Economic link between the US GDP, US oil production, and WTI oil prices

Yanis Calvo

2025-2026

1 Introduction

Our study focuses on the relationship between three key macroeconomic variables of the United States: quarterly real GDP, monthly oil production, and the price of WTI (West Texas Intermediate) oil.

The choice of these variables is explained by our desire to empirically verify the existence of an economic relationship between oil production, oil prices, and economic activity, and to quantify this relationship. Initially, we intended to study this relationship for OPEC countries due to the significant role of oil in their GDP. However, these countries are quite unstable economically and politically, which can lead to extreme observations. Most of them also have a poorly diversified economy that is quite sensitive to exogenous shocks. This led us to choose the United States, which does not have t hese difficulties, as reflected in their economic variables that, unlike those of OPEC countries, appear stationary in difference (as seen in their graph) and are therefore analyzable. Thus, our research question can be formulated as follows: how do oil production in the USA, oil prices, and US GDP mutually influence each other, and what form does this relationship take?

By posing this question, we are assuming that there is a close relationship between these three variables. This hypothesis is based on the considerable weight of the oil sector in the economy of the United States, which is among the largest producers of crude oil globally. The United States has even become the world's leading producer of oil and natural gas, notably due to the spectacular rise of shale oil. Since 2014 (or 2018 according to sources), they have surpassed Saudi Arabia and Russia in production volume. In 2022, American production reached approximately 17.8 million barrels per day, representing nearly 19% of global production.

These figures illustrate how oil production constitutes a fundamental pillar of the A merican economy. According to a study conducted by PricewaterhouseCoopers for the American Petroleum Institute, the total contribution of the oil and gas sector, including upstream, midstream, and downstream activities, as well as indirect and induced effects, accounted for approximately 8% of national GDP in 2019.

This energy hegemony fully justifies our interest in the econometric study of the link between oil production, oil prices, and economic activity. Our objective is to verify whether the theoretically expected correlations are present in the data and to characterize the nature and intensity of these relationships using appropriate econometric tools.

2 Data Selection

This step was the most delicate due to the difficulty of finding aggregated data with the same temporal alignment. We ultimately decided to use data with different temporalities, which we then processed in R to obtain comparable series. We first ensured that the data came from official sources and did not contain outliers. We also selected series with different temporalities: the GDP of the United States is available quarterly, while the oil production in the United States and the price of WTI oil are available monthly. Given that these series are not on the same temporal scale, we will need to adjust them to a quarterly frequency, corresponding to the highest level of aggregation among the studied series. We chose to study the period from the first quarter of 1995 to the fourth quarter of 2007, in order to avoid disturbances related to the 2008 financial crisis and the Covid-19 health crisis. Additionally, we retained seasonally adjusted data to better measure the evolution of the chosen variables, without them being biased by seasonal effects. Also, regarding GDP, we selected real GDP to exclude the effect of inflation.

3 Preliminary step for using the code

Before running the R code provided with this work, it is necessary to define a folder in which the data used for this work is located. It is also in this folder that the data is converted so that it can be used later. The path to this folder must be entered in the setwd() command, l ocated on line 26 of the code.

4 Analysis of the series

4.1 Quarterly WTI price

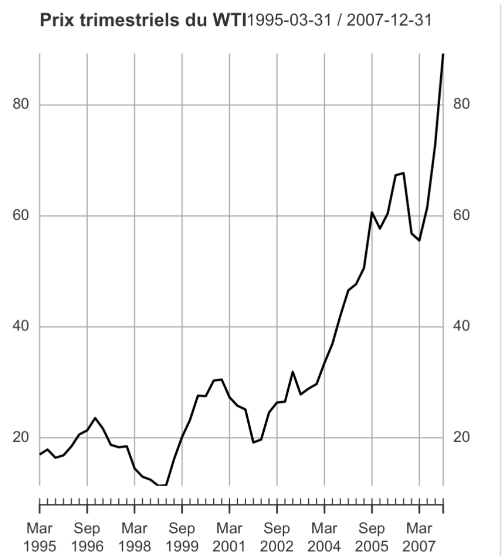


Figure 1 – Quarterly WTI prices (raw series)

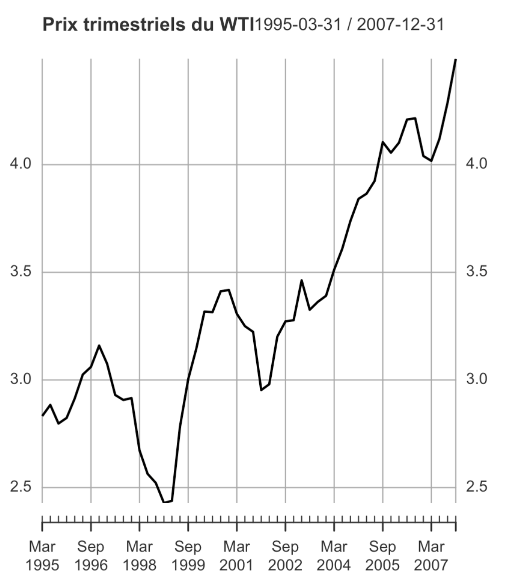
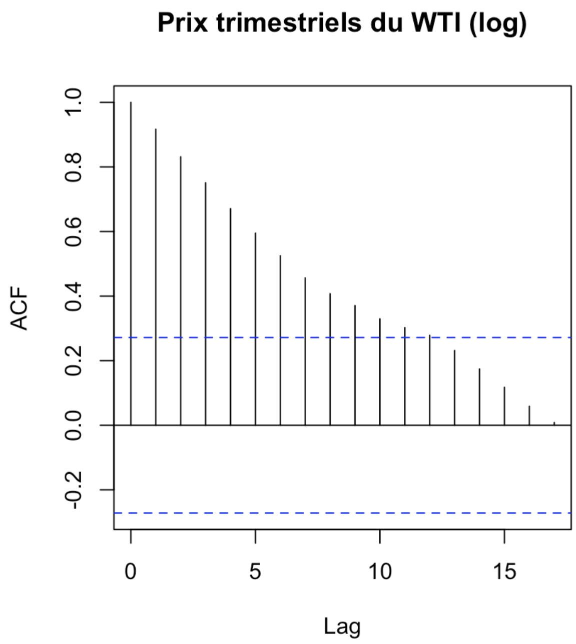


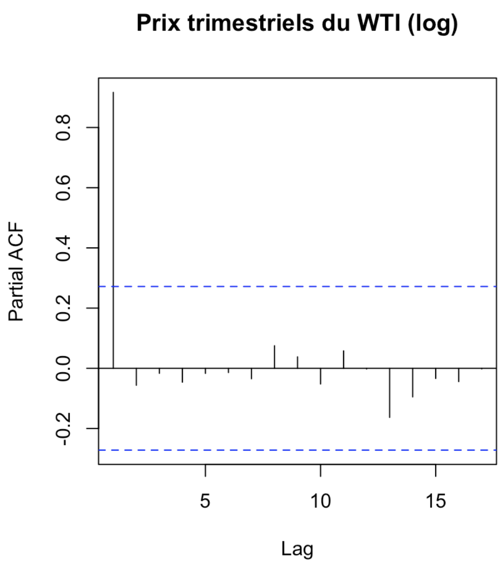
Figure 2 – Quarterly WTI prices (logarithmic series)

We first began by studying the trend of oil prices from the first quarter of 1995 to the last quarter of 2007. This series does not appear to be stationary due to its observable exponential growth behavior in the raw series (see figure 1). Therefore, we applied a natural logarithm transformation to linearize the series and make it more suitable for our analyses (see figure 2). In light of these results, we wanted to better understand the characteristics of the time series. To do this, we examined the autocorrelation function (ACF) and the partial autocorrelation f unction (PACF) of the logarithmic series to gain an initial idea of the dynamic behavior of the series.



