

APPLIDED ECONOMETRICS, PARIS CITE UNIVERSITE

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#### INTRODUCTION

"The world may not be happier now, but the world is more about happiness and well-being than it was ten years ago", said Jeffrey Sachs

For the first time since the creation of the World Happiness Report created by the UN, France is in the Top 20 of this global happiness index, but what are the factors that really impact happiness in the world, how to measure it?

This is an existential question because every human being is looking for his happiness, which he can find in various ways (social, financial, family success, etc.) but can we define places where societies consider themselves happier than elsewhere?

The United States' declaration of independence established the pursuit of happiness as an inalienable human right. But you have to know what happiness is. However, it can have very varied meanings depending on the epochs, countries, cultures or even generations. Many of the proposed determinants are economic, such as GDP per capita, income, unemployment or major economic crises. But we will see throw our study that social factors are important determinants of happiness.

This is the subject of our study where we will try to explain the derived ranking of Gallup weights, which is the ranking of countries where people consider themselves most happy with a survey established in different countries in order to compare them according to economic and social factors

These totally subjective measures nevertheless reflect a strong psychological dimension in the persons concerned and thus make it possible to discuss the issue more in depth.

First of all, we will discuss the notion of happiness and how some variables can explain it thanks to our articles and research. Then we'll explain our database and our variables to make our study lighter and more comprehensive. After these presentations we will start our econometric work by the descriptive analysis of our variable score followed by a part where we'll speak about the relevance of each variable in our estimated model.

Next we'll make our progressive linear regression and talk about , make our multivariate analyze and see how our coefficients impact our explained variable.

It is for these reasons that the choice of our study focuses on the identification of the different factors influencing happiness in different countries in the world. The aim is to construct an econometric model to determine a score of happiness for each country studied.



#### I. LITERATURE REVIEW ON FACTORS IMPACTING GLOBAL HAPPINESS

➤ In a first step through our study we will seek to establish a causal link between variation in a country's happiness and economic factors affecting well-being.

The literature found on this subject shows that economic factors can have a significant impact on happiness represented by the Gallup index, for example GDP per capita, income, unemployment or major economic crisis, are strongly correlated with happiness. ( Dahan Jeremie , 2021)

➤In a second step we will prove that social and non-economic factor such as the social support, can have a real impact on happiness.

A second study, by Esteban Ospina and Max Roser in 2017, on happiness between countries, which takes as a determinant of happiness, income, life event, culture and society shows us that social factors influence happiness.

➤ Policy decisions at the organizational, corporate, and governmental levels should be more heavily influenced by issues related to well-being—people's evaluations and feelings about their lives (ed Dierner et Martin E.P Seligman 2004)



#### II. Presentation of the database

Our database comes from the largest Data Science platform on the internet «Kaggle» is a Database for the analysis of the happiness score across 156 countries in the world, they are ranked from the happiest to the least happy.

Link to our Database below:

HTTPS://WWW.KAGGLE.COM/DATASETS/UNSDSN/WORLD-HAPPINESS?SELECT=2019.CSV

After consulting the necessary documentation concerning happiness in the world, the dependent variable on which our model will be built is the happiness score.

#### 1. DESCRIPTIVE ANALYSIS OF VARIABLE Y: THE HAPPINESS SCORE

The score is calculated based on dystopia.

Dystopia is an imaginary country that has the world's least-happy people. The purpose in establishing Dystopia is to have a benchmark against which all countries can be favorably compared (no country performs more poorly than Dystopia) in terms of each of the six key variables, thus allowing each sub-bar to be of positive width. The lowest scores observed for the six key variables, therefore, characterize Dystopia. Since life would be very unpleasant in a country with the world's lowest incomes, lowest life expectancy, lowest generosity, most corruption, least freedom and least social support, it is referred to as "Dystopia," in contrast to Utopia.

#### Happiness score $Y(score_i)$

It is a quantitative variable that gives a score out of 10 of the country's level of happiness, it is obtained thanks to the data of the Gallup World Poll. It carries out surveys of representative samples in more than 160 countries and in 140 languages, on the basis of standardized questions. The main question asked is: "Imagine a scale with bars numbered from zero at the bottom to ten at the top. The upper bar represents the best possible life for you, the lower bar the worst possible life for you. What bar do you think you stand at this moment in your life?" Known as the Cantril scale, this assessment of well-being, together with the precise formulation of the question, is a common standard for research on subjective well-being.



Table 01: Descriptive statistics of the dependent variable Y

Descriptive statistics of Y			
Moyenne	5,407096154		
Erreur-type	0,089120915		
Médiane	5,3795		
Mode	5,208		
Écart-type	1,113119869		
Variance de l'échantillon	1,239035842		
Kurstosis	-0,60837535		
Coefficient d'asymétrie	0,011449949		
Plage	4,916		
Minimum	2,853		
Maximum	7,769		
Somme	843,507		
Nombre d'échantillons	156		

We can notice that the distribution of happiness scores is fairly homogeneous with a median and a fairly centered mean, we have a small variance of 1.23 so the difference is relatively small to the average.

Representation of the dependent variable 45 40 35 30 25 20 15 10 5 (3,573,4,293) (5,733,6,453) (7,273,7,893) (4,293,5,013) (5,013,5,733) (6,453,7,273)

Figure 01: Representation of the dependent variable

As we can see on this representation of the variable score, the repartition is almost following a Gaussian law and looks good with a variance and a kurtosis coefficient relatively low.



#### 2. DESCRIPTIVE ANALYSIS OF EXPLANATORY VARIABLES

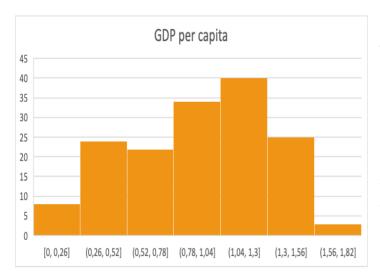
In our study, several variables will play the role of explanatory variables in order to determine happiness score. We have taken into account 2 explanatory variables based on what we could find relevant in the literature and 4 that we found interesting to analyze.

#### Variables from the literature:

#### GDP per capita

According to the literature the work of the Stiglitz Commission GDP is a determinant of happiness, the richer a country is the happier it is, but Esterlin's paradox contradicts this hypothesis and is the subject of a real discussion. We will see in our model through the variable "GDP per capita" impact on happiness.

Figure 02: Representation of the GDP per capita variable



The first graph shows the distribution of the GDP per capita variable across each country surveyed. By graphic reading, we can see that this variable is fairly homogeneously distributed with a Gaussian look; the central values are quite strong compared to the extreme values.

