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Question 1 Return the period of the expansion sequence

1. This question can be categorised into 2 cases:
    a. Base 10
       Calculate a decimal by dividing the numerator by the denominator. To find quotient and remainder, I could use floor division to find quotient and use a modulus to find a remainder. Moreover, I need to multiply remainder by 10 to repeatedly find remainders (Stack Overflow 2015).

    b. Other Bases
       After dividing the numerator by the denominator, use decimal to multiply with base. The digit is the integer part, then repeatedly multiply the result (without the integer part) until the result is zero or reaching desired length (Mad for Math n.d.).
       Example: 7/11 =0.6363…

       $(0.6363… * 4) = 2.5454…$ -> decimal = 0.2
       $(0.5454… * 4) = 2.1818…$ -> decimal = 0.22
       $(0.1818… * 4) = 0.7272…$ -> decimal = 0.220
       $(0.7272… * 4) = 2.9090…$ -> decimal = 0.2202
       $(0.9090… * 4) = 3.6363…$ -> decimal = 0.22023

2. The period of the sequence can be found by subtracting the length of a decimal by the position of the last decimal place before it keep repeating itself and number 2 (len of the string before a sequence starts, '0.').
   Example: 7/11 (base 4) = 0.22023… and the next digit is the repeating digit which is 2
       The length of a decimal = 7
       The position of the last decimal place (0.6363…) = 0
       Number 2 (len('0.')) = 2
       So the period is 5

Question 2 Return the sequence of the expansion digits

After calculating the rational number, it can be included in the sequence of the expansion digits. If the length of the decimal is less than n_steps then the decimal sequence is followed by zero.

Question 3 Create a walk which a rational

According to the sequence of expansion, it can be used to plot a walk. The directions of the walk are as follows: {0: "East", 1: "North", 2: "West", 3: "South"} (COMP1730/6730 2023 Semester 1, Project-1 (Mathematics) 2023).

- X coordinate moves to the right when the sequence is 0
- Y coordinate moves up when the sequence is 1
- X coordinate moves to the left when the sequence is 2
- Y coordinate moves down when the sequence is 3

Question 4 Return a 2-tuple of mutually prime integers to uniquely represent a rational

This question can be categorised into six cases		Prefix		
		Other prefix sequences	Empty sequence ( )	Sequence is (0,)
Period	Other period sequences	Case 1	Case 2	Case 3
	Sequence is (0,)	Case 4	Case 5	Case 6

1. Given the prefix and period sequence, they are used to create the decimal by appending prefix and period (case 1,2,3,4). If they are case 5 and 6 it returns zero because all digits are zero.
2. According to Calculator Soup (n.d.), the decimal needed to be multiplied by 10^(len(prefix)) for case 1,3,4. Whereas case2, decimal needed is multiplied by 10^(len(period_sequence)) to get a decimal with period sequence only.

   <u>Example</u>: Case 1,3,4) 0.113333… * 10^(2) = 11.3333…

         Case 2) 0.2525… * 10^(2) = 25.2525…

3. Convert a decimal number to fraction
   - For case 4, the denominator is 10^(len(prefix))
     <u>Example</u>: numerator = 25, denominator = 10^(2)
   - For case 1,2,3 we need to calculate sequence to find numerator and denominator.
     - For case 3, the new_decimal_of_sequence is sliced using [len(period_sequence) + 2:] where number 2 is the length of '0.'.
     - For case 1 and 2, it is number 1 because it is the length of '.' plus the length of period_sequence or prefix.
   - In each case the numerator and denominator are calculated as follows:
     <u>Example</u>: case 1) Given prefix = (1,1) period = (3,)
          Numerator = (11.333… * 100) - 11.333 = 1122
          Denominator = 9900
        case 2) Given prefix = () period = (2,5)
          Numerator = 25.2525… - 0.2525… = 25
          Denominator = 99
        case 3) Given prefix = (0,) period = (2,5)
          Numerator = (0.2525… * 100) – 0.2525 = 25
          Denominator = 990
        case 4) Given prefix = (2,5) period = (0,)
   Numerator = (0.25* 100) = 25
          Denominator = 100
   - Initially, I find GCD by dividing the number and storing it in the list. However, when it comes to larger numbers the program takes a long time to generate results. To solve this problem, I decided to use math.gcd() method to find the GCD between two numbers (w3schools).
   - To get a rational number, divide the numerator and denominator by GCD

Question 5 Compute rational which generate sample walk

A user can select one of the sample walks that are 6x6 Square, Letter T, and Heart. To generate a specific walk and create a full picture, the number of steps (n_steps) of each walk is 24 steps.

