TD preuves en logique du premier ordre

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Exercices en logique propositionnelle

Propositions à démontrer

1. $A \Rightarrow B \Rightarrow A$
2. $(A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C$
3. $A \wedge B \Rightarrow B$
4. $B \Rightarrow A \vee B$
5. $(A \vee B) \Rightarrow (A \Rightarrow C) \Rightarrow (B \Rightarrow C) \Rightarrow C$
6. $A \Rightarrow \bot \Rightarrow \neg A$
7. $\bot \Rightarrow A$
8. $(A \Leftrightarrow B) \Rightarrow A \Rightarrow B$
9. $(A \Leftrightarrow B) \Rightarrow B \Rightarrow A$
10. $(A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)$

Correction

Preuve (1) dans LJ/LK

▸ Règles LJ

$$A, B \vdash A$$
$$A \vdash B \Rightarrow A$$
$$\vdash A \Rightarrow B \Rightarrow A$$

Correction

Preuve (1) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{A, B \vdash A}{A \vdash B \Rightarrow A}\Rightarrow_{\text{right}}}{\vdash A \Rightarrow B \Rightarrow A}\Rightarrow_{\text{right}}$$

Correction

Preuve (1) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{A, B \vdash A}{A \vdash B \Rightarrow A}\,\Rightarrow_{\text{right}}}{\vdash A \Rightarrow B \Rightarrow A}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (1) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{}{A, B \vdash A}\,\text{ax}}{A \vdash B \Rightarrow A}\,\Rightarrow_{\text{right}}}{\vdash A \Rightarrow B \Rightarrow A}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, B, B \Rightarrow C \vdash B}\quad \dfrac{}{\Gamma, B, B \Rightarrow C, C \vdash C}}{\Gamma, B, B \Rightarrow C \vdash C}}{\Gamma, B \vdash A}\quad \dfrac{}{\Gamma, B \vdash C}}{\Gamma \vdash A}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}$$

$$A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C$$

$$A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C$$

$$\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, B, B \Rightarrow C \vdash B}\quad \dfrac{}{\Gamma, B, B \Rightarrow C, C \vdash C}}{\Gamma, B \vdash A}\quad \dfrac{}{\Gamma, B, B \Rightarrow C \vdash C}}{\Gamma \vdash A}\quad \dfrac{}{\Gamma, B \vdash C}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}$$

$$\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C$$

Correction

Preuve (2) dans LJ

- Règles LJ

$$\Gamma, B, B \Rightarrow C \vdash B \qquad \Gamma, B, B \Rightarrow C, C \vdash C$$

$$\Gamma, B \vdash A \qquad \Gamma, B, B \Rightarrow C \vdash C$$

$$\Gamma \vdash A \qquad \Gamma, B \vdash C$$

$$\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C$$

$$\dfrac{\dfrac{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\ \Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\ \Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

- Règles LJ

$$\Gamma, B, B \Rightarrow C \vdash B \qquad \Gamma, B, B \Rightarrow C, C \vdash C$$

$$\Gamma, B \vdash A \qquad \Gamma, B, B \Rightarrow C \vdash C$$

$$\Gamma \vdash A \qquad \Gamma, B \vdash C$$

$$\dfrac{\dfrac{\dfrac{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\Gamma \vdash A \qquad \Gamma, B \vdash C}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}\Rightarrow_{\text{left}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash A}\,\text{ax} \qquad \Gamma, B \vdash C}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}\,\Rightarrow_{\text{left}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}\,\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\,\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash A}\,\text{ax} \qquad \dfrac{\Gamma, B \vdash A \qquad \Gamma, B, B \Rightarrow C \vdash C}{\Gamma, B \vdash C}\,\Rightarrow_{\text{left}}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}\,\Rightarrow_{\text{left}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}\,\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\,\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash A}\,\text{ax} \qquad \dfrac{\dfrac{}{\Gamma, B \vdash A}\,\text{ax} \qquad \Gamma, B, B \Rightarrow C \vdash B \qquad \Gamma, B, B \Rightarrow C, C \vdash C}{\Gamma, B, B \Rightarrow C \vdash C}\Rightarrow_{\text{left}}}{\Gamma, B \vdash C}\Rightarrow_{\text{left}}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash A}\,\text{ax} \quad \dfrac{\dfrac{}{\Gamma, B \vdash A}\,\text{ax} \quad \dfrac{\Gamma, B, B \Rightarrow C \vdash B \qquad \Gamma, B, B \Rightarrow C, C \vdash C}{\Gamma, B, B \Rightarrow C \vdash C}\Rightarrow_{\text{left}}}{\Gamma, B \vdash C}\Rightarrow_{\text{left}}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}\Rightarrow_{\text{left}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash A}\,\text{ax} \quad \dfrac{\dfrac{}{\Gamma, B \vdash A}\,\text{ax} \quad \dfrac{\dfrac{}{\Gamma, B, B \Rightarrow C \vdash B}\,\text{ax} \quad \Gamma, B, B \Rightarrow C, C \vdash C}{\Gamma, B, B \Rightarrow C \vdash C}\Rightarrow_{\text{left}}}{\Gamma, B \vdash C}\Rightarrow_{\text{left}}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}\Rightarrow_{\text{left}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash A}\,\text{ax} \quad \dfrac{\dfrac{}{\Gamma, B \vdash A}\,\text{ax} \quad \dfrac{\dfrac{}{\Gamma, B, B \Rightarrow C \vdash B}\,\text{ax} \quad \dfrac{}{\Gamma, B, B \Rightarrow C, C \vdash C}\,\text{ax}}{\Gamma, B, B \Rightarrow C \vdash C}\Rightarrow_{\text{left}}}{\Gamma, B \vdash C}\Rightarrow_{\text{left}}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}\Rightarrow_{\text{left}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, B, B \Rightarrow C \vdash C, B}\quad \dfrac{}{\Gamma, B, B \Rightarrow C, C \vdash C}}{\Gamma, B, B \Rightarrow C \vdash C}}{\Gamma, B \vdash C, A}}{\Gamma \vdash C, A}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}$$

$$A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C$$

$$A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C$$

$$\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\Gamma, B, B \Rightarrow C \vdash C, B \qquad \Gamma, B, B \Rightarrow C, C \vdash C$$

$$\Gamma, B \vdash C, A \qquad \Gamma, B, B \Rightarrow C \vdash C$$

$$\Gamma \vdash C, A \qquad \Gamma, B \vdash C$$

$$\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C$$

$$A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C$$

$$\frac{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C} \Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\Gamma, B, B \Rightarrow C \vdash C, B \qquad \Gamma, B, B \Rightarrow C, C \vdash C$$

$$\Gamma, B \vdash C, A \qquad \Gamma, B, B \Rightarrow C \vdash C$$

$$\Gamma \vdash C, A \qquad \Gamma, B \vdash C$$

$$\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C$$

$$\dfrac{\dfrac{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

- Règles LK

$$\Gamma, B, B \Rightarrow C \vdash C, B \qquad \Gamma, B, B \Rightarrow C, C \vdash C$$

$$\Gamma, B \vdash C, A \qquad \Gamma, B, B \Rightarrow C \vdash C$$

$$\Gamma \vdash C, A \qquad \Gamma, B \vdash C$$

$$\dfrac{\dfrac{\dfrac{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\Gamma \vdash C, A \qquad \Gamma, B \vdash C}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}\Rightarrow_{\text{left}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash C, A}\,\text{ax} \qquad \Gamma, B \vdash C}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}\,\Rightarrow_{\text{left}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}\,\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\,\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash C, A}\,\text{ax} \qquad \dfrac{\Gamma, B \vdash C, A \qquad \Gamma, B, B \Rightarrow C \vdash C}{\Gamma, B \vdash C}\Rightarrow_{\text{left}}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}\Rightarrow_{\text{left}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash C, A}\,\text{ax} \quad \dfrac{\dfrac{}{\Gamma, B \vdash C, A}\,\text{ax} \qquad \Gamma, B, B \Rightarrow C \vdash C, B \qquad \Gamma, B, B \Rightarrow C, C \vdash C}{\Gamma, B, B \Rightarrow C \vdash C}\Rightarrow_{\text{left}}}{\Gamma, B \vdash C}\Rightarrow_{\text{left}}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash C, A}\,\text{ax} \quad \dfrac{\dfrac{}{\Gamma, B \vdash C, A}\,\text{ax} \quad \dfrac{\Gamma, B, B \Rightarrow C \vdash C, B \qquad \Gamma, B, B \Rightarrow C, C \vdash C}{\Gamma, B, B \Rightarrow C \vdash C}\Rightarrow_{\text{left}}}{\Gamma, B \vdash C}\Rightarrow_{\text{left}}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}\Rightarrow_{\text{left}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash C, A}\,\text{ax} \quad \dfrac{\dfrac{\dfrac{}{\Gamma, B, B \Rightarrow C \vdash C, B}\,\text{ax} \quad \dfrac{}{\Gamma, B, B \Rightarrow C, C \vdash C}}{\Gamma, B, B \Rightarrow C \vdash C}\Rightarrow_{\text{left}}}{\Gamma, B \vdash C}\Rightarrow_{\text{left}}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}\Rightarrow_{\text{left}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash C, A}\,\text{ax} \quad \dfrac{\dfrac{}{\Gamma, B \vdash C, A}\,\text{ax} \quad \dfrac{\dfrac{}{\Gamma, B, B \Rightarrow C \vdash C, B}\,\text{ax} \quad \dfrac{}{\Gamma, B, B \Rightarrow C, C \vdash C}\,\text{ax}}{\Gamma, B, B \Rightarrow C \vdash C}\Rightarrow_{\text{left}}}{\Gamma, B \vdash C}\Rightarrow_{\text{left}}}{\Gamma = A \Rightarrow B \Rightarrow C, A \Rightarrow B, A \vdash C}\Rightarrow_{\text{left}}}{A \Rightarrow B \Rightarrow C, A \Rightarrow B \vdash A \Rightarrow C}\Rightarrow_{\text{right}}}{A \Rightarrow B \Rightarrow C \vdash (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}}{\vdash (A \Rightarrow B \Rightarrow C) \Rightarrow (A \Rightarrow B) \Rightarrow A \Rightarrow C}\Rightarrow_{\text{right}}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$A, B \vdash B$$
$$A \wedge B \vdash B$$
$$\vdash A \wedge B \Rightarrow B$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{A, B \vdash B}{A \wedge B \vdash B} \Rightarrow_{\text{right}}}{\vdash A \wedge B \Rightarrow B}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{A, B \vdash B}{A \wedge B \vdash B}\,\wedge_{\text{left}}}{\vdash A \wedge B \Rightarrow B}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{}{A, B \vdash B}\,\text{ax}}{A \wedge B \vdash B}\,\wedge_{\text{left}}}{\vdash A \wedge B \Rightarrow B}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (4) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{}{B \vdash B}}{\dfrac{B \vdash A \vee B}{\vdash B \Rightarrow A \vee B}}$$