#### social support

Our second variable chosen through the literature is *social support*, indeed it is a quantitative variable by definition it is the perception and actuality that one is cared for, has assistance available from other people, and most popularly, that one is part of a supportive social network that affects happiness



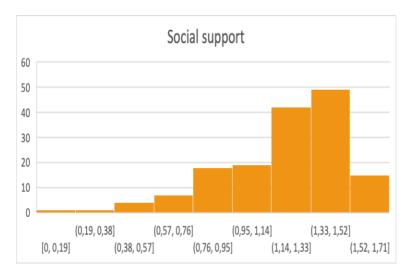


Figure 03: representation of the Social Support variable

This second graph shows the distribution of the variable Social Support for every country in the study. The distribution looks like a Gaussian distribution shift on the right with a peak between 1.44 and 1.52 with more than 80 countries.

#### Variables that we have chosen:

#### Life expectancy

Our third variable is *Life expectancy* which resumes Life expectancy at birth represents the average lifespan of a fictitious generation subject to the age-specific mortality conditions of the year in question.

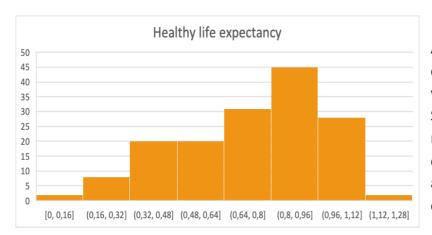


Figure 04: Representation of the healthy life expectancy variable

As the precedent graph we can see a similar distribution with same properties than Social Support graph. The repartition is more concentrated between 0,8 and 1,12 with more than 45 countries in this interval.



#### freedom to make life choices

The fourth variable is the *freedom to make life choices*, it is a quantitative variable describing the freedom of individuals to make life choices freely, without difference of gender, number or social class

Figure 05: Representation of the Freedom to make life choices variable

#### Generosity

*Generosity* describes the rate of action that is benevolent, lenient, forgiving in the country in question. The humanitarian goal rate in the country studied may be a criterion of this variable for example

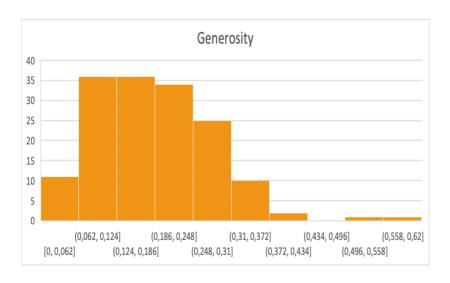


Figure 06: Representation of generosity variable

This distribution of the variable generosity also looks like а Gaussian distribution with a peak on the left means most countries are less generous others but that's interesting because we can imagine countries with the best happiness score are also countries where people are more generous.



#### • The perception of corruption

The *perception of corruption is a variable* that describes whether the inhabitants of the studied country feel in a corrupt country or not.

Perception of corruption 60 50 40 30 20 10 0 (0,061,0,122](0,183, 0,244] (0,305, 0,366] (0,427,0,488][0, 0,061] (0,244, 0,305] (0,122,0,183](0,366, 0,427]

Figure 07: representation of perception of corruption variable

This last graph is about how the distribution of the variable perception of corruption looks also like Gaussian but with a peak on the left that looks more understandable than the distribution of the variable generosity because we can imagine most countries have a less perception of corruption and a better happiness score that looks correlated.

We have done a table to summarize all the variables:

**Table 02: Summary of our variables** 

Name	Nature	Unit	Expected signs
Score	Continuous quantitative	[0,10]	Positive
GDP per capita	Continuous quantitative	[0,10]	Positive
Social support	Continuous quantitative	[0,10]	Positive
Life expectancy	Continuous quantitative	[0,10]	Positive
Freedom to make life choice	Continuous quantitative	[0,10]	Positive
Generosity	Continuous quantitative	[0,10]	Positive
Perception of corruption	Continuous quantitative	[0,10]	Negative



The aim of our study from an econometric point of view is to explain the score by the various explanatory variables according to the following relation:

$$Y = \beta_0 + \beta_1 x 1 + \beta_2 x 2 + \beta_3 x 3 + \beta_4 x 4 + \beta_5 x 5 + \beta_6 x 6 + u_i$$

Our model will be this following relation:

$$Y(score_i) = \beta_0 + \beta_1 GDP + \beta_2 social + \beta_3 health + \beta_4 free + \beta_5 gener + \beta_6 percept + u_i$$

#### III. PRECONCEPTION REGRESSION

An initial exploratory analysis of our data highlights some potential relationships, the relevance of the choice of our variables and adjustments if necessary.

Table 03: table of the initial model

	Coefficients	Erreur-type	Statistique t	Probabilité
Constante	1,795220229	0,211073396	8,505194224	1,7676E-14
GDP per capita	0,775371626	0,218225354	3,553077636	0,0005103
Social support	1,124191579	0,236900074	4,745425193	4,8338E-06
Healthy life expectency	1,078142735	0,334538483	3,222776417	0,0015596
freedom to make choice	1,454832369	0,375337841	3,876061003	0,00015869
Generosity	0,489783351	0,497745477	0,98400362	0,32670886
Perception of corruption	0,972280221	0,542360729	1,792681825	0,07505259

Adjusted coefficient of determination  $R^2 = 0,77$  Fisher P-value (overall model significance) =87.61

#### **Estimated Model Equation:**

$$Y(score_i) = 1,8 + 0,78GDP + 1,12social + 1,08helth + 1,46free + 0,49gener + 0,97precep+u_i$$



If we look at the significance of the coefficients estimated at the threshold of 5% we can notice that all the variables are relatively significant, there is little chance of being mistaken in rejecting the hypothesis H0 for the variables GDP, social support, Health and freedom, In addition, 0 is not included in the 95% confidence intervals of the four coefficients, so they are significantly different from 0 at the 5% threshold.

But the same cannot be said for the variable generosity and perceptions of corruption with a p-value of 0.33 and 0.08 which are > 0.05, these coefficients are not significant.

Following these observations we decided to modify the variables generosity and perception of corruption which are initially quantitative variables in binary quantitative variables, for this we have calculated the average of generosity and perception of corruption, countries with a rate of generosity and a perception of corruption above the average are considered generous and not corrupt.