The observation of a gradual decay of the autocorrelation function (ACF) towards zero suggests that there is a significant dependence between the current observations and the past observations of the time series. This slow and regular decay is typically an indication of the presence of an autoregressive (AR) component in the stochastic process that generates the series. This hypothesis needs to be confirmed with the PACF test.

Figure 3 – ACF of the logarithmic WTI price



Only the first term of the partial autocorrelation function (PACF) is significantly different from zero. This suggests that the series can be modeled by an autoregressive process of order 1 (AR(1)), which confirms the hypothesis formulated based on the analysis of the autocorrelation function (ACF).

Figure 4 – PACF of the logarithmic WTI price

4.2 Quarterly WTI oil production in the USA

The study now focuses on oil production in the United States between the first quarter of 1995 and the last quarter of 2007. Upon examining the series presented below (figure 5 and figure 6), it is observed that they are not stationary. Indeed, the raw series shows a marked and prolonged downward trend throughout the entire period, followed by a significant rebound towards the end. Therefore, the mean of the series is not constant over time.

Figure 5 – Quarterly oil production in the USA (raw series)

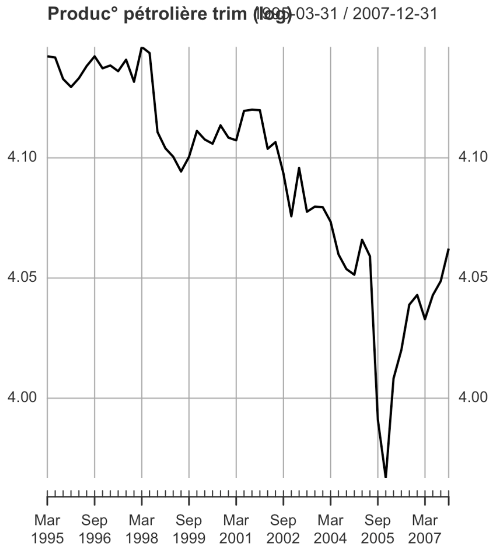


Figure 6 – Quarterly oil production in the USA (logarithmic series)

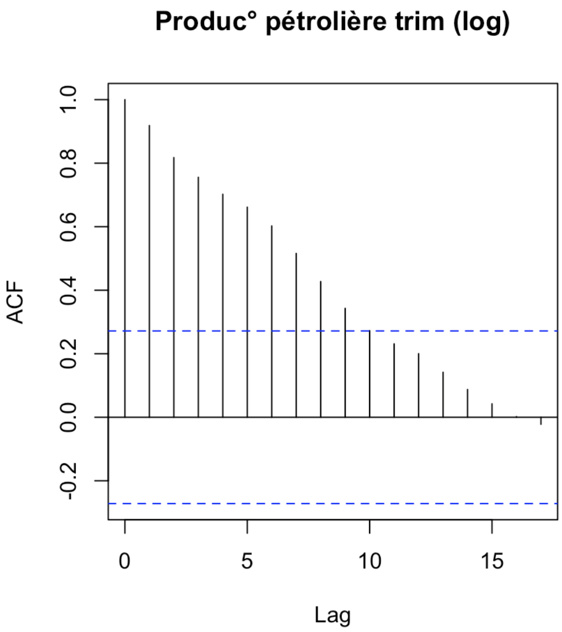
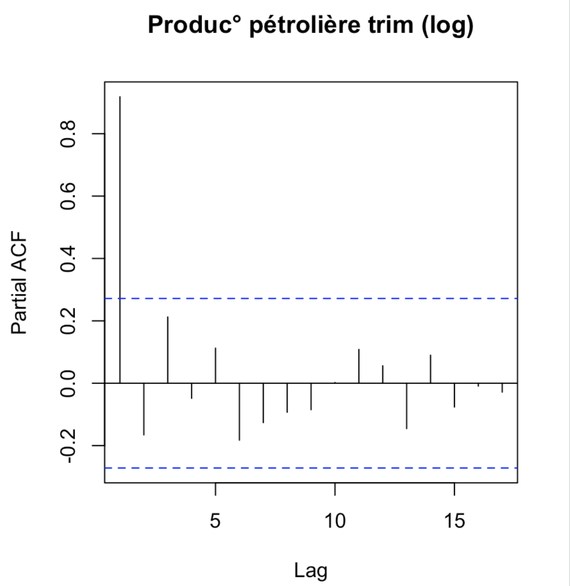


Figure 7 – ACF of logarithmic oil production

The observation of a gradual decline in the autocorrelation function (ACF) of quarterly oil production towards zero suggests the existence of a significant dependency bet ween the present and past observations of the series. This slow and regular decline likely indicates the presence of an autoregressive (AR) component in the stochastic process that generates the series, a hypothesis that should be confirmed using partial autocorrelation function (PACF) analysis.



Only the first term of the partial autocorrelation function (PACF) is significantly different from zero. This suggests that the s series can be modeled by an autoregressive process of order 1 (AR(1)), which confirms the hypothesis formulated from the analysis of the autocorrelation function (ACF).

Figure 8 – PACF of logarithmic oil production

4.3 Quarterly GDP USA

Finally, the last variable we consider is the quarterly GDP of the USA, which is the variable we will study in the univariate model. This series is also not stationary, as seen in the raw series graphs and the series in its logarithmic form (figure 9 and figure 10).

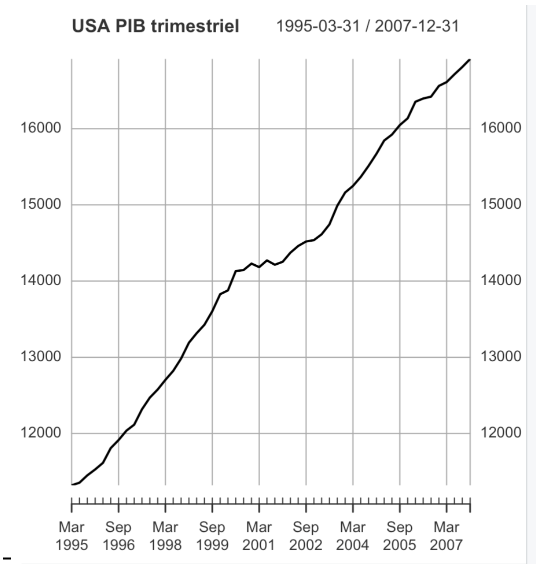


Figure 9 – Quarterly GDP USA (raw series)

