) \triangleq \arg\min_{l\in\mathbb{N}:t\geqslant t_l^m}\{t - t_l^m\}$. In other words, for each $t \in [t_k^i, t_{k+1}^i)$, the latest event time of agent j is $t_{k'(t)}^j$. Hence, the $u_{i2}(t)$ is updated depending on the agent's own event times or the latest information received from its neighbors. For each agent, the event time sequence $\{t_k^i\}$ is defined iteratively by

$$t_{k+1}^i = \inf\left\{t : t > t_k^i, f_i(t) > 0\right\}, \tag{6}$$

where the event-based function $f_i(t)$ is to be designed.

Remark 2. Note that the controller includes two parts. The first part of the control law involves agent's own velocity to guarantee that each agent revolves around the center of a circle. The second part requires information from its neighbors only at event times. This information is used to guarantee that the trajectories of all agents reach the rotating consensus.

This paper aims to find the distributed controller and event-based function, which determine the event times so that the rotating consensus problem from Definition 1 can be solved. The following assumption and lemma are needed to obtain the results.

Assumption 1. The undirected graph $\mathcal{G}$ is connected.

Lemma 2. Under Assumption 1, consider a linear system given by

$$\dot{x}(t) = \bar{A}x(t), \quad x(0) = x_0,$$

where $\bar{A} = \mathrm{diag}\left\{C - \lambda_2 D, \ldots, C - \lambda_n D\right\}$, with $C = \left[\begin{smallmatrix} \mathrm{j} & -\mathrm{j} \\ 0 & 0 \end{smallmatrix}\right], D = \left[\begin{smallmatrix} 0 & 0 \\ -1 & 1+\mathrm{j} \end{smallmatrix}\right]$ and $\lambda_i, i = 2, \ldots, n$ being the nonzero eigenvalues of $\mathcal{G}$ satisfying $0 < \lambda_2(\mathcal{G}) \leqslant \cdots \leqslant \lambda_n(\mathcal{G})$. Then, for $t \geqslant t_0$, we can find positive constants $M > 0$ and $\rho > 0$ such that

$$\left\|\mathrm{e}^{\bar{A}(t-t_0)}\right\| \leqslant M\mathrm{e}^{-\rho(t-t_0)}. \tag{7}$$

Proof. To find the eigenvalues of $\bar{A}$, we calculate the solutions to $\det(\mu I_{2n-2} - \bar{A}) = 0$, where μ is any eigenvalue of $\bar{A}$. The characteristic polynomial of matrix $\bar{A}$ is $\det(\mu I_{2n-2} - \bar{A}) = \prod_{i=1}^n \det(\mu I_2 - C + \lambda_i D) = \prod_{i=2}^n (\mu^2 + \lambda_i + \lambda_i \mathrm{j} - \mathrm{j})\mu + \lambda_i)$. It follows from Lemma 4 of [10] that $\bar{A}$ is Hurwitz. Hence, all eigenvalues of $\bar{A}$ have negative real parts. Thus, for all $t \geqslant t_0$, we can find positive constants $M > 0$ and $\rho > 0$, such that $\|\mathrm{e}^{\bar{A}(t-t_0)}\| \leqslant M\mathrm{e}^{-\rho(t-t_0)}$.

Remark 3. There exits an invertible matrix P such that $\bar{A} = P\bar{J}P^{-1}$, where $\bar{J}$ is in the Jordan normal form. In this case, it is easy to obtain that $\|\mathrm{e}^{\bar{A}(t-t_0)}\| = \|P\mathrm{e}^{\bar{J}(t-t_0)}P^{-1}\| \leqslant \|P\|\|P^{-1}\|\mathrm{e}^{\bar{J}(t-t_0)}\|$. Because $\bar{A}$ is Hurwitz, it can be shown that all eigenvalues of $\bar{A}$ have negative real parts, i.e., $\mathrm{Re}\,(\mu_{1,2}) = \mathrm{Re}(\frac{-h_i \pm \sqrt{h_i^2 - 4\lambda_i}}{2}) < 0, i \in \ell$ where $h_i = \lambda_i + \lambda_i \mathrm{j} - \mathrm{j}$. Then, it follows that $-\rho > \max_{i\in\ell}\{\mathrm{Re}(\frac{-h_i \pm \sqrt{h_i^2 - 4\lambda_i}}{2})\}$ and $M > \|P^{-1}\|\|P\|$.