Correction

Preuve (4) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{}{B \vdash B}}{\dfrac{B \vdash A \vee B}{\vdash B \Rightarrow A \vee B}\Rightarrow_{\text{right}}}$$

Correction

Preuve (4) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{}{B \vdash B}}{B \vdash A \vee B}\vee_{\text{right2}}}{\vdash B \Rightarrow A \vee B}\Rightarrow_{\text{right}}$$

Correction

Preuve (4) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{}{B \vdash B}\,\mathrm{ax}}{B \vdash A \vee B}\,\vee_{\mathrm{right2}}}{\vdash B \Rightarrow A \vee B}\,\Rightarrow_{\mathrm{right}}$$

Correction

Preuve (4) dans LK

▸ Règles LK

$$\dfrac{\dfrac{}{B \vdash A, B}}{B \vdash A \vee B}$$
$$\vdash B \Rightarrow A \vee B$$

Correction

Preuve (4) dans LK

▸ Règles LK

$$\dfrac{\dfrac{B \vdash A, B}{B \vdash A \vee B} \vee_{\text{right}}}{\vdash B \Rightarrow A \vee B} \Rightarrow_{\text{right}}$$

Correction

Preuve (4) dans LK

▸ Règles LK

$$\dfrac{\dfrac{B \vdash A, B}{B \vdash A \vee B}\,\vee_{\text{right}}}{\vdash B \Rightarrow A \vee B}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (4) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{}{B \vdash A, B}\,\text{ax}}{B \vdash A \vee B}\,\vee_{\text{right}}}{\vdash B \Rightarrow A \vee B}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LJ/LK

▸ Règles LJ

$$A, \bot \vdash \neg A$$
$$A \vdash \bot \Rightarrow \neg A$$
$$\vdash A \Rightarrow \bot \Rightarrow \neg A$$

Correction

Preuve (6) dans LJ/LK

▸ Règles LJ

$$A, \bot \vdash \neg A$$

$$\frac{A \vdash \bot \Rightarrow \neg A}{\vdash A \Rightarrow \bot \Rightarrow \neg A} \Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{A, \bot \vdash \neg A}{A \vdash \bot \Rightarrow \neg A}\Rightarrow_{\text{right}}}{\vdash A \Rightarrow \bot \Rightarrow \neg A}\Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{}{A, \bot \vdash \neg A}\,\bot_{\text{left}}}{A \vdash \bot \Rightarrow \neg A}\,\Rightarrow_{\text{right}}}{\vdash A \Rightarrow \bot \Rightarrow \neg A}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (7) dans LJ/LK

▸ Règles LJ

$$\frac{}{\bot \vdash A}$$
$$\vdash \bot \Rightarrow A$$

Correction

Preuve (7) dans LJ/LK

▸ Règles LJ

$$\frac{\bot \vdash A}{\vdash \bot \Rightarrow A} \Rightarrow_{\text{right}}$$

Correction

Preuve (7) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{}{\bot \vdash A}\,\text{ax}}{\vdash \bot \Rightarrow A}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (8) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{A \vdash A \qquad B \vdash B}{A \Leftrightarrow B, A \vdash B}}{A \Leftrightarrow B \vdash A \Rightarrow B}}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B}$$

Correction

Preuve (8) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{}{A \vdash A} \quad \dfrac{}{B \vdash B}}{A \Leftrightarrow B, A \vdash B}}{\dfrac{A \Leftrightarrow B \vdash A \Rightarrow B}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B}\ \Rightarrow_{\text{right}}}$$

Correction

Preuve (8) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{A \vdash A \qquad B \vdash B}{A \Leftrightarrow B, A \vdash B}\Rightarrow_{\text{right}}}{A \Leftrightarrow B \vdash A \Rightarrow B}\Rightarrow_{\text{right}}$$
$$\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B$$

Correction

Preuve (8) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{A \vdash A \qquad B \vdash B}{A \Leftrightarrow B, A \vdash B}\,\Leftrightarrow_{\text{left1}}}{A \Leftrightarrow B \vdash A \Rightarrow B}\,\Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (8) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,\text{ax} \qquad \dfrac{}{B \vdash B}\,\text{ax}}{A \Leftrightarrow B, A \vdash B}\Leftrightarrow_{\text{left1}}}{A \Leftrightarrow B \vdash A \Rightarrow B}\Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B}\Rightarrow_{\text{right}}$$

Correction

Preuve (8) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash A}\,\text{ax} \qquad \dfrac{}{B \vdash B}\,\text{ax}}{A \Leftrightarrow B, A \vdash B}\,\Leftrightarrow_{\text{left1}}}{A \Leftrightarrow B \vdash A \Rightarrow B}\,\Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (8) dans LK

▸ Règles LK

$$\frac{\dfrac{\dfrac{A \vdash B, A, B}{A \Leftrightarrow B, A \vdash B} \qquad \dfrac{A, A, B \vdash B}{A \Leftrightarrow B, A \vdash B}}{A \Leftrightarrow B \vdash A \Rightarrow B}}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B}$$

Correction

Preuve (8) dans LK

▸ Règles LK

$$\dfrac{\dfrac{A \vdash B, A, B \qquad A, A, B \vdash B}{A \Leftrightarrow B, A \vdash B}}{A \Leftrightarrow B \vdash A \Rightarrow B} \Rightarrow_{\text{right}}$$
$$\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B$$

Correction

Preuve (8) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{A \vdash B, A, B \qquad A, A, B \vdash B}{A \Leftrightarrow B, A \vdash B}\Rightarrow_{\text{right}}}{A \Leftrightarrow B \vdash A \Rightarrow B}\Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B}$$

Correction

Preuve (8) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{A \vdash B, A, B \qquad A, A, B \vdash B}{A \Leftrightarrow B, A \vdash B} \Leftrightarrow_{\text{left}}}{A \Leftrightarrow B \vdash A \Rightarrow B} \Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B} \Rightarrow_{\text{right}}$$

Correction

Preuve (8) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash B, A, B}\,\text{ax} \qquad A, A, B \vdash B}{A \Leftrightarrow B, A \vdash B}\,\Leftrightarrow_{\text{left}}}{A \Leftrightarrow B \vdash A \Rightarrow B}\,\Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (8) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vdash B, A, B}\,\text{ax} \qquad \dfrac{}{A, A, B \vdash B}\,\text{ax}}{A \Leftrightarrow B, A \vdash B}\,\Leftrightarrow_{\text{left}}}{A \Leftrightarrow B \vdash A \Rightarrow B}\,\Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow A \Rightarrow B}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (9) dans LJ

▸ Règles LJ

$$\frac{\dfrac{\dfrac{}{B \vdash B} \qquad \dfrac{}{A \vdash A}}{A \Leftrightarrow B, B \vdash A}}{A \Leftrightarrow B \vdash B \Rightarrow A}$$
$$\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A$$

Correction

Preuve (9) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{B \vdash B \qquad A \vdash A}{A \Leftrightarrow B, B \vdash A}}{A \Leftrightarrow B \vdash B \Rightarrow A} \Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A}$$

Correction

Preuve (9) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{}{B \vdash B} \quad \dfrac{}{A \vdash A}}{A \Leftrightarrow B, B \vdash A} \Rightarrow_{\text{right}}}{A \Leftrightarrow B \vdash B \Rightarrow A} \Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A}$$

Correction

Preuve (9) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{B \vdash B \qquad A \vdash A}{A \Leftrightarrow B, B \vdash A}\,\Leftrightarrow_{\text{left2}}}{A \Leftrightarrow B \vdash B \Rightarrow A}\,\Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (9) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{}{B \vdash B}\,\text{ax} \qquad \dfrac{}{A \vdash A}\,\text{ax}}{A \Leftrightarrow B, B \vdash A}\,\Leftrightarrow_{\text{left2}}}{A \Leftrightarrow B \vdash B \Rightarrow A}\,\Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (9) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{}{B \vdash B}\,\text{ax} \qquad \dfrac{}{A \vdash A}\,\text{ax}}{A \Leftrightarrow B, B \vdash A}\,\Leftrightarrow_{\text{left2}}}{A \Leftrightarrow B \vdash B \Rightarrow A}\,\Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (9) dans LK

▸ Règles LK

$$\frac{\dfrac{\dfrac{B \vdash A, A, B \qquad B, A, B \vdash A}{A \Leftrightarrow B, B \vdash A}}{A \Leftrightarrow B \vdash B \Rightarrow A}}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A}$$

Correction

Preuve (9) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{B \vdash A, A, B}{B, A, B \vdash A}}{A \Leftrightarrow B, B \vdash A}}{\dfrac{A \Leftrightarrow B \vdash B \Rightarrow A}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A} \Rightarrow_{\text{right}}}$$

Correction

Preuve (9) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{B \vdash A, A, B \qquad B, A, B \vdash A}{A \Leftrightarrow B, B \vdash A}\Rightarrow_{\text{right}}}{A \Leftrightarrow B \vdash B \Rightarrow A}\Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A}$$