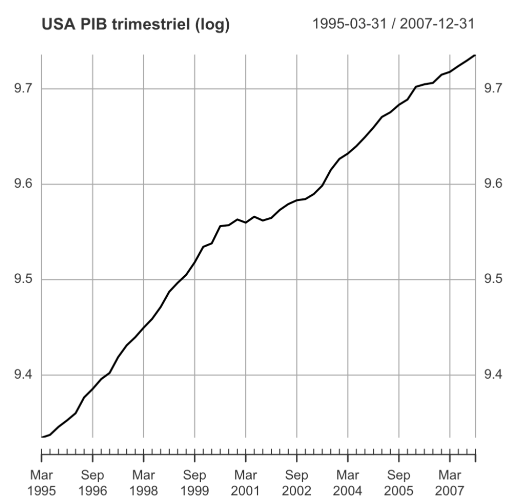
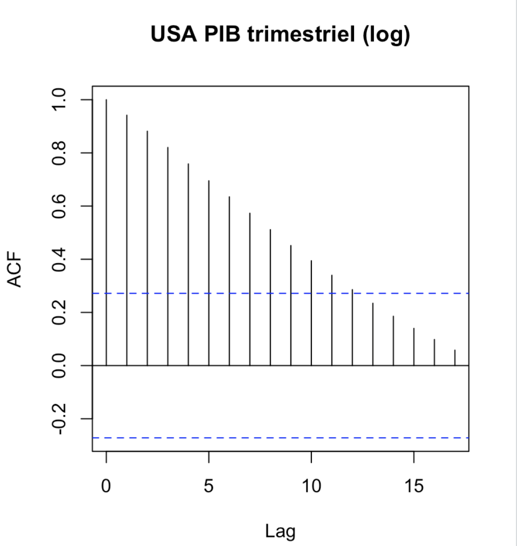
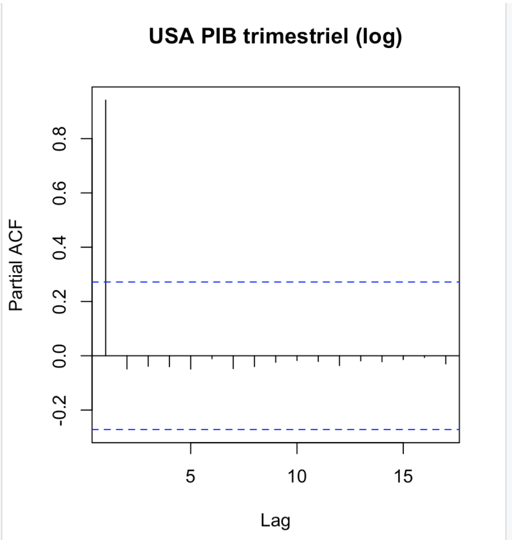


Figure 10 – Quarterly GDP USA (logarithmic series)



The observation of a progressive decrease in the autocorrelation f unction (ACF) of the quarterly GDP of the United States towards zero suggests the existence of a notable dependence between the present and past values of the series. This slow and regular decrease implies the presence of an autoregressive (AR) component in the stochastic process underlying the s eries, a hypothesis that should be verified using the analysis of the partial autocorrelation function (PACF).

Figure 11 – ACF Quarterly GDP USA



Only the first term of the partial autocorrelation function (PACF) of the quarterly GDP of the United States is significantly different from zero. This indicates that the series can be represented by an autoregressive process of order 1 (AR(1)), thus confirming the hypothesis formulated from the analysis of the autocorrelation function.

Figure 12 – PACF Quarterly GDP USA

5 Univariate Modeling

The objective of univariate modeling is to assign a model to one of our variables in order to make forecasts. Here, the variable we have chosen is the GDP of the United States, which is the central variable of our study. The two other variables studied previously can be integrated to enrich the analysis in a multivariate framework. We will first check if the logarithmic GDP series is usable and estimate its characteristics by performing the sequential TRU strategy (unit root test) using the augmented Dickey-Fuller test. If it is not stationary, we will proceed with the logarithmic differential series, to which we will again apply the sequential strategy. This step will then allow us to search for the optimal ARMA model, verify its validity, and make forecasts.

5.1 Study of the log series

We begin by studying the logarithmic series. First, we focus on model 3 of the augmented Dickey-Fuller test to test a series with a trend and a constant:

```
Value of test-statistic is: -1.5193 11.3585 3.0666

Critical values for test statistics:
      1pct  5pct 10pct
tau3 -4.04 -3.45 -3.15
phi2  6.50  4.88  4.16
phi3  8.73  6.49  5.47
```

Figure 13 – ADF Test GDP log model 3

The ADF test gives a statistic of -1.5193, which is above the critical threshold at 5% of -3.45. We do not reject the null hypothesis, indicating that the series is non-stationary. Due to non-stationarity, we conduct the trend test using the Dickey-Fuller tables. The test yields an absolute value of 3.0666, which is below the critical threshold at 5% of 6.49. We do not reject the null hypothesis, meaning that the series does not exhibit a deterministic trend. We then evaluate the series according to model 2.

```
Value of test-statistic is: -3.2032 5.1305

Critical values for test statistics:
      1pct  5pct 10pct
tau2 -3.51 -2.89 -2.58
phi1  6.70  4.71  3.86
```

Figure 14 – ADF test GDP log model 2

The ADF test gives us a test statistic of -2.1325, again exceeding the critical threshold at 5% of -3.45. Once again, the series is not stationary. The constant test (conducted using the Dickey-Fuller tables) yields an absolute value of 16,0881, which is greater than the critical threshold at 5%, allowing us to conclude t hat the series is integrated of order 1 with a constant. In other words, the series follows a random walk with drift:

$$X_t \sim I(1) + c.$$

We verify the result obtained via the KPSS test, which we perform considering the model with a constant.

```
Value of test-statistic is: 1.3703

Critical value for a significance level of:
                10pct  5pct 2.5pct  1pct
critical values 0.347 0.463  0.574 0.739
```

Figure 15 – KPSS test GDP log model 2

The null hypothesis of the test is the absence of a unit root. The test statistic is 1,3703, which is higher than the critical threshold at 5% of 0,463. We reject the null hypothesis, indicating that the series is integrated of order 1 with a constant, thus confirming the result obtained with the ADF test.

We then conduct an ERS test, which allows us to test the specification we obtained previously:

```
Value of test-statistic is: 0.0689

Critical values of DF-GLS are:
                 1pct  5pct 10pct
critical values -2.61 -1.95 -1.62
```

Figure 16 – Test ERS GDP log model 2

The ERS test statistic is 0.0689. The null hypothesis of the test is non-stationarity (presence of a unit root). Since the test statistic 0.0689 is greater than the critical threshold at 5% of -1.95, we accept the null hypothesis, which confirms our specification.

5.2 Study of the non-stationary series

As previously seen, we have an integrated process of order 1 with a constant, modeled in t he form:

$$X_t \sim I(1) + c.$$

To make the series stationary, it is necessary to differentiate it once:

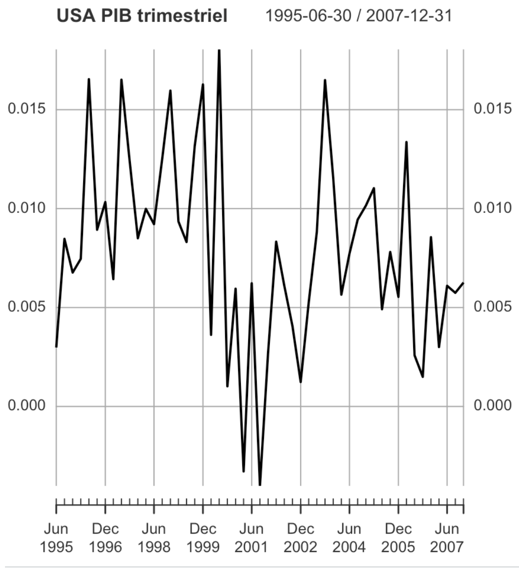


Figure 17 – USA GDP Stationarized

After differentiating the logarithmic series once, the series now visually approaches a stationary series.