We thus obtain our new estimated model:

Table 04: Regression of the model

	Coefficients	Erreur-type	Statistique t	Probabilité
Constante	1,840257464	0,20391399	9,02467485	8,4432E-16
GDP per capita	0,786057446	0,219287	3,58460577	0,00045679
Social support	1,094532906	0,23942859	4,57143771	1,0103E-05
Healthy life expectancy	1,128793783	0,33685257	3,3510024	0,00102041
Freedom to make life choices	1,611462667	0,36943004	4,3620239	2,3934E-05
Generosity quali	0,105587436	0,09400446	1,12321733	0,26315173
Perceptions quali	0,092460973	0,10286504	0,89885712	0,37017905

Adjusted coefficient of determination  $R^2 = 0,77$  Fisher P-value (overall model significance) =85.47

Following the changes on the last two variables, we can see that the explanatory variables are better estimated than in the first case ( table 03) for example for the variables freedom to make choice, the p-value is much smaller than in the first case, the coefficient is more

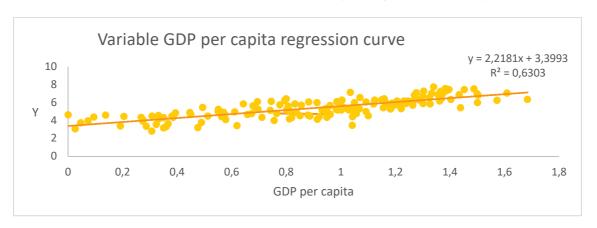


significant at the 5% threshold. However, there is no real impact on the last two explanatory variables, they are not significant because the p-value >0.05.

# V. Progressive linear regression (relationship between each $X_i$ and Y)

#### 1. GDP per capita variable

Figure 08: scatter plot of  $Y(score_i) = \beta_0 + \beta_1 GDP + u_i$ 



**Table 05: Model regression**  $Y(score_i) = \beta_0 + \beta_1 GDP + u_i$ 

	Coefficients	Erreur-type	Statistic t	Probability
Constant	3,399345178	0,135323298	25,12017676	2,4398E-56
GDP per capita	2,218148001	0,136907682	16,20177899	4,3155E-35
		R <sup>2</sup> = <b>0.63</b>		

Fisher P-value (overall model significance) =262,50

The regression gives significant results with an adjusted coefficient of determination  $R^2$ =0.63 (the GDP explains 63% of the variations of the variable Y of the score of happiness all other things being equal).

The GDP per capita variable is significant at 1%, so an increase of 1 unit of the GDP of the studied country increases the happiness score by 2.22 units the happiness score of this country, as seen previously in the literature.



If we want to know the overall significance of the model, the Fisher test gives us a coefficient equal to 262.5 and the table of Fisher's law (appendix 3) gives us 2.1; and as 262.5> 2.1 we reject HO and we can globally conclude that the model is significant at the 5% level.

#### 2. GDP per capita + social support

By adding the social support variable, to the variable Y we obtain the following regression:

Table 06: Model regression  $Y(score_i) = \beta_0 + \beta_1 GDP + \beta_2 social + u_i$ 

	Coefficients	Erreur-type	Statistique t	Probability
Constant	2,329770638	0,211964551	10,99132201	4,43817E-21
GDP per capita	1,346478068	0,187480049	7,181980581	2,80444E-11
Social support	1,537510476	0,249639782	6,158916109	6,17724E-09

Adjusted coefficient of determination  $R^2 = 0,60$ 

By adding the support social variable to our model which was initially bi-varied, we have a coefficient of determination which decreases slightly, which goes from 0.63 to 0.60, which means that the GDP and social support variables explain 60% of the variations in the happiness score, our model with the 2 variables remains relatively well explained.

Moreover, all the coefficients are significant because 0 is not included in the 99% confidence interval. We can also confirm that thanks to the Student p-value, there is little chance of being mistaken in rejecting the hypothesis H0 for the variables GDP and social support.

Thus, an increase of 1 social support unit increases the happiness score by 1.54 units and if GDP per capita and social support equal zero, the happiness score increases by 2,33 units, everything being equal.

For casting according to the literature, the GDP and social support variables are good determinants to explain happiness in the world.



#### 3. GDP per capita + social support+ healthy life expectancy

Table 07: Model regression  $Y(score_i) = \beta_0 + \beta_1 GDP + \beta_2 social + \beta_3 health + u_i$ 

	Coefficients	Erreur-type	Statistique t	Probability
Constant	2,135046981	0,211638497	10,0881787	1,222E-18
GDP per capita	0,809820385	0,235809748	3,43421081	0,000766123
Social support	1,321886267	0,24827968	5,32418227	3,58058E-07
Healthy life expectancy	1,297670988	0,366134165	3,54424993	0,000523454

Adjusted coefficient of determination  $R^2 = 0.84$ 

For the moment it is the model that best explains the variations of the happiness score, the regression gives significant results with an adjusted coefficient of determination  $R^2$ =0.84 that is to say that the variables GDP, social support and health explain 84% of the variations of the variable Y of the score of happiness everything being equal.

We can notice that the *health* variable has a real impact and is an important variable in our model that explains well the happiness score because there is a gap of 21% with the previous model.

All variables are significant at 5%, so an increase of 1 unit of the variable healthy life expectancy increases the happiness score by 1.28 units.

It can be seen that the GDP per capita variable becomes less significant than the previous model seen before.

#### 4. GDP per capita + social support + healthy life expectancy+ freedom to make choice

By adding the variable freedom to make choice to our model we obtain a coefficient of determination equal to 0.76, in this model, the explanatory variables explain 76% of the variations of the score of happiness

However, the significance of the coefficients is present as shown in the table below:



Table 08: Model regression  $Y = \beta_0 + \beta_1 GDP + \beta_2 social + \beta_3 health + \beta_4 free + u_i$ 

	Coefficients	Erreur-type	Statistique t	Probabilité
Constant	1,89209537	0,19936358	9,49067711	4,802E-17
GDP per capita	0,81054939	0,21645095	3,74472553	0,000256291
Social support	1,0166104	0,23474755	4,33065392	2,69697E-05
Healthy life expectancy	1,1414486	0,33730887	3,38398635	0,000910305
Freedom to make life choices	1,84581569	0,34039145	5,42262655	2,28103E-07

# Adjusted coefficient of determination $R^2 = 0.76$ Fisher P-value (overall model significance) =127,05

- Thus, increasing one unit of freedom to make choice increases the happiness score by
   1.85 units all things being equal
- The Fisher P-value (overall model significance) =127,05 shows that it's model globally significant
- 5. GDP per capita + social support + healthy life expectancy + freedom du male choice +generosity

Table 09:  $Y(score_i) = \beta_0 + \beta_1 GDP + \beta_2 social + \beta_3 health + \beta_4 free + \beta_5 gener + u_i$ 

	Coefficients	Erreur-type	Statistique t	Probabilité
Constante	1,833144057	0,20362988	9,002333346	9,23732E-16
GDP per capita	0,81944168	0,215980438	3,794055089	0,000214455
Social support	1,062506532	0,236611019	4,490520084	1,40857E-05
Healthy life expectancy	1,139663072	0,33641988	3,387621065	0,000900568
Freedom to make life choices	1,686462605	0,359653475	4,689131966	6,11816E-06
Generosity quali	0,123291697	0,091858977	1,342184531	0,181564141

Adjusted coefficient of determination  $R^2 = 0,77$ 



By adding the generosity variable, the quality of the model stays sable with a coefficient of determination at 0.77.

however, this same variable is not significant with a p-value of 0.18, we can say that if the country is more generous than the average of all other countries in our observation then it does not have a real impact on its score of happiness

If the country is more generous than the average of all other observations(gener=1) then its happiness score increases by 0.12 points all things being equal.