Correction

Preuve (9) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{B \vdash A, A, B \qquad B, A, B \vdash A}{A \Leftrightarrow B, B \vdash A}\,\Leftrightarrow_{\text{left}}}{A \Leftrightarrow B \vdash B \Rightarrow A}\,\Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (9) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{}{B \vdash A, A, B}\,\text{ax} \qquad B, A, B \vdash A}{A \Leftrightarrow B, B \vdash A}\,\Leftrightarrow_{\text{left}}}{A \Leftrightarrow B \vdash B \Rightarrow A}\,\Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (9) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{}{B \vdash A, A, B}\,\text{ax} \qquad \dfrac{}{B, A, B \vdash A}\,\text{ax}}{A \Leftrightarrow B, B \vdash A}\,\Leftrightarrow_{\text{left}}}{A \Leftrightarrow B \vdash B \Rightarrow A}\,\Rightarrow_{\text{right}}}{\vdash (A \Leftrightarrow B) \Rightarrow B \Rightarrow A}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{B \Rightarrow A, A \vdash A}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \quad \dfrac{B \Rightarrow A, A, B \vdash B}{A \Rightarrow B, B \Rightarrow A, B \vdash A}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B} \quad \dfrac{A \Rightarrow B, B \vdash B}{A \Rightarrow B, B, A \vdash A}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}$$

$$\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B \Rightarrow A, A \vdash A}\quad \dfrac{}{B \Rightarrow A, A, B \vdash B}}{A \Rightarrow B, B \Rightarrow A, A \vdash B}\quad \dfrac{\dfrac{}{A \Rightarrow B, B \vdash B}\quad \dfrac{}{A \Rightarrow B, B, A \vdash A}}{A \Rightarrow B, B \Rightarrow A, B \vdash A}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\Rightarrow_{\textbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\Rightarrow_{\textbf{right}}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$B \Rightarrow A, A \vdash A \qquad B \Rightarrow A, A, B \vdash B \qquad A \Rightarrow B, B \vdash B \qquad A \Rightarrow B, B, A \vdash A$$

$$A \Rightarrow B, B \Rightarrow A, A \vdash B \qquad A \Rightarrow B, B \Rightarrow A, B \vdash A$$

$$\dfrac{\dfrac{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\textbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\textbf{right}}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{B \Rightarrow A, A \vdash A \qquad B \Rightarrow A, A, B \vdash B}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \Rightarrow_{\mathbf{left}} \qquad \dfrac{A \Rightarrow B, B \vdash B \qquad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A} \Rightarrow_{\mathbf{left}}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B} \Leftrightarrow_{\mathbf{right}}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\mathbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\mathbf{right}}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{B \Rightarrow A, A \vdash A \qquad B \Rightarrow A, A, B \vdash B}{A \Rightarrow B, B \Rightarrow A, A \vdash B}\Rightarrow_{\textbf{left}} \qquad A \Rightarrow B, B \vdash B \qquad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A}\Leftrightarrow_{\textbf{right}}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}\Rightarrow_{\textbf{right}}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\Rightarrow_{\textbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B \Rightarrow A, A \vdash A}\,\text{ax} \qquad B \Rightarrow A, A, B \vdash B}{A \Rightarrow B, B \Rightarrow A, A \vdash B}\Rightarrow_{\textbf{left}} \qquad A \Rightarrow B, B \vdash B \qquad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A}\Leftrightarrow_{\textbf{right}}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}\Rightarrow_{\textbf{right}}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\Rightarrow_{\textbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\Rightarrow_{\textbf{right}}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B \Rightarrow A, A \vdash A}\,\text{ax} \qquad \dfrac{}{B \Rightarrow A, A, B \vdash B}\,\text{ax}}{A \Rightarrow B, B \Rightarrow A, A \vdash B}\,\Rightarrow_{\textbf{left}} \qquad A \Rightarrow B, B \vdash B \qquad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A}\,\Leftrightarrow_{\textbf{right}}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}\,\Rightarrow_{\textbf{right}}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{}{B \Rightarrow A, A \vdash A}\,\text{ax} \qquad \dfrac{}{B \Rightarrow A, A, B \vdash B}\,\text{ax}}{A \Rightarrow B, B \Rightarrow A, A \vdash B}\,\Rightarrow_{\textbf{left}} \qquad \dfrac{\dfrac{A \Rightarrow B, B \vdash B \qquad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A}\,\Rightarrow_{\textbf{left}}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}\,\Leftrightarrow_{\textbf{right}}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{}{B \Rightarrow A, A \vdash A}\,\text{ax} \qquad \dfrac{}{B \Rightarrow A, A, B \vdash B}\,\text{ax}}{A \Rightarrow B, B \Rightarrow A, A \vdash B}\,\Rightarrow_{\textbf{left}} \qquad \dfrac{\dfrac{}{A \Rightarrow B, B \vdash B}\,\text{ax} \qquad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A}\,\Rightarrow_{\textbf{left}}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}\,\Leftrightarrow_{\textbf{right}}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}$$

Correction

Preuve (10) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B \Rightarrow A, A \vdash A}\,\text{ax} \quad \dfrac{}{B \Rightarrow A, A, B \vdash B}\,\text{ax}}{A \Rightarrow B, B \Rightarrow A, A \vdash B}\,\Rightarrow_{\textbf{left}} \quad \dfrac{\dfrac{}{A \Rightarrow B, B \vdash B}\,\text{ax} \quad \dfrac{}{A \Rightarrow B, B, A \vdash A}\,\text{ax}}{A \Rightarrow B, B \Rightarrow A, B \vdash A}\,\Rightarrow_{\textbf{left}}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}\,\Leftrightarrow_{\textbf{right}}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{}{B \Rightarrow A, A \vdash B, A} \quad \dfrac{}{B \Rightarrow A, A, B \vdash B}}{A \Rightarrow B, B \Rightarrow A, A \vdash B} \quad \dfrac{\dfrac{}{A \Rightarrow B, B \vdash A, B} \quad \dfrac{}{A \Rightarrow B, B, A \vdash A}}{A \Rightarrow B, B \Rightarrow A, B \vdash A}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}$$

$$\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)$$

Correction

Preuve (10) dans LK

- Règles LK

$$B \Rightarrow A, A \vdash B, A \qquad B \Rightarrow A, A, B \vdash B \qquad A \Rightarrow B, B \vdash A, B \qquad A \Rightarrow B, B, A \vdash A$$

$$A \Rightarrow B, B \Rightarrow A, A \vdash B \qquad A \Rightarrow B, B \Rightarrow A, B \vdash A$$

$$A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B$$

$$\frac{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\textbf{right}}$$

Correction

Preuve (10) dans LK

- Règles LK

$$B \Rightarrow A, A \vdash B, A \qquad B \Rightarrow A, A, B \vdash B \qquad A \Rightarrow B, B \vdash A, B \qquad A \Rightarrow B, B, A \vdash A$$

$$A \Rightarrow B, B \Rightarrow A, A \vdash B \qquad A \Rightarrow B, B \Rightarrow A, B \vdash A$$

$$\dfrac{\dfrac{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\textbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)} \Rightarrow_{\textbf{right}}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$$B \Rightarrow A, A \vdash B, A \qquad B \Rightarrow A, A, B \vdash B \qquad A \Rightarrow B, B \vdash A, B \qquad A \Rightarrow B, B, A \vdash A$$

$$\dfrac{\dfrac{\dfrac{A \Rightarrow B, B \Rightarrow A, A \vdash B \qquad A \Rightarrow B, B \Rightarrow A, B \vdash A}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}\,\Leftrightarrow_{\textbf{right}}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{B \Rightarrow A, A \vdash B, A \qquad B \Rightarrow A, A, B \vdash B}{A \Rightarrow B, B \Rightarrow A, A \vdash B}\Rightarrow_{\textbf{left}} \qquad A \Rightarrow B, B \vdash A, B \qquad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A}\Leftrightarrow_{\textbf{right}}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}\Rightarrow_{\textbf{right}}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\Rightarrow_{\textbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B \Rightarrow A, A \vdash B, A}\,\text{ax} \qquad B \Rightarrow A, A, B \vdash B}{A \Rightarrow B, B \Rightarrow A, A \vdash B}\Rightarrow_{\textbf{left}} \qquad A \Rightarrow B, B \vdash A, B \qquad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A}\Leftrightarrow_{\textbf{right}}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}\Rightarrow_{\textbf{right}}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\Rightarrow_{\textbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\Rightarrow_{\textbf{right}}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B \Rightarrow A, A \vdash B, A}\,\text{ax} \qquad \dfrac{}{B \Rightarrow A, A, B \vdash B}\,\text{ax}}{A \Rightarrow B, B \Rightarrow A, A \vdash B}\,\Rightarrow_{\textbf{left}} \qquad A \Rightarrow B, B \vdash A, B \qquad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A}\,\Leftrightarrow_{\textbf{right}}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}\,\Rightarrow_{\textbf{right}}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}$$

Correction

Preuve (10) dans LK

- Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B \Rightarrow A, A \vdash B, A}\,\text{ax} \qquad \dfrac{}{B \Rightarrow A, A, B \vdash B}\,\text{ax}}{A \Rightarrow B, B \Rightarrow A, A \vdash B}\,\Rightarrow_{\textbf{left}} \qquad \dfrac{A \Rightarrow B, B \vdash A, B \qquad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A}\,\Rightarrow_{\textbf{left}}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}\,\Leftrightarrow_{\textbf{right}}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{}{B \Rightarrow A, A \vdash B, A}\,\text{ax} \qquad \dfrac{}{B \Rightarrow A, A, B \vdash B}\,\text{ax}}{A \Rightarrow B, B \Rightarrow A, A \vdash B}\,\Rightarrow_{\textbf{left}} \qquad \dfrac{\dfrac{}{A \Rightarrow B, B \vdash A, B}\,\text{ax} \qquad A \Rightarrow B, B, A \vdash A}{A \Rightarrow B, B \Rightarrow A, B \vdash A}\,\Rightarrow_{\textbf{left}}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}\,\Leftrightarrow_{\textbf{right}}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}$$