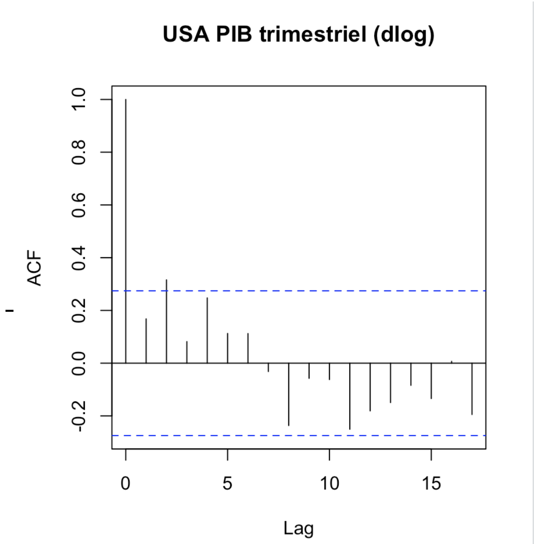


Figure 18 – ACF USA log diff GDP

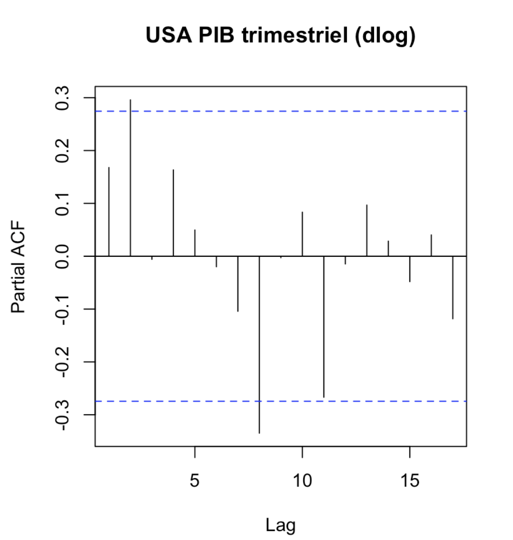


Figure 19 – PACF USA log diff GDP

The ACF (figure 18) allows us to question whether it is a MA(0) or MA(2) since all lags after the first are non-zero. The second is slightly above 0, leading us to hypothesize a MA(2). However, the PACF of the differenced series suggests an AR(2) because the second lag is significantly different from 0 (figure 19). We will now apply the sequential TRU strategy via the Dickey-Fuller test to verify the stationarity of the series and estimate the characteristics of the model.

```
Value of test-statistic is: -3.2032 5.1305

Critical values for test statistics:
      1pct  5pct 10pct
tau2 -3.51 -2.89 -2.58
phi1  6.70  4.71  3.86
```

Figure 20 – ADF test model 2

For model 3, the stationarity test gives us a stationary series, and the trend test indicates that the trend is not significant. We then test model 2. The augmented Dickey-Fuller (ADF) unit root test yields the following results:

$$\begin{aligned} \text{Statistique ADF} = -3.2032 \\ < \text{Seuil à } 5\% = -2.89 \end{aligned}$$

Thus, we reject the null hypothesis, indicating that the series is stationary. Moreover, the constant test (using Student's tables) yields:

$$|\text{Statistique}| = 5.1305 > \text{seuil à } 5\% = 4.71$$

We therefore also reject the null hypothesis.

According to these tests, the differenced logarithmic series is integrated of order zero with a constant, that is to say:

$$Y_t = \Delta X_t \sim I(0) + c$$

This result is confirmed by the KPSS test. It is rejected by the ERS test at the 5% threshold but confirmed at the 10% threshold. Therefore, it can be said that it is rather likely that the model is valid.

5.3 Estimation and validation of the best model

The best model is estimated using the AIC and BIC information criteria.

The values obtained for the different models are presented in figures 21 and 22, and the best models are indicated below:

Model	AIC	BIC
ARMA(2, 0)	-370.4820	-362.9972
ARMA(0, 0)	-369.0790	-365.3366

According to the AIC criterion, the ARMA(2,0) model has the smallest value, making it the best model according to this criterion. In contrast, according to the BIC criterion, the ARMA(0,0) model is slightly preferred.

Overall, the two models yield similar results. By summing the AIC and BIC differences, the ARMA(0,0) model appears to minimize the criteria the most. However, economically, it seems more relevant to retain the ARMA(2,0) model for the continuation of the analysis. Choosing ARMA(0,0) implies that GDP would be memoryless, and thus it would jump from one period to another without any continuity, which is unrealistic: its growth generally depends on the past and reacts to shocks in a lasting manner.

	df	AIC
mod1	3	-368.2678
mod2	4	-370.4820
mod3	5	-368.4929
mod4	3	-367.8170
mod5	4	-369.4586
mod6	5	-367.4615
mod7	4	-368.9054
mod8	5	-368.5722
mod9	6	-366.6802
mod10	5	-367.6913
mod11	6	-365.8276
mod12	6	-367.1274
mod13	7	-365.1513
mod14	7	-365.1904
mod15	8	-367.6539
mod16	2	-369.0790

Figure 21 – AIC Test ARMA

	df	BIC
mod1	3	-362.6542
mod2	4	-362.9972
mod3	5	-359.1369
mod4	3	-362.2034
mod5	4	-361.9738
mod6	5	-358.1055
mod7	4	-361.4206
mod8	5	-359.2162
mod9	6	-355.4530
mod10	5	-358.3353
mod11	6	-354.6004
mod12	6	-355.9002
mod13	7	-352.0529
mod14	7	-352.0920
mod15	8	-352.6843
mod16	2	-365.3366

Figure 22 – BIC Test ARMA

ar1	0.11
	(0.14)
ar2	0.29 *
	(0.14)
intercept	0.01 ***
	(0.00)

Figure 23 – Significance Test of the Coefficients

Below, we focus on the significance of the coefficients. The results indicate that the coefficient of the AR(1) term is not significant, while that of the AR(2) term is weakly significant. In contrast, the constant coefficient c is highly significant. These observations suggest that the ARMA(2,0) model could be well-suited.

Based on the data provided by the code, the constant c is:

$$c = \mu(1 - \phi_1 - \phi_2) = 0{,}0079 \times (1 - 0{,}1095 - 0{,}2903) \approx 0{,}00474.$$

We can thus express the model as follows:

$$x_t = 0.00474 + 0.1095\, x_{t-1} + 0.2903\, x_{t-2} + \varepsilon_t \quad \text{avec } x_t \text{ la croissance du PIB à l'instant } t.$$

We verify the validity of this model through a study of the residuals. First, our analysis focuses on the autocorrelation of the residuals:

```
        Box-Ljung test

data:  residus
X-squared = 9.2314, df = 8, p-value = 0.3232
```

Figure 24 – Autocorrelation Test of the residuals

The obtained p-value ($p = 0{,}3232$) being higher than the significance level of 5%, we do not reject the null hypothesis. Thus, there is no autocorrelation of the residuals.

We then examine the homoscedasticity of the residuals, verifying that their variance is indeed constant over time:

```
	ARCH LM-test; Null hypothesis: no ARCH effects

data:  residus
Chi-squared = 9.8523, df = 10, p-value = 0.4535
```

Figure 25 – Homoscedasticity Test of the residuals

$p = 0.4535 > 0.05 \Rightarrow$ no rejection of H_0 : absence of ARCH effects and absence of heteroscedasticity of the residuals.