We expected a higher value taken the literature into account (table 02)

# 6. GDP per capita + social support healthy life expectancy + freedom du make choice +generosity + perception of corruption

Table10: $Y(score_i) = \beta_0 + \beta_1 GDP + \beta_2 social + \beta_3 health + \beta_4 free + \beta_5 gener + \beta_6 perception + u_i$ 

	Coefficients	Erreur-type	Statistique t	Probability
Constant	1,840257464	0,20391399	9,02467485	8,4432E-16
GDP per capita	0,786057446	0,219287	3,58460577	0,00045679
Social support	1,094532906	0,23942859	4,57143771	1,0103E-05
Healthy life expectancy	1,128793783	0,33685257	3,3510024	0,00102041
Freedom to make life choices	1,611462667	0,36943004	4,3620239	2,3934E-05
Generosity	0,105587436	0,09400446	1,12321733	0,26315173
Perceptions	0,092460973	0,10286504	0,89885712	0,37017905

Adjusted coefficient of determination  $R^2 = 0,77$ 

- As we saw previously in the progressive linear regression, the coefficient of the variable perception of corruption is not significant, there is a very strong chance of being wrong by rejecting the hypothesis H0 (p-value = 0.37).
- there is no real change in the fit quality of the model by adding the variable perception of corruption, the determination coefficient still is  $R^2 = 0.77$



• Countries that have a higher than average perception of corruption (percp=1), increase its happiness score by 0.09 units, everything being equal, which is surprising, we expected more negative value. (table 02)

#### VI. DUMMY VARIABLES STUDY

We will try to see if there is an interaction between variables perception of corruption and generosity which are initially binary quantitative thanks to dummy variables

We will estimate this next model to identify possible interactions:

$$Y(score_i) = \beta_0 + \beta_1 gener + \beta_2 corruption + \beta_3 generous * corruption + u_i$$

Table 11: results of estimation:

$$Y(score_i) = \beta_0 + \beta_1 gener + \beta_2 corruption + \beta_3 generous * corruption + u_i$$

	Coefficients	Erreur-type	Statistique t	Probability
Constant	5,32146875	0,131245893	40,5457924	2,1028E-83
generosity	-0,373273628	0,21003344	-1,7772105	0,07753352
Perception of corruption	0,175766544	0,286486243	0,61352525	0,54044612
generosity* corruption	0,952626569	0,37601512	2,53347942	0,01230681

Adjusted coefficient of determination  $R^2 = 0$ , 11

#### - <u>Interpretation of the interaction term:</u>

The interaction term is 5% significant with a p-value equal to 0.12306 which is a little bit lower than 0.05. So that means there is a significant interaction effect between generosity and perception of corruption.

Generosity's effect on the score depends on the perception of corruption and inversely so the interaction term is useful to explain the happiness score.



#### - coefficient interpretation:

The reference is for generosity = 0 and perception of corruption = 0 so a country is less generous than the average of all other countries and same for the perception of corruption.

The effect that the country is considered more corrupted (perception of corruption = 1) in a ungenerous country (generosity = 0) increases the score by 0.9 units of happiness.

The effect that the country is considered more corrupted (perception of corruption = 1) in a generous country (generosity = 1) increases the score by (0,176+0,953=1,129) 1,13 units of happiness.

The effect that the country is considered more generous (generosity = 1) in a country where there is a low perception of corruption (perception of corruption = 0) decreases the score by 0.37 units of happiness.

The effect that the country is considered more generous (generosity = 1) in a country where there is a high perception of corruption (perception of corruption = 1) decreases the score by (-0.373+0.952=0.579) 0.58 units of happiness.

#### VII. REVERSE CAUSALITY

In this part we will estimate a new model, we will through a bi-varied analysis try to explain the GDP by the score of happiness and thus inverted our variables of our basic model, We found it interesting to analyze this situation knowing the literature cited above, indeed the GDP could vary according to the happiness of the population. The dependent variable on which our model will be built is *GDP per capita*, and the explanatory variable is *score* 

We will estimate this model to identify possible interactions:

$$Y(GDP_i) = \beta_0 + \beta_1 score + ui$$

Table 12: Reverse causality model

	Coefficients	Erreur-type	Statistique t	Probability
Constant	-0,6311894	0,09680105	-6,5204807	9,4989E-10
Score	0,2841334	0,01753718	16,201779	4,3155E-35

$$R^2 = 0.63$$



We obtained satisfactory results, with a model well explained indeed the score of happiness explains 63% of the variations of the GDP, the model to a correct quality of adjustment.

In terms of the significance of the coefficients, they are all significant at 1%, so an increase of one unit of the happiness score increases the GDP by 0.28 units and if the happiness score is zero then the GDP falls by 0.63 units everything being equal.

The coefficients are expected signs and are in agreement with the other studies carried out and cited in our literature. We can say that the GDP plays a primordial role in our model is a variable that explains well the variations of the happiness score.

#### VIII.MULTIVARIATE ANALYSIS

#### 1. The significance of the variables

Estimated (General) Model Equation:

 $Y(score_i) = 1,840257464 + 0,786057446GDP + \\ 1,094532906social + 1,128793783health + 1,611462667free + 0,105587436generous + \\ 0,105587436perception + u_i$ 

Table 13: Estimation of the general model

	Coefficients	Erreur-type	Statistique t	Probability
Constant	1,840257464	0,20391399	9,02467485	8,4432E-16
GDP per capita	0,786057446	0,219287	3,58460577	0,0004567 9
Social support	1,094532906	0,23942859	4,57143771	1,0103E-05
Healthy life expectancy	1,128793783	0,33685257	3,3510024	0,0010204 1
Freedom to make life choices	1,611462667	0,36943004	4,3620239	2,3934E-05
Generosity	0,105587436	0,09400446	1,12321733	0,26315173
Perceptions	0,092460973	0,10286504	0,89885712	0,37017905

Thanks to table 13, we can see that GDP, social support, healthy life expectancy, freedom to make life choices and the constant are significant at the 99% level. Indeed, their T-stat is over



2.58 and their P-value is under 0.01. So we can reject the hypothesis H0 of non-significance for these 4 variables and the constant at 90%, 95% and 99%.