Correction

Preuve (10) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{B \Rightarrow A, A \vdash B, A}\,\text{ax} \quad \dfrac{}{B \Rightarrow A, A, B \vdash B}\,\text{ax}}{A \Rightarrow B, B \Rightarrow A, A \vdash B}\,\Rightarrow_{\textbf{left}} \quad \dfrac{\dfrac{}{A \Rightarrow B, B \vdash A, B}\,\text{ax} \quad \dfrac{}{A \Rightarrow B, B, A \vdash A}\,\text{ax}}{A \Rightarrow B, B \Rightarrow A, B \vdash A}\,\Rightarrow_{\textbf{left}}}{A \Rightarrow B, B \Rightarrow A \vdash A \Leftrightarrow B}\,\Leftrightarrow_{\textbf{right}}}{A \Rightarrow B \vdash (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}}{\vdash (A \Rightarrow B) \Rightarrow (B \Rightarrow A) \Rightarrow (A \Leftrightarrow B)}\,\Rightarrow_{\textbf{right}}$$

Exercices en logique du premier ordre

Propositions à démontrer

1. $\forall x.P(x) \Rightarrow \exists y.P(y) \vee Q(y)$
2. $(\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))$
3. $(\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)$
4. $(\forall x.P(x) \wedge Q(x)) \Rightarrow (\forall x.P(x)) \wedge (\forall x.Q(x))$
5. $(\forall x.\neg P(x)) \Rightarrow \neg(\exists x.P(x))$
6. $\neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)$

Correction

Preuve (1) dans LJ/LK

▸ Règles LJ

$$P(x) \vdash P(x), Q(x)$$
$$P(x) \vdash P(x) \vee Q(x)$$
$$P(x) \vdash \exists y.P(y) \vee Q(y)$$
$$\vdash P(x) \Rightarrow \exists y.P(y) \vee Q(y)$$
$$\vdash \forall x.P(x) \Rightarrow \exists y.P(y) \vee Q(y)$$

Correction

Preuve (1) dans LJ/LK

▸ Règles LJ

$$P(x) \vdash P(x), Q(x)$$
$$P(x) \vdash P(x) \vee Q(x)$$
$$P(x) \vdash \exists y.P(y) \vee Q(y)$$
$$\frac{\vdash P(x) \Rightarrow \exists y.P(y) \vee Q(y)}{\vdash \forall x.P(x) \Rightarrow \exists y.P(y) \vee Q(y)} \forall_{\text{right}}$$

Correction

Preuve (1) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{}{P(x) \vdash P(x), Q(x)}}{P(x) \vdash P(x) \vee Q(x)}}{\dfrac{P(x) \vdash \exists y.P(y) \vee Q(y)}{\vdash P(x) \Rightarrow \exists y.P(y) \vee Q(y)} \Rightarrow_{\text{right}}}{\vdash \forall x.P(x) \Rightarrow \exists y.P(y) \vee Q(y)} \forall_{\text{right}}$$

Correction

Preuve (1) dans LJ/LK

▸ Règles LJ

$$P(x) \vdash P(x), Q(x)$$

$$\dfrac{\dfrac{\dfrac{P(x) \vdash P(x) \vee Q(x)}{P(x) \vdash \exists y.P(y) \vee Q(y)}\,\exists_{\text{right}}}{\vdash P(x) \Rightarrow \exists y.P(y) \vee Q(y)}\,\Rightarrow_{\text{right}}}{\vdash \forall x.P(x) \Rightarrow \exists y.P(y) \vee Q(y)}\,\forall_{\text{right}}$$

Correction

Preuve (1) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(x) \vdash P(x), Q(x)}{P(x) \vdash P(x) \vee Q(x)} \vee_{\text{right}}}{P(x) \vdash \exists y.P(y) \vee Q(y)} \exists_{\text{right}}}{\vdash P(x) \Rightarrow \exists y.P(y) \vee Q(y)} \Rightarrow_{\text{right}}}{\vdash \forall x.P(x) \Rightarrow \exists y.P(y) \vee Q(y)} \forall_{\text{right}}$$

Correction

Preuve (1) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x) \vdash P(x), Q(x)}\,\text{ax}}{P(x) \vdash P(x) \vee Q(x)}\,\vee_{\text{right}}}{P(x) \vdash \exists y.P(y) \vee Q(y)}\,\exists_{\text{right}}}{\vdash P(x) \Rightarrow \exists y.P(y) \vee Q(y)}\,\Rightarrow_{\text{right}}}{\vdash \forall x.P(x) \Rightarrow \exists y.P(y) \vee Q(y)}\,\forall_{\text{right}}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{P(x) \vdash P(x)}{P(x) \vdash \exists x.P(x)}}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \quad \dfrac{\dfrac{Q(x) \vdash Q(x)}{Q(x) \vdash \exists x.Q(x)}}{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}}{\dfrac{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}}$$

$$\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{P(x) \vdash P(x)}{P(x) \vdash \exists x.P(x)}}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \quad \dfrac{\dfrac{Q(x) \vdash Q(x)}{Q(x) \vdash \exists x.Q(x)}}{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \Rightarrow_{\text{right}}$$

$$\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{P(x) \vdash P(x)}{P(x) \vdash \exists x.P(x)}}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \quad \dfrac{\dfrac{Q(x) \vdash Q(x)}{Q(x) \vdash \exists x.Q(x)}}{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}}{\dfrac{\dfrac{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\ \exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))}\ \Rightarrow_{\text{right}}}$$

Correction

Preuve (2) dans LJ

- Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{P(x) \vdash P(x)}{P(x) \vdash \exists x.P(x)}}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \quad \dfrac{\dfrac{Q(x) \vdash Q(x)}{Q(x) \vdash \exists x.Q(x)}}{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{\text{left}}}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \exists_{\text{left}}}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \Rightarrow_{\text{right}}$$

$$\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))$$

Correction

Preuve (2) dans LJ

- Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(x) \vdash P(x)}{P(x) \vdash \exists x.P(x)}\vee_{\text{right1}}}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \quad \dfrac{\dfrac{Q(x) \vdash Q(x)}{Q(x) \vdash \exists x.Q(x)}\vee_{\text{right2}}}{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\vee_{\text{left}}}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))}\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(x) \vdash P(x)}{P(x) \vdash \exists x.P(x)}\,\exists_{\text{right}}}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{right1}} \qquad \dfrac{\dfrac{Q(x) \vdash Q(x)}{Q(x) \vdash \exists x.Q(x)}\,\exists_{\text{right}}}{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{right2}}}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{left}}}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x) \vdash P(x)}\,\text{ax}}{P(x) \vdash \exists x.P(x)}\,\exists_{\text{right}}}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{right1}} \qquad \dfrac{\dfrac{Q(x) \vdash Q(x)}{Q(x) \vdash \exists x.Q(x)}\,\exists_{\text{right}}}{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{right2}}}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{left}}}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x) \vdash P(x)}\,\text{ax}}{P(x) \vdash \exists x.P(x)}\,\exists_{\text{right}}}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{right1}} \qquad \dfrac{\dfrac{Q(x) \vdash Q(x)}{Q(x) \vdash \exists x.Q(x)}}{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{right2}}}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{left}}}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x) \vdash P(x)}\,\text{ax}}{P(x) \vdash \exists x.P(x)}\,\exists_{\text{right}}}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{right1}} \qquad \dfrac{\dfrac{\dfrac{Q(x) \vdash Q(x)}{Q(x) \vdash \exists x.Q(x)}\,\exists_{\text{right}}}{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{right2}}}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{left}}}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LJ

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x) \vdash P(x)}\,\text{ax}}{P(x) \vdash \exists x.P(x)}\,\exists_{\text{right}}}{P(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{right1}} \quad \dfrac{\dfrac{\dfrac{}{Q(x) \vdash Q(x)}\,\text{ax}}{Q(x) \vdash \exists x.Q(x)}\,\exists_{\text{right}}}{Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{right2}}}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{left}}}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\frac{\dfrac{\dfrac{\dfrac{\dfrac{P(x) \vdash P(x), \exists x.Q(x)}{P(x) \vdash \exists x.P(x), \exists x.Q(x)} \quad \dfrac{Q(x) \vdash \exists x.P(x), Q(x)}{Q(x) \vdash \exists x.P(x), \exists x.Q(x)}}{P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x)}}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x) \vdash P(x), \exists x.Q(x)} \quad \dfrac{}{Q(x) \vdash \exists x.P(x), Q(x)}}{P(x) \vdash \exists x.P(x), \exists x.Q(x)} \quad \dfrac{}{Q(x) \vdash \exists x.P(x), \exists x.Q(x)}}{P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x)}}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\dfrac{\dfrac{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))}\,\Rightarrow_{\text{right}}$$

$$P(x) \vdash P(x), \exists x.Q(x) \qquad Q(x) \vdash \exists x.P(x), Q(x)$$

$$P(x) \vdash \exists x.P(x), \exists x.Q(x) \qquad Q(x) \vdash \exists x.P(x), \exists x.Q(x)$$

$$P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x)$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x)}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \vee_{\text{right}}}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))} \exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))} \Rightarrow_{\text{right}}$$

