

TD preuves en logique du premier ordre

David Delahaye

Faculté des Sciences
David.Delahaye@lirmm.fr

Master Informatique M2 2021-2022

Exercices en logique propositionnelle

Propositions à démontrer

- ❶ $A \Rightarrow B \Rightarrow A$
- ❷ $(A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C$
- ❸ $A \wedge B \Rightarrow B$
- ❹ $B \Rightarrow A \vee B$
- ❺ $(A \vee B) \Rightarrow (A \Rightarrow C) \Rightarrow (B \Rightarrow C) \Rightarrow C$
- ❻ $A \Rightarrow \perp \Rightarrow \neg A$
- ❼ $\perp \Rightarrow A$
- ❽ $(A \Leftrightarrow B) \Rightarrow A \Rightarrow B$
- ❾ $(A \Leftrightarrow B) \Rightarrow B \Rightarrow A$
- ❿ $(A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)$

Correction

Preuve (1) dans LJ/LK

▸ Règles LJ

$$\begin{array}{c} A, B \vdash A \\ A \vdash B \Rightarrow A \\ \vdash A \Rightarrow B \Rightarrow A \end{array}$$

Correction

Preuve (1) dans LJ/LK

► Règles LJ

$$\frac{A, B \vdash A}{\vdash A \Rightarrow B \Rightarrow A} \Rightarrow_{\text{right}}$$

Preuve (1) dans LJ/LK

► Règles LJ

$$\frac{\frac{A, B \vdash A}{A \vdash B \Rightarrow A} \Rightarrow_{\text{right}}}{\vdash A \Rightarrow B \Rightarrow A} \Rightarrow_{\text{right}}$$

Preuve (1) dans LJ/LK

► Règles LJ

$$\frac{\frac{\overline{A, B \vdash A} \text{ ax}}{A \vdash B \Rightarrow A} \Rightarrow_{\text{right}}}{\vdash A \Rightarrow B \Rightarrow A} \Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

► Règles LJ

$$\begin{array}{c} \Gamma \vdash A \qquad \Gamma, B, B \Rightarrow C \vdash B \qquad \Gamma, B, B \Rightarrow C, C \vdash C \\ \Gamma, B \vdash A \qquad \Gamma, B, B \Rightarrow C \vdash C \\ \Gamma \vdash A \qquad \Gamma, B \vdash C \\ \Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C \\ A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C \\ A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C \\ \vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C \end{array}$$

Correction

Preuve (2) dans LJ

► Règles LJ

$$\frac{\begin{array}{c} \Gamma \vdash A \\ \Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C \\ A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C \\ A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C \end{array} \quad \begin{array}{c} \Gamma, B, B \Rightarrow C \vdash B \quad \Gamma, B, B \Rightarrow C, C \vdash C \\ \Gamma, B, B \Rightarrow C \vdash C \\ \Gamma, B \vdash C \end{array}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

► Règles LJ

$$\frac{\frac{\frac{\Gamma \vdash A \quad \Gamma, B \vdash A}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C} \quad \frac{\frac{\Gamma, B, B \Rightarrow C \vdash B \quad \Gamma, B, B \Rightarrow C, C \vdash C}{\Gamma, B, B \Rightarrow C \vdash C}}{\Gamma, B \vdash C}}{\frac{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}} \Rightarrow_{\text{right}} \vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C$$

Correction

Preuve (2) dans LJ

► Règles LJ

$$\begin{array}{c} \Gamma \vdash A \qquad \Gamma, B \vdash A \qquad \Gamma, B, B \Rightarrow C \vdash B \qquad \Gamma, B, B \Rightarrow C, C \vdash C \\ \Gamma \vdash C \qquad \Gamma, B \vdash C \qquad \Gamma, B, B \Rightarrow C \vdash C \\ \hline \frac{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C} \Rightarrow_{\text{right}} \\ \hline \frac{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}} \\ \hline \vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\frac{\frac{\frac{\Gamma \vdash A}{\Gamma \Rightarrow A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C} \Rightarrow_{\text{left}} \quad \frac{\Gamma, B \vdash C}{\Gamma, B, B \Rightarrow C, C \vdash C} \Rightarrow_{\text{right}}}{\frac{\Gamma, B \vdash A \quad \Gamma, B, B \Rightarrow C \vdash C}{\Gamma, B, B \Rightarrow C \vdash B} \Rightarrow_{\text{right}}}{\frac{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\begin{array}{c} \frac{}{\Gamma \vdash A} \text{ax} \qquad \frac{}{\Gamma, B \vdash A} \text{ax} \qquad \frac{}{\Gamma, B, B \Rightarrow C \vdash B} \text{ax} \qquad \frac{}{\Gamma, B, B \Rightarrow C, C \vdash C} \text{ax} \\ \frac{\Gamma \vdash A \quad \Gamma, B \vdash C}{\Gamma \vdash A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C} \Rightarrow_{\text{left}} \\ \frac{\Gamma \vdash A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C} \Rightarrow_{\text{right}} \\ \frac{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}} \\ \frac{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{\Gamma \vdash A}{\Gamma \vdash A} \text{ ax}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C} \Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C} \Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}}{\frac{\frac{\Gamma, B \vdash A}{\Gamma, B \vdash C} \Rightarrow_{\text{left}} \quad \frac{\Gamma, B, B \Rightarrow C \vdash B \quad \Gamma, B, B \Rightarrow C, C \vdash C}{\Gamma, B, B \Rightarrow C \vdash C} \Rightarrow_{\text{left}}}{\Gamma, B \vdash C} \Rightarrow_{\text{left}}}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{\frac{\Gamma \vdash A}{\Gamma \vdash A} \text{ ax}}{\Gamma, B \vdash A} \text{ ax}}{\Gamma, B, B \Rightarrow C \vdash B} \text{ ax}}{\Gamma, B, B \Rightarrow C, C \vdash C} \text{ ax}}{\Gamma, B \vdash C} \Rightarrow_{\text{left}}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C} \Rightarrow_{\text{left}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C} \Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

► Règles LJ

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{\frac{\Gamma \vdash A}{\Gamma \vdash A} \text{ ax}}{\Gamma, B \vdash A} \text{ ax}}{\Gamma, B, B \Rightarrow C \vdash B} \text{ ax}}{\Gamma, B, B \Rightarrow C, C \vdash C} \Rightarrow \text{left}}{\Gamma, B, B \Rightarrow C \vdash C} \Rightarrow \text{left}}{\Gamma, B \vdash C} \Rightarrow \text{left}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C} \Rightarrow \text{right}}{\frac{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow \text{right}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow \text{right}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{\frac{\Gamma \vdash A}{\Gamma \vdash A} \text{ ax}}{\Gamma, B \vdash A} \text{ ax}}{\Gamma, B, B \Rightarrow C \vdash B} \text{ ax}}{\Gamma, B, B \Rightarrow C, C \vdash C} \text{ ax}}{\Gamma, B, B \Rightarrow C \vdash C} \Rightarrow_{\text{left}}}{\Gamma, B \vdash C} \Rightarrow_{\text{left}}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C} \Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C} \Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}$$

Correction

Preuve (2) dans LK

► Règles LK

$$\begin{array}{c} \Gamma \vdash C, A \qquad \Gamma, B \vdash C, A \qquad \Gamma, B, B \Rightarrow C \vdash C, B \qquad \Gamma, B, B \Rightarrow C, C \vdash C \\ \Gamma, B \vdash C, A \qquad \Gamma, B \vdash C \qquad \Gamma, B, B \Rightarrow C \vdash C \\ \Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C \\ A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C \\ A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C \\ \vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C \end{array}$$

Correction

Preuve (2) dans LK

► Règles LK

$$\frac{\begin{array}{c} \Gamma \vdash C, A \\ \Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C \\ A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C \\ A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C \end{array} \quad \frac{\Gamma, B, B \Rightarrow C \vdash C, B \quad \Gamma, B, B \Rightarrow C, C \vdash C}{\Gamma, B, B \Rightarrow C \vdash C} \quad \Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}$$

Correction

Preuve (2) dans LK

► Règles LK

$$\frac{\frac{\frac{\Gamma \vdash C, A \quad \Gamma, B \vdash C, A}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C} \quad \frac{\frac{\Gamma, B, B \Rightarrow C \vdash C, B \quad \Gamma, B, B \Rightarrow C, C \vdash C}{\Gamma, B, B \Rightarrow C \vdash C} \quad \Gamma, B \vdash C}{\frac{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

► Règles LK

$$\frac{\frac{\frac{\Gamma \vdash C, A \quad \Gamma, B \vdash C, A}{\Gamma, B \vdash C, A} \quad \frac{\Gamma, B, B \Rightarrow C \vdash C, B \quad \Gamma, B, B \Rightarrow C, C \vdash C}{\Gamma, B, B \Rightarrow C \vdash C}}{\Gamma, B \vdash C} \Rightarrow_{\text{right}} \quad \frac{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C} \Rightarrow_{\text{right}}}{\frac{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\frac{\frac{\frac{\Gamma \vdash C, A}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C} \Rightarrow_{\text{left}} \quad \frac{\frac{\frac{\frac{\Gamma, B, B \Rightarrow C \vdash C, B}{\Gamma, B, B \Rightarrow C, C \vdash C} \Rightarrow_{\text{right}}}{\Gamma, B, B \Rightarrow C \vdash C} \Rightarrow_{\text{right}}}{\Gamma, B \vdash C} \Rightarrow_{\text{right}}}{\Gamma \vdash C, A} \Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\frac{\frac{\frac{\frac{\frac{\Gamma \vdash C, A}{\Gamma \vdash C, A} \text{ ax}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C} \Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C} \Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}} \quad \frac{\Gamma, B \vdash C, A \quad \frac{\frac{\Gamma, B, B \Rightarrow C \vdash C, B \quad \Gamma, B, B \Rightarrow C, C \vdash C}{\Gamma, B, B \Rightarrow C \vdash C} \Rightarrow_{\text{left}}}{\Gamma, B \vdash C} \Rightarrow_{\text{left}}}{\Gamma, B \vdash C} \Rightarrow_{\text{left}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\frac{\frac{\frac{\frac{\frac{\frac{\Gamma \vdash C, A}{\Gamma \vdash C, A} \text{ ax}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C} \Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C} \Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}}{\frac{\frac{\Gamma, B \vdash C, A}{\Gamma, B \vdash C} \Rightarrow_{\text{left}} \quad \frac{\Gamma, B, B \Rightarrow C \vdash C, B \quad \Gamma, B, B \Rightarrow C, C \vdash C}{\Gamma, B, B \Rightarrow C \vdash C} \Rightarrow_{\text{left}}}{\Gamma, B \vdash C} \Rightarrow_{\text{left}}}$$

