AIA Wavelet Problem

Yannick PORTO

Index Terms—**Wavelet, 2D, gaussian white noise**

I. Introduction

THE wavelet decomposition owe its creation to the drawbacks of the Fourier transform. As Fourier does not embed any notion of chronology, a new transform has to take place. That is why the Wavelet transform uses some filter banks and the frequency space to detect some phase and temporal information in the signal to analyze.

II. Decomposition

The decomposition is performed thanks to two kinds of filter : a low-pass filter and a high-pass filter built from the opposite one. Applying these filters in different orders leads to different information of the signal and different scales.

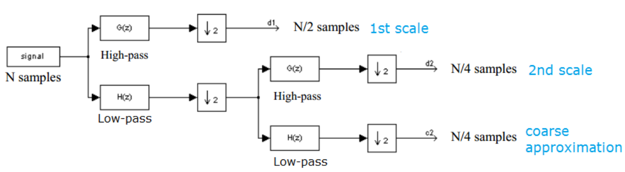

Fig. 1: Framework of the wavelet decomposition (1D)

What we obtain after two applications of a high-pass filter is a first scale, after one application of a low-pass filter and one application of a high-pass filter is a second scale, and after two applications of a low-pass filter is a coarse approximation.

In order to perform a wavelet decomposition on a 2D image, we need to apply this process once in horizontal for each row and once in vertical for each column. The vertical decomposition has to be applied on the result of the horizontal wavelet decomposition.

For multiple levels, the result of the next level has to be apply on the coarse approximation of the previous one. So for a 2D image, the result of the next level will always be located on the top left corner of the wavelet coefficients image.

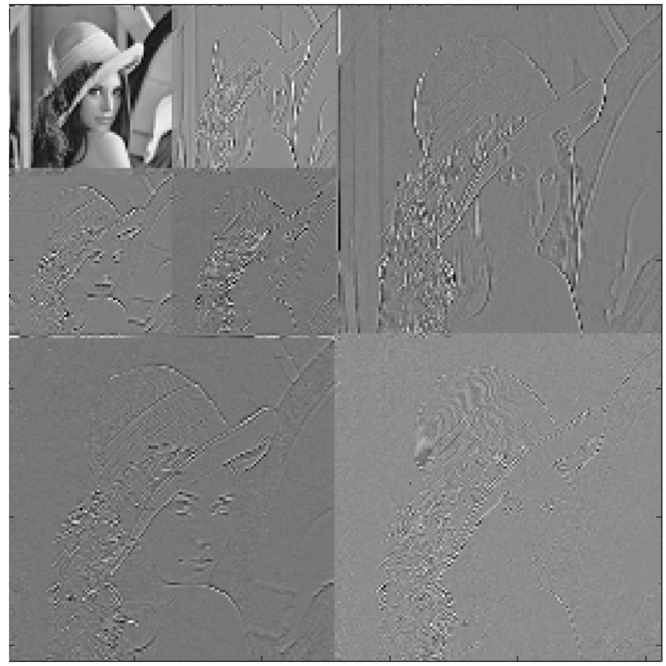

Fig. 2: Wavelet coefficients of the decomposition with 2 levels

We can therefore observe that we have different regions in the image of the coefficients. On the bottom right corner, the image displays the high frequencies of the original image. This is the first scale. On the top right corner and bottom left corner, we can see some lower frequencies which are the second scales in horizontal and vertical. Then we have a totally different region on the top left corner which represents the next level. This next level also have the same scales than the previous one, and if it is the last level, we can see the coarse approximation.

III. Reconstruction

The reconstruction is simply the reverse process of the decomposition using the coefficients. By applying upsampling and the same filters than before we can get the original image. We first reconstruct the last level of the decomposition and replace its coefficients by the reconstruction in the coefficient image, and so on until the first level.

Fig. 3: Reconstruction from 10% of the largest coefficients of the wavelet decomposition

By taking more or less coefficients we can reconstruct properly the original image. If we take all the coefficients, the reconstruction is perfect, but if the coefficients are too few, we can really see the degradation in the reconstructed image.