Generosity and Perception variables have no T-stat above or P-value below the relevant thresholds, so we cannot reject the H0 hypothesis of no significance

#### 2. Interpretation and analysis of coefficients

All things being equal:

- If all explanatory variables are equal to zero, the happiness score increase by 1.84 units
- An increase of 1 unit of country's GDP increase happiness score by 0.78 units
- An increase of 1 unit of Social Support increase happiness score by 1.09 units
- An increase of 1 unit of Healthy Life Expectancy increase happiness score by 1.13 units
- An increase of 1 unit of Freedom to make life choices increase happiness score by 1.61 units
- If the country is considered generous his score increase by 0.1 units
- If inhabitants feel like living in a corrupted country the happiness score increases by 0.09 units.

In our final model all coefficients positively influence the happiness score which is surprising as we expected a negative coefficient for the last variable, the perception of corruption influences happiness positively but very weakly

#### 3. Fisher and overall model significance:

The following hypothesis are used:

H0 -> the model is not significant

H1 -> the model is significant

If we calculate the coefficient of Fisher  $F=(R^2/k)/[(1-R^2)/(N-(k+1))]$  we get 85.47 (which corresponds to our analysis table of variance of our regression) and  $F_{1-\alpha}(k,N-1-k)$  we get 2.10.

here  $F > F_{1-\alpha}(k, N-1-k)$  we accept H0



According to the Fisher test, after comparing the F of the model and the table in (appendix 3), we find that we reject the hypothesis H0 meaning that the model is poorly estimated. Thus we conclude here that the model is globally significant

#### IX - LES TESTS

#### 1-test de multi colinéarité

**Tableau 14: correlation matrix** 

	Score	GDP per capita	Social support	Health	Freedom	Generosity	Perception
Score	1						5
GDP per capita	0,79388287	1					
Social support	0,77705779	0,754905727	1				
Health	0,77988315	0,835462115	0,719009459	1			
Freedom	0,56674183	0,379079072	0,447333164	0,390395	1		
Generosity	0,04564942	-0,079157446	-0,095163554	-0,056417	0,2510867	1	
Perceptions	0,29922192	0,267431552	0,145092332	0,242428	0,33916177	0,25930763	1

Let remind that this table shows the correlation links between variables on the scale from 0 to 1, 0 showing an absence of link and 1 showing a perfect collinearity. Negative values having the same scale but an inverse ratio, that is, when one variable increases, the other decreases

- There is a strong correlation between the score and GDP as we see in the study of the precedent model
- We can see also a strong correlation between variables GDP and Social Support (with a correlation coefficient equal to 0.84) this result is predictable thanks to the literature
- However, GDP and generosity have a negative correlation which is understandable in our study, there is a close link between perception of corruption and generosity of inhabitants which is a social factor and GDP which is a variables.



#### 2-Klein test and VIF

#### Klein test

The approach of this test is as follows:

We compare the  $\mathbb{R}^2$  calculated from the regression of the general model and the auxiliary regressions

the regression of our general model is :  $Y(score_i) = \beta_0 + \beta_1 GDP + \beta_2 social + \beta_3 health + \beta_4 free + \beta_5 gener + \beta_6 percept$ 

Auxiliary regressions are:

$$Y(GDP) = \gamma_0 + \gamma_1 social + \gamma_2 health + \gamma_3 free + \gamma_4 gener + \gamma_5 percept + v_i$$

$$Y(social) = \gamma_0 + \gamma_1 GDP + \gamma_2 health + \gamma_3 free + \gamma_4 gener + \gamma_5 percept + v_i$$

$$Y(health) = \gamma_0 + \gamma_1 GDP + \gamma_2 social + \gamma_3 free + \gamma_4 gener + \gamma_5 percept + v_i$$

$$Y(free) = \gamma_0 + \gamma_1 GDP + \gamma_2 social + \gamma_3 health + \gamma_4 gener + \gamma_5 percept + v_i$$

$$Y(gener) = \gamma_0 + \gamma_1 GDP + \gamma_2 social + \gamma_3 health + \gamma_4 free + \gamma_5 percept + v_i$$

$$Y(percept) = \gamma_0 + \gamma_1 GDP + \gamma_2 social + \gamma_3 health + \gamma_4 gener + \gamma_5 gener + v_i$$

#### • <u>VIF</u>

For each auxiliary regression, the tolerance is calculated:  $TOL = 1 - R^2$ 

Then we calculate VIF = 1/TOL

Finally, all higher values of VIF (>10) indicate the presence of multi colinearity.



### Tableau 15: multicollinearity tests:

	$R^2$	Tol	VIF
General model: score =f(GDP per Capita, Family, Life Expectancy, Freedom, Generosity, Trust Government Corruption)	0,77	0,23	4,35
Auxiliar model  GDP per capita =f( social support, Life Expectancy, Freedom, Generosity, Trust Government Corruption)	0,75	0,25	4
social support =f( GDP per capita , Life Expectancy, Freedom, Generosity, Trust Government Corruption)	0,62	0,38	2,63
Life Expectancy,=f( GDP per capita ,social support, Freedom, Generosity, Trust Government Corruption)	0,71	0,29	3,45
freedom =f( GDP per capita ,social support, Life Expectancy, Generosity, Trust Government Corruption)	0,31	0,69	1,45
Generosity=f( GDP per capita ,social support, Life Expectancy , freedom, Generosity, Trust Government Corruption)	0,13	0,87	1,15
Trust Government Corruption=f( GDP per capita ,social support, Life Expectancy , freedom, Generosity,)	0,16	0,84	1,19

#### commentaries:

- $R^2$  of auxiliary models are lower than the  $Ra^2$  so we can say there is no collinearity problems
- All the VIFs of the auxiliary models are strictly less than 10
- So, we can conclude that our model presents an absence of multicollinearity



#### 3. Detection of heteroscedasticity

#### • Breush-Pagan test

The approach is estimate the model used by the OLS :  $Y(score_i) = \beta_0 + \beta_1 GDP + \beta_2 social + \beta_3 health + \beta_4 free + \beta_5 gener + \beta_6 percept + u_i$  and we take  $u_i^2$ 

- -we estimated the auxiliary model  $u_i^2 = \lambda_0 + \lambda_1 GDP + \lambda_2 social + \lambda_3 health + \lambda_4 free + \lambda_5 gener + \lambda_6 percept + vi$
- -We calulate the  ${\it R}^{2}$  of the auxiliary model  ${\it R}a^{2}$
- We calculate Breush-Pagan stat : BPc=Nx $Ra^2$  ~Chi2(Ka); with Ka the number of explanatory variables of the auxiliary model, here we have 6 variables
- We conclude, at the 5% threshold if BPc<Chi2 we do not reject the null hypothesis of homoscedasticity
- In our case Bpc= 9,31468197 and Chi2=12,592 (appendix 2), Chi2>Bpc therefore, H0 is not rejected, there is no risk of heteroscedasticity



#### CONCLUSION

Thanks to the documentation and our study, we can define some factors with more or less impact on happiness.