4 Main results

To design the event-based function, we define two measurement errors as follows:

$$\begin{aligned} \varepsilon_i(t) &= r_i\left(t_k^i\right) - r_i(t), \\ e_i(t) &= v_i\left(t_k^i\right) - v_i(t), \quad t \in \left[t_k^i, t_{k+1}^i\right), \end{aligned} \tag{8}$$

where $\varepsilon_i(t)$ and $e_i(t)$ are the sampling-incurred measurement errors with respect to position and velocity, respectively.

This definition of $k'(t)$ implies that $r_m(t_{k'(t)}^m) = r_m(t) + \varepsilon_m(t)$ and $v_m(t_{k'(t)}^m) = v_m(t) + e_m(t)$. As a result,

$$\begin{aligned} u_{i2} = &- \sum_{m\in N_i} a_{im}\left[v_i(t) - v_m(t)\right] - \sum_{m\in N_i} a_{im}\left[e_i(t) - e_m(t)\right] \\ &- \sum_{m\in N_i} a_{im}\left[r_i(t) + \mathrm{j}v_i(t) - (r_m(t) + \mathrm{j}v_m(t))\right] - \sum_{m\in N_i} a_{im}\left[\varepsilon_i(t) + \mathrm{j}e_i(t) - (\varepsilon_m(t) + \mathrm{j}e_m(t))\right]. \end{aligned}$$

Define $c_i = r_i(t)+\mathrm{j}v_i(t)$, $\xi_i(t) = [r_i(t), c_i(t)]^{\mathrm{T}}$ and $\tilde{e}_i(t) = [\varepsilon_i(t), \varepsilon_i(t) + \mathrm{j}e_i(t)]^{\mathrm{T}}, \forall i \in \ell$. Then, we define $\xi(t) = [\xi_1(t), \ldots, \xi_n(t)]^{\mathrm{T}}$ and $\tilde{e}(t) = [\tilde{e}_1^{\mathrm{T}}(t), \ldots, \tilde{e}_n^{\mathrm{T}}(t)]^{\mathrm{T}}$. It follows from (1) and (5) that, the closed-loop system can be written as

$$\dot{\xi}(t) = (I_n \otimes A - L \otimes B)\, \xi(t) - (L \otimes B)\tilde{e}(t), \tag{9}$$

where $A = \left[\begin{smallmatrix} \mathrm{j} & -\mathrm{j} \\ 0 & 0 \end{smallmatrix}\right]$ and $B = \left[\begin{smallmatrix} 0 & 0 \\ -1 & 1+\mathrm{j} \end{smallmatrix}\right]$.

We define the disagreement vector as $\delta(t) = \xi(t) - \mathbf{1} \otimes \left[(\alpha(t), \alpha(t) + \mathrm{j}\beta(t)\right]^{\mathrm{T}},$ where $\alpha(t) = \frac{1}{n}\sum_{i=1}^{n} r_i(t)$ and $\beta(t) = \frac{1}{n}\sum_{i=1}^{n} v_i(t)$. Then $\left(\mathbf{1}^{\mathrm{T}} \otimes I_2\right)\delta(t) = 0$ and the dynamics of the disagreement system are given by

$$\dot{\delta}(t) = (I_n \otimes A - L \otimes B)\, \delta(t) - (L \otimes B)\tilde{e}(t). \tag{10}$$

Remark 4. Note that $\alpha(t)$ and $\beta(t)$ are used to represent the average values of all agents' positions and velocities, respectively. In this case, $\alpha(t) + \mathrm{j}\beta(t)$ is the average value of the centers of all agents' circles. Because $\mathcal{G}$ is an undirected graph, then $\dot{\alpha}(t) = \beta(t)$ and $\dot{\beta}(t) = \mathrm{j}\frac{1}{n}\sum_{i=1}^{n} v_i(t) = \mathrm{j}\beta(t)$. It can be obtained that $\dot{\alpha}(t) + \mathrm{j}\dot{\beta}(t) = 0$. Then $\alpha(t) + \mathrm{j}\beta(t)$ is an invariant quantity, which equals to $\frac{1}{n}\sum_{i=1}^{n} r_i(0) + \mathrm{j}\frac{1}{n}\sum_{i=1}^{n} v_i(0)$. In this case, $\alpha(t) + \mathrm{j}\beta(t)$ is also known as the common center for all agents if $\lim_{t\to+\infty} \delta(t) = 0$.