Finally, we check the normality of the residuals:

```
	Jarque Bera Test

data:  residus
X-squared = 1.0225, df = 2, p-value = 0.5997
```

Figure 26 – Normality Test of the residuals

The obtained p-value ($p = 0.5997$) being greater than the significance level of 5%, we do not reject the null hypothesis. We conclude that the residuals are normally distributed. Thus, the residuals meet all the criteria: the model can therefore be used for forecasting.

5.4 Forecasts

5.4.1 In-sample forecasts

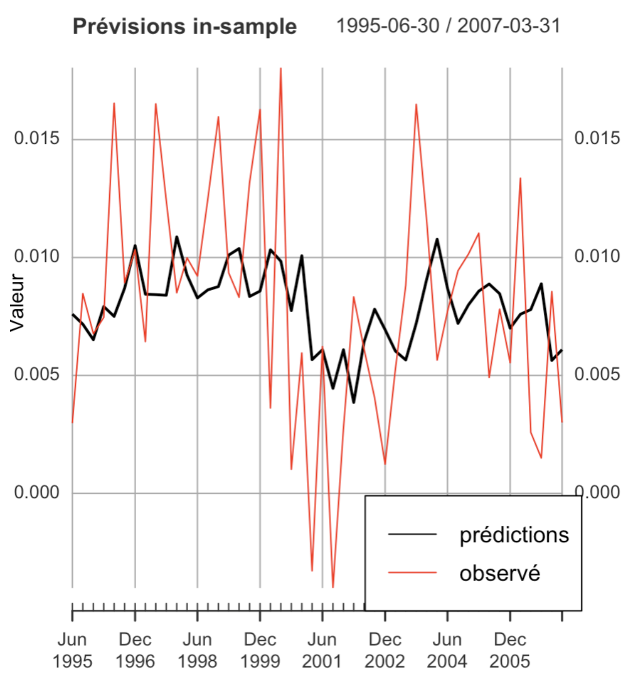


Figure 27 – In-sample forecasts

In-sample forecasts are made, which allow us to measure the model performance via its ability to re produce known data. The graph shows that the model captures the general trend and medium-term movements of the observed values relatively well. The black line (predictions) generally follows the red line (observed). The gap between the two curves represents the prediction error (residual). This error appears significant during extreme market movements, indicating that the model struggles to anticipate or react to shocks or unexpected events that cause strong variations. However, the model still manages to reproduce the general trend well.

```
Call:
arima(x = dlog_PIB_ts_new, order = c(2, 0, 0))

Coefficients:
         ar1     ar2  intercept
      0.1095  0.2903     0.0079
s.e.  0.1384  0.1377     0.0011

sigma^2 estimated as 2.194e-05:  log likelihood = 189.24,  aic = -370.48

Training set error measures:
                       ME       RMSE        MAE       MPE     MAPE     MASE        ACF1
Training set 4.599766e-05 0.00468427 0.00376319 -35.52419 84.59503 0.721545 0.004725834
```

Figure 28 – evaluation of the forecast

Thanks to the evaluation of the forecasts, we see that the RMSE and MAE are low in absolute value. This confirms that the model predicts GDP growth well. The MAPE is high (84.6%), but this may be related to two factors. First, the values of the logarithmic differential are close to 0, which means that a small absolute variation i n these values leads to a large percentage variation. Second, even if the observed values are not close to 0, the MAPE can increase significantly when there are very poor predictions, which aligns with the fact that the model struggles to anticipate shocks that cause strong variations.

5.4.2 Calculation of forecasts (out-of-sample forecasts

Using the obtained model, forecasts can be calculated for horizons 1, 2, and 3. The calculation involves using the model and applying it to the values known up to time T to obtain the value at time T$+1$. With this value, we can then obtain the value at time T$+2$, and so on.

As a reminder, our model is:

$$x_t = 0.00474 + 0.1095\, x_{t-1} + 0.2903\, x_{t-2} + \varepsilon_t$$

Here we set T to correspond to the 48th date. Indeed, we have 51 data points (the first value being zero due to the fact that we are in logarithmic differential). The last values of the logarithmic differential of the GDP obtained, which we will use for the forecast, are 0.003003841 for the 48th value and 0.008557776 for the 47th. Thus, the 49th value is:

$$\hat{x}_{T+1} = \hat{x}_{49} = 0.00474 + 0.1095 \times 0.003003841 + 0.2903 \times 0.008557776 = 0.007553$$

To obtain the 50th value, we use the 48th value that we had in the data and the 49th value that we calculated previously:

$$\hat{x}_{T+2} = \hat{x}_{50} = 0.00474 + 0.1095 \times 0.007553 + 0.2903 \times 0.003003841 = 0.006439$$

To obtain the 51st value, we use the 50th and the 49th values that we calculated previously:

$$\hat{x}_{T+3} = \hat{x}_{51} = 0.00474 + 0.1095 \times 0.006439 + 0.2903 \times 0.007553 = 0.007638$$

We are here in the case of out-of-sample forecasts, where the model's performance is measured on data not used during the estimation.

```
Time Series:
Start = 49
End = 51
Frequency = 1
[1] 0.007535690 0.006419329 0.007612912
```

Figure 29 – Values of out-of-sample forecasts

We see that the values we calculated are almost equal to the values obtained by running the code on R.

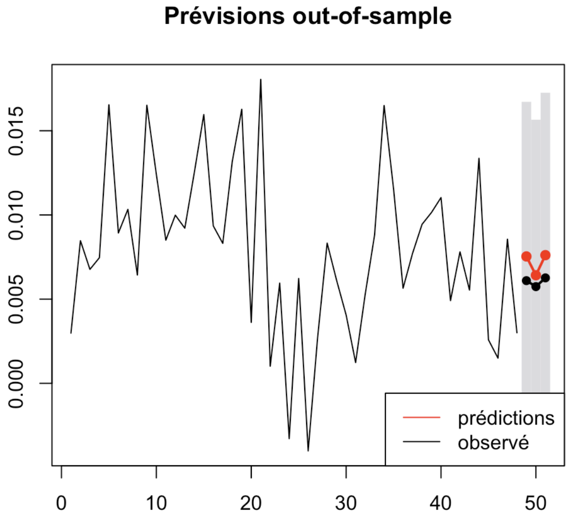


Figure 30 – Out-of-sample forecast graph

Through the application on R, by comparing the actual values and the predicted values, we see that the model correctly anticipates the recent dynamics of the series, with the predicted values (in red) close to the forecasts (in black). However, the accuracy interval (the gray band) appears narrower than the volatility observed across all observations, which suggests that the model may underestimate future uncertainty. Despite this, the general direction (a decline followed by a rise) and the level of the latest observations are well captured.

6 Multivariate Modeling

In this section, we will extend our analysis by moving to a multivariate modeling approach using the VAR (Vector AutoRegression) model. This model will allow us to study the dynamic interactions between economic variables, examine the causal relationships between them, and analyze the impulse responses (IRF) to better understand the impact of shocks. We will also check for the presence of cointegration to identify potential long-term equilibria.