The study of correlation between variables in our econometric model allowed us to say that some of these variables can explain why there are some countries where the happiness score is higher than somewhere else. Indeed, in our third regression (table 07) for example we can see for the variables' combination GDP per capita, Social support and Healthy life expectancy a R2 = 0.84 which means the combination of these 3 variables explain 84% of the happiness score variation. We can also see in our correlation matrix a strong link between score and variable GDP per capita. That means some variables have a heavier impact on happiness score than others so countries with better GDP per capita, Social support and Healthy life expectancy have a better happiness score than others.

However it's also very important to remind ourselves there are a lot of other factors which can explain this score and this study can't define an universal ranking of "Where people are the happiest in the world?" because every data of this study is based on the dystopia's concept which is subjective.

So is it really possible to measure happiness significantly and to compare with each other?



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3)Diener, E., & Seligman, M. E. P. (2004). Beyond Money: Toward an Economy of Well-Being. *Psychological Science in the Public Interest*, *5*(1), 1–31.

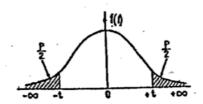


# **APPENDIX**

Appendix 01

### TABLE DE LA LOI DE STUDENT

Valeurs de T ayant la probabilité P d'être dépassées en valeur absolue

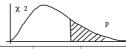


, ,	°=0,90	0,80	0,70	0,60	0,50	0,40	0,30	0,2	,   (	. 10	20,0	0,02	10,0	1
1	158	0,325	0.519	0.727	1.000	1,376	1,963	3,078	6.	314	12,706	31,821	63,657	1
- 1	1142	0.289	0.445		0.816	1.061	1,386	1,886		920	4,303	6,965	9,925	
-1-	•						1.250	1.63		353	3,182	4,541	5,841	
	),137 ),134	0.271	0.414	0.569		0.941	1.190	1.53		132	2,776	3,747	4,604	
-	1,134 1,132	0.267	0.408	0.559			1,156	1.47		015	2,571	3,365	4,032	
_	0.131	0.265	0.404	0.553		0.906	1,134	1.44	- 1	943	2,447	3,143	3,707	
-	0.130	0.263	0,402	0.549	0.711	0.896	1.119	11.41		\$95:	2,365	2,998	3,499	
-	0.130	0.262	0.399	0.546	0.706	0.889	1,108	1,39		\$60	2,306	2,896	3,355	
٦.	0.129	0.261	0.398	0,543	0.703	0.883	1.100	1.38		.833	2,262	2,821	3,250	
- 1		0.260	0.397	0.542	0.700	0.879	1.093	1,37		.812	2,228	2,764	3,16	, I
14	0,129	10,250	0.237	4,542	9.700	10,0,,	1	1	- ľ	•	1	1	Ì	. 1
		0.00	10.700	0.540	0,697	0.876	1.088	11.30	53 ls	.796	2,201	2,718	3.10	
	0.129	0.260		0,539	10.695	0.873	1.083	1.3	- 1	.782	2,179	2,681	3,05	
	0,128	0,259	0.395		0,694	0.870	1.079	1.3		1,771	2,160	2,650		
	0,128	0,259	0,394		0,692	0,868	1.076	113		1,761	2,145	2,624	2,97	Π.
	0,128	0,258	0,393	0,537			1,074	11.3		1.753	2,131	2,602		
	0,128	0,258	0,393	0,336		0,866	1.071	113	1	1,746	2,120	2,583		
	0,128	0,258	0,392	0,535	0,690	0,865	1.069			1.740	2,110		2,8	98
	0,128	0,257	0,392	0,534	0,689	0,863				1,734	2,101		2 _ 28	<u>78</u>
	0,127	0,257	0,392		0,688	0,862	1,067		28_	1.729	2.093		2,8	
	0,127		0,391	0,533	0.688	0,861	11,064		25	1,725			8 2.8	45
ĮΧ	0,127	0,257	0,391	0,533	0,687	0,860	1,00	'   '*		ļ'''~~	1		1	
١	l	l		l	10.00	1	1,06	, 1, ,	323	1,721	2.08	oʻl 2.51		181
	0,127								321	1.717				<b>119</b>
	40,127								319	1.71				<b>207</b>
	30,127								318	1.71				,797
	40,127								316	1.70				,787
	50,127								315	1.70		6 2.4		,779
	60,127								314	1.70	3 2,05			.771
	70,127								313	1,70		18 . 2.4		,763
	80.127								311	1.69	9 2,0	15 72.4		756
	90,12								310	1,69		12 2,4	57   2	2,750
1	×0,12	s66 0,25			1 :			- 1				١	2634	2,575

Nota. - v est le nombre de degrés de liberté.