Finally, the main results of this paper are described below.

Theorem 1. Under Assumption 1, consider multi-agent system (1) with control law (5). If the event time sequence is determined by (6) with the following event-based function:

$$\begin{aligned} f_i(t) = \|\tilde{e}_i(t)\| &- \phi_1 \left\| \sum_{m\in N_i} a_{im}\left(r_i\left(t_k^i\right) - r_m\left(t_{k'(t)}^m\right)\right)\right\| \\ &- \phi_2 \left\| \sum_{m\in N_i} a_{im}\left(r_i\left(t_k^i\right) + \mathrm{j}v_i\left(t_k^i\right) - \left(r_m\left(t_{k'(t)}^m\right) + \mathrm{j}v_m\left(t_{k'(t)}^m\right)\right)\right)\right\| - \mathrm{e}^{-\lambda(t-t_0)}, \end{aligned}$$

where $\tilde{e}_i(t) = [\varepsilon_i(t), \varepsilon_i(t) + \mathrm{j}e_i(t)]^{\mathrm{T}}, 0 < \lambda < \rho$ and $\phi_1, \phi_2 \in (0, \frac{\rho-\lambda}{[M\|L\otimes B\|+(\rho-\lambda)]\sqrt{2N}(d+N)})$ with M, ρ defined in Lemma 2 and $d = \max_{i\in\ell}\{|N_i|\}$, then multi-agent system (1) can reach the rotating consensus with the common center at $\frac{1}{n}\sum_{i=1}^{n} r_i(0) + \mathrm{j}\frac{1}{n}\sum_{i=1}^{n} v_i(0)$.

Proof. Owing to the symmetry of L, there exists an orthogonal matrix $W \in \mathbb{R}^{n\times n}$ whose first column is $\frac{1}{\sqrt{n}}$, such that $W^{\mathrm{T}}LW = J = \operatorname{diag}\{0, \lambda_2, \ldots, \lambda_n\}$ where $0 < \lambda_2 \leqslant \cdots \leqslant \lambda_n$. It follows that

$$(W \otimes I_2)^{\mathrm{T}}\, \dot{\delta}(t) = (I_n \otimes A - J \otimes B)\,(W \otimes I_2)^{\mathrm{T}}\, \delta(t) - (J \otimes B)\,(W \otimes I_2)^{\mathrm{T}}\, \tilde{e}(t). \tag{11}$$

Then, we define $(W \otimes I_2)^{\mathrm{T}}\, \delta(t) = \left[0, \bar{\delta}^{\mathrm{T}}(t)\right]^{\mathrm{T}}$ and $(J \otimes B)\,(W \otimes I_2)^{\mathrm{T}}\, \tilde{e}(t) = \left[0, \bar{e}^{\mathrm{T}}(t)\right]^{\mathrm{T}}$, where $\bar{\delta}(t) \in C^{2n-2}$ and $\bar{e}(t) \in C^{2n-2}$. Eq. (11) can now be transformed into the following system:

$$\dot{\bar{\delta}}(t) = \operatorname{diag}\{A - \lambda_2 B, \ldots, A - \lambda_n B\}\, \bar{\delta}(t) - \bar{e}(t). \tag{12}$$

Through direct calculation, we have

$$\bar{\delta}(t) = \mathrm{e}^{\operatorname{diag}\{A-\lambda_2 B,\ldots,A-\lambda_n B\}(t-t_0)}\bar{\delta}\left(t_0\right) - \int_{t_0}^{t} \bar{e}(\tau)\mathrm{e}^{\operatorname{diag}\{A-\lambda_2 B,\ldots,A-\lambda_n B\}(t-\tau)}\mathrm{d}\tau.$$

It follows from Lemma 2 that there exist positive constants $M > 0$ and $\rho > 0$ such that

$$\left\|\mathrm{e}^{\operatorname{diag}\{A-\lambda_2 B,\ldots,A-\lambda_n B\}(t-t_0)}\right\| \leqslant M\mathrm{e}^{-\rho(t-t_0)}. \tag{13}$$