Correction

Preuve (2) dans LK

► Règles LK

$$\begin{array}{c}
 \frac{}{\Gamma \vdash C, A} \text{ ax} \quad \frac{}{\Gamma, B \vdash C, A} \text{ ax} \quad \frac{}{\Gamma, B, B \Rightarrow C \vdash C, B} \quad \frac{}{\Gamma, B, B \Rightarrow C, C \vdash C} \\
 \frac{}{\Gamma \vdash C, A} \text{ ax} \quad \frac{\frac{}{\Gamma, B \vdash C, A} \text{ ax} \quad \frac{}{\Gamma, B, B \Rightarrow C \vdash C, B} \quad \frac{}{\Gamma, B, B \Rightarrow C, C \vdash C}}{\Gamma, B \vdash C} \Rightarrow_{\text{left}} \\
 \frac{\frac{\frac{}{\Gamma \vdash C, A} \text{ ax} \quad \frac{}{\Gamma, B \vdash C} \Rightarrow_{\text{left}}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C} \Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C} \Rightarrow_{\text{right}} \\
 \frac{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}
 \end{array}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\frac{\frac{\frac{\frac{\frac{\frac{\frac{\Gamma \vdash C, A}{\Gamma \vdash C, A} \text{ ax}}{\Gamma, B \vdash C, A} \text{ ax}}{\Gamma, B \vdash C} \Rightarrow_{\text{left}}}{\Gamma, B, B \Rightarrow C \vdash C, B} \quad \frac{\Gamma, B, B \Rightarrow C, C \vdash C}{\Gamma, B, B \Rightarrow C \vdash C} \Rightarrow_{\text{left}}}{\Gamma, B \vdash C} \Rightarrow_{\text{left}}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C} \Rightarrow_{\text{right}}}{\frac{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\begin{array}{c}
 \frac{}{\Gamma \vdash C, A} \text{ax} \quad \frac{}{\Gamma, B \vdash C, A} \text{ax} \quad \frac{}{\Gamma, B, B \Rightarrow C \vdash C, B} \text{ax} \quad \frac{}{\Gamma, B, B \Rightarrow C, C \vdash C} \Rightarrow_{\text{left}} \\
 \frac{}{\Gamma, B \vdash C} \Rightarrow_{\text{left}} \quad \frac{}{\Gamma, B, B \Rightarrow C \vdash C} \Rightarrow_{\text{left}} \\
 \frac{}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C} \Rightarrow_{\text{right}} \\
 \frac{}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C} \Rightarrow_{\text{right}} \\
 \frac{}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}} \\
 \frac{}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}
 \end{array}$$

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\begin{array}{c} A, B \vdash B \\ A \wedge B \vdash B \\ \hline \vdash A \wedge B \Rightarrow B \end{array}$$

Correction

Preuve (3) dans LJ/LK

► Règles LJ

$$\frac{A, B \vdash B}{\vdash A \wedge B \Rightarrow B} \Rightarrow_{\text{right}}$$

Preuve (3) dans LJ/LK

► Règles LJ

$$\frac{\frac{A, B \vdash B}{A \wedge B \vdash B} \wedge_{\text{left}}}{\vdash A \wedge B \Rightarrow B} \Rightarrow_{\text{right}}$$

Preuve (3) dans LJ/LK

► Règles LJ

$$\frac{\frac{\overline{A, B \vdash B}^{\text{ax}}}{A \wedge B \vdash B}^{\wedge_{\text{left}}}}{\vdash A \wedge B \Rightarrow B}^{\Rightarrow_{\text{right}}}$$

Preuve (4) dans LJ

▸ Règles LJ

$$\begin{array}{c} B \vdash B \\ B \vdash A \vee B \\ \vdash B \Rightarrow A \vee B \end{array}$$

Correction

Preuve (4) dans LJ

► Règles LJ

$$\frac{B \vdash B \quad B \vdash A \vee B}{\vdash B \Rightarrow A \vee B} \Rightarrow_{\text{right}}$$

Correction

Preuve (4) dans LJ

► Règles LJ

$$\frac{\frac{B \vdash B}{B \vdash A \vee B} \vee_{\text{right2}}}{\vdash B \Rightarrow A \vee B} \Rightarrow_{\text{right}}$$

Correction

Preuve (4) dans LJ

▸ Règles LJ

$$\frac{\frac{\overline{B \vdash B}^{\text{ax}}}{B \vdash A \vee B} \vee_{\text{right2}}}{\vdash B \Rightarrow A \vee B} \Rightarrow_{\text{right}}$$

Preuve (4) dans LK

▸ Règles LK

$$\begin{array}{c} B \vdash A, B \\ B \vdash A \vee B \\ \vdash B \Rightarrow A \vee B \end{array}$$

Preuve (4) dans LK

► Règles LK

$$\frac{B \vdash A, B}{\vdash B \Rightarrow A \vee B} \Rightarrow_{\text{right}}$$

Preuve (4) dans LK

► Règles LK

$$\frac{\frac{B \vdash A, B}{B \vdash A \vee B} \vee_{\text{right}}}{\vdash B \Rightarrow A \vee B} \Rightarrow_{\text{right}}$$

Preuve (4) dans LK

► Règles LK

$$\frac{\frac{\overline{B \vdash A, B}^{\text{ax}}}{B \vdash A \vee B} \vee_{\text{right}}}{\vdash B \Rightarrow A \vee B} \Rightarrow_{\text{right}}$$

Preuve (6) dans LJ/LK

▸ Règles LJ

$$\begin{array}{l} A, \perp \vdash \neg A \\ A \vdash \perp \Rightarrow \neg A \\ \vdash A \Rightarrow \perp \Rightarrow \neg A \end{array}$$

Correction

Preuve (6) dans LJ/LK

► Règles LJ

$$\frac{A, \perp \vdash \neg A}{\vdash A \Rightarrow \perp \Rightarrow \neg A} \Rightarrow_{\text{right}}$$

Preuve (6) dans LJ/LK

► Règles LJ

$$\frac{\frac{A, \perp \vdash \neg A}{A \vdash \perp \Rightarrow \neg A} \Rightarrow_{\text{right}}}{\vdash A \Rightarrow \perp \Rightarrow \neg A} \Rightarrow_{\text{right}}$$

Preuve (6) dans LJ/LK

► Règles LJ

$$\frac{\frac{\overline{A, \perp \vdash \neg A} \quad \perp_{\text{left}}}{A \vdash \perp \Rightarrow \neg A} \Rightarrow_{\text{right}}}{\vdash A \Rightarrow \perp \Rightarrow \neg A} \Rightarrow_{\text{right}}$$

Correction

Preuve (7) dans LJ/LK

► Règles LJ

$$\frac{\perp \vdash A}{\vdash \perp \Rightarrow A}$$

Correction

Preuve (7) dans LJ/LK

► Règles LJ

$$\frac{\perp \vdash A}{\vdash \perp \Rightarrow A} \Rightarrow_{\text{right}}$$

Correction

Preuve (7) dans LJ/LK

► Règles LJ

$$\frac{\overline{\perp \vdash A} \text{ ax}}{\vdash \perp \Rightarrow A} \Rightarrow_{\text{right}}$$

Correction

Preuve (8) dans LJ

▸ Règles LJ

$$\begin{array}{c} A \vdash A \quad B \vdash B \\ A \Leftrightarrow B, A \vdash B \\ A \Leftrightarrow B \vdash A \Rightarrow B \\ \vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B \end{array}$$

Correction

Preuve (8) dans LJ

▸ Règles LJ

$$\frac{\begin{array}{c} A \vdash A \quad B \vdash B \\ A \Leftrightarrow B, A \vdash B \\ A \Leftrightarrow B \vdash A \Rightarrow B \end{array}}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B} \Rightarrow_{\text{right}}$$

Correction

Preuve (8) dans LJ

▸ Règles LJ

$$\frac{\frac{A \vdash A \quad B \vdash B}{A \Leftrightarrow B, A \vdash B} \Rightarrow_{\text{right}}}{A \Leftrightarrow B \vdash A \Rightarrow B} \Rightarrow_{\text{right}} \vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B$$

Correction

Preuve (8) dans LJ

▸ Règles LJ

$$\frac{\frac{\frac{A \vdash A \quad B \vdash B}{A \Leftrightarrow B, A \vdash B} \Leftrightarrow_{\text{left1}}}{A \Leftrightarrow B \vdash A \Rightarrow B} \Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B} \Rightarrow_{\text{right}}$$

Correction

Preuve (8) dans LJ

▸ Règles LJ

$$\frac{\frac{\frac{}{A \vdash A} \text{ax} \quad B \vdash B}{A \Leftrightarrow B, A \vdash B} \Leftrightarrow_{\text{left1}}}{\frac{A \Leftrightarrow B \vdash A \Rightarrow B}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B} \Rightarrow_{\text{right}} \Rightarrow_{\text{right}}}$$

Correction

Preuve (8) dans LJ

▸ Règles LJ

$$\frac{\frac{\frac{}{A \vdash A} \text{ax} \quad \frac{}{B \vdash B} \text{ax}}{A \Leftrightarrow B, A \vdash B} \Leftrightarrow_{\text{left1}}}{\frac{A \Leftrightarrow B \vdash A \Rightarrow B}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B} \Rightarrow_{\text{right}} \Rightarrow_{\text{right}}}$$

Preuve (8) dans LK

► Règles LK

$$\begin{array}{c} A \vdash B, A, B \quad A, A, B \vdash B \\ A \Leftrightarrow B, A \vdash B \\ A \Leftrightarrow B \vdash A \Rightarrow B \\ \vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B \end{array}$$

Correction

Preuve (8) dans LK

► Règles LK

$$\frac{\begin{array}{c} A \vdash B, A, B \quad A, A, B \vdash B \\ A \Leftrightarrow B, A \vdash B \\ A \Leftrightarrow B \vdash A \Rightarrow B \end{array}}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B} \Rightarrow_{\text{right}}$$

Correction

Preuve (8) dans LK

► Règles LK

$$\begin{array}{c} A \vdash B, A, B \quad A, A, B \vdash B \\ \hline A \Leftrightarrow B, A \vdash B \Rightarrow_{\text{right}} \\ \hline A \Leftrightarrow B \vdash A \Rightarrow B \Rightarrow_{\text{right}} \\ \hline \vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B \end{array}$$

Correction

Preuve (8) dans LK

▸ Règles LK

$$\frac{\frac{\frac{A \vdash B, A, B \quad A, A, B \vdash B}{A \Leftrightarrow B, A \vdash B} \Leftrightarrow_{\text{left}}}{A \Leftrightarrow B \vdash A \Rightarrow B} \Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B} \Rightarrow_{\text{right}}$$

Correction

Preuve (8) dans LK

► Règles LK

$$\frac{\frac{\frac{A \vdash B, A, B}{A \Leftrightarrow B, A \vdash B} \Rightarrow_{\text{right}}}{A \Leftrightarrow B \vdash A \Rightarrow B} \Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B} \Leftrightarrow_{\text{left}} \frac{\frac{A \vdash B, A, B}{A \Leftrightarrow B, A \vdash B} \Rightarrow_{\text{right}}}{A \Leftrightarrow B \vdash A \Rightarrow B} \Rightarrow_{\text{right}}$$

Correction

Preuve (8) dans LK

► Règles LK

$$\frac{\frac{\overline{A \vdash B, A, B} \text{ ax} \quad \overline{A, A, B \vdash B} \text{ ax}}{A \Leftrightarrow B, A \vdash B} \Leftrightarrow_{\text{left}}}{\frac{A \Leftrightarrow B, A \vdash B}{A \Leftrightarrow B \vdash A \Rightarrow B} \Rightarrow_{\text{right}}} \Rightarrow_{\text{right}} \vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B$$

Correction

Preuve (9) dans LJ

▸ Règles LJ

$$\begin{array}{c} B \vdash B \quad A \vdash A \\ A \Leftrightarrow B, B \vdash A \\ A \Leftrightarrow B \vdash B \Rightarrow A \\ \vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A \end{array}$$