6.1 Selection of the optimal lag length and estimation of the VAR

```
$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     1      1      1      1

$criteria
                   1             2             3             4             5
AIC(n) -2.289304e+01 -2.281408e+01 -2.257571e+01 -2.242978e+01 -2.241380e+01
HQ(n)  -2.266967e+01 -2.245668e+01 -2.208429e+01 -2.180432e+01 -2.165432e+01
SC(n)  -2.229675e+01 -2.186001e+01 -2.126386e+01 -2.076015e+01 -2.038639e+01
FPE(n)  1.144988e-10  1.249179e-10  1.613596e-10  1.926707e-10  2.058906e-10
```

Figure 31 – Selection of the optimal number of lags

The AIC information criterion reaches its minimum for the order $n = 1$. The most appropriate model according to this selection criterion is the VAR(1).

The estimated VAR is as follows:

$$\begin{pmatrix} \Delta PIB_t \\ \Delta Prix_t \\ \Delta Prod_t \end{pmatrix} =$$
$$\begin{pmatrix} 0.009415 \\ -0.086667 \\ -0.00333 \end{pmatrix} + \begin{pmatrix} -0.00009391 \\ 0.001937 \\ 0.0005554 \end{pmatrix} t + \begin{pmatrix} 0.1082 & 0.002536 & -0.06261 \\ 7.377326 & 0.302928 & -0.171229 \\ 0.05586 & 0.1059 & 0.1094 \end{pmatrix} \begin{pmatrix} \Delta PIB_{t-1} \\ \Delta Prix_{t-1} \\ \Delta Prod_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{PIB,t} \\ \varepsilon_{Prix,t} \\ \varepsilon_{Prod,t} \end{pmatrix}$$

6.2 Residual diagnostics in multivariate modeling

We now check the validity of the model through several tests on the residuals.

6.2.1 Autocorrelation Test (Portmanteau) of multivariate residuals

```
        Portmanteau Test (asymptotic)

data:  Residuals of VAR object var_model
Chi-squared = 26.117, df = 27, p-value = 0.5121
```

Figure 32 – Autocorrelation Test of the residuals in multivariate analysis

We do not reject the null hypothesis, which means that the residuals are collectively uncorrelated.

6.2.2 Homoscedasticity Test of multivariate residuals

```
        ARCH (multivariate)

data:  Residuals of VAR object var_model
Chi-squared = 146.2, df = 144, p-value = 0.4332
```

Figure 33 – ARCH Test in multivariate

We do not reject the null hypothesis: the residuals are homoscedastic, meaning that their variance is constant over time.

6.2.3 Normality Test of multivariate residuals

```
        JB-Test (multivariate)

data:  Residuals of VAR object var_model
Chi-squared = 106.99, df = 6, p-value < 2.2e-16
```

Figure 34 – Jarque-Bera Test for normality of residuals in multivariate analysis

We reject the null hypothesis, which means that the residuals do not follow a normal distribution. This result is very common with financial or macroeconomic time series (log differences, returns), which often exhibit thicker tails and more pronounced peaks than the normal distribution.

6.2.4 Stability Test in multivariate

```
[1] 0.36416320 0.09640838 0.05994333
```

Figure 35 – Stability Test in multivariate

All the inverse roots of the VAR model are less than 1. Therefore, we conclude that the model is stable (stationary) and that the dynamics of the system do not diverge over time.

6.3 Granger Causality Test

As we have 3 variables, we conduct 9 Granger causality tests: 6 in bivariate and 3 in multivariate, in order to evaluate and quantify the causal relationships between the variables.

6.3.1 Oil Prices and GDP

```
$Granger

        Granger causality H0: dlog_prix_trim do not Granger-cause dlog_PIB_trim

data:  VAR object var_bi
F-Test = 0.21516, df1 = 1, df2 = 92, p-value = 0.6439
```

Figure 36 – WTI price causing US GDP

For the test of WTI price causing US GDP, the obtained p-value is equal to 0,6439, which is above the significance threshold of 0,05. Therefore, we do not reject the null hypothesis and conclude that oil prices do not cause GDP in the Granger sense.

```
$Granger

        Granger causality H0: dlog_PIB_trim do not Granger-cause dlog_prix_trim

data:  VAR object var_bi
F-Test = 4.7697, df1 = 1, df2 = 92, p-value = 0.03151
```

Figure 37 – US GDP causing WTI price

For the test of US GDP causing the WTI price, the obtained p-value is equal to 0,03151, which is lower than the significance threshold of 0,05. Therefore, we reject the null hypothesis and conclude that US GDP causes the WTI price in the Granger sense. Thus, among the two variables, only the WTI price is caused by US GDP.

6.3.2 Oil Production and GDP

For the two tests conducted here (oil production causing GDP and vice versa), the obtained p-value is consistently well above the significance threshold of 0.05. Thus, there is no causal relationship between these variables.

6.3.3 Oil Production and Oil Prices

For the two tests conducted here (oil production causing oil prices and vice versa), the obtained p-value is also consistently well above the significance threshold of 0.05. Therefore, there is likewise no causal relationship between these variables.

6.3.4 Granger Causality in Trivariate

In trivariate analysis, the p-value is consistently higher than the threshold of 0.05, i ndicating that there is also no causal relationship, in the sense that one of the three variables does not simultaneously cause the other two.

The Granger causality test has therefore allowed us to identify only one causal relationship: that the US GDP causes oil prices.

6.4 Impulse Response Analysis

We now focus on impulse response analysis, which means we seek to understand how an economic variable reacts when there is a shock to another. The previously conducted causality test allowed us to see that there would be a causal relationship between WTI price and US GDP, in the sense that the GDP would cause the prices. It is then interesting to observe how prices would react to a shock on the GDP. Furthermore, we conducted shocks on all the relationships between our variables (a total of 6 shocks), which allowed us to identify other relationships worth noting.

6.4.1 Response of WTI price variation to a shock on US GDP growth

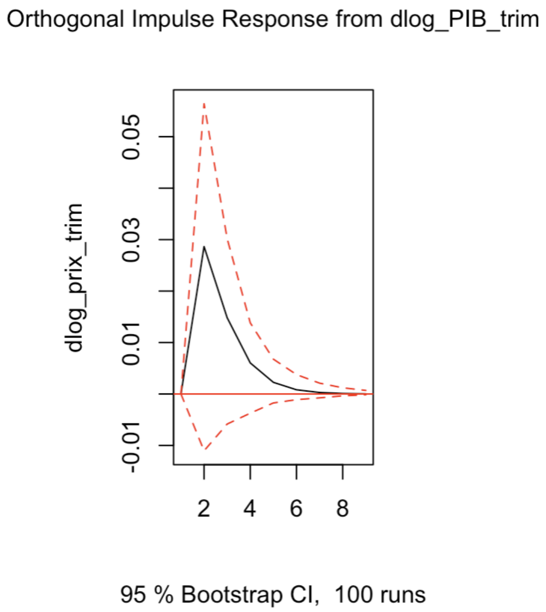


Figure 38 – Response of WTI price variation to a shock on US GDP growth

We see that a positive shock of one unit on economic growth ($\Delta\log(\text{GDP})_{\text{trim}}$) leads to a positive and te mporary increase in the variation of the WTI price ($\Delta\log(\text{price})_{\text{trim}}$). There is a peak after two quarters, and the effect completely dissipates in 4 to 6 quarters. Such a shock is typical of a price reaction to economic activity, but we notice that the red bands constantly frame the x-axis, which would imply that the effect is not statistically significant at the 95% confidence level.