Invoking (13) and given that $\|\tilde{\delta}(t)\| = \|\delta(t)\|$, one can obtain

$$\|\delta(t)\| \leqslant M\mathrm{e}^{-\rho(t-t_0)}\left\|\delta\left(t_0\right)\right\| + M\int_{t_0}^{t} \mathrm{e}^{-\rho(t-\tau)}\|\bar{e}(\tau)\|\mathrm{d}\tau$$

$$\leqslant M\mathrm{e}^{-\rho(t-t_0)}\left\|\delta\left(t_0\right)\right\|+M\|J\otimes B\|\int_{t_0}^{t}\mathrm{e}^{-\rho(t-\tau)}\|\tilde{e}(\tau)\|\mathrm{d}\tau, \tag{14}$$

where $\|\bar{e}(\tau)\|\leqslant\|J\otimes B\|\tilde{e}(t)\|$.

The event-based condition (6) enforces that

$$\begin{aligned}\|\tilde{e}_i(t)\| \leqslant{}& \phi_1\left\|\sum_{m\in N_i}a_{im}\left(r_i(t)-r_m(t)\right)\right\|+\phi_2\left\|\sum_{m\in N_i}a_{im}\left(r_i(t)+\mathrm{j}v_i(t)-\left(r_m(t)+\mathrm{j}v_m(t)\right)\right)\right\| \\ &+\phi_1\left\|\sum_{m\in N_i}a_{im}\left(\varepsilon_i(t)-\varepsilon_m(t)\right)\right\|+\phi_2\left\|\sum_{m\in N_i}a_{im}\left(\varepsilon_i(t)+\mathrm{j}e_i(t)-\left(\varepsilon_m(t)+\mathrm{j}e_m(t)\right)\right)\right\|+\mathrm{e}^{-\lambda(t-t_0)} \\ \leqslant{}& \sqrt{2}\phi\left\|\sum_{m\in N_i}a_{im}\left(\xi_i(t)-\xi_m(t)\right)\right\|+\sqrt{2}\phi\left\|\sum_{m\in N_i}a_{im}\left(\tilde{e}_i(t)-\tilde{e}_m(t)\right)\right\|+\mathrm{e}^{-\lambda(t-t_0)} \\ ={}& \sqrt{2}\phi\left\|\sum_{m\in N_i}a_{im}\left(\delta_i(t)-\delta_m(t)\right)\right\|+\sqrt{2}\phi\left\|\sum_{m\in N_i}a_{im}\left(\tilde{e}_i(t)-\tilde{e}_m(t)\right)\right\|+\mathrm{e}^{-\lambda(t-t_0)} \\ \leqslant{}& \sqrt{2}\phi d_i\left\|\delta_i(t)\right\|+\sqrt{2}\phi\sum_{m=1}^{N}a_{im}\left\|\delta_m(t)\right\|+\sqrt{2}\phi d_i\left\|\tilde{e}_i(t)\right\|+\sqrt{2}\phi\sum_{m=1}^{N}a_{im}\left\|\tilde{e}_m(t)\right\|+\mathrm{e}^{-\lambda(t-t_0)},\end{aligned}$$

where $\phi=\max\left\{\phi_1,\phi_2\right\}$. Define $a=\max_{i,m\in\ell}\left\{a_{im}\right\}=1$ and $d=\max_{i\in\ell}\left\{d_i\right\}$ with $d_i=|N_i|$. Denote $\phi\in(0,\frac{\rho-\lambda}{[M\|L\otimes B\|+(\rho-\lambda)]\sqrt{2N}(d+N)})$. Owing to $\lambda\in(0,\rho)$, we obtain that $\frac{\rho-\lambda}{[M\|L\otimes B\|+(\rho-\lambda)]\sqrt{2N}(d+N)}<\frac{1}{\sqrt{2N}(d+N)}$. Then, it follows that

$$\|\tilde{e}(t)\|\leqslant\frac{\sqrt{2N}\phi(d+N)\|\delta(t)\|+N\mathrm{e}^{-\lambda(t-t_0)}}{1-\sqrt{2N}\phi(d+N)}. \tag{15}$$