$P(x) \vdash P(x), \exists x.Q(x)$ $\quad$ $Q(x) \vdash \exists x.P(x), Q(x)$

$P(x) \vdash \exists x.P(x), \exists x.Q(x)$ $\quad$ $Q(x) \vdash \exists x.P(x), \exists x.Q(x)$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(x) \vdash P(x), \exists x.Q(x) \qquad Q(x) \vdash \exists x.P(x), Q(x)}{P(x) \vdash \exists x.P(x), \exists x.Q(x) \qquad Q(x) \vdash \exists x.P(x), \exists x.Q(x)}\ \vee_{\text{left}}}{P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x)}\ \vee_{\text{right}}}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\ \exists_{\text{left}}}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\ \Rightarrow_{\text{right}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(x) \vdash P(x), \exists x.Q(x)}{P(x) \vdash \exists x.P(x), \exists x.Q(x)}\exists_{\text{right}} \quad \dfrac{Q(x) \vdash \exists x.P(x), Q(x)}{Q(x) \vdash \exists x.P(x), \exists x.Q(x)}\exists_{\text{right}}}{P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x)}\vee_{\text{left}}}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\vee_{\text{right}}}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))}\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x) \vdash P(x), \exists x.Q(x)}\,\text{ax}}{P(x) \vdash \exists x.P(x), \exists x.Q(x)}\,\exists_{\text{right}} \qquad Q(x) \vdash \exists x.P(x), Q(x)}{Q(x) \vdash \exists x.P(x), \exists x.Q(x)}\,\vee_{\text{left}}}{P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x)}\,\vee_{\text{right}}}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\exists_{\text{left}}}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x) \vdash P(x), \exists x.Q(x)}\,\text{ax}}{P(x) \vdash \exists x.P(x), \exists x.Q(x)}\,\exists_{\text{right}} \quad \dfrac{Q(x) \vdash \exists x.P(x), Q(x)}{Q(x) \vdash \exists x.P(x), \exists x.Q(x)}\,\exists_{\text{right}}}{P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x)}\,\vee_{\text{left}}}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{right}}}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (2) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x) \vdash P(x), \exists x.Q(x)}\,\text{ax}}{P(x) \vdash \exists x.P(x), \exists x.Q(x)}\,\exists_{\text{right}} \quad \dfrac{\dfrac{}{Q(x) \vdash \exists x.P(x), Q(x)}\,\text{ax}}{Q(x) \vdash \exists x.P(x), \exists x.Q(x)}\,\exists_{\text{right}}}{P(x) \vee Q(x) \vdash \exists x.P(x), \exists x.Q(x)}\,\vee_{\text{left}}}{P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\vee_{\text{right}}}{\exists x.P(x) \vee Q(x) \vdash (\exists x.P(x)) \vee (\exists x.Q(x))}\,\exists_{\text{left}}}{\vdash (\exists x.P(x) \vee Q(x)) \Rightarrow (\exists x.P(x)) \vee (\exists x.Q(x))}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\frac{\dfrac{\dfrac{\dfrac{}{P(x), \forall x.Q(x) \vdash P(x)}}{\forall x.P(x), \forall x.Q(x) \vdash P(x)} \qquad \dfrac{\dfrac{}{\forall x.P(x), Q(x) \vdash Q(x)}}{\forall x.P(x), \forall x.Q(x) \vdash Q(x)}}{\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)}}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)}}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)}}{\vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{P(x), \forall x.Q(x) \vdash P(x)}{\forall x.P(x), \forall x.Q(x) \vdash P(x)} \quad \dfrac{\forall x.P(x), Q(x) \vdash Q(x)}{\forall x.P(x), \forall x.Q(x) \vdash Q(x)}}{\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)}}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)}}{\dfrac{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)}{\vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)} \Rightarrow_{\text{right}}}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$P(x), \forall x.Q(x) \vdash P(x) \qquad \forall x.P(x), Q(x) \vdash Q(x)$$
$$\forall x.P(x), \forall x.Q(x) \vdash P(x) \qquad \forall x.P(x), \forall x.Q(x) \vdash Q(x)$$
$$\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)$$

$$\dfrac{\dfrac{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)}\ \forall_{\text{right}}}{\vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)}\ \Rightarrow_{\text{right}}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$P(x), \forall x.Q(x) \vdash P(x) \qquad \forall x.P(x), Q(x) \vdash Q(x)$$
$$\forall x.P(x), \forall x.Q(x) \vdash P(x) \qquad \forall x.P(x), \forall x.Q(x) \vdash Q(x)$$

$$\dfrac{\dfrac{\dfrac{\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)} \wedge_{\text{left}}}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)} \forall_{\text{right}}}{\vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)} \Rightarrow_{\text{right}}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(x), \forall x.Q(x) \vdash P(x) \qquad \forall x.P(x), Q(x) \vdash Q(x)}{\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)}\,\wedge_{\text{right}}}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)}\,\wedge_{\text{left}}}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)}\,\forall_{\text{right}}}{\vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{P(x), \forall x.Q(x) \vdash P(x)}{\forall x.P(x), \forall x.Q(x) \vdash P(x)} \forall_{\text{left}} \quad \forall x.P(x), Q(x) \vdash Q(x)}{\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)} \wedge_{\text{right}}}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)} \wedge_{\text{left}}}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)} \forall_{\text{right}}}{\vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)} \Rightarrow_{\text{right}}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x), \forall x.Q(x) \vdash P(x)}\,\text{ax}}{\forall x.P(x), \forall x.Q(x) \vdash P(x)}\,\forall_{\text{left}} \qquad \forall x.P(x), Q(x) \vdash Q(x)}{\forall x.P(x), \forall x.Q(x) \vdash Q(x)}\,\wedge_{\text{right}}}{\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)}\,\wedge_{\text{left}}}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)}\,\forall_{\text{right}}}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)}\,\Rightarrow_{\text{right}}}{\vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x), \forall x.Q(x) \vdash P(x)}\,\text{ax}}{\forall x.P(x), \forall x.Q(x) \vdash P(x)}\,\forall_{\text{left}} \qquad \dfrac{\forall x.P(x), Q(x) \vdash Q(x)}{\forall x.P(x), \forall x.Q(x) \vdash Q(x)}\,\forall_{\text{left}}}{\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)}\,\wedge_{\text{right}}}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)}\,\wedge_{\text{left}}}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)}\,\forall_{\text{right}}}{\vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (3) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{}{P(x), \forall x.Q(x) \vdash P(x)}\,\text{ax}}{\forall x.P(x), \forall x.Q(x) \vdash P(x)}\,\forall_{\text{left}} \qquad \dfrac{\dfrac{}{\forall x.P(x), Q(x) \vdash Q(x)}\,\text{ax}}{\forall x.P(x), \forall x.Q(x) \vdash Q(x)}\,\forall_{\text{left}}}{\forall x.P(x), \forall x.Q(x) \vdash P(x) \wedge Q(x)}\,\wedge_{\text{right}}}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash P(x) \wedge Q(x)}\,\wedge_{\text{left}}}{(\forall x.P(x)) \wedge (\forall x.Q(x)) \vdash \forall x.P(x) \wedge Q(x)}\,\forall_{\text{right}}}{\vdash (\forall x.P(x)) \wedge (\forall x.Q(x)) \Rightarrow \forall x.P(x) \wedge Q(x)}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (4) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x), Q(x) \vdash P(x)}}{P(x) \wedge Q(x) \vdash P(x)}}{\forall x.P(x) \wedge Q(x) \vdash P(x)}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.P(x)} \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x), Q(x) \vdash Q(x)}}{P(x) \wedge Q(x) \vdash Q(x)}}{\forall x.P(x) \wedge Q(x) \vdash Q(x)}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.Q(x)}}{\forall x.P(x) \wedge Q(x) \vdash (\forall x.P(x)) \wedge (\forall x.Q(x))}}{\vdash (\forall x.P(x) \wedge Q(x)) \Rightarrow (\forall x.P(x)) \wedge (\forall x.Q(x))}$$

Correction

Preuve (4) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{}{P(x), Q(x) \vdash P(x)}}{P(x) \wedge Q(x) \vdash P(x)}}{\forall x.P(x) \wedge Q(x) \vdash P(x)}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.P(x)} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{P(x), Q(x) \vdash Q(x)}}{P(x) \wedge Q(x) \vdash Q(x)}}{\forall x.P(x) \wedge Q(x) \vdash Q(x)}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.Q(x)}$$

$$\dfrac{\forall x.P(x) \wedge Q(x) \vdash (\forall x.P(x)) \wedge (\forall x.Q(x))}{\vdash (\forall x.P(x) \wedge Q(x)) \Rightarrow (\forall x.P(x)) \wedge (\forall x.Q(x))} \Rightarrow_{\text{right}}$$

Correction

Preuve (4) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{P(x), Q(x) \vdash P(x)}{P(x) \wedge Q(x) \vdash P(x)}}{\forall x.P(x) \wedge Q(x) \vdash P(x)} \quad \dfrac{\dfrac{\dfrac{P(x), Q(x) \vdash Q(x)}{P(x) \wedge Q(x) \vdash Q(x)}}{\forall x.P(x) \wedge Q(x) \vdash Q(x)}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.P(x) \quad \forall x.P(x) \wedge Q(x) \vdash \forall x.Q(x)} \wedge_{\text{right}}}{\forall x.P(x) \wedge Q(x) \vdash (\forall x.P(x)) \wedge (\forall x.Q(x))} \Rightarrow_{\text{right}}}{\vdash (\forall x.P(x) \wedge Q(x)) \Rightarrow (\forall x.P(x)) \wedge (\forall x.Q(x))}$$

Correction

Preuve (4) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{P(x), Q(x) \vdash P(x)}{P(x) \wedge Q(x) \vdash P(x)}}{\forall x.P(x) \wedge Q(x) \vdash P(x)}\,\forall_{\text{right}} \qquad \dfrac{\dfrac{P(x), Q(x) \vdash Q(x)}{P(x) \wedge Q(x) \vdash Q(x)}}{\forall x.P(x) \wedge Q(x) \vdash Q(x)}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.Q(x)}\,\wedge_{\text{right}}}{\forall x.P(x) \wedge Q(x) \vdash (\forall x.P(x)) \wedge (\forall x.Q(x))}\,\Rightarrow_{\text{right}}$$

$$\vdash (\forall x.P(x) \wedge Q(x)) \Rightarrow (\forall x.P(x)) \wedge (\forall x.Q(x))$$