6.4.2 Response of WTI price variation to a shock on production variation

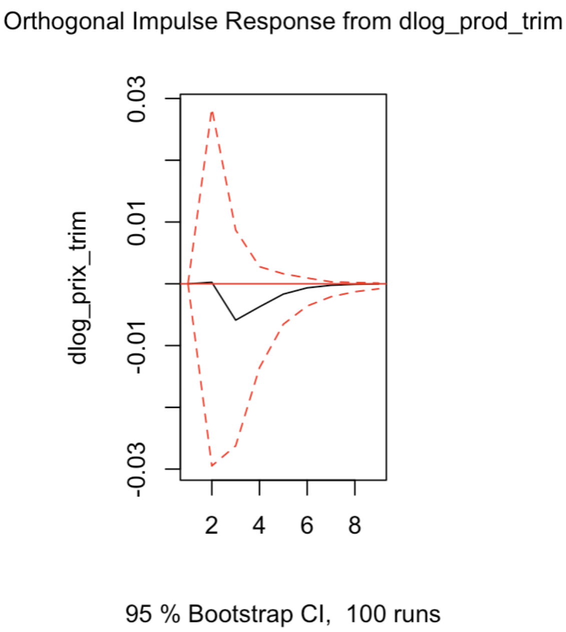


Figure 39 – Response of WTI price variation to a shock on production variation

We also conducted a shock on production growth and observed a notable effect on price variation. A positive shock of one unit on the variation of production ($\Delta\log(\text{prod})_{\text{trim}}$) leads to a brief decrease in the variation of prices ($\Delta\log(\text{prix})_{\text{trim}}$), which begins in the second quarter and reaches its lowest point in the third quarter, after which prices rise and the effect fades after 5 quarters. This effect is not statistically significant at the 95% confidence level.

6.4.3 Response of GDP growth to a shock on the variation of production

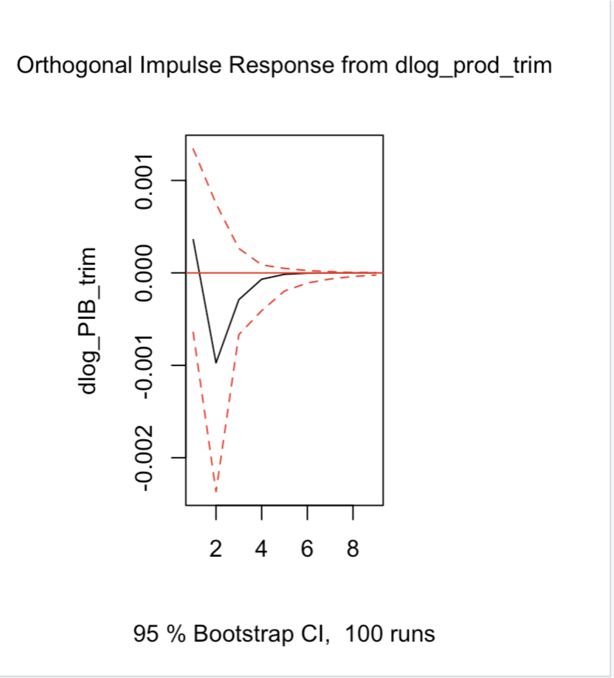


Figure 40 – Response of GDP growth to a shock on the variation of production

We also conducted a shock on the variation of production and observed a notable effect on GDP growth. It is seen that a positive shock of one unit on the growth of production ($\Delta \log(prod)_{trim}$) leads to a decrease in GDP growth ($\Delta \log(GDP)_{trim}$), reaching its lowest point in the second period (a negative trough), and then fades after 4 quarters. Again, this effect is not statistically significant, as the 95% confidence band still encompasses the horizontal axis.

The results of the impulse response functions analysis show several relationships where a shock on one variable can impact another. In all these situations, the impacts remain framed within the confidence interval, leading to the conclusion that they would be insignificant. However, if we focus on the magnitude of the shocks and try to rank these responses from the largest to the smallest magnitude, we first have the response (positive) of price variation to a shock on GDP growth, then a response (negative) of GDP growth to a shock on the variation of production, and finally a response (negative) of prices to a shock on the variation of production. The first result confirms that GDP does indeed have an impact on oil prices, as established by the Granger causality test. The last result is also interesting and coherent. Economically, an increase in production leads to an increase in supply. With demand unchanged, a supply greater than demand leads to a decrease in prices, and thus to the response we obtained here. If we consider the relationship in the opposite direction, a decrease in production leads to an increase in prices. If we had established a causal relationship between GDP and production, in the sense that an increase in GDP would lead to a decrease in production, then production would play the role of an intermediate variable in the relationship between GDP and prices. We will examine this point further.

6.5 Cointegration Test

We have not previously established a relationship between oil production and US GDP. It would be interesting to see if there is a long-term link between these variables, through a cointegration test with production as the explained variable and GDP as the explanatory variable.

```
Call:
lm(formula = log_production_trimestrielle ~ log_PIB_trimestriel)

Residuals:
      Min        1Q    Median        3Q       Max
-0.083137 -0.008445  0.001733  0.013761  0.031524

Coefficients:
                    Estimate Std. Error t value Pr(>|t|)
(Intercept)          7.05023    0.24212   29.12   <2e-16 ***
log_PIB_trimestriel -0.30966    0.02534  -12.22   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.02174 on 50 degrees of freedom
Multiple R-squared:  0.7491,	Adjusted R-squared:  0.7441
F-statistic: 149.3 on 1 and 50 DF,  p-value: < 2.2e-16
```

Figure 41 – Analysis of the residuals of explained production and explanatory GDP

The Ordinary Least Squares (OLS) regression model establishes a statistically significant long-term relationship between oil production and GDP. The model explains 74.9% of the variation in oil production, which is high, and the adjusted R^2 is extre mely close to R^2, confirming that the model is not over-parameterized. The mean error on production is nearly zero (RMSE: 0.02). Finally, the elasticity is -0.31, indicating that oil production reacts quite weakly (and negatively) to an increase in GDP.

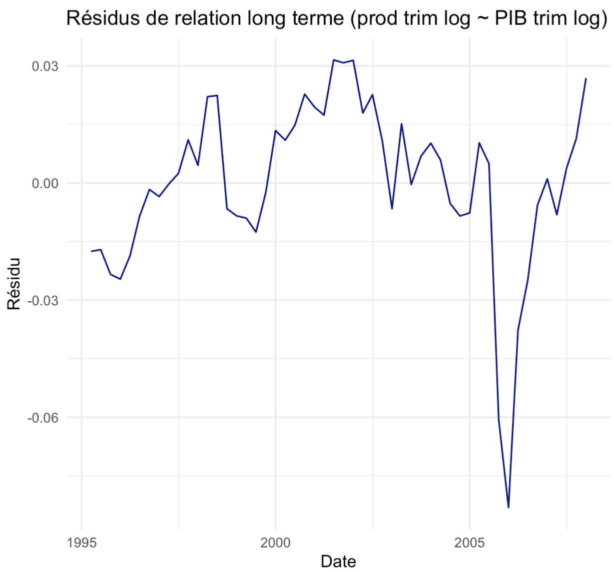


Figure 42 – Graph of residuals for explained production and explanatory GDP

The graph strongly suggests the stationarity of the residuals. Indeed, they oscillate weakly around 0 (between -0.03 and 0.03). The most striking point is the sharp drop in residuals around 2006, followed by a rapid rise back into the aforementioned fluctuation range (suggesting the existence of an error correction mechanism). However, the graph still ends with an increase towards the upper bound of 0.03, raising questions about what happens after our study period. Nevertheless, focusing on our study period, it appears that there is indeed a long-term relationship between oil production and GDP, which is consistent with a cointegration relationship between production and GDP.

```
Residual standard error: 0.01438 on 48 degrees of freedom
Multiple R-squared:  0.1797,	Adjusted R-squared:  0.1455
F-statistic: 5.256 on 2 and 48 DF,  p-value: 0.008627


Value of test-statistic is: -3.1622

Critical values for test statistics:
      1pct  5pct 10pct
tau1 -2.6 -1.95 -1.61
```

Figure 43– ADF Test on Residuals of Explained Prices and Explanatory GDP

An ADF test is conducted on the residuals to verify the observed cointegration relationship through graphical analysis. The t-statistic is equal to $-3{,}1622$, which is less than the critical negative value from the tables of Engle and Yoo. Co nsequently, we reject the null hypothesis and conclude that production and GDP are cointegrated.