#### **TABLE DU CHI-DEUX** : χ<sup>2</sup>(n)



n P         0.90         0.80         0.70         0.50         0.30         0.20         0.10         0.05         0.02           1         0,0158         0,0642         0,148         0,455         1,074         1,642         2,706         3,841         5,412           2         0,211         0,446         0,713         1,386         2,408         3,219         4,605         5,991         7,824           3         0,584         1,005         1,424         2,366         3,665         4,642         6,251         7,815         9,837           4         1,064         1,649         2,195         3,357         4,878         5,989         7,779         9,488         11,668           5         1,610         2,343         3,000         4,351         6,064         7,289         9,236         11,070         13,388           6         2,204         3,070         3,828         5,348         7,231         8,558         10,645         12,592         15,033           7         2,833         3,822         4,671         6,346         8,383         9,803         12,017         14,067         16,622           8         3,490         4,594	0.01 6,635 9,210 11,341
2         0,211         0,446         0,713         1,386         2,408         3,219         4,605         5,991         7,824           3         0,584         1,005         1,424         2,366         3,665         4,642         6,251         7,815         9,837           4         1,064         1,649         2,195         3,357         4,878         5,989         7,779         9,488         11,668           5         1,610         2,343         3,000         4,351         6,064         7,289         9,236         11,070         13,388           6         2,204         3,070         3,828         5,348         7,231         8,558         10,645         12,592         15,033           7         2,833         3,822         4,671         6,346         8,383         9,803         12,017         14,067         16,622           8         3,490         4,594         5,527         7,344         9,524         11,030         13,362         15,507         18,168           9         4,168         5,380         6,393         8,343         10,656         12,242         14,684         16,919         19,679           10         4,865         6,	9,210
3         0,584         1,005         1,424         2,366         3,665         4,642         6,251         7,815         9,837           4         1,064         1,649         2,195         3,357         4,878         5,989         7,779         9,488         11,668           5         1,610         2,343         3,000         4,351         6,064         7,289         9,236         11,070         13,388           6         2,204         3,070         3,828         5,348         7,231         8,558         10,645         12,592         15,033           7         2,833         3,822         4,671         6,346         8,383         9,803         12,017         14,067         16,622           8         3,490         4,594         5,527         7,344         9,524         11,030         13,362         15,507         18,168           9         4,168         5,380         6,393         8,343         10,656         12,242         14,684         16,919         19,679           10         4,865         6,179         7,267         9,342         11,781         13,442         15,987         18,307         21,161           11         5,578	-
4         1,064         1,649         2,195         3,357         4,878         5,989         7,779         9,488         11,668           5         1,610         2,343         3,000         4,351         6,064         7,289         9,236         11,070         13,388           6         2,204         3,070         3,828         5,348         7,231         8,558         10,645         12,592         15,033           7         2,833         3,822         4,671         6,346         8,383         9,803         12,017         14,067         16,622           8         3,490         4,594         5,527         7,344         9,524         11,030         13,362         15,507         18,168           9         4,168         5,380         6,393         8,343         10,656         12,242         14,684         16,919         19,679           10         4,865         6,179         7,267         9,342         11,781         13,442         15,987         18,307         21,161           11         5,578         6,989         8,148         10,341         12,899         14,631         17,275         19,675         22,618           12         6,304	11 341
5         1,610         2,343         3,000         4,351         6,064         7,289         9,236         11,070         13,388           6         2,204         3,070         3,828         5,348         7,231         8,558         10,645         12,592         15,033           7         2,833         3,822         4,671         6,346         8,383         9,803         12,017         14,067         16,622           8         3,490         4,594         5,527         7,344         9,524         11,030         13,362         15,507         18,168           9         4,168         5,380         6,393         8,343         10,656         12,242         14,684         16,919         19,679           10         4,865         6,179         7,267         9,342         11,781         13,442         15,987         18,307         21,161           11         5,578         6,989         8,148         10,341         12,899         14,631         17,275         19,675         22,618           12         6,304         7,807         9,034         11,340         14,011         15,812         18,549         21,026         24,054           13         7,042<	11,511
6         2,204         3,070         3,828         5,348         7,231         8,558         10,645         12,592         15,033           7         2,833         3,822         4,671         6,346         8,383         9,803         12,017         14,067         16,622           8         3,490         4,594         5,527         7,344         9,524         11,030         13,362         15,507         18,168           9         4,168         5,380         6,393         8,343         10,656         12,242         14,684         16,919         19,679           10         4,865         6,179         7,267         9,342         11,781         13,442         15,987         18,307         21,161           11         5,578         6,989         8,148         10,341         12,899         14,631         17,275         19,675         22,618           12         6,304         7,807         9,034         11,340         14,011         15,812         18,549         21,026         24,054           13         7,042         8,634         9,926         12,340         15,119         16,985         19,812         22,362         25,472           14         7	13,277
7         2,833         3,822         4,671         6,346         8,383         9,803         12,017         14,067         16,622           8         3,490         4,594         5,527         7,344         9,524         11,030         13,362         15,507         18,168           9         4,168         5,380         6,393         8,343         10,656         12,242         14,684         16,919         19,679           10         4,865         6,179         7,267         9,342         11,781         13,442         15,987         18,307         21,161           11         5,578         6,989         8,148         10,341         12,899         14,631         17,275         19,675         22,618           12         6,304         7,807         9,034         11,340         14,011         15,812         18,549         21,026         24,054           13         7,042         8,634         9,926         12,340         15,119         16,985         19,812         22,362         25,472           14         7,790         9,467         10,821         13,339         16,222         18,151         21,064         23,685         26,873           15	15,086
8         3,490         4,594         5,527         7,344         9,524         11,030         13,362         15,507         18,168           9         4,168         5,380         6,393         8,343         10,656         12,242         14,684         16,919         19,679           10         4,865         6,179         7,267         9,342         11,781         13,442         15,987         18,307         21,161           11         5,578         6,989         8,148         10,341         12,899         14,631         17,275         19,675         22,618           12         6,304         7,807         9,034         11,340         14,011         15,812         18,549         21,026         24,054           13         7,042         8,634         9,926         12,340         15,119         16,985         19,812         22,362         25,472           14         7,790         9,467         10,821         13,339         16,222         18,151         21,064         23,685         26,873           15         8,547         10,307         11,721         14,339         17,322         19,311         22,307         24,996         28,259           16	16,812
9         4,168         5,380         6,393         8,343         10,656         12,242         14,684         16,919         19,679           10         4,865         6,179         7,267         9,342         11,781         13,442         15,987         18,307         21,161           11         5,578         6,989         8,148         10,341         12,899         14,631         17,275         19,675         22,618           12         6,304         7,807         9,034         11,340         14,011         15,812         18,549         21,026         24,054           13         7,042         8,634         9,926         12,340         15,119         16,985         19,812         22,362         25,472           14         7,790         9,467         10,821         13,339         16,222         18,151         21,064         23,685         26,873           15         8,547         10,307         11,721         14,339         17,322         19,311         22,307         24,996         28,259           16         9,312         11,152         12,624         15,338         18,418         20,465         23,542         26,296         29,633           17 <th>18,475</th>	18,475
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	20,090
11         5,578         6,989         8,148         10,341         12,899         14,631         17,275         19,675         22,618           12         6,304         7,807         9,034         11,340         14,011         15,812         18,549         21,026         24,054           13         7,042         8,634         9,926         12,340         15,119         16,985         19,812         22,362         25,472           14         7,790         9,467         10,821         13,339         16,222         18,151         21,064         23,685         26,873           15         8,547         10,307         11,721         14,339         17,322         19,311         22,307         24,996         28,259           16         9,312         11,152         12,624         15,338         18,418         20,465         23,542         26,296         29,633           17         10,085         12,002         13,531         16,338         19,511         21,615         24,769         27,587         30,995           18         10,865         12,857         14,440         17,338         20,601         22,760         25,989         28,869         32,346	21,666
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	23,209
13         7,042         8,634         9,926         12,340         15,119         16,985         19,812         22,362         25,472           14         7,790         9,467         10,821         13,339         16,222         18,151         21,064         23,685         26,873           15         8,547         10,307         11,721         14,339         17,322         19,311         22,307         24,996         28,259           16         9,312         11,152         12,624         15,338         18,418         20,465         23,542         26,296         29,633           17         10,085         12,002         13,531         16,338         19,511         21,615         24,769         27,587         30,995           18         10,865         12,857         14,440         17,338         20,601         22,760         25,989         28,869         32,346           19         11,651         13,716         15,352         18,338         21,689         23,900         27,204         30,144         33,687	24,725
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	26,217
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	27,688
16         9,312         11,152         12,624         15,338         18,418         20,465         23,542         26,296         29,633           17         10,085         12,002         13,531         16,338         19,511         21,615         24,769         27,587         30,995           18         10,865         12,857         14,440         17,338         20,601         22,760         25,989         28,869         32,346           19         11,651         13,716         15,352         18,338         21,689         23,900         27,204         30,144         33,687	29,141
17     10,085     12,002     13,531     16,338     19,511     21,615     24,769     27,587     30,995       18     10,865     12,857     14,440     17,338     20,601     22,760     25,989     28,869     32,346       19     11,651     13,716     15,352     18,338     21,689     23,900     27,204     30,144     33,687	30,578
18         10,865         12,857         14,440         17,338         20,601         22,760         25,989         28,869         32,346           19         11,651         13,716         15,352         18,338         21,689         23,900         27,204         30,144         33,687	32,000
19         11,651         13,716         15,352         18,338         21,689         23,900         27,204         30,144         33,687	33,409
	34,805
20   12,443   14,578   16,266   19,337   22,775   25,038   28,412   31,410   35,020	36,191
	37,566
21   13,240   15,445   17,182   20,337   23,858   26,171   29,615   32,671   36,343	38,932
22   14,041   16,314   18,101   21,337   24,939   27,301   30,813   33,924   37,659	40,289
23   14,848   17,187   19,021   22,337   26,018   28,429   32,007   35,172   38,968	41,638
24   15,659   18,062   19,943   23,337   27,096   29,553   33,196   36,415   40,270	42,980
25   16,473   18,940   20,867   24,337   28,172   30,675   34,382   37,652   41,566	44,314
26   17,292   19,820   21,792   25,336   29,246   31,795   35,563   38,885   42,856	45,642
27   18,114   20,703   22,719   26,336   30,319   32,912   36,741   40,113   44,140	46,963
28   18,939   21,588   23,647   27,336   31,391   34,027   37,916   41,337   45,419	48,278
29   19,768   22,475   24,577   28,336   32,461   35,139   39,087   42,557   46,693	49,588
30         20,599         23,364         25,508         29,336         33,530         36,250         40,256         43,773         47,962	50,892