Fig. 4: Reconstruction from 5% of the largest coefficients of the wavelet decomposition

IV. Denoising

It is possible to detect a certain type of noise in an image using the wavelet transform. As we have information about frequency, a noise of the type of the white gaussian noise can easily be discarded.

Fig. 5: Additive white gaussian noise with standard deviation of 15

This kind of noise is really spread over the image but not in the frequency space. In this domain, it will take only the highest frequency part.

So by taking only the first and second scale of the decomposition and computing the standard deviation of their magnitude, we can keep enough information to remove all the noise. The number of standard deviations below which all the noise is removed is usually 3σ but we can tune this number in order to have a trade-off between the noise to remove and the image information to keep.

Using the hard thresholding, our experiments show that it is effectively a threshold of 3σ which is the more suitable to remove noise, in term of signal to noise ration (snr). Below this threshold, too much noise is left, and above this threshold the image is too much degraded (figure 6).

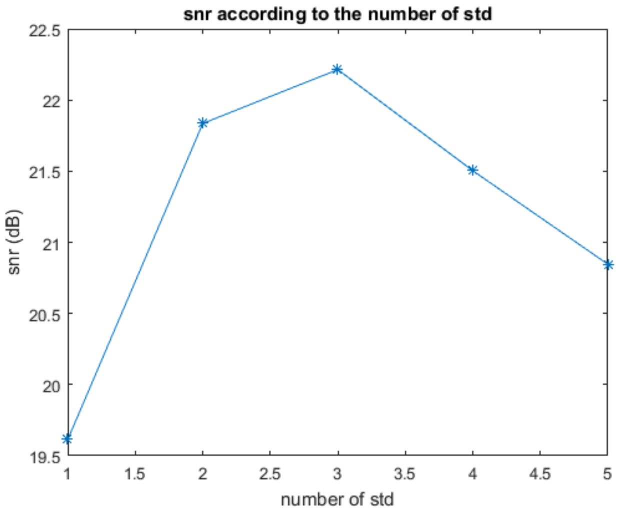

Fig. 6: SNR in function of the number of std for hard thresholding

Using a soft thresholding, it is showing that a number of 1σ is better to get a better result. After this threshold the snr is decreasing showing a bad reconstruction of the image.

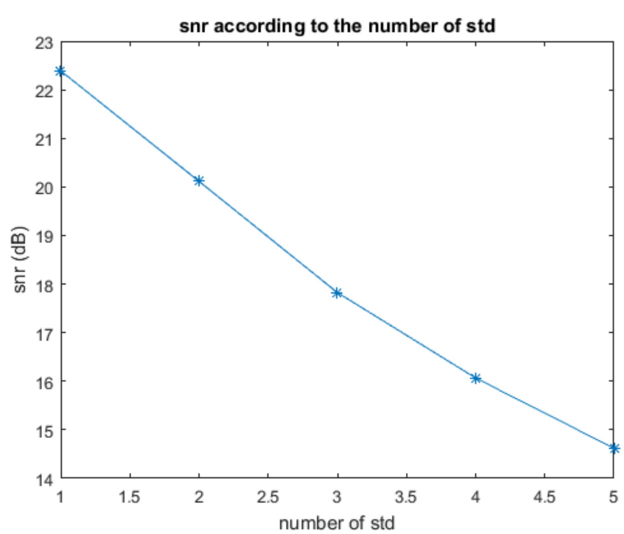

Fig. 7: SNR in function of the number of std for soft thresholding

V. Conclusion

Fig. 8: Reconstruction using the hard thresholding with a threshold of 3σ

VI. Conclusion

The wavelet decomposition in packet is a useful and powerful tool to get strong information about frequency and chronology of a 1D signal as well as a 2D image. Using it, we can compress an image easily using few coefficients of its decomposition. In order to reconstruct the original image, the only low-pass filter analysis is needed, what is really light.