Correction

Preuve (9) dans LJ

▸ Règles LJ

$$\frac{\begin{array}{c} B \vdash B \quad A \vdash A \\ A \Leftrightarrow B, B \vdash A \end{array}}{A \Leftrightarrow B \vdash B \Rightarrow A} \Rightarrow_{\text{right}} \vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A$$

Correction

Preuve (9) dans LJ

▸ Règles LJ

$$\frac{\frac{B \vdash B \quad A \vdash A}{A \Leftrightarrow B, B \vdash A} \Rightarrow_{\text{right}}}{A \Leftrightarrow B \vdash B \Rightarrow A} \Rightarrow_{\text{right}} \vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A$$

Correction

Preuve (9) dans LJ

▸ Règles LJ

$$\frac{\frac{\frac{B \vdash B \quad A \vdash A}{A \Leftrightarrow B, B \vdash A} \Leftrightarrow_{\text{left2}}}{A \Leftrightarrow B \vdash B \Rightarrow A} \Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A} \Rightarrow_{\text{right}}$$

Correction

Preuve (9) dans LJ

▸ Règles LJ

$$\frac{\frac{\frac{\overline{B \vdash B}^{\text{ax}} \quad A \vdash A}{A \Leftrightarrow B, B \vdash A} \Leftrightarrow_{\text{left2}}}{A \Leftrightarrow B \vdash B \Rightarrow A} \Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A} \Rightarrow_{\text{right}}$$

Correction

Preuve (9) dans LJ

▸ Règles LJ

$$\frac{\frac{\overline{B \vdash B} \text{ ax} \quad \overline{A \vdash A} \text{ ax}}{A \Leftrightarrow B, B \vdash A} \Leftrightarrow_{\text{left2}}}{\frac{A \Leftrightarrow B \vdash B \Rightarrow A}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A} \Rightarrow_{\text{right}} \Rightarrow_{\text{right}}}$$

Preuve (9) dans LK

► Règles LK

$$B \vdash A, A, B \quad B, A, B \vdash A$$

$$A \Leftrightarrow B, B \vdash A$$

$$A \Leftrightarrow B \vdash B \Rightarrow A$$

$$\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A$$

Correction

Preuve (9) dans LK

► Règles LK

$$\frac{\begin{array}{c} B \vdash A, A, B \quad B, A, B \vdash A \\ A \Leftrightarrow B, B \vdash A \\ A \Leftrightarrow B \vdash B \Rightarrow A \end{array}}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A} \Rightarrow_{\text{right}}$$

Correction

Preuve (9) dans LK

► Règles LK

$$\begin{array}{c} B \vdash A, A, B \quad B, A, B \vdash A \\ \frac{A \Leftrightarrow B, B \vdash A}{A \Leftrightarrow B \vdash B \Rightarrow A} \Rightarrow_{\text{right}} \\ \frac{A \Leftrightarrow B \vdash B \Rightarrow A}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A} \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (9) dans LK

▸ Règles LK

$$\frac{\frac{\frac{B \vdash A, A, B \quad B, A, B \vdash A}{A \Leftrightarrow B, B \vdash A} \Leftrightarrow_{\text{left}}}{A \Leftrightarrow B \vdash B \Rightarrow A} \Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A} \Rightarrow_{\text{right}}$$

Correction

Preuve (9) dans LK

▸ Règles LK

$$\frac{\frac{\frac{}{B \vdash A, A, B} \text{ax} \quad B, A, B \vdash A}{A \Leftrightarrow B, B \vdash A} \Leftrightarrow \text{left}}{\frac{A \Leftrightarrow B \vdash B \Rightarrow A}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A} \Rightarrow \text{right}} \Rightarrow \text{right}$$

Correction

Preuve (9) dans LK

► Règles LK

$$\frac{\frac{\overline{B \vdash A, A, B} \text{ ax} \quad \overline{B, A, B \vdash A} \text{ ax}}{A \Leftrightarrow B, B \vdash A} \Leftrightarrow \text{left}}{\frac{A \Leftrightarrow B \vdash B \Rightarrow A}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A} \Rightarrow \text{right}} \Rightarrow \text{right}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\begin{array}{cccc} B \Rightarrow A, A \vdash A & B \Rightarrow A, A, B \vdash B & A \Rightarrow B, B \vdash B & A \Rightarrow B, B, A \vdash A \\ A \Rightarrow B, B \Rightarrow A, A \vdash B & & A \Rightarrow B, B \Rightarrow A, B \vdash A & \\ A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B & & & \\ A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B) & & & \\ \vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B) & & & \end{array}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\begin{array}{cccc} B \Rightarrow A, A \vdash A & B \Rightarrow A, A, B \vdash B & A \Rightarrow B, B \vdash B & A \Rightarrow B, B, A \vdash A \\ A \Rightarrow B, B \Rightarrow A, A \vdash B & & A \Rightarrow B, B \Rightarrow A, B \vdash A & \\ A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B & & & \\ \hline \vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B) & \Rightarrow_{\text{right}} & & \end{array}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\begin{array}{c} B \Rightarrow A, A \vdash A \qquad B \Rightarrow A, A, B \vdash B \qquad A \Rightarrow B, B \vdash B \qquad A \Rightarrow B, B, A \vdash A \\ A \Rightarrow B, B \Rightarrow A, A \vdash B \qquad A \Rightarrow B, B \Rightarrow A, B \vdash A \\ \hline \frac{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}} \\ \hline \vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B) \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\frac{\begin{array}{c} B \Rightarrow A, A \vdash A \quad B \Rightarrow A, A, B \vdash B \quad A \Rightarrow B, B \vdash B \quad A \Rightarrow B, B, A \vdash A \\ A \Rightarrow B, B \Rightarrow A, A \vdash B \quad A \Rightarrow B, B \Rightarrow A, B \vdash A \end{array}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B} \Leftrightarrow_{\text{right}}$$
$$\frac{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}}$$
$$\frac{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\frac{\frac{B \Rightarrow A, A \vdash A \quad B \Rightarrow A, A, B \vdash B}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \Rightarrow_{\text{left}} \quad \frac{A \Rightarrow B, B \vdash B \quad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A} \Leftrightarrow_{\text{right}}}{\frac{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\begin{array}{c}
 \frac{}{B \Rightarrow A, A \vdash A} \text{ax} \quad \frac{B \Rightarrow A, A, B \vdash B}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \Rightarrow_{\text{left}} \quad \frac{A \Rightarrow B, B \vdash B \quad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A} \Leftrightarrow_{\text{right}} \\
 \frac{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}} \\
 \frac{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}}
 \end{array}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\begin{array}{c}
 \frac{}{B \Rightarrow A, A \vdash A} \text{ax} \quad \frac{}{B \Rightarrow A, A, B \vdash B} \text{ax} \quad A \Rightarrow B, B \vdash B \quad A \Rightarrow B, B, A \vdash A \\
 \hline
 \frac{}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \Rightarrow_{\text{left}} \quad \frac{}{A \Rightarrow B, B \Rightarrow A, B \vdash A} \Rightarrow_{\text{right}} \\
 \hline
 \frac{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}} \quad \frac{}{A \Rightarrow B, B \Rightarrow A, B \vdash A} \Leftrightarrow_{\text{right}} \\
 \hline
 \frac{}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}}
 \end{array}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\frac{\frac{B \Rightarrow A, A \vdash A}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \text{ax} \quad \frac{B \Rightarrow A, A, B \vdash B}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \text{ax}}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \Rightarrow \text{left} \quad \frac{A \Rightarrow B, B \vdash B \quad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A} \Rightarrow \text{left}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B} \Leftrightarrow \text{right}$$
$$\frac{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow \text{right}$$
$$\frac{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow \text{right}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\frac{\frac{\frac{}{B \Rightarrow A, A \vdash A} \text{ax}}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \Rightarrow \text{left} \quad \frac{\frac{\frac{}{B \Rightarrow A, A, B \vdash B} \text{ax}}{A \Rightarrow B, B \Rightarrow A, B \vdash A} \Rightarrow \text{left} \quad \frac{\frac{\frac{}{A \Rightarrow B, B \vdash B} \text{ax}}{A \Rightarrow B, B \Rightarrow A, B \vdash A} \Rightarrow \text{left}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B} \Leftrightarrow \text{right} \Rightarrow \text{right}}{\frac{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow \text{right}} \Rightarrow \text{right}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\begin{array}{c}
 \frac{}{B \Rightarrow A, A \vdash A} \text{ax} \quad \frac{}{B \Rightarrow A, A, B \vdash B} \text{ax} \quad \frac{}{A \Rightarrow B, B \vdash B} \text{ax} \quad \frac{}{A \Rightarrow B, B, A \vdash A} \text{ax} \\
 \frac{}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \Rightarrow_{\text{left}} \quad \frac{}{A \Rightarrow B, B \Rightarrow A, B \vdash A} \Rightarrow_{\text{left}} \\
 \frac{}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B} \Leftrightarrow_{\text{right}} \\
 \frac{}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}} \\
 \frac{}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}}
 \end{array}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$$\begin{array}{cccc} B \Rightarrow A, A \vdash B, A & B \Rightarrow A, A, B \vdash B & A \Rightarrow B, B \vdash A, B & A \Rightarrow B, B, A \vdash A \\ A \Rightarrow B, B \Rightarrow A, A \vdash B & & A \Rightarrow B, B \Rightarrow A, B \vdash A & \\ A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B & & & \\ A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B) & & & \\ \vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B) & & & \end{array}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$$\begin{array}{cccc} B \Rightarrow A, A \vdash B, A & B \Rightarrow A, A, B \vdash B & A \Rightarrow B, B \vdash A, B & A \Rightarrow B, B, A \vdash A \\ A \Rightarrow B, B \Rightarrow A, A \vdash B & & A \Rightarrow B, B \Rightarrow A, B \vdash A & \\ A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B & & & \\ \hline \vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B) & \Rightarrow_{\text{right}} & & \end{array}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$B \Rightarrow A, A \vdash B, A$