Pour n > 30, on peut admettre que  $\sqrt{2\chi^2}$  -  $\sqrt{2n-1} \approx N(0,1)$ 

# Appendix 02



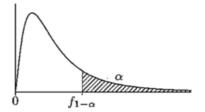
#### Appendix 3

Lot de Fisher-Snedecor ( $\alpha = 0,05$ )

Si F est une variable aléatoire suivant la loi de Fisher–Snedecor à  $(\nu_1,\nu_2)$  degrés de liberté, la table donne la valeur  $f_{1-\alpha}$  telle que

$$\mathbb{P}\{F \geqslant f_{1-\alpha}\} = \alpha = 0.05.$$

Ainsi,  $f_{1-\alpha}$  est le quantile d'ordre  $1-\alpha=0.95$  de la loi de Fisher–Snedecor à  $(\nu_1,\nu_2)$  degrés de liberté.



$\nu_2^{\nu_1}$	1	2	3	4	5	6	8	10	15	20	30	$\infty$
1	161	200	216	225	230	234	239	242	246	248	250	254
2	18,5	19,0	19,2	19,2	19,3	19,3	19,4	19,4	19,4	19,4	19,5	19,5
3	10,1	9,55	9,28	9,12	9,01	8,94	8,85	8,79	8,70	8,66	8,62	8,53
4	7,71	6,94	6,59	6,39	6,26	6,16	6,04	5,96	5,86	5,80	5,75	5,63
5	6,61	5,79	5,41	5,19	5,05	4,95	4,82	4,74	4,62	4,56	4,50	4,36
6	5,99	5,14	4,76	4,53	4,39	4,28	4,15	4,06	3,94	3,87	3,81	3,67
7	5,59	4,74	4,35	4,12	3,97	3,87	3,73	3,64	3,51	3,44	3,38	3,23
8	5,32	4,46	4,07	3,84	3,69	3,58	3,44	3,35	3,22	3,15	3,08	2,93
9	5,12	4,26	3,86	3,63	3,48	3,37	3,23	3,14	3,01	2,94	2,86	2,71
10	4,96	4,10	3,71	3,48	3,33	3,22	3,07	2,98	2,85	2,77	2,70	2,54
11	4,84	3,98	3,59	3,36	3,20	3,09	2,95	2,85	2,72	2,65	2,57	2,40
12	4,75	3,89	3,49	3,26	3,11	3,00	2,85	2,75	2,62	2,54	2,47	2,30
13	4,67	3,81	3,41	3,18	3,03	2,92	2,77	2,67	2,53	2,46	2,38	2,21
14	4,60	3,74	3,34	3,11	2,96	2,85	2,70	2,60	2,46	2,39	2,31	2,13
15	4,54	3,68	3,29	3,06	2,90	2,79	2,64	2,54	2,40	2,33	2,25	2,07
16	4,49	3,63	3,24	3,01	2,85	2,74	2,59	2,49	2,35	2,28	2,19	2,01
17	4,45	3,59	3,20	2,96	2,81	2,70	2,55	2,45	2,31	2,23	2,15	1,96
18	4,41	3,55	3,16	2,93	2,77	2,66	2,51	2,41	2,27	2,19	2,11	1,92
19	4,38	3,52	3,13	2,90	2,74	2,63	2,48	2,38	2,23	2,16	2,07	1,88
20	4,35	3,49	3,10	2,87	2,71	2,60	2,45	2,35	2,20	2,12	2,04	1,84
22	4,30	3,44	3,05	2,82	2,66	2,55	2,40	2,30	2,15	2,07	1,98	1,78
24	4,26	3,40	3,01	2,78	2,62	2,51	2,36	2,25	2,11	2,03	1,94	1,73
26	4,23	3,37	2,98	2,74	2,59	2,47	2,32	2,22	2,07	1,99	1,90	1,69
28	4,20	3,34	2,95	2,71	2,56	2,45	2,29	2,19	2,04	1,96	1,87	1,65
30	4,17	3,32	2,92	2,69	2,53	2,42	2,27	2,16	2,01	1,93	1,84	1,62
40	4,08	3,23	2,84	2,61	2,45	2,34	2,18	2,08	1,92	1,84	1,74	1,51
50	4,03	3,18	2,79	2,56	2,40	2,29	2,13	2,03	1,87	1,78	1,69	1,44
60	4,00	3,15	2,76	2,53	2,37	2,25	2,10	1,99	1,84	1,75	1,65	1,39
80	3,96	3,11	2,72	2,49	2,33	2,21	2,06	1,95	1,79	1,70	1,60	1,32
: 100	3,94	3,09	2,70	2,46	2,31	2,19	2,03	1,93	1,77	1,68	1,57	1,28
00	3,84	3,00	2,60	2,37	2,21	2,10	1,94	1,83	1,67	1,57	1,46	1,00