Correction

Preuve (4) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{P(x), Q(x) \vdash P(x)}{P(x) \wedge Q(x) \vdash P(x)}\,\forall_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash P(x)}\,\forall_{\text{right}} \qquad \dfrac{P(x), Q(x) \vdash Q(x)}{P(x) \wedge Q(x) \vdash Q(x)}\,\forall_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.Q(x)}\,\forall_{\text{right}}}{\forall x.P(x) \wedge Q(x) \vdash (\forall x.P(x)) \wedge (\forall x.Q(x))}\,\wedge_{\text{right}}}{\vdash (\forall x.P(x) \wedge Q(x)) \Rightarrow (\forall x.P(x)) \wedge (\forall x.Q(x))}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (4) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(x), Q(x) \vdash P(x)}{P(x) \wedge Q(x) \vdash P(x)} \wedge_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash P(x)} \forall_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.P(x)} \forall_{\text{right}} \qquad \dfrac{\dfrac{\dfrac{P(x), Q(x) \vdash Q(x)}{P(x) \wedge Q(x) \vdash Q(x)} \wedge_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash Q(x)} \forall_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.Q(x)} \forall_{\text{right}}}{\forall x.P(x) \wedge Q(x) \vdash (\forall x.P(x)) \wedge (\forall x.Q(x))} \wedge_{\text{right}}}{\vdash (\forall x.P(x) \wedge Q(x)) \Rightarrow (\forall x.P(x)) \wedge (\forall x.Q(x))} \Rightarrow_{\text{right}}$$

Correction

Preuve (4) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x), Q(x) \vdash P(x)}\,\text{ax}}{P(x) \wedge Q(x) \vdash P(x)}\,\wedge_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash P(x)}\,\forall_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.P(x)}\,\forall_{\text{right}} \qquad \dfrac{\dfrac{\dfrac{P(x), Q(x) \vdash Q(x)}{P(x) \wedge Q(x) \vdash Q(x)}\,\wedge_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash Q(x)}\,\forall_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.Q(x)}\,\forall_{\text{right}}}{\forall x.P(x) \wedge Q(x) \vdash (\forall x.P(x)) \wedge (\forall x.Q(x))}\,\wedge_{\text{right}}}{\vdash (\forall x.P(x) \wedge Q(x)) \Rightarrow (\forall x.P(x)) \wedge (\forall x.Q(x))}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (4) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x), Q(x) \vdash P(x)}\,\text{ax}}{P(x) \wedge Q(x) \vdash P(x)}\,\wedge_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash P(x)}\,\forall_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.P(x)}\,\forall_{\text{right}} \qquad \dfrac{\dfrac{P(x), Q(x) \vdash Q(x)}{P(x) \wedge Q(x) \vdash Q(x)}}{\forall x.P(x) \wedge Q(x) \vdash Q(x)}\,\forall_{\text{right}}}{\forall x.P(x) \wedge Q(x) \vdash (\forall x.P(x)) \wedge (\forall x.Q(x))}\,\wedge_{\text{right}}}{\vdash (\forall x.P(x) \wedge Q(x)) \Rightarrow (\forall x.P(x)) \wedge (\forall x.Q(x))}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (4) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x), Q(x) \vdash P(x)}\,\text{ax}}{P(x) \wedge Q(x) \vdash P(x)}\,\wedge_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash P(x)}\,\forall_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.P(x)}\,\forall_{\text{right}} \qquad \dfrac{\dfrac{\dfrac{P(x), Q(x) \vdash Q(x)}{P(x) \wedge Q(x) \vdash Q(x)}\,\forall_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash Q(x)}\,\forall_{\text{right}}}{\forall x.P(x) \wedge Q(x) \vdash (\forall x.P(x)) \wedge (\forall x.Q(x))}\,\wedge_{\text{right}}}{\vdash (\forall x.P(x) \wedge Q(x)) \Rightarrow (\forall x.P(x)) \wedge (\forall x.Q(x))}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (4) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x), Q(x) \vdash P(x)}\,\text{ax}}{P(x) \wedge Q(x) \vdash P(x)}\,\wedge_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash P(x)}\,\forall_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.P(x)}\,\forall_{\text{right}} \qquad \dfrac{\dfrac{\dfrac{\dfrac{P(x), Q(x) \vdash Q(x)}{P(x) \wedge Q(x) \vdash Q(x)}\,\wedge_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash Q(x)}\,\forall_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.Q(x)}\,\forall_{\text{right}}}{\forall x.P(x) \wedge Q(x) \vdash (\forall x.P(x)) \wedge (\forall x.Q(x))}\,\wedge_{\text{right}}}{\vdash (\forall x.P(x) \wedge Q(x)) \Rightarrow (\forall x.P(x)) \wedge (\forall x.Q(x))}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (4) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x), Q(x) \vdash P(x)}\,\text{ax}}{P(x) \wedge Q(x) \vdash P(x)}\,\wedge_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash P(x)}\,\forall_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.P(x)}\,\forall_{\text{right}} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{P(x), Q(x) \vdash Q(x)}\,\text{ax}}{P(x) \wedge Q(x) \vdash Q(x)}\,\wedge_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash Q(x)}\,\forall_{\text{left}}}{\forall x.P(x) \wedge Q(x) \vdash \forall x.Q(x)}\,\forall_{\text{right}}}{\forall x.P(x) \wedge Q(x) \vdash (\forall x.P(x)) \wedge (\forall x.Q(x))}\,\wedge_{\text{right}}}{\vdash (\forall x.P(x) \wedge Q(x)) \Rightarrow (\forall x.P(x)) \wedge (\forall x.Q(x))}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (5) dans LJ/LK

▸ Règles LJ

$$P(x) \vdash P(x)$$
$$\neg P(x), P(x) \vdash \bot$$
$$\forall x.\neg P(x), P(x) \vdash \bot$$
$$\forall x.\neg P(x), \exists x.P(x) \vdash \bot$$
$$\forall x.\neg P(x) \vdash \neg(\exists x.P(x))$$
$$\vdash (\forall x.\neg P(x)) \Rightarrow \neg(\exists x.P(x))$$

Correction

Preuve (5) dans LJ/LK

▸ Règles LJ

$$P(x) \vdash P(x)$$
$$\neg P(x), P(x) \vdash \bot$$
$$\forall x.\neg P(x), P(x) \vdash \bot$$
$$\forall x.\neg P(x), \exists x.P(x) \vdash \bot$$
$$\frac{\forall x.\neg P(x) \vdash \neg(\exists x.P(x))}{\vdash (\forall x.\neg P(x)) \Rightarrow \neg(\exists x.P(x))} \Rightarrow_{\text{right}}$$

Correction

Preuve (5) dans LJ/LK

▸ Règles LJ

$$P(x) \vdash P(x)$$

$$\neg P(x), P(x) \vdash \bot$$

$$\forall x.\neg P(x), P(x) \vdash \bot$$

$$\dfrac{\dfrac{\forall x.\neg P(x), \exists x.P(x) \vdash \bot}{\forall x.\neg P(x) \vdash \neg(\exists x.P(x))}\ \neg_{\text{right}}}{\vdash (\forall x.\neg P(x)) \Rightarrow \neg(\exists x.P(x))}\ \Rightarrow_{\text{right}}$$

Correction

Preuve (5) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{}{P(x) \vdash P(x)}}{\neg P(x), P(x) \vdash \bot}}{\forall x. \neg P(x), P(x) \vdash \bot}\,\exists_{\text{left}}}{\forall x. \neg P(x), \exists x. P(x) \vdash \bot}\,\neg_{\text{right}}}{\forall x. \neg P(x) \vdash \neg(\exists x. P(x))}\,\Rightarrow_{\text{right}}$$
$$\vdash (\forall x. \neg P(x)) \Rightarrow \neg(\exists x. P(x))$$

Correction

Preuve (5) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x) \vdash P(x)}}{\neg P(x), P(x) \vdash \bot}\neg_{\text{left}}}{\forall x.\neg P(x), P(x) \vdash \bot}\forall_{\text{left}}}{\forall x.\neg P(x), \exists x.P(x) \vdash \bot}\exists_{\text{left}}}{\forall x.\neg P(x) \vdash \neg(\exists x.P(x))}\neg_{\text{right}}}{\vdash (\forall x.\neg P(x)) \Rightarrow \neg(\exists x.P(x))}\Rightarrow_{\text{right}}$$

Correction

Preuve (5) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(x) \vdash P(x)}{\neg P(x), P(x) \vdash \bot}\,\neg_{\text{left}}}{\forall x.\neg P(x), P(x) \vdash \bot}\,\forall_{\text{left}}}{\forall x.\neg P(x), \exists x.P(x) \vdash \bot}\,\exists_{\text{left}}}{\forall x.\neg P(x) \vdash \neg(\exists x.P(x))}\,\neg_{\text{right}}}{\vdash (\forall x.\neg P(x)) \Rightarrow \neg(\exists x.P(x))}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (5) dans LJ/LK

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x) \vdash P(x)}\,\text{ax}}{\neg P(x), P(x) \vdash \bot}\,\neg_{\text{left}}}{\forall x.\neg P(x), P(x) \vdash \bot}\,\forall_{\text{left}}}{\forall x.\neg P(x), \exists x.P(x) \vdash \bot}\,\exists_{\text{left}}}{\forall x.\neg P(x) \vdash \neg(\exists x.P(x))}\,\neg_{\text{right}}}{\vdash (\forall x.\neg P(x)) \Rightarrow \neg(\exists x.P(x))}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LJ_{em}

▸ Règles LJ

$$\neg P(x) \vdash \neg P(x)$$
$$\neg P(x) \vdash \exists x.\neg P(x)$$
$$\neg\exists x.\neg P(x), \neg P(x) \vdash \bot$$
$$\neg\exists x.\neg P(x) \vdash \neg\neg P(x)$$
$$\neg\exists x.\neg P(x) \vdash P(x)$$
$$\neg\exists x.\neg P(x) \vdash \forall x.P(x)$$
$$\neg\forall x.P(x), \neg\exists x.\neg P(x) \vdash \bot$$
$$\neg\forall x.P(x) \vdash \neg\neg\exists x.\neg P(x)$$
$$\neg\forall x.P(x) \vdash \exists x.\neg P(x)$$
$$\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)$$