$B \Rightarrow A, A, B \vdash B$

$A \Rightarrow B, B \vdash A, B$

$A \Rightarrow B, B, A \vdash A$

$A \Rightarrow B, B \Rightarrow A, A \vdash B$

$A \Rightarrow B, B \Rightarrow A, B \vdash A$

$$\frac{\frac{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$$\frac{\frac{B \Rightarrow A, A \vdash B, A \quad B \Rightarrow A, A, B \vdash B \quad A \Rightarrow B, B \vdash A, B \quad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \quad \frac{A \Rightarrow B, B \Rightarrow A, B \vdash A}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B} \Leftrightarrow_{\text{right}}}{\frac{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$$\begin{array}{c}
 \frac{B \Rightarrow A, A \vdash B, A \quad B \Rightarrow A, A, B \vdash B}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \Rightarrow_{\text{left}} \quad \frac{A \Rightarrow B, B \vdash A, B \quad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A} \Leftrightarrow_{\text{right}} \\
 \frac{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}} \\
 \frac{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}}
 \end{array}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$$\begin{array}{c}
 \frac{}{B \Rightarrow A, A \vdash B, A} \text{ax} \quad \frac{B \Rightarrow A, A, B \vdash B}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \Rightarrow_{\text{left}} \quad \frac{A \Rightarrow B, B \vdash A, B \quad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A} \Leftrightarrow_{\text{right}} \\
 \frac{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}} \\
 \frac{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}}
 \end{array}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$$\begin{array}{c}
 \frac{}{B \Rightarrow A, A \vdash B, A} \text{ax} \quad \frac{}{B \Rightarrow A, A, B \vdash B} \text{ax} \\
 \hline
 A \Rightarrow B, B \Rightarrow A, A \vdash B \quad \Rightarrow_{\text{left}} \quad A \Rightarrow B, B \vdash A, B \quad A \Rightarrow B, B, A \vdash A \\
 \hline
 A \Rightarrow B, B \Rightarrow A, B \vdash A \quad \Leftrightarrow_{\text{right}} \\
 \hline
 A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B \quad \Rightarrow_{\text{right}} \\
 \hline
 A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B) \quad \Rightarrow_{\text{right}} \\
 \hline
 \vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)
 \end{array}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$$\begin{array}{c}
 \frac{}{B \Rightarrow A, A \vdash B, A} \text{ax} \quad \frac{}{B \Rightarrow A, A, B \vdash B} \text{ax} \quad \frac{A \Rightarrow B, B \vdash A, B \quad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A} \Rightarrow_{\text{left}} \Rightarrow_{\text{right}} \\
 \frac{}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \Rightarrow_{\text{left}} \quad \frac{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}} \\
 \frac{}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}}
 \end{array}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$$\frac{\frac{\frac{}{B \Rightarrow A, A \vdash B, A} \text{ ax} \quad \frac{}{B \Rightarrow A, A, B \vdash B} \text{ ax}}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \Rightarrow \text{left} \quad \frac{\frac{\frac{}{A \Rightarrow B, B \vdash A, B} \text{ ax} \quad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A} \Rightarrow \text{left}}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \Leftrightarrow \text{right}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B} \Rightarrow \text{right}$$
$$\frac{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow \text{right}$$
$$\frac{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow \text{right}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$$\frac{\frac{B \Rightarrow A, A \vdash B, A}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \text{ax}}{\frac{B \Rightarrow A, A, B \vdash B}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \Rightarrow_{\text{left}}} \quad \frac{\frac{A \Rightarrow B, B \vdash A, B}{A \Rightarrow B, B \Rightarrow A, B \vdash A} \text{ax}}{\frac{A \Rightarrow B, B \Rightarrow A, B \vdash A}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \Rightarrow_{\text{left}}} \quad \frac{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}} \quad \frac{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\text{right}}$$

Exercices en logique du premier ordre

Propositions à démontrer

- ❶ $\forall x.P(x) \Rightarrow \exists y.P(y) \vee Q(y)$
- ❷ $(\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))$
- ❸ $(\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)$
- ❹ $(\forall x.P(x) \wedge Q(x)) \Rightarrow (\forall x.P(x)) \wedge (\forall x.Q(x))$
- ❺ $(\forall x.\neg P(x)) \Rightarrow \neg(\exists x.P(x))$
- ❻ $\neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)$

Preuve (1) dans LJ/LK

▸ Règles LJ

$$P(x) \vdash P(x), Q(x)$$

$$P(x) \vdash P(x) \vee Q(x)$$

$$P(x) \vdash \exists y.P(y) \vee Q(y)$$

$$\vdash P(x) \Rightarrow \exists y.P(y) \vee Q(y)$$

$$\vdash \forall x.P(x) \Rightarrow \exists y.P(y) \vee Q(y)$$

Preuve (1) dans LJ/LK

► Règles LJ

$$\frac{\begin{array}{l} P(x) \vdash P(x), Q(x) \\ P(x) \vdash P(x) \vee Q(x) \\ P(x) \vdash \exists y. P(y) \vee Q(y) \\ \vdash P(x) \Rightarrow \exists y. P(y) \vee Q(y) \end{array}}{\vdash \forall x. P(x) \Rightarrow \exists y. P(y) \vee Q(y)} \forall_{\text{right}}$$

Preuve (1) dans LJ/LK

▸ Règles LJ

$$\frac{\frac{P(x) \vdash P(x), Q(x)}{P(x) \vdash P(x) \vee Q(x)} \quad \frac{P(x) \vdash \exists y.P(y) \vee Q(y)}{\vdash P(x) \Rightarrow \exists y.P(y) \vee Q(y)} \Rightarrow_{\text{right}}}{\vdash \forall x.P(x) \Rightarrow \exists y.P(y) \vee Q(y)} \forall_{\text{right}}$$

Correction

Preuve (1) dans LJ/LK

▸ Règles LJ

$$\frac{\frac{\frac{P(x) \vdash P(x), Q(x)}{P(x) \vdash P(x) \vee Q(x)} \exists_{\text{right}}}{\vdash P(x) \Rightarrow \exists y. P(y) \vee Q(y)} \Rightarrow_{\text{right}}}{\vdash \forall x. P(x) \Rightarrow \exists y. P(y) \vee Q(y)} \forall_{\text{right}}$$

Preuve (1) dans LJ/LK

▸ Règles LJ

$$\frac{\frac{\frac{P(x) \vdash P(x), Q(x)}{P(x) \vdash P(x) \vee Q(x)} \vee_{\text{right}}}{P(x) \vdash \exists y. P(y) \vee Q(y)} \exists_{\text{right}}}{\vdash P(x) \Rightarrow \exists y. P(y) \vee Q(y)} \Rightarrow_{\text{right}} \frac{}{\vdash \forall x. P(x) \Rightarrow \exists y. P(y) \vee Q(y)} \forall_{\text{right}}$$

Preuve (1) dans LJ/LK

► Règles LJ

$$\frac{\frac{\frac{\overline{P(x) \vdash P(x), Q(x)}}{P(x) \vdash P(x) \vee Q(x)} \vee_{\text{right}}}{P(x) \vdash \exists y.P(y) \vee Q(y)} \exists_{\text{right}}}{\vdash P(x) \Rightarrow \exists y.P(y) \vee Q(y)} \Rightarrow_{\text{right}} \frac{}{\vdash \forall x.P(x) \Rightarrow \exists y.P(y) \vee Q(y)} \forall_{\text{right}}$$

Correction

Preuve (2) dans LJ

► Règles LJ

$$\begin{array}{l} P(x) \vdash P(x) \\ P(x) \vdash \exists x.P(x) \\ P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x)) \\ P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x)) \\ \exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x)) \\ \vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x)) \end{array} \qquad \begin{array}{l} Q(x) \vdash Q(x) \\ Q(x) \vdash \exists x.Q(x) \\ Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x)) \end{array}$$

Correction

Preuve (2) dans LJ

► Règles LJ

$$\begin{array}{c} P(x) \vdash P(x) \qquad Q(x) \vdash Q(x) \\ P(x) \vdash \exists x.P(x) \qquad Q(x) \vdash \exists x.Q(x) \\ P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x)) \qquad Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x)) \\ P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x)) \\ \frac{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (2) dans LJ

► Règles LJ

$$\begin{array}{c} \begin{array}{cc} P(x) \vdash P(x) & Q(x) \vdash Q(x) \\ P(x) \vdash \exists x.P(x) & Q(x) \vdash \exists x.Q(x) \\ P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x)) & Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x)) \end{array} \\ \frac{\frac{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (2) dans LJ

► Règles LJ

$$\frac{\frac{\frac{P(x) \vdash P(x)}{P(x) \vdash \exists x.P(x)} \quad \frac{\frac{Q(x) \vdash Q(x)}{Q(x) \vdash \exists x.Q(x)}}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x)) \quad Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{\text{left}}}{\frac{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

► Règles LJ

$$\frac{\frac{P(x) \vdash P(x)}{P(x) \vdash \exists x.P(x)} \quad \frac{Q(x) \vdash Q(x)}{Q(x) \vdash \exists x.Q(x)} \quad \frac{P(x) \vdash \exists x.P(x) \quad Q(x) \vdash \exists x.Q(x)}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{\text{right1}} \quad \frac{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{\text{left}}}{\frac{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \exists_{\text{left}} \quad \frac{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \Rightarrow_{\text{right}}}$$

Correction

Preuve (2) dans LJ

► Règles LJ

$$\frac{\frac{\frac{P(x) \vdash P(x)}{P(x) \vdash \exists x.P(x)} \exists_{\text{right}}}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{\text{right1}} \quad \frac{\frac{Q(x) \vdash Q(x)}{Q(x) \vdash \exists x.Q(x)} \quad Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{\text{left}}}{\frac{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

► Règles LJ

$$\frac{\frac{\frac{}{P(x) \vdash P(x)}{ax} \quad \frac{}{P(x) \vdash \exists x.P(x)}{\exists_{right}}}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{right1} \quad \frac{\frac{Q(x) \vdash Q(x)}{Q(x) \vdash \exists x.Q(x)} \quad Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{left}}{\frac{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \exists_{left}} \Rightarrow_{right} \vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))$$

Correction

Preuve (2) dans LJ

► Règles LJ

$$\frac{\frac{\frac{}{P(x) \vdash P(x)}{ax}}{P(x) \vdash \exists x.P(x)} \exists_{right}}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{right1} \quad \frac{\frac{Q(x) \vdash Q(x)}{Q(x) \vdash \exists x.Q(x)}}{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{right2}$$
$$\frac{\frac{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \exists_{left}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \Rightarrow_{right}$$

Correction

Preuve (2) dans LJ

► Règles LJ

$$\frac{\frac{\frac{}{P(x) \vdash P(x)}{ax}}{P(x) \vdash \exists x.P(x)} \exists_{right}}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{right1} \quad \frac{\frac{\frac{}{Q(x) \vdash Q(x)}}{Q(x) \vdash \exists x.Q(x)} \exists_{right}}{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{right2}$$
$$\frac{\frac{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \exists_{left}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \Rightarrow_{right}$$

Correction

Preuve (2) dans LJ

► Règles LJ

$$\frac{\frac{\frac{}{P(x) \vdash P(x)}{ax}}{P(x) \vdash \exists x.P(x)} \exists_{right}}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{right1} \quad \frac{\frac{\frac{}{Q(x) \vdash Q(x)}{ax}}{Q(x) \vdash \exists x.Q(x)} \exists_{right}}{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{right2}$$
$$\frac{\frac{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \exists_{left}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \Rightarrow_{right}$$

Preuve (2) dans LK

▸ Règles LK

$$\begin{array}{l} P(x) \vdash P(x), \exists x.Q(x) \qquad Q(x) \vdash \exists x.P(x), Q(x) \\ P(x) \vdash \exists x.P(x), \exists x.Q(x) \qquad Q(x) \vdash \exists x.P(x), \exists x.Q(x) \\ P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x) \\ P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x)) \\ \exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x)) \\ \vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x)) \end{array}$$

Correction

Preuve (2) dans LK

► Règles LK

$$\begin{array}{c} P(x) \vdash P(x), \exists x.Q(x) \qquad Q(x) \vdash \exists x.P(x), Q(x) \\ P(x) \vdash \exists x.P(x), \exists x.Q(x) \qquad Q(x) \vdash \exists x.P(x), \exists x.Q(x) \\ P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x) \\ P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x)) \\ \frac{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\begin{array}{c} P(x) \vdash P(x), \exists x.Q(x) \qquad Q(x) \vdash \exists x.P(x), Q(x) \\ P(x) \vdash \exists x.P(x), \exists x.Q(x) \qquad Q(x) \vdash \exists x.P(x), \exists x.Q(x) \\ P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x) \\ \frac{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\frac{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \exists_{\text{left}}}} \Rightarrow_{\text{right}} \end{array}$$