We verify the result with the Phillips-Ouliaris test: the p-value being greater than 0.05, the null hypothesis cannot be rejected, which means that no cointegration is detected according to this test, contradicting the results of the Phillips-Ouliaris test. Given the graphical analysis, it is rather likely that such a relationship exists, but it may be fragile.

6.6 Global Interpretation

Through the Granger causality test, we identified only one causal relationship: that the US GDP would cause oil prices. Through impulse-response analyses, we then identified several shock impacts. The GDP would lead to a significant increase in oil prices, and a decrease in production would lead to a rise in prices. The first result confirms what was observed through Granger causality, and the second is quite economically coherent. The third impact we i dentified describes a decrease in GDP growth following an increase in production variation. The Engle-Granger cointegration test establishes a long-term relationship between oil production and GDP, which would manifest in that an increase in GDP would lead to a decrease in production. However, as we saw through Granger causality, a decrease in production leads to an increase in price. Thus, the decrease in production due to the increase in GDP would lead to an increase in GDP, which confirms the causality observed through Granger.

Economically, it is usually the oil prices that cause a variation in the US GDP. This is a well-documented relationship. Indeed, oil is used by all citizens and, from an economic standpoint, by all sectors of activity. An increase in oil prices directly impacts companies' margins, which must face higher costs. Regarding the relationship observed through the causality test, where an increase in production growth would cause a decrease in GDP growth, we currently do not have a plausible economic explanation, and this relationship seems to be the least relevant. Nevertheless, it seems logical that when economic activity increases, it demands more and more oil, especially considering that the economy is becoming increasingly energy-intensive. This is where production can come into play. An increase in GDP growth can be associated with a rise in oil needs across various sectors (to cope with the increase in economic activity). This increase in oil demand leads to a depletion of oil reserves and consequently a rise in oil prices, which may provide an explanation for the long-term relationship.

However, these relationships must be nuanced. One factor to consider here is the origin of the oil used. Indeed, since the 2000s, the United States has experienced the Shale Revolution, with the development of processes allowing them to extract shale oil. If until 2008 their national oil production had decreased (due to resource depletion and an increasingly energy-intensive economy), it then exploded, reaching an unprecedented peak in 2019. Since 2006, due to this increasing independence from abroad, oil imports have seen a nearly continuous decline, while exports have experienced a much greater increase than before. When oil is imported, the impact of the exchange rate depending on its country of origin also comes into play. A decrease in imports would then be associated with a weaker impact of the exchange rate (given equal variation). Thus, the analysis of these factors leads to the conclusion that their impact on the American economy was more significant during the early stages of the shale revolution than it i s in the more recent period. There are also other factors whose effects persist despite the gre ater American energy independence, such as geopolitics and ecology. Therefore, other factors also come into play.

Furthermore, during our study period, the United States had a greater energy dependence on external sources. This means two things: - the share of domestic oil used was lower. Thus, an impact on this oil (production or price) was less likely to affect GDP and vice versa. - the share of foreign oil was larger, and the U.S. GDP (and indirectly, domestic production and WTI prices) could then be influenced more significantly by external factors, such as exchange rates. Additionally, today the country has an increasingly significant domestic production, with a GDP that continues to rise. Theoretically, the negative relationship between GDP and production would no longer hold today; it would be rather positive. Therefore, the temporal dimension must also be taken into account, as well as the spatial dimension, considering elements such as internal demand in the country, etc.

Thus, it appears that the interaction between our study variables is more complex than it seems. These nuances could help explain, among other things, why the observed responses via the IRF functions are not significant. Temporal elements, spatiotemporal factors, and other elements could be considered. One could build an ECM between production and GDP, where we would also add another factor, which would be imports (since, as previously mentioned, the depletion of internal resources would lead to the need for imports):

$$\Delta PROD_t = \gamma_0 + \lambda(PROD_{t-1} - \hat{\alpha} - \hat{\beta}PIB_{t-1}) + \gamma_1 \Delta PROD_{t-1} + \gamma_2 \Delta PIB_t + \gamma_3 \Delta IMP_t + \varepsilon_t$$

Even if we bring the study of the interactions between these three variables to a more recent period, the difficulties are not less significant as this period is marked by various crises (Covid, war in Ukraine, etc.) that lead to even less stationary data, especially regarding oil prices, which are inherently very volatile. We would then face the challenges we encountered when selecting our economic variables.

7 Conclusion

Our study focused on three variables: US GDP, WTI prices, and US oil production.

The first part of this work centered on the study of GDP through univariate modeling. The study of GDP (as well as all the variables in our study in general) required us to take the logarithm and perform differentiation to make it stationary. We were thus able to model its differentiated form using an ARMA(2,0). The resulting model was well-fitted and captured the general trends well, although it might struggle to anticipate shocks and unexpected events. We were able to make forecasts that also captured the general trend, where the difference from the actual observations lay in the amplitude. Anticipating shocks and unexpected events would likely require estimating GDP not only based on its past values but also considering other f actors that could signal the trajectory of GDP.

The second part of this work focused on studying the relationships between our three variables. We were able to establish a causal relationship, which is manifested by the fact that the US GDP would cause WTI prices. Additionally, the analysis of the IRFs allowed us to confirm this causal relationship but also potentially identify others. However, all these response functions remained within the considered confidence interval, leading us to think that the observed relationships may not be significant. Finally, we were able to establish a potential long-term relationship between oil production (the explained variable) and the US GDP (the explanatory variable). These relationships may potentially find an economic explanation. Nevertheless, more robust and conclusive results could be obtained by adding other variables that may have a link with oil (such as oil imports, etc.). Furthermore, it is also important to consider the temporal and spatial dimensions: these r elationships may not be observed in another period (especially the recent period when the US oil system has changed) and in another country. It is difficult to obtain the existence of a general economic relationship between these three variables.

8 Sources

https://www.pwc.com/us/en/industries/energy-utilities-resources/library/energy-trends.html

https://geoconfluences.ens-lyon.fr/scientific-information/thematic-files/critical-geography-of-resources/articles/gas-and-shale-oil-united-states

https://fred.stlouisfed.org/series/GDPC1 https://fred.stlouisfed.org/series/IPG211S
https://www.eia.gov/dnav/pet/hist/LeafHandler.ashx?n=PET&s=F003048623&f=M
Course by Ms. Chloé Derouen and Ms. Sixtine Bigot