Correction

Preuve (6) dans LJ_{em}

▸ Règles LJ

$$\neg P(x) \vdash \neg P(x)$$
$$\neg P(x) \vdash \exists x.\neg P(x)$$
$$\neg\exists x.\neg P(x), \neg P(x) \vdash \bot$$
$$\neg\exists x.\neg P(x) \vdash \neg\neg P(x)$$
$$\neg\exists x.\neg P(x) \vdash P(x)$$
$$\neg\exists x.\neg P(x) \vdash \forall x.P(x)$$
$$\neg\forall x.P(x), \neg\exists x.\neg P(x) \vdash \bot$$
$$\neg\forall x.P(x) \vdash \neg\neg\exists x.\neg P(x)$$
$$\frac{\neg\forall x.P(x) \vdash \exists x.\neg P(x)}{\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)} \Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LJ_{em}

▸ Règles LJ

$$\neg P(x) \vdash \neg P(x)$$
$$\neg P(x) \vdash \exists x.\neg P(x)$$
$$\neg\exists x.\neg P(x), \neg P(x) \vdash \bot$$
$$\neg\exists x.\neg P(x) \vdash \neg\neg P(x)$$
$$\neg\exists x.\neg P(x) \vdash P(x)$$
$$\neg\exists x.\neg P(x) \vdash \forall x.P(x)$$
$$\neg\forall x.P(x), \neg\exists x.\neg P(x) \vdash \bot$$

$$\dfrac{\dfrac{\neg\forall x.P(x) \vdash \neg\neg\exists x.\neg P(x)}{\neg\forall x.P(x) \vdash \exists x.\neg P(x)}\,\text{em}}{\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LJ_{em}

▸ Règles LJ

$$\neg P(x) \vdash \neg P(x)$$
$$\neg P(x) \vdash \exists x.\neg P(x)$$
$$\neg\exists x.\neg P(x), \neg P(x) \vdash \bot$$
$$\neg\exists x.\neg P(x) \vdash \neg\neg P(x)$$
$$\neg\exists x.\neg P(x) \vdash P(x)$$
$$\neg\exists x.\neg P(x) \vdash \forall x.P(x)$$

$$\dfrac{\dfrac{\dfrac{\neg\forall x.P(x), \neg\exists x.\neg P(x) \vdash \bot}{\neg\forall x.P(x) \vdash \neg\neg\exists x.\neg P(x)}\ \neg_{\text{right}}}{\neg\forall x.P(x) \vdash \exists x.\neg P(x)}\ \text{em}}{\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)}\ \Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LJ_{em}

▸ Règles LJ

$$\neg P(x) \vdash \neg P(x)$$
$$\neg P(x) \vdash \exists x.\neg P(x)$$
$$\neg\exists x.\neg P(x), \neg P(x) \vdash \bot$$
$$\neg\exists x.\neg P(x) \vdash \neg\neg P(x)$$
$$\neg\exists x.\neg P(x) \vdash P(x)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\neg\exists x.\neg P(x) \vdash \forall x.P(x)}{\neg\forall x.P(x), \neg\exists x.\neg P(x) \vdash \bot}\,\neg_{\text{left}}}{\neg\forall x.P(x) \vdash \neg\neg\exists x.\neg P(x)}\,\neg_{\text{right}}}{\neg\forall x.P(x) \vdash \exists x.\neg P(x)}\,\text{em}}{\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LJ_{em}

▸ Règles LJ

$$\neg P(x) \vdash \neg P(x)$$
$$\neg P(x) \vdash \exists x.\neg P(x)$$
$$\neg\exists x.\neg P(x), \neg P(x) \vdash \bot$$
$$\neg\exists x.\neg P(x) \vdash \neg\neg P(x)$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\neg\exists x.\neg P(x) \vdash P(x)}{\neg\exists x.\neg P(x) \vdash \forall x.P(x)}\,\forall_{\text{right}}}{\neg\forall x.P(x), \neg\exists x.\neg P(x) \vdash \bot}\,\neg_{\text{left}}}{\neg\forall x.P(x) \vdash \neg\neg\exists x.\neg P(x)}\,\neg_{\text{right}}}{\neg\forall x.P(x) \vdash \exists x.\neg P(x)}\,\text{em}}{\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LJ_{em}

▸ Règles LJ

$$\neg P(x) \vdash \neg P(x)$$
$$\neg P(x) \vdash \exists x.\neg P(x)$$
$$\neg\exists x.\neg P(x), \neg P(x) \vdash \bot$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\neg\exists x.\neg P(x) \vdash \neg\neg P(x)}{\neg\exists x.\neg P(x) \vdash P(x)}\,\text{em}}{\neg\exists x.\neg P(x) \vdash \forall x.P(x)}\,\forall_{\text{right}}}{\neg\forall x.P(x), \neg\exists x.\neg P(x) \vdash \bot}\,\neg_{\text{left}}}{\neg\forall x.P(x) \vdash \neg\neg\exists x.\neg P(x)}\,\neg_{\text{right}}}{\neg\forall x.P(x) \vdash \exists x.\neg P(x)}\,\text{em}}{\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LJ_{em}

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg P(x) \vdash \neg P(x)}}{\neg P(x) \vdash \exists x.\neg P(x)}}{\neg\exists x.\neg P(x), \neg P(x) \vdash \bot}\,\neg_{\text{right}}}{\neg\exists x.\neg P(x) \vdash \neg\neg P(x)}\,\text{em}}{\neg\exists x.\neg P(x) \vdash P(x)}\,\forall_{\text{right}}}{\neg\exists x.\neg P(x) \vdash \forall x.P(x)}\,\neg_{\text{left}}}{\neg\forall x.P(x), \neg\exists x.\neg P(x) \vdash \bot}\,\neg_{\text{right}}}{\neg\forall x.P(x) \vdash \neg\neg\exists x.\neg P(x)}\,\text{em}}{\neg\forall x.P(x) \vdash \exists x.\neg P(x)}\,\Rightarrow_{\text{right}}}{\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)}$$

Correction

Preuve (6) dans LJ_{em}

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg P(x) \vdash \neg P(x)}}{\neg P(x) \vdash \exists x.\neg P(x)}\,\exists_{\text{right}}}{\neg\exists x.\neg P(x), \neg P(x) \vdash \bot}\,\neg_{\text{left}}}{\neg\exists x.\neg P(x) \vdash \neg\neg P(x)}\,\neg_{\text{right}}}{\neg\exists x.\neg P(x) \vdash P(x)}\,\text{em}}{\neg\exists x.\neg P(x) \vdash \forall x.P(x)}\,\forall_{\text{right}}}{\neg\forall x.P(x), \neg\exists x.\neg P(x) \vdash \bot}\,\neg_{\text{left}}}{\neg\forall x.P(x) \vdash \neg\neg\exists x.\neg P(x)}\,\neg_{\text{right}}}{\neg\forall x.P(x) \vdash \exists x.\neg P(x)}\,\text{em}}{\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LJ_{em}

‣ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\neg P(x) \vdash \neg P(x)}{\neg P(x) \vdash \exists x. \neg P(x)}\ \exists_{\text{right}}}{\neg \exists x. \neg P(x), \neg P(x) \vdash \bot}\ \neg_{\text{left}}}{\neg \exists x. \neg P(x) \vdash \neg\neg P(x)}\ \neg_{\text{right}}}{\neg \exists x. \neg P(x) \vdash P(x)}\ \text{em}}{\neg \exists x. \neg P(x) \vdash \forall x. P(x)}\ \forall_{\text{right}}}{\neg \forall x. P(x), \neg \exists x. \neg P(x) \vdash \bot}\ \neg_{\text{left}}}{\neg \forall x. P(x) \vdash \neg\neg \exists x. \neg P(x)}\ \neg_{\text{right}}}{\neg \forall x. P(x) \vdash \exists x. \neg P(x)}\ \text{em}}{\vdash \neg(\forall x. P(x)) \Rightarrow \exists x. \neg P(x)}\ \Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LJ_{em}

▸ Règles LJ

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg P(x) \vdash \neg P(x)}\,\text{ax}}{\neg P(x) \vdash \exists x.\neg P(x)}\,\exists_{\text{right}}}{\neg\exists x.\neg P(x), \neg P(x) \vdash \bot}\,\neg_{\text{left}}}{\neg\exists x.\neg P(x) \vdash \neg\neg P(x)}\,\neg_{\text{right}}}{\neg\exists x.\neg P(x) \vdash P(x)}\,\text{em}}{\neg\exists x.\neg P(x) \vdash \forall x.P(x)}\,\forall_{\text{right}}}{\neg\forall x.P(x), \neg\exists x.\neg P(x) \vdash \bot}\,\neg_{\text{left}}}{\neg\forall x.P(x) \vdash \neg\neg\exists x.\neg P(x)}\,\neg_{\text{right}}}{\neg\forall x.P(x) \vdash \exists x.\neg P(x)}\,\text{em}}{\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LK

▸ Règles LK

$$P(x) \vdash P(x)$$
$$\vdash P(x), \neg P(x)$$
$$\vdash P(x), \exists x.\neg P(x)$$
$$\vdash \forall x.P(x), \exists x.\neg P(x)$$
$$\neg\forall x.P(x) \vdash \exists x.\neg P(x)$$
$$\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)$$

Correction

Preuve (6) dans LK

▸ Règles LK

$$P(x) \vdash P(x)$$
$$\vdash P(x), \neg P(x)$$
$$\vdash P(x), \exists x.\neg P(x)$$
$$\vdash \forall x.P(x), \exists x.\neg P(x)$$
$$\frac{\neg \forall x.P(x) \vdash \exists x.\neg P(x)}{\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)} \Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LK