Preuve (2) dans LK

▸ Règles LK

$$\begin{array}{c} \frac{P(x) \vdash P(x), \exists x.Q(x) \quad Q(x) \vdash \exists x.P(x), Q(x)}{P(x) \vdash \exists x.P(x), \exists x.Q(x) \quad Q(x) \vdash \exists x.P(x), \exists x.Q(x)} \\ \frac{P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x)}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{\text{right}} \\ \frac{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \exists_{\text{left}} \\ \frac{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\frac{\frac{\frac{P(x) \vdash P(x), \exists x.Q(x) \quad Q(x) \vdash \exists x.P(x), Q(x)}{P(x) \vdash \exists x.P(x), \exists x.Q(x) \quad Q(x) \vdash \exists x.P(x), \exists x.Q(x)} \vee_{\text{left}}}{\frac{P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x)}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{\text{right}}}{\frac{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \exists_{\text{left}} \Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

► Règles LK

$$\frac{\frac{P(x) \vdash P(x), \exists x.Q(x)}{P(x) \vdash \exists x.P(x), \exists x.Q(x)} \exists_{\text{right}} \quad \frac{Q(x) \vdash \exists x.P(x), Q(x)}{Q(x) \vdash \exists x.P(x), \exists x.Q(x)} \vee_{\text{left}}}{\frac{P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x)}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{\text{right}} \quad \frac{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \exists_{\text{left}} \Rightarrow_{\text{right}}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\frac{\frac{\frac{}{P(x) \vdash P(x), \exists x.Q(x)}{P(x) \vdash \exists x.P(x), \exists x.Q(x)} \text{ax}}{\frac{Q(x) \vdash \exists x.P(x), Q(x)}{Q(x) \vdash \exists x.P(x), \exists x.Q(x)} \text{ax}}{\frac{P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x)}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{\text{left}} \text{ax}} \vee_{\text{right}} \text{ax}$$
$$\frac{\frac{\frac{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \Rightarrow_{\text{right}} \text{ax}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\frac{\frac{\frac{}{P(x) \vdash P(x), \exists x.Q(x)}{P(x) \vdash \exists x.P(x), \exists x.Q(x)} \text{ ax}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \begin{array}{l} \exists_{\text{right}} \\ \exists_{\text{right}} \\ \vee_{\text{left}} \\ \vee_{\text{right}} \\ \exists_{\text{left}} \\ \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (2) dans LK

► Règles LK

$$\frac{\frac{\frac{P(x) \vdash P(x), \exists x.Q(x)}{P(x) \vdash \exists x.P(x), \exists x.Q(x)} \text{ ax}}{\exists_{\text{right}}} \quad \frac{\frac{\frac{Q(x) \vdash \exists x.P(x), Q(x)}{Q(x) \vdash \exists x.P(x), \exists x.Q(x)} \text{ ax}}{\exists_{\text{right}}} \quad \frac{\exists_{\text{right}} \quad \exists_{\text{right}}}{\vee_{\text{left}}}$$
$$\frac{P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x)}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{\text{right}}$$
$$\frac{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \exists_{\text{left}}$$
$$\frac{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \Rightarrow_{\text{right}}$$

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\begin{array}{c} P(x), \forall x.Q(x) \vdash P(x) \qquad \forall x.P(x), Q(x) \vdash Q(x) \\ \forall x.P(x), \forall x.Q(x) \vdash P(x) \qquad \forall x.P(x), \forall x.Q(x) \vdash Q(x) \\ \forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x) \\ (\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x) \\ (\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x) \\ \vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x) \end{array}$$

Preuve (3) dans LJ/LK

► Règles LJ

$$\begin{array}{c} P(x), \forall x.Q(x) \vdash P(x) \qquad \forall x.P(x), Q(x) \vdash Q(x) \\ \forall x.P(x), \forall x.Q(x) \vdash P(x) \qquad \forall x.P(x), \forall x.Q(x) \vdash Q(x) \\ \forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x) \\ (\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x) \\ \hline \vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x) \quad \Rightarrow_{\text{right}} \end{array}$$

Preuve (3) dans LJ/LK

► Règles LJ

$$\begin{array}{c} P(x), \forall x.Q(x) \vdash P(x) \qquad \forall x.P(x), Q(x) \vdash Q(x) \\ \forall x.P(x), \forall x.Q(x) \vdash P(x) \qquad \forall x.P(x), \forall x.Q(x) \vdash Q(x) \\ \forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x) \\ \frac{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)} \forall_{\text{right}} \\ \frac{}{\vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)} \Rightarrow_{\text{right}} \end{array}$$

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\begin{array}{c} P(x), \forall x.Q(x) \vdash P(x) \qquad \forall x.P(x), Q(x) \vdash Q(x) \\ \forall x.P(x), \forall x.Q(x) \vdash P(x) \qquad \forall x.P(x), \forall x.Q(x) \vdash Q(x) \\ \hline \forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x) \\ \hline (\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x) \quad \wedge_{\text{left}} \\ \hline (\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x) \quad \forall_{\text{right}} \\ \hline \vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x) \quad \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\frac{\frac{\frac{P(x), \forall x.Q(x) \vdash P(x)}{\forall x.P(x), \forall x.Q(x) \vdash P(x)} \quad \frac{\frac{\forall x.P(x), Q(x) \vdash Q(x)}{\forall x.P(x), \forall x.Q(x) \vdash Q(x)}}{\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)} \wedge_{\text{right}}}{\frac{\frac{\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)} \wedge_{\text{left}}}{\frac{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)} \forall_{\text{right}}}{\vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)} \Rightarrow_{\text{right}}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\frac{\frac{P(x), \forall x.Q(x) \vdash P(x)}{\forall x.P(x), \forall x.Q(x) \vdash P(x)} \forall_{\text{left}} \quad \frac{\forall x.P(x), Q(x) \vdash Q(x)}{\forall x.P(x), \forall x.Q(x) \vdash Q(x)} \wedge_{\text{right}}}{\frac{\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)} \wedge_{\text{left}} \quad \frac{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)} \forall_{\text{right}}}{\vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)} \Rightarrow_{\text{right}}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\frac{\frac{\overline{P(x), \forall x.Q(x) \vdash P(x)}^{\text{ax}}}{\forall x.P(x), \forall x.Q(x) \vdash P(x)} \forall_{\text{left}} \quad \frac{\forall x.P(x), Q(x) \vdash Q(x)}{\forall x.P(x), \forall x.Q(x) \vdash Q(x)} \wedge_{\text{right}}}{\frac{\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)} \wedge_{\text{left}} \quad \frac{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)} \forall_{\text{right}}}{\vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)} \Rightarrow_{\text{right}}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\frac{\frac{\frac{}{P(x), \forall x.Q(x) \vdash P(x)}{ax}}{\forall x.P(x), \forall x.Q(x) \vdash P(x)} \forall_{left} \quad \frac{\forall x.P(x), Q(x) \vdash Q(x)}{\forall x.P(x), \forall x.Q(x) \vdash Q(x)} \forall_{left}}{\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)} \wedge_{right}$$
$$\frac{\frac{\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)} \wedge_{left}}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)} \forall_{right}$$
$$\frac{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)}{\vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)} \Rightarrow_{right}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\frac{\frac{\frac{P(x), \forall x.Q(x) \vdash P(x)}{\forall x.P(x), \forall x.Q(x) \vdash P(x)} \text{ax}}{\forall x.P(x), \forall x.Q(x) \vdash P(x)} \forall_{\text{left}} \quad \frac{\frac{\frac{\forall x.P(x), Q(x) \vdash Q(x)}{\forall x.P(x), \forall x.Q(x) \vdash Q(x)} \text{ax}}{\forall x.P(x), \forall x.Q(x) \vdash Q(x)} \forall_{\text{left}}}{\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)} \wedge_{\text{right}}$$
$$\frac{\frac{\frac{\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)} \wedge_{\text{left}}}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)} \forall_{\text{right}}}{\vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)} \Rightarrow_{\text{right}}$$

Preuve (4) dans LJ/LK

▸ Règles LJ

$$P(x), Q(x) \vdash P(x)$$

$$P(x), Q(x) \vdash Q(x)$$

$$P(x) \wedge Q(x) \vdash P(x)$$

$$P(x) \wedge Q(x) \vdash Q(x)$$

$$\forall x. P(x) \wedge Q(x) \vdash P(x)$$

$$\forall x. P(x) \wedge Q(x) \vdash Q(x)$$

$$\forall x. P(x) \wedge Q(x) \vdash \forall x. P(x)$$

$$\forall x. P(x) \wedge Q(x) \vdash \forall x. Q(x)$$

$$\forall x. P(x) \wedge Q(x) \vdash (\forall x. P(x)) \wedge (\forall x. Q(x))$$

$$\vdash (\forall x. P(x) \wedge Q(x)) \Rightarrow (\forall x. P(x)) \wedge (\forall x. Q(x))$$

Correction

Preuve (4) dans LJ/LK

► Règles LJ

$$\begin{array}{ll} P(x), Q(x) \vdash P(x) & P(x), Q(x) \vdash Q(x) \\ P(x) \wedge Q(x) \vdash P(x) & P(x) \wedge Q(x) \vdash Q(x) \\ \forall x. P(x) \wedge Q(x) \vdash P(x) & \forall x. P(x) \wedge Q(x) \vdash Q(x) \\ \forall x. P(x) \wedge Q(x) \vdash \forall x. P(x) & \forall x. P(x) \wedge Q(x) \vdash \forall x. Q(x) \\ \hline \forall x. P(x) \wedge Q(x) \vdash (\forall x. P(x)) \wedge (\forall x. Q(x)) & \Rightarrow_{\text{right}} \\ \vdash (\forall x. P(x) \wedge Q(x)) \Rightarrow (\forall x. P(x)) \wedge (\forall x. Q(x)) \end{array}$$

Correction

Preuve (4) dans LJ/LK

▸ Règles LJ

$$\frac{\begin{array}{l} P(x), Q(x) \vdash P(x) \\ P(x) \wedge Q(x) \vdash P(x) \\ \forall x. P(x) \wedge Q(x) \vdash P(x) \\ \forall x. P(x) \wedge Q(x) \vdash \forall x. P(x) \end{array} \quad \begin{array}{l} P(x), Q(x) \vdash Q(x) \\ P(x) \wedge Q(x) \vdash Q(x) \\ \forall x. P(x) \wedge Q(x) \vdash Q(x) \\ \forall x. P(x) \wedge Q(x) \vdash \forall x. Q(x) \end{array}}{\frac{\forall x. P(x) \wedge Q(x) \vdash (\forall x. P(x)) \wedge (\forall x. Q(x))}{\vdash (\forall x. P(x) \wedge Q(x)) \Rightarrow (\forall x. P(x)) \wedge (\forall x. Q(x))}} \Rightarrow_{\text{right}} \wedge_{\text{right}}$$