Because $\|J\otimes B\|=\|L\otimes B\|$, it follows from (14) and (15) that

$$\|\delta(t)\|\leqslant M\mathrm{e}^{-\rho(t-t_0)}\left\|\delta\left(t_0\right)\right\|+Mc'\int_{t_0}^{t}\mathrm{e}^{-\rho(t-\tau)}\|\delta(\tau)\|\mathrm{d}\tau+M\alpha'\int_{t_0}^{t}\mathrm{e}^{-\rho(t-\tau)-\lambda(\tau-t_0)}\mathrm{d}\tau, \tag{16}$$

where $c'=\frac{\sqrt{2N}\phi(d+N)\|L\otimes B\|}{1-\sqrt{2N}\phi(d+N)}$ and $\alpha'=\frac{N\phi\|L\otimes B\|}{1-\sqrt{2N}\phi(d+N)}$.

Because $\phi\in(0,\frac{1}{[M\|L\otimes B\|+(\rho-\lambda)]\sqrt{2N}(d+N)})$, then $\frac{Mc'}{\rho-\lambda}<1$. We can claim that

$$\|\delta(t)\|\leqslant Z\mathrm{e}^{-\lambda(t-t_0)},\quad t>t_0, \tag{17}$$

where $Z=\max\{\frac{M\alpha'}{(\rho-\lambda)-Mc'},M\left\|\delta\left(t_0\right)\right\|\}$.

To prove (17), we first show that for any $\eta>1$, the following inequality is true:

$$\|\delta(t)\|<\eta Z\mathrm{e}^{-\lambda(t-t_0)}\doteq v(t). \tag{18}$$

Suppose (18) does not hold, there exists a $t^*>t_0$ such that $\left\|\delta\left(t^*\right)\right\|=v\left(t^*\right)$ and $\|\delta(t)\|<v(t)$ for $t\in(t_0,t^*)$. From (16) and (18) we have that

$$v\left(t^*\right)=\left\|\delta\left(t^*\right)\right\|\leqslant\eta M\left\|\delta\left(t_0\right)\right\|\mathrm{e}^{-\rho(t^*-t_0)}+\frac{\eta M\left(Zc'+\alpha'\right)}{\rho-\lambda}\left(\mathrm{e}^{-\lambda(t^*-t_0)}-\mathrm{e}^{-\rho(t^*-t_0)}\right). \tag{19}$$

Case 1. $Z=M\left\|\delta\left(t_0\right)\right\|$, which means that $\left\|\delta\left(t_0\right)\right\|\geqslant\frac{\alpha'}{-(Mc'-(\rho-\lambda))}$, that is, $\left[(\rho-\lambda)-Mc'\right]\left\|\delta\left(t_0\right)\right\|\geqslant\alpha'$. Then, we obtain that $\eta M\left\|\delta\left(t_0\right)\right\|-\eta M(\frac{Zc'+\alpha'}{\rho-\lambda})\geqslant0$. Because $\mathrm{e}^{-\lambda(t^*-t_0)}-\mathrm{e}^{-\rho(t^*-t_0)}>0$, inequality (19) can be transformed into the following form:

$$v\left(t^*\right)=\left\|\delta\left(t^*\right)\right\|\leqslant\eta M\left\|\delta\left(t_0\right)\right\|\mathrm{e}^{-\rho(t^*-t_0)}+\eta M\left\|\delta\left(t_0\right)\right\|\left(\mathrm{e}^{-\lambda(t^*-t_0)}-\mathrm{e}^{-\rho(t^*-t_0)}\right).$$

It implies that $\|\delta(t^*)\| < \eta M \|\delta(t_0)\| \mathrm{e}^{-\lambda(t^*-t_0)} = v(t^*)$.