6x6 Square, n_steps = 24   Letter T, n_steps = 24   Heart, n_steps = 24
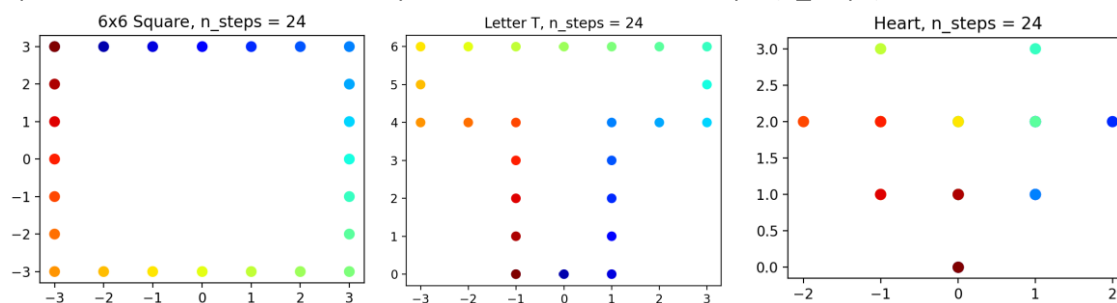
- Since it is the sequence of digits in base 4. The directions of the walk are as follows: {0: "East", 1: "North", 2: "West", 3: "South"} (COMP1730/6730 2023 Semester 1, Project-1 (Mathematics) 2023).
- After generating a walk, the directions can be converted back to decimal base 10. And converting back to base 10 by dividing 4^(place value) and adding the products.
  <u>Example</u>: 1) 6x6 square: [0, 0, 0, 0, 0, 0, 3, 3, 3, 3, 3, 3, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1]

$$\frac{0}{4^1} + \frac{0}{4^2} + \cdots + \frac{1}{4^{24}}$$

2) Letter T: [0, 0, 1, 1, 1, 1, 0, 0, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 0, 0, 3, 3, 3, 3]

$$\frac{0}{4^1} + \frac{0}{4^2} + \cdots + \frac{3}{4^{24}}$$

3) Heart: [1, 0, 1, 0, 2, 3, 2, 1, 0, 1, 3, 2, 2, 1, 3, 0, 3, 2, 1, 2, 0, 3, 0, 3]

$$\frac{1}{4^1} + \frac{0}{4^2} + \cdots + \frac{3}{4^{24}}$$

- Then multiply the result by 10^(number of place value) to get numerator, while the denominator is 10^(number of place value).
  <u>Example</u>: numerator of 6x6 square = 0.00024412075193325222 * 10^(20)
              denominator = 10^(20)
- Consequently, we need to find GCD and the numerator and denominator are divided by GCD.
- Fraction of 6x6 square: $\frac{3051509399165653}{12500000000000000000}$
- Fraction of Letter T: $\frac{5189339335855969}{250000000000000000}$
- Fraction of Heart: $\frac{6711242516171279}{25000000000000000}$

To execute program, __name__ == "__main__" is used to run question 1 to question5 codes. Where a user can change the input such as fraction, base, n_steps, prefix (could be () or (0,)) period_sequence (could be () or (0,)).

Reference list

— n.d., DECIMAL TO BASE 4 CONVERTER (WITH STEPS), madformath.com, viewed 1 May 2023, <https://madformath.com/calculators/basic-math/base-converters/decimal-to-base-4-converter-with-steps/decimal-to-base-4-converter-with-steps>.

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Stack Overflow. (n.d.). Implementing long division using a generator function in python. [online] Available at: https://stackoverflow.com/questions/29424349/implementing-long-division-using-a-generator-function-in-python [Accessed 27 April 2023].