Correction

Preuve (4) dans LJ/LK

► Règles LJ

$$\frac{\frac{\frac{P(x), Q(x) \vdash P(x)}{P(x) \wedge Q(x) \vdash P(x)} \quad \forall x.P(x) \wedge Q(x) \vdash P(x)}{\forall x.P(x) \wedge Q(x) \vdash \forall x.P(x)} \forall_{\text{right}} \quad \frac{\frac{\frac{P(x), Q(x) \vdash Q(x)}{P(x) \wedge Q(x) \vdash Q(x)} \quad \forall x.P(x) \wedge Q(x) \vdash Q(x)}{\forall x.P(x) \wedge Q(x) \vdash \forall x.Q(x)} \forall_{\text{right}}}{\frac{\forall x.P(x) \wedge Q(x) \vdash (\forall x.P(x)) \wedge (\forall x.Q(x))}{\vdash (\forall x.P(x) \wedge Q(x)) \Rightarrow (\forall x.P(x)) \wedge (\forall x.Q(x))} \Rightarrow_{\text{right}}} \wedge_{\text{right}}$$

Correction

Preuve (4) dans LJ/LK

► Règles LJ

$$\frac{\frac{\frac{P(x), Q(x) \vdash P(x)}{P(x) \wedge Q(x) \vdash P(x)} \quad \forall_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash P(x)} \quad \forall_{\text{right}}}{\forall x. P(x) \wedge Q(x) \vdash \forall x. P(x)} \quad \wedge_{\text{right}} \Rightarrow_{\text{right}} \frac{\forall x. P(x) \wedge Q(x) \vdash (\forall x. P(x)) \wedge (\forall x. Q(x))}{\vdash (\forall x. P(x) \wedge Q(x)) \Rightarrow (\forall x. P(x)) \wedge (\forall x. Q(x))}$$

Correction

Preuve (4) dans LJ/LK

► Règles LJ

$$\frac{\frac{\frac{P(x), Q(x) \vdash P(x)}{P(x) \wedge Q(x) \vdash P(x)} \wedge_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash P(x)} \forall_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash \forall x. P(x)} \forall_{\text{right}} \quad \frac{\frac{\frac{P(x), Q(x) \vdash Q(x)}{P(x) \wedge Q(x) \vdash Q(x)} \wedge_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash Q(x)} \forall_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash \forall x. Q(x)} \forall_{\text{right}}}{\frac{\forall x. P(x) \wedge Q(x) \vdash (\forall x. P(x)) \wedge (\forall x. Q(x))}{\vdash (\forall x. P(x) \wedge Q(x)) \Rightarrow (\forall x. P(x)) \wedge (\forall x. Q(x))} \Rightarrow_{\text{right}} \wedge_{\text{right}}}$$

Correction

Preuve (4) dans LJ/LK

► Règles LJ

$$\frac{\frac{\frac{P(x), Q(x) \vdash P(x)}{P(x) \wedge Q(x) \vdash P(x)} \wedge_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash P(x)} \forall_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash \forall x. P(x)} \forall_{\text{right}} \quad \frac{\frac{\frac{P(x), Q(x) \vdash Q(x)}{P(x) \wedge Q(x) \vdash Q(x)} \wedge_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash Q(x)} \forall_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash \forall x. Q(x)} \forall_{\text{right}}}{\frac{\forall x. P(x) \wedge Q(x) \vdash (\forall x. P(x)) \wedge (\forall x. Q(x))}{\vdash (\forall x. P(x) \wedge Q(x)) \Rightarrow (\forall x. P(x)) \wedge (\forall x. Q(x))} \Rightarrow_{\text{right}}} \wedge_{\text{right}}$$

Correction

Preuve (4) dans LJ/LK

► Règles LJ

$$\frac{\frac{\frac{\overline{P(x), Q(x) \vdash P(x)}{ax}}{P(x) \wedge Q(x) \vdash P(x)}{\wedge_{left}}}{\forall x. P(x) \wedge Q(x) \vdash P(x)}{\forall_{left}} \quad \frac{\frac{P(x), Q(x) \vdash Q(x)}{P(x) \wedge Q(x) \vdash Q(x)}{\wedge_{left}}}{\forall x. P(x) \wedge Q(x) \vdash Q(x)}{\forall_{left}} \quad \frac{\forall x. P(x) \wedge Q(x) \vdash P(x)}{\forall x. P(x) \wedge Q(x) \vdash \forall x. P(x)}{\forall_{right}} \quad \frac{\forall x. P(x) \wedge Q(x) \vdash Q(x)}{\forall x. P(x) \wedge Q(x) \vdash \forall x. Q(x)}{\forall_{right}} \quad \frac{\forall x. P(x) \wedge Q(x) \vdash \forall x. P(x)}{\forall x. P(x) \wedge Q(x) \vdash \forall x. Q(x)}{\wedge_{right}} \quad \frac{\forall x. P(x) \wedge Q(x) \vdash (\forall x. P(x)) \wedge (\forall x. Q(x))}{\vdash (\forall x. P(x) \wedge Q(x)) \Rightarrow (\forall x. P(x)) \wedge (\forall x. Q(x))} \Rightarrow_{right}$$

Correction

Preuve (4) dans LJ/LK

▸ Règles LJ

$$\frac{\frac{\frac{\overline{P(x), Q(x) \vdash P(x)}}{P(x) \wedge Q(x) \vdash P(x)} \wedge_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash P(x)} \forall_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash \forall x. P(x)} \forall_{\text{right}} \quad \frac{\frac{\frac{P(x), Q(x) \vdash Q(x)}{P(x) \wedge Q(x) \vdash Q(x)} \wedge_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash Q(x)} \forall_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash \forall x. Q(x)} \forall_{\text{right}} \wedge_{\text{right}}}{\forall x. P(x) \wedge Q(x) \vdash (\forall x. P(x)) \wedge (\forall x. Q(x))} \Rightarrow_{\text{right}} \vdash (\forall x. P(x) \wedge Q(x)) \Rightarrow (\forall x. P(x)) \wedge (\forall x. Q(x))$$

Correction

Preuve (4) dans LJ/LK

► Règles LJ

$$\frac{\frac{\frac{\overline{P(x), Q(x) \vdash P(x)}}{P(x) \wedge Q(x) \vdash P(x)} \wedge_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash P(x)} \forall_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash \forall x. P(x)} \forall_{\text{right}} \quad \frac{\frac{\frac{P(x), Q(x) \vdash Q(x)}{P(x) \wedge Q(x) \vdash Q(x)} \wedge_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash Q(x)} \forall_{\text{left}}}{\forall x. P(x) \wedge Q(x) \vdash \forall x. Q(x)} \forall_{\text{right}}}{\frac{\forall x. P(x) \wedge Q(x) \vdash (\forall x. P(x)) \wedge (\forall x. Q(x))}{\vdash (\forall x. P(x) \wedge Q(x)) \Rightarrow (\forall x. P(x)) \wedge (\forall x. Q(x))} \Rightarrow_{\text{right}}}$$

Correction

Preuve (4) dans LJ/LK

► Règles LJ

$$\frac{\frac{\frac{\overline{P(x), Q(x) \vdash P(x)}{ax}}{P(x) \wedge Q(x) \vdash P(x)}{\wedge_{left}}}{\forall x. P(x) \wedge Q(x) \vdash P(x)}{\forall_{left}}}{\forall x. P(x) \wedge Q(x) \vdash \forall x. P(x)}{\forall_{right}} \quad \frac{\frac{\frac{\overline{P(x), Q(x) \vdash Q(x)}{ax}}{P(x) \wedge Q(x) \vdash Q(x)}{\wedge_{left}}}{\forall x. P(x) \wedge Q(x) \vdash Q(x)}{\forall_{left}}}{\forall x. P(x) \wedge Q(x) \vdash \forall x. Q(x)}{\forall_{right}}}{\forall x. P(x) \wedge Q(x) \vdash (\forall x. P(x)) \wedge (\forall x. Q(x))}{\wedge_{right}}}{\vdash (\forall x. P(x) \wedge Q(x)) \Rightarrow (\forall x. P(x)) \wedge (\forall x. Q(x))}{\Rightarrow_{right}}$$

Preuve (5) dans LJ/LK

▸ Règles LJ

$$\begin{array}{c} P(x) \vdash P(x) \\ \neg P(x), P(x) \vdash \perp \\ \forall x. \neg P(x), P(x) \vdash \perp \\ \forall x. \neg P(x), \exists x. P(x) \vdash \perp \\ \forall x. \neg P(x) \vdash \neg(\exists x. P(x)) \\ \vdash (\forall x. \neg P(x)) \Rightarrow \neg(\exists x. P(x)) \end{array}$$

Correction

Preuve (5) dans LJ/LK

▸ Règles LJ

$$\begin{array}{c} P(x) \vdash P(x) \\ \neg P(x), P(x) \vdash \perp \\ \forall x. \neg P(x), P(x) \vdash \perp \\ \forall x. \neg P(x), \exists x. P(x) \vdash \perp \\ \forall x. \neg P(x) \vdash \neg(\exists x. P(x)) \\ \hline \vdash (\forall x. \neg P(x)) \Rightarrow \neg(\exists x. P(x)) \end{array} \Rightarrow_{\text{right}}$$

Preuve (5) dans LJ/LK

► Règles LJ

$$\begin{array}{c} P(x) \vdash P(x) \\ \neg P(x), P(x) \vdash \perp \\ \forall x. \neg P(x), P(x) \vdash \perp \\ \frac{\forall x. \neg P(x), \exists x. P(x) \vdash \perp}{\forall x. \neg P(x) \vdash \neg(\exists x. P(x))} \neg_{\text{right}} \\ \frac{\forall x. \neg P(x) \vdash \neg(\exists x. P(x))}{\vdash (\forall x. \neg P(x)) \Rightarrow \neg(\exists x. P(x))} \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (5) dans LJ/LK

► Règles LJ

$$\frac{\frac{\frac{P(x) \vdash P(x)}{\neg P(x), P(x) \vdash \perp}}{\forall x. \neg P(x), P(x) \vdash \perp}}{\forall x. \neg P(x), \exists x. P(x) \vdash \perp} \exists_{\text{left}} \quad \frac{\forall x. \neg P(x), \exists x. P(x) \vdash \perp}{\forall x. \neg P(x) \vdash \neg(\exists x. P(x))} \neg_{\text{right}}}{\vdash (\forall x. \neg P(x)) \Rightarrow \neg(\exists x. P(x))} \Rightarrow_{\text{right}}$$

Correction

Preuve (5) dans LJ/LK

► Règles LJ

$$\frac{\frac{\frac{P(x) \vdash P(x)}{\neg P(x), P(x) \vdash \perp} \forall_{\text{left}}}{\forall x. \neg P(x), P(x) \vdash \perp} \exists_{\text{left}}}{\forall x. \neg P(x), \exists x. P(x) \vdash \perp} \neg_{\text{right}}}{\vdash (\forall x. \neg P(x)) \Rightarrow \neg(\exists x. P(x))} \Rightarrow_{\text{right}}$$