Case 2. $Z = \frac{M\alpha'}{-(Mc'-(\rho-\lambda))}$, which means that $\|\delta(t_0)\| \leqslant \frac{\alpha'}{-(Mc'-(\rho-\lambda))}$, that is, $[(\rho-\lambda)-Mc']\|\delta(t_0)\| \leqslant \alpha'$. Then, we obtain that $\eta M\|\delta(t_0)\| - \eta M(\frac{Zc'+\alpha'}{\rho-\lambda}) \leqslant 0$. Because $\mathrm{e}^{-\lambda(t^*-t_0)} - \mathrm{e}^{-\rho(t^*-t_0)} > 0$, inequality (19) can be transformed into the following form:

$$v(t^*) = \|\delta(t^*)\| \leqslant \eta M\left(\frac{Zc'+\alpha'}{\rho-\lambda}\right)\mathrm{e}^{-\rho(t^*-t_0)} + \eta M\left(\frac{Zc'+\alpha'}{\rho-\lambda}\right)\left(\mathrm{e}^{-\lambda(t^*-t_0)} - \mathrm{e}^{-\rho(t^*-t_0)}\right).$$

It implies that $\|\delta(t^*)\| < \eta M(\frac{Zc'+\alpha'}{\rho-\lambda})\mathrm{e}^{-\lambda(t^*-t_0)} = v(t^*)$.

This contradiction shows that Eq. (17) holds for any $\eta > 1$. Therefore, letting $\eta \to 1$, the inequality (17) holds and $\delta(t)$ exponentially converges to zero. Then it follows that $\lim_{t\to+\infty}\|\delta(t)\| = 0$. It means that $\lim_{t\to+\infty}[r_i(t) - \alpha(t)] = 0$ and $\lim_{t\to+\infty}[(r_i(t) + \mathrm{j}v_i(t)) - (\alpha(t) + \mathrm{j}\beta(t))] = 0$ for any $i \in \ell$. Thus, the multi-agent system (1) with protocol (5) reaches a rotating consensus.

To guarantee that the event-based control law can be implemented in practice, it is essential to exclude Zeno behavior for each agent. Zeno behavior implies that the infinite number of events appears when the time interval between the previous and the next event time goes to zero. To exclude Zeno behavior, we summarize the following theorem.

Theorem 2. Under Assumption 1, consider a multi-agent system (1) with control law (5). If the event time sequence is determined by (6) as given in Theorem 1, then for each agent Zeno behavior can be strictly avoided.

Proof. First, we calculate the upper right hand Dini derivative. That is, $D^+\|\tilde{e}_i(t)\|$ over the interval $[t_k^i, t_{k+1}^i)$:

$$\begin{aligned} D^+\|\tilde{e}_i(t)\| &\leqslant \left\|\dot{\tilde{e}}_i(t)\right\| = \left\|\dot{\xi}_i(t)\right\| = \left\|\dot{\delta}_i(t)\right\| + \frac{1}{n}\left\|\sum_{i=1}^n v_i(0)\right\| \\ &\leqslant \|I_n \otimes A - L \otimes B\|\|\delta(t)\| + \|L \otimes B\|\|\tilde{e}_i(t)\| + \frac{1}{n}\left\|\sum_{i=1}^n v_i(0)\right\|. \end{aligned}$$

Using (14) and (15), it follows that

$$D^+\|\tilde{e}_i(t)\| \leqslant (\varphi_1 + \varphi_2)\mathrm{e}^{-\lambda(t-t_0)} + c,$$

where $\varphi_1 = \|I_n \otimes A - L \otimes B\| Z$, $\varphi_2 = \frac{\sqrt{2N}\phi(d+N)Z + N\mathrm{e}^{-\lambda(t-t_0)}}{1-\sqrt{2N}\phi(d+N)}$ and $c = \frac{1}{n}\|\sum_{i=1}^n v_i(0)\|$. Then, it can be shown that

$$\|\tilde{e}_i(t)\| \leqslant (\varphi_1 + \varphi_2)\left(\mathrm{e}^{-\lambda(t_k^i - t_0)} - \mathrm{e}^{-\lambda(t-t_0)}\right) + c\left(t_{k+1}^i - t_k^i\right), \quad t \in \left[t_k^i, t_{k+1}^i\right).$$