▸ Règles LK

$$P(x) \vdash P(x)$$
$$\vdash P(x), \neg P(x)$$
$$\vdash P(x), \exists x.\neg P(x)$$
$$\dfrac{\dfrac{\vdash \forall x.P(x), \exists x.\neg P(x)}{\neg\forall x.P(x) \vdash \exists x.\neg P(x)}\ \neg_{\text{left}}}{\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)}\ \Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{P(x) \vdash P(x)}{\vdash P(x), \neg P(x)}}{\vdash P(x), \exists x.\neg P(x)}\,\forall_{\text{right}}}{\vdash \forall x.P(x), \exists x.\neg P(x)}\,\neg_{\text{left}}}{\neg\forall x.P(x) \vdash \exists x.\neg P(x)}\,\Rightarrow_{\text{right}}$$

$$\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)$$

Correction

Preuve (6) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(x) \vdash P(x)}{\vdash P(x), \neg P(x)}\,\exists_{\text{right}}}{\vdash P(x), \exists x.\neg P(x)}\,\forall_{\text{right}}}{\vdash \forall x.P(x), \exists x.\neg P(x)}\,\neg_{\text{left}}}{\neg\forall x.P(x) \vdash \exists x.\neg P(x)}\,\Rightarrow_{\text{right}}}{\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)}$$

Correction

Preuve (6) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{P(x) \vdash P(x)}{\vdash P(x), \neg P(x)}\,\neg_{\text{right}}}{\vdash P(x), \exists x.\neg P(x)}\,\exists_{\text{right}}}{\vdash \forall x.P(x), \exists x.\neg P(x)}\,\forall_{\text{right}}}{\neg\forall x.P(x) \vdash \exists x.\neg P(x)}\,\neg_{\text{left}}}{\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)}\,\Rightarrow_{\text{right}}$$

Correction

Preuve (6) dans LK

▸ Règles LK

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P(x) \vdash P(x)}\,\text{ax}}{\vdash P(x), \neg P(x)}\,\neg_{\text{right}}}{\vdash P(x), \exists x.\neg P(x)}\,\exists_{\text{right}}}{\vdash \forall x.P(x), \exists x.\neg P(x)}\,\forall_{\text{right}}}{\neg\forall x.P(x) \vdash \exists x.\neg P(x)}\,\neg_{\text{left}}}{\vdash \neg(\forall x.P(x)) \Rightarrow \exists x.\neg P(x)}\,\Rightarrow_{\text{right}}$$

Exercices en Coq

Propositions à démontrer

- Exercices en logique propositionnelle ;
- Exercices en logique du premier ordre.
- À faire chez soi.

Installation de Coq

- Tout est indiqué ici : https://coq.inria.fr/.

Calcul des séquents intuitionniste (LJ)

Règles

$$\frac{}{\Gamma, A \vdash A}\ \text{ax} \qquad \frac{\Gamma, A, A \vdash B}{\Gamma, A \vdash B}\ \text{cont}$$

$$\frac{\Gamma \vdash A \qquad \Gamma, B \vdash C}{\Gamma, A \Rightarrow B \vdash C} \Rightarrow_{\text{left}} \qquad \frac{\Gamma, A \vdash B}{\Gamma \vdash A \Rightarrow B} \Rightarrow_{\text{right}}$$

$$\frac{\Gamma \vdash A \qquad \Gamma, B \vdash C}{\Gamma, A \Leftrightarrow B \vdash C} \Leftrightarrow_{\text{left1}}$$

$$\frac{\Gamma \vdash B \qquad \Gamma, A \vdash C}{\Gamma, A \Leftrightarrow B \vdash C} \Leftrightarrow_{\text{left2}} \qquad \frac{\Gamma, A \vdash B \qquad \Gamma, B \vdash A}{\Gamma \vdash A \Leftrightarrow B} \Leftrightarrow_{\text{right}}$$

Calcul des séquents intuitionniste (LJ)

Règles

$$\frac{\Gamma, A, B \vdash C}{\Gamma, A \wedge B \vdash C} \wedge_{\text{left}} \qquad \frac{\Gamma \vdash A \qquad \Gamma \vdash B}{\Gamma \vdash A \wedge B} \wedge_{\text{right}}$$

$$\frac{\Gamma \vdash A}{\Gamma \vdash A \vee B} \vee_{\text{right1}}$$

$$\frac{\Gamma, A \vdash C \qquad \Gamma, B \vdash C}{\Gamma, A \vee B \vdash C} \vee_{\text{left}} \qquad \frac{\Gamma \vdash B}{\Gamma \vdash A \vee B} \vee_{\text{right2}}$$

$$\frac{\Gamma \vdash A}{\Gamma, \neg A \vdash B} \neg_{\text{left}} \qquad \frac{\Gamma, A \vdash \bot}{\Gamma \vdash \neg A} \neg_{\text{right}}$$

$$\frac{}{\Gamma, \bot \vdash A} \bot_{\text{left}} \qquad \frac{}{\Gamma \vdash \top} \top_{\text{right}}$$

Calcul des séquents intuitionniste (LJ)

Règles

$$\frac{\Gamma, A(t) \vdash B}{\Gamma, \forall x.A(x) \vdash B} \forall_{\text{left}} \qquad \frac{\Gamma \vdash A(x)}{\Gamma \vdash \forall x.A(x)} \forall_{\text{right}},\ x \notin \Gamma$$

$$\frac{\Gamma, A(x) \vdash B}{\Gamma, \exists x.A(x) \vdash B} \exists_{\text{left}},\ x \notin \Gamma, B \qquad \frac{\Gamma \vdash A(t)}{\Gamma \vdash \exists x.A(x)} \exists_{\text{right}}$$

$$\frac{\Gamma \vdash A \qquad \Gamma, A \vdash B}{\Gamma \vdash B} \text{cut}$$

Calcul des séquents classique (LJ_{em})

Règles

$$\frac{\Gamma, A(t) \vdash B}{\Gamma, \forall x.A(x) \vdash B}\ \forall_{\text{left}} \qquad \frac{\Gamma \vdash A(x)}{\Gamma \vdash \forall x.A(x)}\ \forall_{\text{right}},\ x \notin \Gamma$$

$$\frac{\Gamma, A(x) \vdash B}{\Gamma, \exists x.A(x) \vdash B}\ \exists_{\text{left}},\ x \notin \Gamma, B \qquad \frac{\Gamma \vdash A(t)}{\Gamma \vdash \exists x.A(x)}\ \exists_{\text{right}}$$

$$\frac{\Gamma \vdash A \qquad \Gamma, A \vdash B}{\Gamma \vdash B}\ \text{cut} \qquad \frac{\Gamma \vdash \neg\neg A}{\Gamma \vdash A}\ \text{em}$$

Calcul des séquents classique (LK)

Règles

$$\frac{}{\Gamma, A \vdash \Delta, A}\ \text{ax} \qquad \frac{\Gamma \vdash \Delta, A \qquad \Gamma, A \vdash \Delta, B}{\Gamma \vdash \Delta, B}\ \text{cut}$$

$$\frac{\Gamma, A, A \vdash \Delta}{\Gamma, A \vdash \Delta}\ \text{cont}_{\text{left}} \qquad \frac{\Gamma \vdash \Delta, A, A}{\Gamma \vdash \Delta, A}\ \text{cont}_{\text{right}}$$

Calcul des séquents classique (LK)

Règles

$$\frac{\Gamma \vdash \Delta, A \qquad \Gamma, B \vdash \Delta}{\Gamma, A \Rightarrow B \vdash \Delta} \Rightarrow_{\text{left}} \qquad \frac{\Gamma, A \vdash \Delta, B}{\Gamma \vdash \Delta, A \Rightarrow B} \Rightarrow_{\text{right}}$$

$$\frac{\Gamma \vdash \Delta, A, B \qquad \Gamma, A, B \vdash \Delta}{\Gamma, A \Leftrightarrow B \vdash \Delta} \Leftrightarrow_{\text{left}}$$

$$\frac{\Gamma, A \vdash \Delta, B \qquad \Gamma, B \vdash \Delta, A}{\Gamma \vdash \Delta, A \Leftrightarrow B} \Leftrightarrow_{\text{right}}$$

Calcul des séquents classique (LK)

Règles

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} \wedge_{\text{left}} \qquad \frac{\Gamma \vdash \Delta, A \qquad \Gamma \vdash \Delta, B}{\Gamma \vdash \Delta, A \wedge B} \wedge_{\text{right}}$$

$$\frac{\Gamma, A \vdash \Delta \qquad \Gamma, B \vdash \Delta}{\Gamma, A \vee B \vdash \Delta} \vee_{\text{left}} \qquad \frac{\Gamma \vdash \Delta, A, B}{\Gamma \vdash \Delta, A \vee B} \vee_{\text{right}}$$

$$\frac{\Gamma \vdash \Delta, A}{\Gamma, \neg A \vdash \Delta} \neg_{\text{left}} \qquad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash \Delta, \neg A} \neg_{\text{right}}$$

$$\frac{}{\Gamma, \bot \vdash \Delta} \bot_{\text{left}} \qquad \frac{}{\Gamma \vdash \Delta, \top} \top_{\text{right}}$$

Calcul des séquents classique (LK)

Règles

$$\frac{\Gamma, A(t) \vdash \Delta}{\Gamma, \forall x.A(x) \vdash \Delta} \forall_{\text{left}} \qquad \frac{\Gamma \vdash \Delta, A(x)}{\Gamma \vdash \Delta, \forall x.A(x)} \forall_{\text{right}},\ x \notin \Gamma, \Delta$$

$$\frac{\Gamma, A(x) \vdash \Delta}{\Gamma, \exists x.A(x) \vdash \Delta} \exists_{\text{left}},\ x \notin \Gamma, \Delta \qquad \frac{\Gamma \vdash \Delta, A(t)}{\Gamma \vdash \Delta, \exists x.A(x)} \exists_{\text{right}}$$