Correction

Preuve (5) dans LJ/LK

► Règles LJ

$$\frac{\frac{\frac{P(x) \vdash P(x)}{\neg P(x), P(x) \vdash \perp} \neg\text{left}}{\forall x. \neg P(x), P(x) \vdash \perp} \forall\text{left}}{\frac{\forall x. \neg P(x), \exists x. P(x) \vdash \perp} \exists\text{left}}{\frac{\forall x. \neg P(x) \vdash \neg(\exists x. P(x))}{\vdash (\forall x. \neg P(x)) \Rightarrow \neg(\exists x. P(x))} \Rightarrow\text{right}} \neg\text{right}$$

Preuve (5) dans LJ/LK

► Règles LJ

$$\frac{\frac{\frac{\frac{\frac{\frac{\overline{P(x) \vdash P(x)}}{ax}}{\neg P(x), P(x) \vdash \perp}{\neg left}}{\forall x. \neg P(x), P(x) \vdash \perp}{\forall left}}{\forall x. \neg P(x), \exists x. P(x) \vdash \perp}{\exists left}}{\forall x. \neg P(x) \vdash \neg(\exists x. P(x))}{\neg right}}{\vdash (\forall x. \neg P(x)) \Rightarrow \neg(\exists x. P(x))}{\Rightarrow right}}$$

Correction

Preuve (6) dans LJ_{em}

▸ Règles LJ

$$\neg P(x) \vdash \neg P(x)$$

$$\neg P(x) \vdash \exists x. \neg P(x)$$

$$\neg \exists x. \neg P(x), \neg P(x) \vdash \perp$$

$$\neg \exists x. \neg P(x) \vdash \neg \neg P(x)$$

$$\neg \exists x. \neg P(x) \vdash P(x)$$

$$\neg \exists x. \neg P(x) \vdash \forall x. P(x)$$

$$\neg \forall x. P(x), \neg \exists x. \neg P(x) \vdash \perp$$

$$\neg \forall x. P(x) \vdash \neg \neg \exists x. \neg P(x)$$

$$\neg \forall x. P(x) \vdash \exists x. \neg P(x)$$

$$\vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x)$$

Correction

Preuve (6) dans LJ_{em}

▸ Règles LJ

$$\begin{array}{l} \neg P(x) \vdash \neg P(x) \\ \neg P(x) \vdash \exists x. \neg P(x) \\ \neg \exists x. \neg P(x), \neg P(x) \vdash \perp \\ \neg \exists x. \neg P(x) \vdash \neg \neg P(x) \\ \neg \exists x. \neg P(x) \vdash P(x) \\ \neg \exists x. \neg P(x) \vdash \forall x. P(x) \\ \neg \forall x. P(x), \neg \exists x. \neg P(x) \vdash \perp \\ \neg \forall x. P(x) \vdash \neg \neg \exists x. \neg P(x) \\ \neg \forall x. P(x) \vdash \exists x. \neg P(x) \end{array} \quad \frac{}{\vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x)} \Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LJ_{em}

► Règles LJ

$$\begin{array}{l} \neg P(x) \vdash \neg P(x) \\ \neg P(x) \vdash \exists x. \neg P(x) \\ \neg \exists x. \neg P(x), \neg P(x) \vdash \perp \\ \neg \exists x. \neg P(x) \vdash \neg \neg P(x) \\ \neg \exists x. \neg P(x) \vdash P(x) \\ \neg \exists x. \neg P(x) \vdash \forall x. P(x) \\ \neg \forall x. P(x), \neg \exists x. \neg P(x) \vdash \perp \\ \frac{\neg \forall x. P(x) \vdash \neg \neg \exists x. \neg P(x)}{\neg \forall x. P(x) \vdash \exists x. \neg P(x)} \text{em} \\ \frac{}{\vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x)} \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (6) dans LJ_{em}

► Règles LJ

$$\neg P(x) \vdash \neg P(x)$$

$$\neg P(x) \vdash \exists x. \neg P(x)$$

$$\neg \exists x. \neg P(x), \neg P(x) \vdash \perp$$

$$\neg \exists x. \neg P(x) \vdash \neg \neg P(x)$$

$$\neg \exists x. \neg P(x) \vdash P(x)$$

$$\neg \exists x. \neg P(x) \vdash \forall x. P(x)$$

$$\frac{\neg \forall x. P(x), \neg \exists x. \neg P(x) \vdash \perp}{\neg \forall x. P(x) \vdash \neg \neg \exists x. \neg P(x)} \neg_{\text{right}}$$

$$\frac{\neg \forall x. P(x) \vdash \neg \neg \exists x. \neg P(x)}{\neg \forall x. P(x) \vdash \exists x. \neg P(x)} \text{em}$$

$$\frac{\neg \forall x. P(x) \vdash \exists x. \neg P(x)}{\vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x)} \Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LJ_{em}

► Règles LJ

$$\begin{array}{l} \neg P(x) \vdash \neg P(x) \\ \neg P(x) \vdash \exists x. \neg P(x) \\ \neg \exists x. \neg P(x), \neg P(x) \vdash \perp \\ \neg \exists x. \neg P(x) \vdash \neg \neg P(x) \\ \neg \exists x. \neg P(x) \vdash P(x) \\ \neg \exists x. \neg P(x) \vdash \forall x. P(x) \\ \hline \neg \forall x. P(x), \neg \exists x. \neg P(x) \vdash \perp \quad \neg_{\text{left}} \\ \hline \neg \forall x. P(x) \vdash \neg \neg \exists x. \neg P(x) \quad \neg_{\text{right}} \\ \hline \neg \forall x. P(x) \vdash \exists x. \neg P(x) \quad \text{em} \\ \hline \vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x) \quad \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (6) dans LJ_{em}

► Règles LJ

$$\begin{array}{c} \neg P(x) \vdash \neg P(x) \\ \neg P(x) \vdash \exists x. \neg P(x) \\ \neg \exists x. \neg P(x), \neg P(x) \vdash \perp \\ \neg \exists x. \neg P(x) \vdash \neg \neg P(x) \\ \frac{\neg \exists x. \neg P(x) \vdash P(x)}{\neg \exists x. \neg P(x) \vdash \forall x. P(x)} \forall_{\text{right}} \\ \frac{\neg \exists x. \neg P(x) \vdash \forall x. P(x)}{\neg \forall x. P(x), \neg \exists x. \neg P(x) \vdash \perp} \neg_{\text{left}} \\ \frac{\neg \forall x. P(x), \neg \exists x. \neg P(x) \vdash \perp}{\neg \forall x. P(x) \vdash \neg \neg \exists x. \neg P(x)} \neg_{\text{right}} \\ \frac{\neg \forall x. P(x) \vdash \neg \neg \exists x. \neg P(x)}{\neg \forall x. P(x) \vdash \exists x. \neg P(x)} \text{em} \\ \frac{\neg \forall x. P(x) \vdash \exists x. \neg P(x)}{\vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x)} \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (6) dans LJ_{em}

► Règles LJ

$$\begin{array}{c} \neg P(x) \vdash \neg P(x) \\ \neg P(x) \vdash \exists x. \neg P(x) \\ \neg \exists x. \neg P(x), \neg P(x) \vdash \perp \\ \hline \neg \exists x. \neg P(x) \vdash \neg \neg P(x) \\ \hline \neg \exists x. \neg P(x) \vdash P(x) \quad \text{em} \\ \hline \neg \exists x. \neg P(x) \vdash \forall x. P(x) \quad \forall_{\text{right}} \\ \hline \neg \forall x. P(x), \neg \exists x. \neg P(x) \vdash \perp \quad \neg_{\text{left}} \\ \hline \neg \forall x. P(x) \vdash \neg \neg \exists x. \neg P(x) \quad \neg_{\text{right}} \\ \hline \neg \forall x. P(x) \vdash \exists x. \neg P(x) \quad \text{em} \\ \hline \vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x) \quad \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (6) dans LJ_{em}

► Règles LJ

$$\begin{array}{c} \neg P(x) \vdash \neg P(x) \\ \neg P(x) \vdash \exists x. \neg P(x) \\ \hline \neg \exists x. \neg P(x), \neg P(x) \vdash \perp \\ \hline \neg \exists x. \neg P(x) \vdash \neg \neg P(x) \quad \neg_{right} \\ \hline \neg \exists x. \neg P(x) \vdash P(x) \quad em \\ \hline \neg \exists x. \neg P(x) \vdash \forall x. P(x) \quad \forall_{right} \\ \hline \neg \forall x. P(x), \neg \exists x. \neg P(x) \vdash \perp \quad \neg_{left} \\ \hline \neg \forall x. P(x) \vdash \neg \neg \exists x. \neg P(x) \quad \neg_{right} \\ \hline \neg \forall x. P(x) \vdash \exists x. \neg P(x) \quad em \\ \hline \vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x) \quad \Rightarrow_{right} \end{array}$$

Correction

Preuve (6) dans LJ_{em}

► Règles LJ

$$\begin{array}{c} \neg P(x) \vdash \neg P(x) \\ \hline \neg P(x) \vdash \exists x. \neg P(x) \\ \hline \neg \exists x. \neg P(x), \neg P(x) \vdash \perp \quad \neg_{\text{left}} \\ \hline \neg \exists x. \neg P(x) \vdash \neg \neg P(x) \quad \neg_{\text{right}} \\ \hline \neg \exists x. \neg P(x) \vdash P(x) \quad \text{em} \\ \hline \neg \exists x. \neg P(x) \vdash \forall x. P(x) \quad \forall_{\text{right}} \\ \hline \neg \forall x. P(x), \neg \exists x. \neg P(x) \vdash \perp \quad \neg_{\text{left}} \\ \hline \neg \forall x. P(x) \vdash \neg \neg \exists x. \neg P(x) \quad \neg_{\text{right}} \\ \hline \neg \forall x. P(x) \vdash \exists x. \neg P(x) \quad \text{em} \\ \hline \vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x) \quad \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (6) dans LJ_{em}

► Règles LJ

$$\begin{array}{c} \frac{\neg P(x) \vdash \neg P(x)}{\neg P(x) \vdash \exists x. \neg P(x)} \exists_{\text{right}} \\ \frac{\neg P(x) \vdash \exists x. \neg P(x)}{\neg \exists x. \neg P(x), \neg P(x) \vdash \perp} \neg_{\text{left}} \\ \frac{\neg \exists x. \neg P(x), \neg P(x) \vdash \perp}{\neg \exists x. \neg P(x) \vdash \neg \neg P(x)} \neg_{\text{right}} \\ \frac{\neg \exists x. \neg P(x) \vdash \neg \neg P(x)}{\neg \exists x. \neg P(x) \vdash P(x)} \text{em} \\ \frac{\neg \exists x. \neg P(x) \vdash P(x)}{\neg \exists x. \neg P(x) \vdash \forall x. P(x)} \forall_{\text{right}} \\ \frac{\neg \forall x. P(x), \neg \exists x. \neg P(x) \vdash \perp}{\neg \forall x. P(x) \vdash \neg \neg \exists x. \neg P(x)} \neg_{\text{left}} \\ \frac{\neg \forall x. P(x) \vdash \neg \neg \exists x. \neg P(x)}{\neg \forall x. P(x) \vdash \exists x. \neg P(x)} \neg_{\text{right}} \\ \frac{\neg \forall x. P(x) \vdash \exists x. \neg P(x)}{\vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x)} \text{em} \\ \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (6) dans LJ_{em}