If the event-based function is greater than zero, the next event occurs, which implies that

$$\begin{aligned} &\phi_1\left\|\sum_{m\in N_i} a_{im}\left(r_i\left(t_k^i\right) - r_m\left(t_{k'(t)}^m\right)\right)\right\| + \mathrm{e}^{-\lambda\left(t_{k+1}^i - t_0\right)} \\ &\quad + \phi_2\left\|\sum_{m\in N_i} a_{im}\left(r_i\left(t_k^i\right) + \mathrm{j}v_i\left(t_k^i\right) - \left(r_m\left(t_{k'(t)}^m\right) + \mathrm{j}v_m\left(t_{k'(t)}^m\right)\right)\right)\right\| \\ &= \left\|\tilde{e}_i\left(t_{k+1}^i\right)\right\| \leqslant (\varphi_1 + \varphi_2)\left(\mathrm{e}^{-\lambda\left(t_k^i - t_0\right)} - \mathrm{e}^{-\lambda\left(t_{k+1}^i - t_0\right)}\right) + c\left(t_{k+1}^i - t_k^i\right). \end{aligned} \tag{20}$$

Define $T_{k+1}^i = t_{k+1}^i - t_k^i$ and $\hat{c} = c\mathrm{e}^{\lambda\left(t_k^i - t_0\right)}$. Using (20), we can obtain that

$$\mathrm{e}^{-\lambda T_{k+1}^i} \leqslant (\varphi_1 + \varphi_2)\left(1 - \mathrm{e}^{-\lambda T_{k+1}^i}\right) + \hat{c}T_{k+1}^i. \tag{21}$$

Define $f(x) = \hat{c}x - (1 + \varphi_1 + \varphi_2)\mathrm{e}^{-\lambda x} + (\varphi_1 + \varphi_2)$; then it follows from (21) that $f(x) \geqslant 0$ should have a positive solution. Given $f'(x) = \hat{c} + \lambda(1 + \varphi_1 + \varphi_2)\mathrm{e}^{-\lambda x} > 0, \forall x \in \mathbb{R}$, $f(0) = -(1 + \varphi_1 + \varphi_2) < 0$, and that $f(x)$ is monotonous, we can find that an $x' > 0$ makes $f(x') \geqslant 0$ hold. Therefore, $T_k^i \geqslant x' > 0$, which implies that each individual agent will not exhibit Zeno behavior.

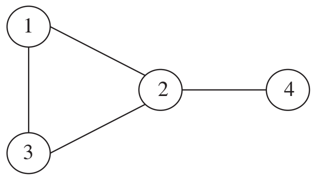

Figure 1 The communication graph of the multi-agent system.

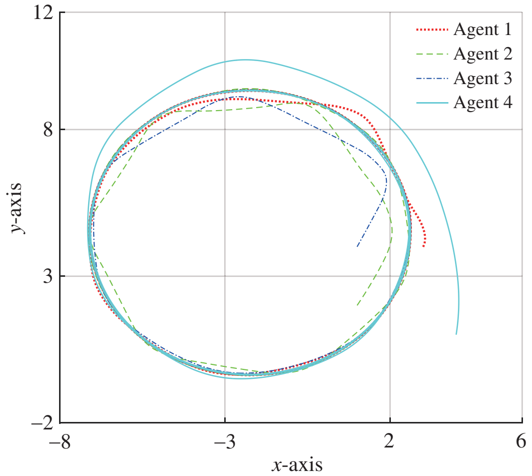

Figure 2 (Color online) The trajectory of four agents with $\phi_1 = 0.1$ and $\phi_2 = 0.05$.

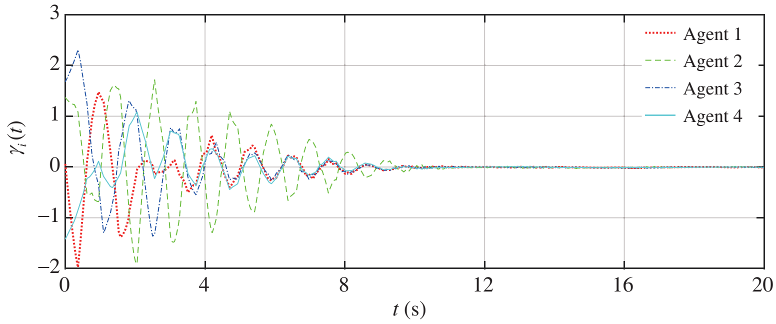

Figure 3 (Color online) The evolution of $\gamma_i(t)$ for all agents.