► Règles LJ

$$\frac{\frac{\frac{\overline{\neg P(x) \vdash \neg P(x)}}{ax}}{\neg P(x) \vdash \exists x. \neg P(x)} \exists_{right}}{\frac{\neg \exists x. \neg P(x), \neg P(x) \vdash \perp}{\neg \exists x. \neg P(x) \vdash \neg \neg P(x)} \neg_{left}} \neg_{right} \frac{\neg \exists x. \neg P(x) \vdash \neg \neg P(x)}{em} \frac{\neg \exists x. \neg P(x) \vdash P(x)}{\neg \exists x. \neg P(x) \vdash \forall x. P(x)} \forall_{right} \frac{\neg \forall x. P(x), \neg \exists x. \neg P(x) \vdash \perp}{\neg \forall x. P(x) \vdash \neg \neg \exists x. \neg P(x)} \neg_{left} \neg_{right} \frac{\neg \forall x. P(x) \vdash \neg \neg \exists x. \neg P(x)}{em} \frac{\neg \forall x. P(x) \vdash \exists x. \neg P(x)}{\vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x)} \Rightarrow_{right}$$

Preuve (6) dans LK

▸ Règles LK

$$\begin{array}{l} P(x) \vdash P(x) \\ \vdash P(x), \neg P(x) \\ \vdash P(x), \exists x. \neg P(x) \\ \vdash \forall x. P(x), \exists x. \neg P(x) \\ \neg \forall x. P(x) \vdash \exists x. \neg P(x) \\ \vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x) \end{array}$$

Correction

Preuve (6) dans LK

▸ Règles LK

$$\begin{array}{c} P(x) \vdash P(x) \\ \vdash P(x), \neg P(x) \\ \vdash P(x), \exists x. \neg P(x) \\ \vdash \forall x. P(x), \exists x. \neg P(x) \\ \neg \forall x. P(x) \vdash \exists x. \neg P(x) \\ \hline \vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x) \end{array} \Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LK

► Règles LK

$$\begin{array}{c} P(x) \vdash P(x) \\ \vdash P(x), \neg P(x) \\ \vdash P(x), \exists x. \neg P(x) \\ \hline \vdash \forall x. P(x), \exists x. \neg P(x) \\ \hline \neg \forall x. P(x) \vdash \exists x. \neg P(x) \quad \neg\text{left} \\ \hline \vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x) \quad \Rightarrow\text{right} \end{array}$$

Correction

Preuve (6) dans LK

► Règles LK

$$\begin{array}{c} P(x) \vdash P(x) \\ \vdash P(x), \neg P(x) \\ \vdash P(x), \exists x. \neg P(x) \\ \hline \vdash \forall x. P(x), \exists x. \neg P(x) \quad \forall_{\text{right}} \\ \hline \neg \forall x. P(x) \vdash \exists x. \neg P(x) \quad \neg_{\text{left}} \\ \hline \vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x) \quad \Rightarrow_{\text{right}} \end{array}$$

Correction

Preuve (6) dans LK

► Règles LK

$$\frac{\frac{\frac{P(x) \vdash P(x)}{\vdash P(x), \neg P(x)} \exists_{\text{right}}}{\vdash P(x), \exists x. \neg P(x)} \forall_{\text{right}}}{\vdash \forall x. P(x), \exists x. \neg P(x)} \neg_{\text{left}}}{\vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x)} \Rightarrow_{\text{right}}$$

Preuve (6) dans LK

► Règles LK

$$\frac{\frac{\frac{P(x) \vdash P(x)}{\vdash P(x), \neg P(x)} \neg_{\text{right}}}{\vdash P(x), \exists x. \neg P(x)} \exists_{\text{right}}}{\vdash \forall x. P(x), \exists x. \neg P(x)} \forall_{\text{right}} \quad \frac{\vdash \forall x. P(x), \exists x. \neg P(x)}{\vdash \neg(\forall x. P(x)) \vdash \exists x. \neg P(x)} \neg_{\text{left}}}{\vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x)} \Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LK

► Règles LK

$$\frac{\frac{\frac{\frac{\overline{P(x) \vdash P(x)}}{ax}}{\vdash P(x), \neg P(x)}{\neg_{right}}}{\vdash P(x), \exists x. \neg P(x)}{\exists_{right}}}{\vdash \forall x. P(x), \exists x. \neg P(x)}{\forall_{right}}}{\vdash \neg \forall x. P(x) \vdash \exists x. \neg P(x)}{\neg_{left}}}{\vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x)}{\Rightarrow_{right}}$$

Exercices en Coq

Propositions à démontrer

- Exercices en logique propositionnelle ;
- Exercices en logique du premier ordre.
- À faire chez soi.

Installation de Coq

- Tout est indiqué ici : <https://coq.inria.fr/>.

Calcul des séquents intuitionniste (LJ)

Règles

$$\frac{}{\Gamma, A \vdash A} \text{ ax}$$

$$\frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B} \text{ cont}$$

$$\frac{\Gamma \vdash A \quad \Gamma, B \vdash C}{\Gamma, A \Rightarrow B \vdash C} \Rightarrow_{\text{left}}$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \Rightarrow B} \Rightarrow_{\text{right}}$$

$$\frac{\Gamma \vdash A \quad \Gamma, B \vdash C}{\Gamma, A \Leftrightarrow B \vdash C} \Leftrightarrow_{\text{left1}}$$

$$\frac{\Gamma \vdash B \quad \Gamma, A \vdash C}{\Gamma, A \Leftrightarrow B \vdash C} \Leftrightarrow_{\text{left2}}$$

$$\frac{\Gamma, A \vdash B \quad \Gamma, B \vdash A}{\Gamma \vdash A \Leftrightarrow B} \Leftrightarrow_{\text{right}}$$

Calcul des séquents intuitionniste (LJ)

Règles

$$\frac{\Gamma, A, B \vdash C}{\Gamma, A \wedge B \vdash C} \wedge_{\text{left}}$$

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \wedge_{\text{right}}$$

$$\frac{\Gamma, A \vdash C \quad \Gamma, B \vdash C}{\Gamma, A \vee B \vdash C} \vee_{\text{left}}$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \vee_{\text{right1}}$$

$$\frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} \vee_{\text{right2}}$$

$$\frac{\Gamma \vdash A}{\Gamma, \neg A \vdash B} \neg_{\text{left}}$$

$$\frac{\Gamma, A \vdash \perp}{\Gamma \vdash \neg A} \neg_{\text{right}}$$

$$\frac{}{\Gamma, \perp \vdash A} \perp_{\text{left}}$$

$$\frac{}{\Gamma \vdash \top} \top_{\text{right}}$$

Calcul des séquents intuitionniste (LJ)

Règles

$$\frac{\Gamma, A(t) \vdash B}{\Gamma, \forall x. A(x) \vdash B} \forall_{\text{left}}$$

$$\frac{\Gamma \vdash A(x)}{\Gamma \vdash \forall x. A(x)} \forall_{\text{right}}, x \notin \Gamma$$

$$\frac{\Gamma, A(x) \vdash B}{\Gamma, \exists x. A(x) \vdash B} \exists_{\text{left}}, x \notin \Gamma, B$$

$$\frac{\Gamma \vdash A(t)}{\Gamma \vdash \exists x. A(x)} \exists_{\text{right}}$$

$$\frac{\Gamma \vdash A \quad \Gamma, A \vdash B}{\Gamma \vdash B} \text{cut}$$

Calcul des séquents classique (LJ_{em})

Règles

$$\frac{\Gamma, A(t) \vdash B}{\Gamma, \forall x. A(x) \vdash B} \forall_{\text{left}}$$

$$\frac{\Gamma \vdash A(x)}{\Gamma \vdash \forall x. A(x)} \forall_{\text{right}}, x \notin \Gamma$$

$$\frac{\Gamma, A(x) \vdash B}{\Gamma, \exists x. A(x) \vdash B} \exists_{\text{left}}, x \notin \Gamma, B$$

$$\frac{\Gamma \vdash A(t)}{\Gamma \vdash \exists x. A(x)} \exists_{\text{right}}$$

$$\frac{\Gamma \vdash A \quad \Gamma, A \vdash B}{\Gamma \vdash B} \text{cut}$$

$$\frac{\Gamma \vdash \neg\neg A}{\Gamma \vdash A} \text{em}$$

Calcul des séquents classique (LK)

Règles

$$\frac{}{\Gamma, A \vdash \Delta, A} \text{ ax}$$

$$\frac{\Gamma \vdash \Delta, A \quad \Gamma, A \vdash \Delta, B}{\Gamma \vdash \Delta, B} \text{ cut}$$

$$\frac{\Gamma, A, A \vdash \Delta}{\Gamma, A \vdash \Delta} \text{ cont}_{\text{left}}$$

$$\frac{\Gamma \vdash \Delta, A, A}{\Gamma \vdash \Delta, A} \text{ cont}_{\text{right}}$$

Calcul des séquents classique (LK)

Règles

$$\frac{\Gamma \vdash \Delta, A \quad \Gamma, B \vdash \Delta}{\Gamma, A \Rightarrow B \vdash \Delta} \Rightarrow_{\text{left}} \qquad \frac{\Gamma, A \vdash \Delta, B}{\Gamma \vdash \Delta, A \Rightarrow B} \Rightarrow_{\text{right}}$$

$$\frac{\Gamma \vdash \Delta, A, B \quad \Gamma, A, B \vdash \Delta}{\Gamma, A \Leftrightarrow B \vdash \Delta} \Leftrightarrow_{\text{left}}$$

$$\frac{\Gamma, A \vdash \Delta, B \quad \Gamma, B \vdash \Delta, A}{\Gamma \vdash \Delta, A \Leftrightarrow B} \Leftrightarrow_{\text{right}}$$

Calcul des séquents classique (LK)

Règles

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} \wedge_{\text{left}}$$

$$\frac{\Gamma \vdash \Delta, A \quad \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \wedge B} \wedge_{\text{right}}$$

$$\frac{\Gamma, A \vdash \Delta \quad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta} \vee_{\text{left}}$$

$$\frac{\Gamma \vdash \Delta, A, B}{\Gamma \vdash \Delta, A \vee B} \vee_{\text{right}}$$

$$\frac{\Gamma \vdash \Delta, A}{\Gamma, \neg A \vdash \Delta} \neg_{\text{left}}$$

$$\frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \Delta, \neg A} \neg_{\text{right}}$$

$$\frac{}{\Gamma, \perp \vdash \Delta} \perp_{\text{left}}$$

$$\frac{}{\Gamma \vdash \Delta, \top} \top_{\text{right}}$$

Calcul des séquents classique (LK)

Règles

$$\frac{\Gamma, A(t) \vdash \Delta}{\Gamma, \forall x.A(x) \vdash \Delta} \forall_{\text{left}}$$

$$\frac{\Gamma \vdash \Delta, A(x)}{\Gamma \vdash \Delta, \forall x.A(x)} \forall_{\text{right}}, x \notin \Gamma, \Delta$$

$$\frac{\Gamma, A(x) \vdash \Delta}{\Gamma, \exists x.A(x) \vdash \Delta} \exists_{\text{left}}, x \notin \Gamma, \Delta$$

$$\frac{\Gamma \vdash \Delta, A(t)}{\Gamma \vdash \Delta, \exists x.A(x)} \exists_{\text{right}}$$