5 Numerical example

In this section, we carry out several numerical simulations to validate the effectiveness of our theoretical results. We assume that there are four agents moving in the plane, and their communication topology is described by Figure 1. The initial conditions are arbitrarily chosen as follows: $[r_1(0), v_1(0), \ldots, r_4(0), v_4(0)] = [3 + 4\mathrm{j}, 1 + 2\mathrm{j}, 1 + 2\mathrm{j}, 3 + 5\mathrm{j}, 1 + 4\mathrm{j}, 2 + 4\mathrm{j}, 4 + \mathrm{j}, 1 + 7\mathrm{j}]$.

According to Lemma 2, we choose $\rho = 0.5$ and $M = 2.6$ for this example. According to Theorem 1, the design parameters can be selected as follows: $\lambda = 0.05 < \rho = 0.5$, $\phi_1 = 0.1$ and $\phi_2 = 0.05 \in (0, 0.12)$. The trajectories of all agents are shown in Figure 2, where x-axis and y-axis represent the real and imaginary axes of the complex plane, respectively. Moreover, we denote $\gamma_i(t) = \|r_i(t) + \mathrm{j}v_i(t)\| - \|\alpha(t) + \mathrm{j}\beta(t)\|$ as the disagreement with respect to the common center for each agent. The evolution of $\gamma_i(t)$ with the proposed control law is shown in Figure 3, which demonstrates that the rotating consensus is reached. For comparison, the numbers of event times with the event-based control law and the periodic sampling control law are illustrated in Figures 4 and 5, respectively. It is observed that after incorporating the event-based control strategy, the multi-agent system still achieves the rotating consensus while the communication load is significantly reduced.

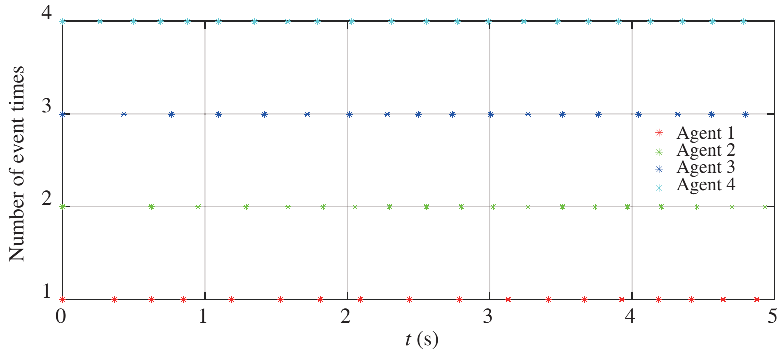

Figure 4 (Color online) The number of event times for all agents with $\phi_1 = 0.1$ and $\phi_2 = 0.05$.

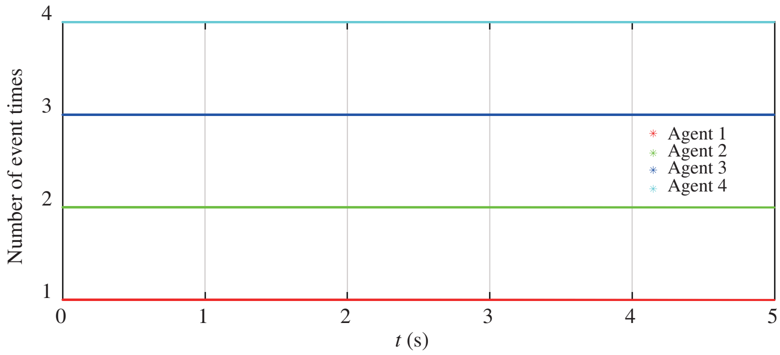

Figure 5 (Color online) The number of event times for all agents with sampling period $\tau = 0.001$.

6 Conclusion

In this paper, we address the rotating consensus problem for the double-integrator multi-agent system described by complex systems. We propose a distributed event-based law, which relies on the information at the event times of the agent itself and its neighbors such that the communication load can be reduced. Moreover, Zeno behavior is excluded for each agent. Future research will focus on the event-based rotating consensus problem for multi-agent systems with switching topologies and time delay.

Acknowledgements This work was supported in part by National Natural Science Foundation of China (Grant Nos. 61803392, 61790572, 618909302), China Postdoctoral Science Foundation in Central South University (Grant Nos. 2018M6-30912, 2019T120714), and Hunan Provincial Natural Science Foundation (Grant No. 2018JJ3685).

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